EGN-5439  The Design of Tall Buildings

Lecture #14

The Design of Reinforced Concrete Slabs

Via the Direct Method as per ACI 318-05

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Reinforced concrete floor systems provide an economical solution for virtually any span and loading condition.
Introduction.
Selecting the most effective floor system can be vital to achieving overall economy, especially for low- and mid-rise buildings and for buildings subjected to relatively low lateral forces where the cost of the lateral-force-resisting system is minimal.

Concrete, reinforcement, and formwork are the three primary expenses in cast-in-place concrete floor construction to consider throughout the design process, but especially during the initial planning stages. Of these three, formwork comprises about 55 percent of the total cost and has the greatest influence on the overall cost of the floor system. The cost of the concrete, including placing and finishing, typically accounts for about 30 percent of the overall cost. The reinforcing steel has the lowest influence on overall cost (15%). To achieve overall economy, designers should satisfy the following three basic principles of formwork economy:

1) **Specify readily available standard form sizes.** Rarely will custom forms be economical, unless they are required in a quantity that allows for mass production.

2) **Repeat sizes and shapes of the concrete members wherever possible.** Repetition allows reuse of forms from bay-to-bay and from floor-to-floor.

3) **Strive for simple formwork.** In cast-in-place concrete construction, economy is rarely achieved by reducing quantities of materials.
For example, varying the depth of a beam with the loading and span variations would give a moderate savings in materials, but would create substantial additional costs in formwork, resulting in a more expensive structure. The simplest and most cost-effective solution would be providing a constant beam depth and varying the reinforcement along the span. Simple formwork can make construction time shorter, resulting in a building that can be occupied sooner.

Additional parameters must be considered when selecting an economical floor system. In general, span lengths, floor loads, and geometry of a floor panel all play a key role in the selection process. Detailed information on how to select economical concrete floor systems for a wide variety of situations can be found in the following Portland Cement Association (PCA) publications:

1) Concrete Floor Systems - Guide to Estimating and Economizing (SP041), and
2) Long-Span Concrete Floor Systems (SP339).
Preliminary sizing of the slab.

Before analyzing the floor system, designers must assume preliminary member sizes. Typically, the slab and/or beam thickness is determined first to ensure that the deflection requirements of ACI 318-05, Section 9.5 are satisfied.

For solid, one-way slabs and beams that are not supporting or attached to partitions or other construction likely to be damaged by large deflections, Table 9.5(a) may be used to determine minimum thickness $h$. For continuous one-way slabs and beams, determine $h$ based on one-end continuous, since this thickness will satisfy deflection criteria for all spans. The preliminary thickness of a solid one-way slab with normal weight concrete and Grade 60 reinforcement is $l/24$, where $l$ is the span length in inches. Similarly, for beams, minimum $h$ is $l/18.5$. Deflections need not be computed when a thickness at least equal to the minimum is provided.

For non-prestressed, two-way slabs, minimum thickness requirements are given in Section 9.5.3. By satisfying these minimum requirements, which are illustrated in the figure on the next slide for Grade 60 reinforcement, deflections need not be computed. Deflection calculations for two-way slabs are complex, even when linear elastic behavior is assumed. In the figure $\alpha_f$ is the ratio of the flexural stiffness of a beam section to the flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (Section 13.6.1.6), and $\alpha_{fm}$ is the average value of $\alpha_f$ for all beams on the edges of a panel. For two-way construction, $l_n$ is the clear span length in the long direction measured face-to-face of supports.
**Spandrel beam-to-slab stiffness ratio** $\alpha \geq 0.8$

**$\alpha_m > 2.0$**
When two-way slab systems are supported directly on columns, shear around the columns is critically important, especially at exterior slab-column connections where the total exterior slab moment must be transferred directly to the column. Minimum slab thickness for flat plates often is governed by this condition.

Once the depth of a beam has been computed, the beam width $b_w$ can be determined based on moment strength. The following equation, which is derived in the PCA’s Simplified Design (EB104) Handbook can be used to determine $b_w$ for the typical case when $f'_c = 4$ ksi and $f_y = 60$ ksi:

$$b_w = \frac{20M_u}{d^2}$$

In this equation, $M_u$ is the largest factored moment along the span (in foot-kips) and $d$ is the required effective depth of the beam (inches), based on deflection criteria. For beams with one layer of reinforcement, $d$ can be taken equal to $h – 2.5$ inches, while for joists and slabs, $d$ can be taken as $h – 1.25$ inches. Similar sizing equations can be derived for other concrete strengths and grades of reinforcement.
In the preliminary design stage, it is important for the engineer to consider fire resistance. Building codes regulate the fire resistance of the various elements and assemblies of a building structure. Fire resistance must be considered when choosing a slab thickness. Table 721.2.2.1 in the International Code Council’s 2003 International Building Code (IBC) contains minimum reinforced concrete slab thickness for fire-resistance ratings of one to four hours, based on the type of aggregate used in the concrete mix. In general, concrete member thickness required for structural purposes is usually adequate to provide at least a two-hour fire-resistance rating. Adequate cover to the reinforcing steel is required to protect it from the effects of fire. Cover thicknesses for reinforced concrete floor slabs and beams are given in IBC Tables 721.2.3(1) and 721.2.3(3) respectively.

The minimum cover requirements in Section 7.7.1 of ACI 318-05 will provide at least a two-hour fire resistance rating. In all cases, the local building code governing the specific project must be consulted to ensure that minimum fire resistance requirements are met.
The Direct Design Method of Slabs.

Section 8.3 contains criteria for analyzing continuous beams and one-way slabs. In general, all members of frames or continuous construction must be designed for the maximum effects of factored loads, per Section 9.2, using an elastic analysis. Even though numerous computer programs exist that can accomplish this task (e.g., CSI’s SAFE), the set of approximate coefficients in Section 8.3.3 can be used to determine moments and shear forces, provided the limitations in the figure below satisfied. These coefficients, which are given in the figure on the next slide, provide a quick and conservative way of determining design forces for beams and one-way slabs, and can be used to check output from a computer program.

\[ W_u \quad \text{Uniformly Distributed Load} \quad (L/D \leq 3) \]

\[ \leq 1.2 \ell_n \]

\[ \ell_n \]

Prismatic Members

Conditions for analysis by coefficients of ACI Section 8.3.3
Analysis by coefficients of ACI Section 8.3.3
In lieu of an analysis procedure satisfying equilibrium and geometric compatibility, the Direct Design Method of Section 13.6 or the Equivalent Frame Method of Section 13.7 can be used to obtain design moments for two-way slab systems. If the limitations of the Direct Design Method in Section 13.6.1 are met (see the figure below), then the total factored static moment $M_o$ for a span can be distributed as negative and positive moments in the column and middle strips in accordance with Sections 13.6.3, 13.6.4, and 13.6.6.
The total factored static moment $M_o$ is given by,

$$M_o = \frac{q_u l_2 l_n^2}{8}$$

where $q_u =$ factored load per unit area; $l_2 =$ length of span, measured center-to-center of supports in the direction perpendicular to the direction moments are being determined, and $l_n =$ length of clear span, measured face-to-face of supports, in the direction moments are being determined (Section 13.6.2.5).

The figures on the next two slides summarize the moments in the column and middle strips along the span of flat plates or flat slabs supported directly on columns and flat plates or flat slabs with spandrel beams, respectively.
Flat Plate or Flat Slab Supported Directly on Columns

<table>
<thead>
<tr>
<th>Slab Moments</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exterior Negative</td>
<td>Positive</td>
<td>First Interior Negative</td>
<td>Positive</td>
<td>Interior Negative</td>
</tr>
<tr>
<td>Total Moment</td>
<td>0.26 M₀</td>
<td>0.52 M₀</td>
<td>0.70 M₀</td>
<td>0.35 M₀</td>
<td>0.65 M₀</td>
</tr>
<tr>
<td>Column Strip</td>
<td>0.26 M₀</td>
<td>0.31 M₀</td>
<td>0.53 M₀</td>
<td>0.21 M₀</td>
<td>0.49 M₀</td>
</tr>
<tr>
<td>Middle Strip</td>
<td>0</td>
<td>0.21 M₀</td>
<td>0.17 M₀</td>
<td>0.14 M₀</td>
<td>0.16 M₀</td>
</tr>
</tbody>
</table>

Note: All negative moments are at face of support.

Design moment coefficients used with the Direct Design Method for flat plates or flat slabs supported directly on columns.
Flat Plate or Flat Slab with Spandrel Beams

<table>
<thead>
<tr>
<th>Slab Moments</th>
<th>End Span</th>
<th>Interior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 Exterior Negative</td>
<td>2 Positive</td>
</tr>
<tr>
<td>Total Moment</td>
<td>0.30 $M_o$</td>
<td>0.50 $M_o$</td>
</tr>
<tr>
<td>Column Strip</td>
<td>0.23 $M_o$</td>
<td>0.30 $M_o$</td>
</tr>
<tr>
<td>Middle Strip</td>
<td>0.07 $M_o$</td>
<td>0.20 $M_o$</td>
</tr>
</tbody>
</table>

Note:

1. All negative moments are at face of support.
2. Torsional stiffness of spandrel beams $\beta_t \geq 2.5$. For values of $\beta_t$ less than 2.5, exterior negative column strip moment increases to $(0.30 + 0.03\beta_t) M_o$.

Design moment coefficients used with the Direct Design Method for flat plates or flat slabs with spandrel beams.
Design for Flexural Reinforcement.

The required amount of flexural reinforcement is calculated using the design assumptions of Section 10.2 and the general principles and requirements of Section 10.3, based on the factored moments from the analysis. In typical cases, beams, one-way slabs, and two-way slabs will be tension controlled sections, so that the strength reduction factor $\varphi$ is equal to 0.9 in accordance with Section 9.3. In such cases, the required amount of flexural reinforcement $A_s$ at a section can be determined from the following equation, which is derived in PCA’s Simplified Design for $f'c = 4$ ksi and $f_y = 60$ ksi:

$$A_s = \frac{M_u}{4d}$$

where $M_u$ is the factored bending moment at the section (foot-kips) and $d$ is the distance from the extreme compression fiber to the centroid of the longitudinal tension reinforcement (inches).

For greater concrete strengths, this equation yields slightly conservative results. The required $A_s$ must be greater than or equal to the minimum area of steel and less than or equal to the maximum area of steel.
For beams, the minimum area of steel $A_{s,\text{min}}$ is given in Section 10.5.1:

$$A_s = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200b_w d}{f_y}$$

The equation for $A_{s,\text{min}}$ need not be satisfied where the provided $A_s$ at every section is greater than one-third that required by analysis.

For one-way slabs, $A_{s,\text{min}}$ in the direction of the span is the same as the minimum area of steel for shrinkage and temperature reinforcement, which is $(0.0216)h$ per foot width of slab for Grade 60 reinforcement (Section 10.5.4). The maximum spacing of the reinforcement is $3h$ or 18 inches, whichever is less.

For two-way slabs, the minimum reinforcement ratio in each direction is 0.0018 for Grade 60 reinforcement (Section 13.3). In this case, the maximum spacing is $2h$ or 18 inches.

A maximum reinforcement ratio for beams and slabs is not directly given in ACI 318-05. Instead, Section 10.3.5 requires that non-prestressed flexural members must be designed such that the net tensile strain in the extreme layer of longitudinal tension steel at nominal strength $\varepsilon_t$ is greater than or equal to 0.004. In essence, this requirement limits the amount of flexural reinforcement that can be provided at a section. Using a strain compatibility analysis for 4 ksi concrete and Grade 60 reinforcement, the maximum reinforcement ratio is 0.0206.
When selecting bar sizes, it is important to consider the minimum and maximum number of reinforcing bars that are permitted in a cross-section. The limits are a function of the following requirements for cover and spacing:

- Sections 7.6.1 and 3.3.2 (minimum spacing for concrete placement);
- Section 7.7.1 (minimum cover for protection of reinforcement); and
- Section 10.6 (maximum spacing for control of flexural cracking).

The maximum spacing of reinforcing bars is limited to the value given by Equation (10-4) in Section 10.6.4. The following equation can be used to determine the minimum number of bars $n_{\text{min}}$ required in a single layer:

$$n_{\text{min}} = \frac{b_w - 2(c_c + 0.5d_b)}{s} + 1$$

The bar spacing $s$ is given by Equation (10-4):

$$s = 15\left(\frac{40,000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40,000}{f_s}\right)$$
In these equations, $c_c$ is the least distance from the surface of the reinforcement to the tension face of the section, $d_b$ is the nominal diameter of the reinforcing bar, and $f_s$ is the calculated tensile stress in the reinforcement at service loads, which can be taken equal to $2f_y / 3$. The values obtained from the above equation for $n_{min}$ should be rounded up to the next whole number.

The maximum number of bars $n_{max}$ permitted in a section can be computed from the following equation:

$$n_{max} = \frac{b_w - 2(c_s + d_s + r)}{(\text{minimum clear space}) + d_b} + 1$$

where $c_s$ = clear cover to the stirrups; $d_s$ = diameter of stirrup reinforcing bar; $r$ = 0.75 inch for No. 3 stirrups, or 1.0 inch for No. 4 stirrups; and clear space is the largest of 1 inch, $d_b$, or 1.33 (maximum aggregate size).

The computed values of $n_{max}$ from this equation should be rounded down to the next whole number.
Design for Shear Reinforcement.

Design provisions for shear are given in Chapter 11. A summary of the one-way shear provisions is given on the Table 1 below. These provisions are applicable to normal-weight concrete members subjected to shear and flexure only with Grade 60 shear reinforcement. The strength reduction factor $\phi = 0.75$ per Section 9.3.2 and $V_u \leq 2\phi f'_{c} ld$ per Equation (11-3).

### Table 1: ACI 318-05 provisions for shear design

<table>
<thead>
<tr>
<th>Required area area of stirrups, $A_v$</th>
<th>$V_u \leq \phi V_c / 2$</th>
<th>$\phi V_c \geq V_u &gt; \phi V_c / 2$</th>
<th>$V_u &gt; \phi V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required</td>
<td>None</td>
<td>$0.75 \sqrt{f'<em>{c} b_w s} f</em>{yt}$</td>
<td>$\frac{V_u - \phi V_c}{s} \phi f_{yt} d$</td>
</tr>
<tr>
<td>Stirrup spacing, s</td>
<td>Maximum</td>
<td>$\frac{A_v f_{yt}}{0.75 \sqrt{f'<em>{c} b_w}} \leq \frac{A_v f</em>{yt}}{50 b_w}$</td>
<td>$\frac{\phi A_v f_{yt} d}{V_u - \phi V_c}$</td>
</tr>
</tbody>
</table>

$d / 2 \leq 24$ inches for

$\frac{(V_u - \phi V_c)}{d} \leq 4\phi \sqrt{f'_{c} b_w d}$

$d / 4 \leq 12$ inches for

$(V_u - \phi V_c) > 4\phi \sqrt{f'_{c} b_w d}$
Both one-way shear and two-way shear must be investigated in two-way floor systems.

**One-way shear** — Design for one-way shear, which rarely governs, consists of checking that the following equation is satisfied at critical sections located a distance $d$ from the face of the support (as seen in the figure below):

$$V_u \leq 2 \phi \sqrt{f_c} l d$$

where $l$ is equal to $l_1$ or $l_2$ and $V_u$ is the corresponding shear force at the critical section.

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Wide beam shear or one-way shear.

Two-way shear or punching shear.
Two-way shear — As noted previously, two-way or punching shear usually is more critical than one-way shear in slab systems supported directly on columns. As shown in the figure, the critical section for two-way action is at a distance of \( d/2 \) from edges or corners of columns, concentrated loads, reaction areas, and changes in slab thickness, such as edges of column capitals or drop panels. For non-prestressed slabs of normal-weight concrete without shear reinforcement, the following must be satisfied (Section 11.12.2):

\[
v_u \leq \phi v_c = \text{the smallest of } \begin{cases} 
\phi (2 + \frac{4}{\beta}) \sqrt{f_c'} \\
\phi (\frac{\alpha_s d}{b_o} + 2) \sqrt{f_c'} \\
\phi 4 \sqrt{f_c'}
\end{cases}
\]

where \( v_u \) is the maximum factored shear stress at the critical section and all other variables are defined in Chapter 2.
**Moment transfer.** The transfer of moment in the slab-column connections takes place by a combination of flexure (Section 13.5.3) and eccentricity of shear (Section 11.12.6). The portion of total unbalanced moment $M_u$ transferred by flexure is $\gamma_f M_u$, where $\gamma_f$ is defined in Equation (13-1) as a function of the critical section dimensions $b_1$ and $b_2$. It is assumed that $\gamma_f M_u$ is transferred within an effective slab width equal to $c_2 + 1.5$ (slab or drop panel thickness on each side of the column or capital). Reinforcement is concentrated in the effective slab width such that $\varphi M_n \geq \gamma_f M_u$. The portion of $M_u$ transferred by eccentricity of shear is $\gamma_v M_u = (1 - \gamma_f) M_u$ (Sections 13.5.3.1 and 11.12.6). When the Direct Design Method is used, the gravity load moment $M_u$ to be transferred between slab and edge column must be $0.3 M_o$ (Section 13.6.3.6). The factored shear forces on the faces of the critical section AB and CD are as follows (Section 11.12.6.2):

$$v_u (AB) = \frac{V_u}{A_c} + \frac{\gamma_v M_u c_{AB}}{J_c}$$

$$v_u (CD) = \frac{V_u}{A_c} - \frac{\gamma_v M_u c_{CD}}{J_c}$$

where $A_c$ is the area of the critical section and $J_c / c_{AB}$ and $J_c / c_{CD}$ are the section modulii of the critical section. Numerous resources are available that give equations for $A_c$, $J_c / c_{AB}$, and $J_c / c_{CD}$, including PCA’s Simplified Design.
Summary.

The above discussion summarized the design requirements of concrete floor systems with non-prestressed reinforcement according to ACI 318-05. It is important to note that once the required flexure and shear reinforcement have been determined, the reinforcing bars must be developed properly in accordance with the provisions in Chapters 12 and 13. The structural integrity requirements of Section 7.13 must be satisfied as well.
References.

1. ACI 318-05 Code and Commentary;