FEM due to q_{Du} + $3/4q_{Lu}$ = 0.0842 (0.221 × 22) 17.5^2 = 125.4 ft-kips

FEM due to q_{Du} only = 0.0842 (0.101 × 22) 17.5^2 = 57.3 ft-kips

b. Moment distribution for the five loading conditions is shown in Table 20-3. Counter-clockwise rotational moments acting on member ends are taken as positive. Positive span moments are determined from the equation:

$M_{u(midspan)} = M_o - (M_{uL} + M_{uR})/2$

where M_o is the moment at midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at midspan, but its value is close to that at midspan.

Positive moment in span 1-2 for loading (1):

$+M_u = (0.261 \times 22)\ 17.5^2/8 - (93.1 + 167.7)/2 = 89.4$ ft-kips

The following moment values for the slab-beams are obtained from Table 20-3. Note that according to 13.7.6.3, the design moments shall be taken not less than those occurring with full factored live load on all spans.

Maximum positive moment in end span

= the larger of 89.4 or 83.3 = 89.4 ft-kips

Maximum positive moment in interior span*

= the larger of 66.2 or 71.3 = 71.3 ft-kips

Maximum negative moment at end support

= the larger of 93.1 or 86.7 = 93.1 ft-kips

Maximum negative moment at interior support of end span

= the larger of 167.7 or 145.6 = 167.7 ft-kips

Maximum negative moment at interior support of interior span

= the larger of 153.6 or 139.2 = 153.6 ft-kips

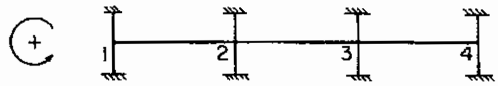
4. Design moments.

Positive and negative factored moments for the slab system in the transverse direction are plotted in Fig. 20-20. The negative factored moments are taken at the face of rectilinear supports at distances not greater than $0.175\ell_1$ from the center of supports. *13.7.7.1*

$\frac{18 \text{ in.}}{2} = 0.75 \text{ ft} < 0.175 \times 17.5 = 3.1 \text{ ft}$ (Use face of support location).

* This is the only moment governed by the pattern loading with partial live load. All other maximum moments occur with full factored live load on all spans.

Table 20-3 Moment Distribution for Partial Frame (Transverse Direction)



Joint	1	2		3		4
Member	1-2	2-1	2-3	3-2	3-4	4-3
DF	0.394	0.306	0.306	0.306	0.306	0.394
COF	0.507	0.507	0.507	0.507	0.507	0.507

(1) All spans loaded with full factored live load

FEM	148.1	-148.1	148.1	-148.1	148.1	-148.1
Dist	-58.4	0.0	0.0	0.0	0.0	58.4
CO	0.0	-29.6	0.0	0.0	29.6	0.0
Dist	0.0	9.1	9.1	-9.1	-9.1	0.0
CO	4.6	0.0	-4.6	4.6	0.0	-4.6
Dist	-1.8	1.4	1.4	-1.4	-1.4	1.8
CO	0.7	-0.9	-0.7	0.7	0.9	-0.7
Dist	-0.3	0.5	0.5	-0.5	-0.5	0.3
CO	0.3	-0.1	-0.3	0.3	0.1	-0.3
Dist	-0.1	0.1	0.1	-0.1	-0.1	0.1
M	93.1	-167.7	153.6	-153.6	167.7	-93.1

(2) First and third spans loaded with 3/4 factored live load

FEM	125.4	-125.4	57.3	-57.3	125.4	-125.4
Dist	-49.4	20.8	20.8	-20.8	-20.8	49.4
CO	10.6	-25.1	-10.6	10.6	25.1	-10.6
Dist	-4.2	10.9	10.9	-10.9	-10.9	4.2
CO	5.5	-2.1	-5.5	5.5	2.1	-5.5
Dist	-2.2	2.3	2.3	-2.3	-2.3	2.2
CO	1.2	-1.1	-1.2	1.2	1.1	-1.2
Dist	-0.5	0.7	0.7	-0.7	-0.7	0.5
CO	0.4	-0.2	-0.4	0.4	0.2	-0.4
Dist	-0.1	0.2	0.2	-0.2	-0.2	0.1
M	86.7	-119.0	74.6	-74.6	119.0	-86.7
Midspan M	83.3				83.3	

(3) Center span loaded with 3/4 factored live load

FEM	57.3	-57.3	125.4	-125.4	57.3	-57.3
Dist	-22.6	-20.8	-20.8	20.8	20.8	22.6
CO	-10.6	-11.4	10.6	-10.6	11.4	10.6
Dist	4.2	0.3	0.3	-0.3	-0.3	-4.2
CO	0.1	2.1	-0.1	0.1	-2.1	-0.1
Dist	-0.1	-0.6	-0.6	0.6	0.6	0.1
CO	-0.3	0.0	0.3	-0.3	0.0	0.3
Dist	0.1	-0.1	-0.1	0.1	0.1	-0.1
CO	0.0	0.1	0.0	0.0	-0.1	0.0
Dist	0.0	0.0	0.0	0.0	0.0	0.0
M	28.2	-87.9	114.9	-114.9	87.9	-28.2
Midspan M			71.2			

Table cont'd on next page

Table 20-3 Moment Distribution for Partial Frame (Transverse Direction) — continued —

(4) First span loaded with 3/4 factored live load and beam-slab assumed fixed st support two spans away

FEM	125.4	-125.4	57.3	-57.3
Dist	-49.4	20.8	20.8	0.0
CO	10.6	-25.0	0.0	10.6
Dist	-4.2	7.7	7.7	0.0
CO	3.9	-2.1	0.0	3.9
Dist	-1.5	0.6	0.6	0.0
CO	0.3	-0.8	0.0	0.3
Dist	-0.1	0.2	0.2	0.0
CO	0.1	-0.1	0.0	0.1
Dist	0.0	0.0	0.0	0.0
M	85.0	-124.0	86.7	-42.4

(5) First and second span loaded with 3/4 factored live load

FEM	125.4	-125.4	125.4	-125.4	57.3	-57.3
Dist	-49.4	0.0	0.0	20.8	20.8	22.6
CO	0.0	-25.1	10.6	0.0	11.4	10.6
Dist	0.0	4.4	4.4	-3.5	-3.5	-4.2
CO	2.2	0.0	-1.8	2.2	-2.1	-1.8
Dist	-0.9	0.5	0.5	0.0	0.0	0.7
CO	0.3	-0.4	0.0	0.3	0.4	0.0
Dist	-0.1	0.1	0.1	-0.2	-0.2	0.0
CO	0.1	-0.1	-0.1	0.1	0.0	-0.1
Dist	0.0	0.0	0.0	0.0	0.0	0.0
M	77.6	-145.8	139.2	-105.7	84.1	-29.5

5. Total factored moment per span. *13.7.7.4*

Slab systems within the limitations of 13.6.1 may have the resulting moments reduced in such proportion that the numerical sum of the positive and average negative moments are not greater than the total static moment M_o given by Eq. (13-3). Check limitations of 13.6.1.6 for relative stiffness of beams in two perpendicular directions.

For interior panel (see Example 19.2):

$$\frac{\alpha_{f1}\ell_2^2}{\alpha_{f2}\ell_1^2} = \frac{316\,(22)^2}{3.98\,(17.5)^2} = 1.25$$ *13.6.1.6*

$0.2 < 1.25 < 5.0$ O.K.

For exterior panel (see Example 19.2):

$$\frac{3.16\,(22)^2}{16.45\,(17.5)^2} = 0.30$$

$0.2 < 0.30 < 5.0$ O.K.

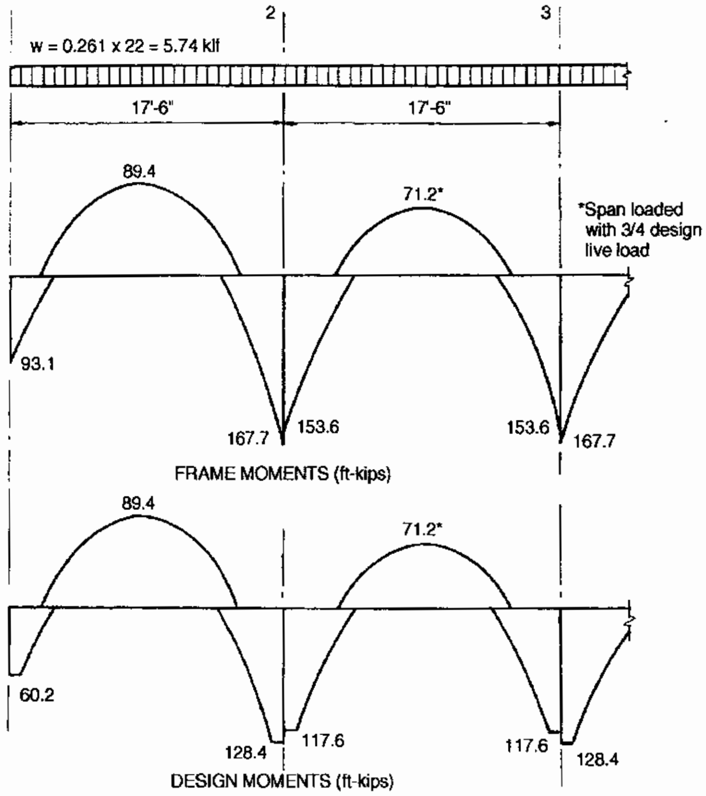


Figure 20-20 Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load Except as Noted)

All limitations of 13.6.1 are satisfied and the provisions of 13.7.7.4 may be applied.

$$M_o = \frac{q_u \ell_2 \ell_n^2}{8} = \frac{0.261 \times 22 \times 16^2}{8} = 183.7 \text{ ft-kips}$$ *Eq. (13-3)*

End span: $89.4 + (60.2 + 128.4)/2 = 183.7$ ft-kips

Interior span: $71.2 + (117.6 + 117.6)/2 = 188.8$ ft-kips

To illustrate proper procedure, the interior span factored moments may be reduced as follows:

Permissible reduction $= 183.7/188.8 = 0.973$

Adjusted negative design moment $= 117.6 \times 0.973 = 114.3$ ft-kips
Adjusted positive design moment $= 71.2 \times 0.973 = 69.3$ ft-kips
$M_o = 183.7$ ft-kips

6. Distribution of design moments across slab-beam strip. *13.7.7.5*

Negative and positive factored moments at critical sections may be distributed to the column strip, beam and two-half middle strips of the slab-beam according to the proportions specified in 13.6.4, 13.6.5 and 13.6.6, if requirement of 13.6.1.6 is satisfied.

a. Since the relative stiffnesses of beams are between 0.2 and 5.0 (see step No. 5), the moments can be distributed across slab-beams as specified in 13.6.4, 13.6.5 and 13.6.6.

b. Distribution of factored moments at critical section:

$$\frac{\ell_2}{\ell_1} = \frac{22}{17.5} = 1.257$$

$$\frac{\alpha_{f1}\ell_2}{\ell_1} = 3.16 \times 1.257 = 3.97$$

$$\beta_t = \frac{C}{2I_s} = \frac{17{,}868}{(2 \times 4752)} = 1.88$$

where $I_s = \dfrac{22 \times 12 \times 6^3}{12} = 4{,}752 \text{ in.}^4$

$C = 17{,}868 \text{ in.}^4$ (see Fig. 22-18)

Factored moments at critical sections are summarized in Table 20-4.

Table 20-4 Distribution of Design Moments

	Factored Moment (ft-kips)	Column Strip		Moment (ft-kips) in Two Half-Middle Strips**
		Percent*	Moment (ft-kips)	
End Span:				
Exterior Negative	60.2	75	45.2	15.0
Positive	89.4	67	59.9	29.5
Interior Negative	128.4	67	86.0	42.4
Interior Span:				
Negative	117.6	67	78.8	38.8
Positive	71.2	67	47.8	23.5

* *Since $\alpha_1\ell_2/\ell_1$ > 1.0 beams must be proportioned to resist 85 percent of column strip moment per 13.6.5.1.*
** *That portion of the factored moment not resisted by the column strip is assigned to the two half-middle strips.*

7. Calculations for shear in beams and slab are performed in Example 19.2, Part 19.

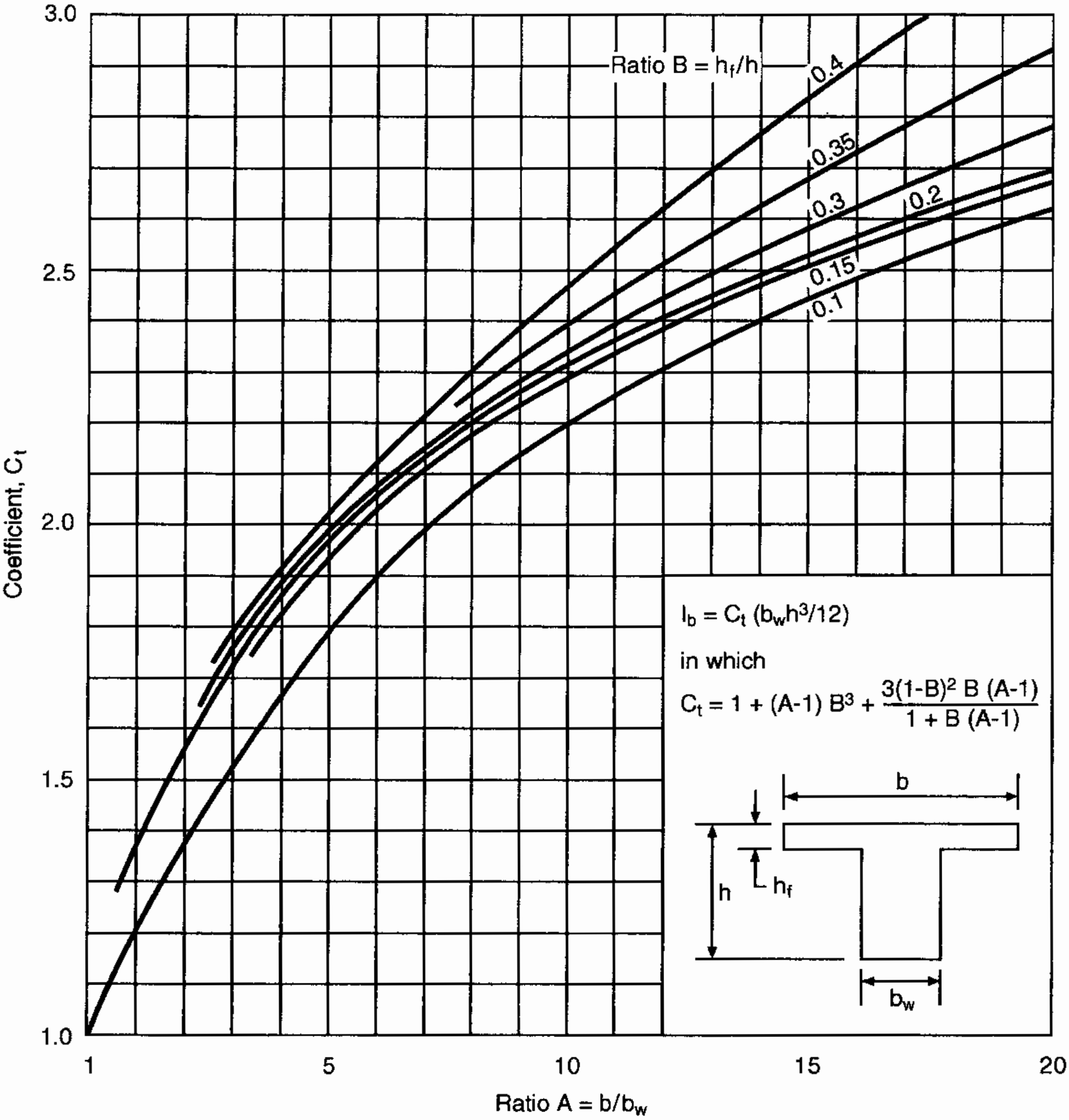


Figure 20-21 Coefficient C_t for Gross Moment of Inertia of Flanged Sections (Flange on One or Two Sides)

Blank

21

Walls

UPDATE FOR THE '05 CODE

Section 14.8.2.3 is updated to reflect the change in design approach that was introduced in 10.3 of the 2002 code. The previous requirement that the reinforcement ratio should not exceed 0.6 ρ_{bal} was replaced by the requirement that the wall be tension-controlled, leading to approximately the same reinforcement ratio.

14.1 SCOPE

Chapter 14 contains the provisions for the design of walls subjected to axial loads, with or without flexure (14.1.1). Cantilever retaining walls with minimum horizontal reinforcement according to 14.3.3 are designed according to the flexural design provisions of Chapter 10 (14.1.2).

14.2 GENERAL

According to 14.2.2, walls shall be designed in accordance with the provisions of 14.2, 14.3, and either 14.4, 14.5, or 14.8. Section 14.4 contains the requirements for walls designed as compression members using the strength design provisions for flexure and axial loads of Chapter 10. Any wall may be designed by this method, and no minimum wall thicknesses are prescribed.

Section 14.5 contains the Empirical Design Method which applies to walls of solid rectangular cross-section with resultant loads for all applicable load combinations falling within the middle third of the wall thickness at all sections along the height of the wall. Minimum thicknesses of walls designed by this method are contained in 14.5.3. Walls of nonrectangular cross-section, such as ribbed wall panels, must be designed by the provisions of 14.4, or if applicable, 14.8.

Section 14.8 contains the provisions of the Alternate Design Method, which are applicable to simply supported, axially loaded members subjected to out-of-plane uniform lateral loads, with maximum moments and deflections occurring at mid-height. Also, the wall cross-section must be constant over the height of the panel. No minimum wall thicknesses are prescribed for walls designed by this method.

All walls must be designed for the effects of shear forces. Section 14.2.3 requires that the design for shear must be in accordance with 11.10, the special shear provisions for walls. The required shear reinforcement may exceed the minimum wall reinforcement prescribed in 14.3.

For rectangular walls containing uniformly distributed vertical reinforcement and subjected to an axial load smaller than that producing balanced failure, the following approximate equation can be used to determine the design moment capacity of the wall (Ref. 21.7and 21.8):

$$\phi M_n = \phi\left[0.5 A_{st} f_y \ell_w \left(1+\frac{P_u}{A_{st} f_y}\right)\left(1-\frac{c}{\ell_w}\right)\right]$$

where

A_{st} = total area of vertical reinforcement, in.^{2}
ℓ_w = horizontal length of wall, in.
P_u = factored axial compressive load, kips
f_y = yield strength of reinforcement, ksi

$$\frac{c}{\ell_w} = \frac{\omega + \alpha}{2\omega + 0.85\beta_1}$$

β_1 = factor relating depth of equivalent rectangular compressive stress block to the neutral axis depth (10.2.7.3)

$$\omega = \left(\frac{A_{st}}{\ell_w h}\right)\frac{f_y}{f_c'}$$

f_c' = compressive strength of concrete, ksi

$$\alpha = \frac{P_u}{\ell_w h f_c'}$$

h = thickness of wall, in.
ϕ = 0.90 (strength primarily controlled by flexure with low axial load)

For a wall subjected to a series of point loads, the horizontal length of the wall that is considered effective for each concentrated load is the least of the center-to-center distance between loads and width of bearing plus four times the wall thickness (14.2.4). Columns built integrally with walls shall conform to 10.8.2 (14.2.5). Walls shall be properly anchored into all intersecting elements, such as floors, columns, other walls, and footings (14.2.6).

Section 15.8 provides the requirements for force transfer between a wall and a footing. Note that for cast-in-place walls, the required area of reinforcement across the interface shall not be less than the minimum vertical reinforcement given in 14.3.2 (15.8.2.2).

14.3 MINIMUM WALL REINFORCEMENT

The minimum wall reinforcement provisions apply to walls designed according to 14.4, 14.5, or 14.8, unless a greater amount is required to resist horizontal shear forces in the plane of the wall according to 11.10.9.

Walls must contain both vertical and horizontal reinforcement. The minimum ratio of vertical reinforcement area to gross concrete area is (1) 0.0012 for deformed bars not larger than No. 5 with $f_y \geq 60{,}000$ psi, or for welded wire reinforcement (plain or deformed) not larger than W31 or D31, or (2) 0.0015 for all other deformed bars (14.3.2). The minimum ratio of horizontal reinforcement is (1) 0.0020 for deformed bars not larger than No. 5 with $f_y \geq 60{,}000$ psi, or for welded wire reinforcement (plain or deformed) not larger than W31 or D31, or (2) 0.0025 for all other deformed bars (14.3.3).

The minimum wall reinforcement required by 14.3 is provided primarily for control of cracking due to shrinkage and temperature stresses. Also, the minimum vertical wall reinforcement required by 14.3.2 does not substantially increase the strength of a wall above that of a plain concrete wall. It should be noted that the reinforcement and minimum thickness requirements of 14.3 and 14.5.3 may be waived where structural analysis shows adequate strength and wall stability (14.2.7). This required condition may be satisfied by a design using the structural plain concrete provisions in Chapter 22 of the code.

For walls thicker than 10 in., except for basement walls, reinforcement in each direction shall be placed in two layers (14.3.4).

Spacing of vertical and horizontal reinforcement shall not exceed 18 in. nor three times the wall thickness (14.3.5).

According to 14.3.6, lateral ties for vertical reinforcement are not required as long as the vertical reinforcement is not required as compression reinforcement or the area of vertical reinforcement does not exceed 0.01 times the gross concrete area.

A minimum of two No. 5 bars shall be provided around all window and door openings, with minimum bar extension beyond the corner of opening equal to the greater of bar development length or 24 in. (14.3.7).

14.4 WALLS DESIGNED AS COMPRESSION MEMBERS

When the limitations of 14.5 or 14.8 are not satisfied, walls must be designed as compression members by the strength design provisions in Chapter 10 for flexure and axial loads. The minimum reinforcement requirements of 14.3 apply to walls designed by this method. Vertical wall reinforcement need not be enclosed by lateral ties (as for columns) when the conditions of 14.3.6 are satisfied. All other code provisions for compression members apply to walls designed by Chapter 10.

As with columns, the design of walls is usually difficult without the use of design aids. Wall design is further complicated by the fact that slenderness is a consideration in practically all cases. A second-order analysis, which takes into account variable wall stiffness, as well as the effects of member curvature and lateral drift, duration of the loads, shrinkage and creep, and interaction with the supporting foundation, is specified in 10.10.1. In lieu of that procedure, the approximate evaluation of slenderness effects prescribed in 10.11 may be used (10.10.2).

It is important to note that Eqs. (10-11) and (10-12) for EI in the approximate slenderness method were not originally derived for members with a single layer of reinforcement. For members with a single layer of reinforcement, the following expression for EI has been suggested in Ref. 21.2:

$$EI = \frac{E_c I_g}{\beta}\left(0.5 - \frac{e}{h}\right) \geq 0.1\,\frac{E_c I_g}{\beta} \qquad \textit{Eq. (1)}$$

$$\leq 0.4\,\frac{E_c I_g}{\beta}$$

where

E_c = modulus of elasticity of concrete
I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting reinforcement
e = eccentricity of the axial loads and lateral forces for all applicable load combinations
h = overall thickness of wall
$\beta = 0.9 + 0.5\beta_d^2 - 12\rho \geq 1.0$
β_d = ratio of dead load to total load
ρ = ratio of area of vertical reinforcement to gross concrete area

The definition of β_d, included in Eqs. (10-11) and (10-12) for EI, depends on the frame being non-sway or sway. According to 10.0, β_d for non-sway frames is the ratio of the maximum factored axial sustained load to the maximum factored axial load associated with the same load combination. For consistency, the same definition of β_d seems appropriate for the EI expressions for walls in Eq. (1). Note that if it is determined by the provisions of 10.11.4 that a sway condition exists, $\beta_d = 0$ for the case of lateral loads that are not sustained (10.0).

Figure 21-1 shows the comparison of flexural stiffness (EI) by Code Eq. (10-12) and Eq. (1) in terms of E_cI_g. The ratio of EI/E_cI_g is plotted as a function of e/h for several values of β_d, for a constant reinforcement ratio ρ of 0.0015. Note that Code Eq. (10-12) assumes EI to be independent of e/h and appears to overestimate the wall stiffness for larger eccentricities. For walls designed by Chapter 10 with slenderness evaluation by 10.11, Eq. (1) is recommended in lieu of Code Eq. (10-12) for determining wall stiffness. Example 21.1 illustrates this method for a tilt-up wall panel.

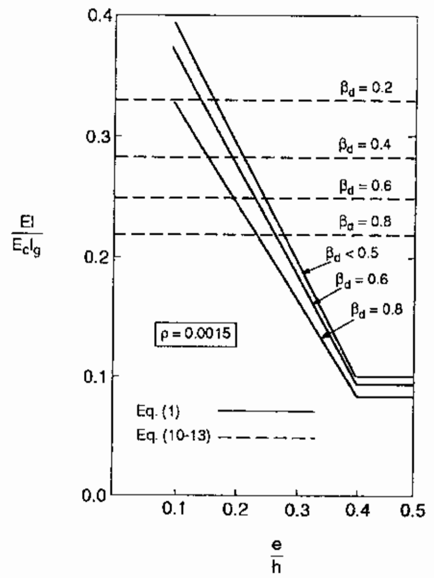


Figure 21-1 Stiffness EI of Walls

When wall slenderness exceeds the limit for application of the approximate slenderness evaluation method of 10.11 ($k\ell_u/r > 100$, i.e. $k\ell_u/h > 30$), 10.10.1 must be used to determine the slenderness effects (10.11.5). The wall panels currently used in some building systems, especially in tilt-up wall construction, usually fall in this high slenderness category. The slenderness analysis must account for the influence of variable wall stiffness, the effects of deflections on the moments and forces, and the effects of load duration.

14.5 EMPIRICAL DESIGN METHOD

The Empirical Design Method may be used for the design of walls if the resultant of all applicable loads falls within the middle one-third of the wall thickness (eccentricity $e \leq h/6$), and the thickness is at least the minimum prescribed in 14.5.3 (see Fig. 21-2). Note that in addition to any eccentric axial loads, the effect of any lateral loads on the wall must be included to determine the total eccentricity of the resultant load. The method applies only to walls of solid rectangular cross-section.

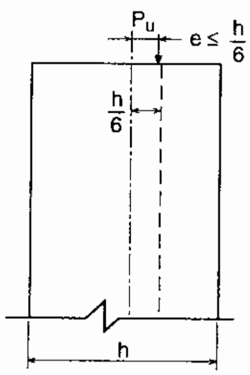


Figure 21-2 Design of Walls by Empirical Design Method (14.5)

Primary application of this method is for relatively short or squat walls subjected to vertical loads only. Application becomes extremely limited when lateral loads need to be considered, because the total load eccentricity must not exceed h/6. Walls not meeting these criteria must be designed as compression members for axial load and flexure by the provisions of Chapter 10 (14.4) or, if applicable, by the Alternate Design Method of 14.8.

When the total eccentricity e does not exceed h/6, the design is performed considering P_u as a concentric axial load. The factored axial load P_u must be less than or equal to the design axial load strength ϕP_n computed by Eq. (14-1):

$$P_u \le \phi P_n$$

$$\le 0.55\phi f_c' A_g \left[1 - \left(\frac{k\ell_c}{32h}\right)^2\right] \qquad \textit{Eq. (14-1)}$$

where

ϕ = strength reduction factor 0.65(h) corresponding to compression-controlled sections in accordance with 9.3.2.2.
A_g = gross area of wall section
k = effective length factor defined in 14.5.2
ℓ_c = vertical distance between supports

Equation (14-1) takes into consideration both load eccentricity and slenderness effects. The eccentricity factor 0.55 was originally selected to give strengths comparable to those given by Chapter 10 for members with axial load applied at an eccentricity not to exceed h/6.

In order to use Eq. (14-1), the wall thickness h must not be less than 1/25 times the supported length or height, whichever is shorter, nor less than 4 in. (14.5.3.1). Exterior basement walls and foundation walls must be at least 7-1/2 in. thick (14.5.3.2).

With the publication of the 1980 supplement of ACI 318, Eq. (14-1) was modified to reflect the general range of end conditions encountered in wall design, and to allow for a wider range of design applications. The wall strength equation in previous codes was based on the assumption that the top and bottom ends of the wall are restrained against lateral movement, and that rotation restraint exists at one end, so as to have an effective length factor between 0.8 and 0.9. Axial load strength values could be unconservative for pinned-pinned end conditions, which can exist in certain walls, particularly of precast and tilt-up applications. Axial strength could also be overestimated where the top end of the wall is free and not braced against translation. In these cases, it is necessary to reflect the proper effective length in the design equation. Equation (14-1) allows the use of different effective length factors k to address this situation. The values of k have been specified in 14.5.2 for commonly occurring wall end conditions. Equation (14-1) will give the same results as the 1977 Code Eq. (14-1) for walls braced against translation at both ends and with reasonable base restraint against rotation. Reasonable base restraint against rotation implies attachment to a member having a flexural stiffness EI/ℓ at least equal to that of the wall. Selection of the proper k for a particular set of support end conditions is left to the judgment of the engineer.

Figure 21-3 shows typical axial load-moment strength curves for 8-, 10-, and 12-in. walls with $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.[21.3] The curves yield eccentricity factors (ratios of strength under eccentric loading to that under concentric loading) of 0.562, 0.568, and 0.563 for the 8-, 10-, and 12-in. walls with e = h/6 and $\rho = 0.0015$.

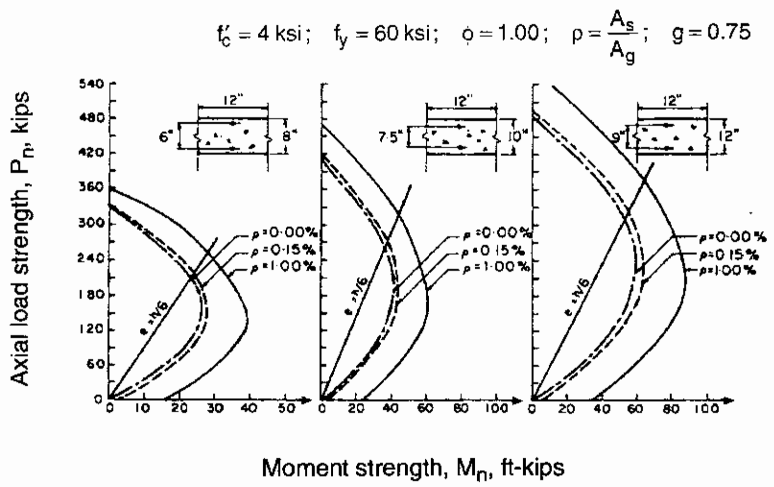


Figure 21-3 Typical Load-Moment Strength Curves for 8-, 10-, and 12-in. Walls

Figure R14.5 in the Commentary shows a comparison of the strengths obtained from the Empirical Design Method and Sect. 14.4 for members loaded at the middle third of the thickness with different end conditions.

Example 21.2 illustrates application of the Empirical Design Method to a bearing wall supporting precast floor beams.

14.8 ALTERNATE DESIGN OF SLENDER WALLS

The alternate design method for walls is based on the experimental research reported in Ref. 21.4. This method has appeared in the Uniform Building Code (UBC) since 1988, and is contained in the 2003 International Building Code (IBC)[21.5]. It is important to note that the provisions of 14.8 differ from those in the UBC and IBC in the following ways: (1) nomenclature and wording has been changed to comply with ACI 318 style, (2) the procedure has been limited to out-of-plane flexural effects on simply supported wall panels with maximum moments and deflections occurring at midspan, and (3) the procedure has been made as compatible as possible with the provisions of 9.5.2.3 for obtaining the cracking moment and the effective moment of inertia.

According to 14.8.1, the provisions of 14.8 are considered to satisfy 10.10 when flexural tension controls the design of a wall. The following limitations apply to the alternate design method (14.8.2):

1. The wall panel shall be simply supported, axially loaded, and subjected to an out-of-plane uniform lateral load. The maximum moments and deflections shall occur at the mid-height of the wall (14.8.2.1).
2. The cross-section is constant over the height of the panel (14.8.2.2).
3. The wall cross sections shall be tension-controlled.
4. Reinforcement shall provide a design moment strength ϕM_n greater than or equal to M_{cr}, where M_{cr} is the moment causing flexural cracking due to the applied lateral and vertical loads. Note that M_{cr} shall be obtained using the modulus of rupture f_r given by Eq. (9-10) (14.8.2.4).
5. Concentrated gravity loads applied to the wall above the design flexural section shall be distributed over a width equal to the lesser of (a) the bearing width plus a width on each side that increases at a slope of 2 vertical to 1 horizontal down to the design flexural section or (b) the spacing of the concentrated loads. Also, the distribution width shall not extend beyond the edges of the wall panel (14.8.2.5) (see Fig. 21-4).
6. The vertical stress P_u/A_g at the mid-height section shall not exceed $0.06f'_c$ (14.8.2.6).

When one or more of these conditions are not satisfied, the wall must be designed by the provisions of 14.4.

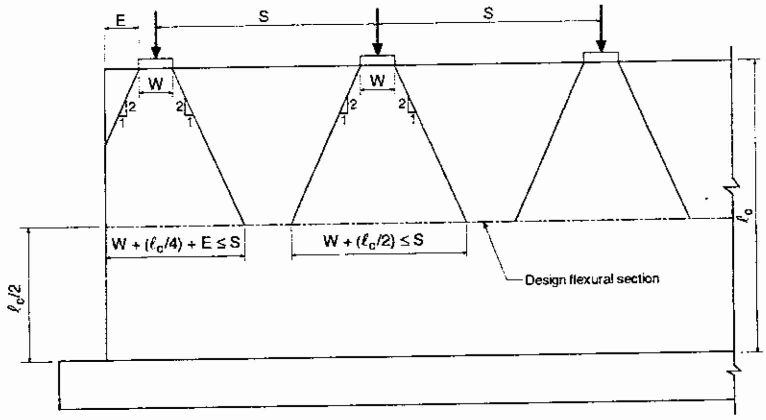


Figure 21-4 Distribution Width of Concentrated Gravity Loads (14.8.2.5)

According to 14.8.3, the design moment strength ϕM_n for combined flexure and axial loads at the mid-height cross-section must be greater than or equal to the total factored moment M_u at this section. The factored moment M_u includes P-Δ effects and is defined as follows:

$$M_u = M_{ua} + P_u \Delta_u \qquad \textit{Eq. (14-4)}$$

where M_{ua} = factored moment at the mid-height section of the wall due to factored lateral and eccentric vertical loads

P_u = factored axial load

Δ_u = deflection at the mid-height of the wall due to the factored loads

$= 5M_u \ell_c^2/(0.75)48E_cI_{cr}$ *Eq. (14-5)*

ℓ_c = vertical distance between supports

E_c = modulus of elasticity of concrete (8.5)

I_{cr} = moment of inertia of cracked section transformed to concrete

$= nA_{se}(d - c)^2 + (\ell_w c^3/3)$ *Eq. (14-7)*

n = modular ratio of elasticity = $E_s/E_c \geq 6$

E_s = modulus of elasticity of nonprestressed reinforcement

A_{se} = area of effective longitudinal tension reinforcement in the wall segment

$= (P_u + A_s f_y)/f_y$ *Eq. (14-8)*

A_s = area of longitudinal tension reinforcement in the wall segment

f_y = specified yield stress of nonprestressed reinforcement

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

c = distance from extreme compression fiber to neutral axis

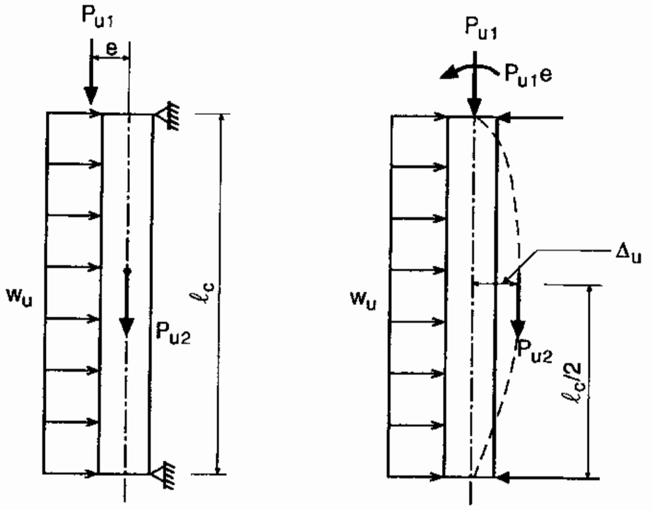
ℓ_w = horizontal length of the wall

Note that Eq. (14-4) includes the effects of the factored axial loads and lateral load (M_{ua}), as well as the P-Δ effects ($P_u \Delta_u$).

Substituting Eq. (14-5) for Δ_u into Eq. (14-4) results in the following equation for M_u:

$$M_u = \frac{M_{ua}}{1 - \dfrac{5P_u \ell_c^2}{(0.75)48E_c I_{cr}}} \qquad \textit{Eq. (14-6)}$$

Figure 21-5 shows the analysis of the wall according to the provisions of 14.8 for the case of additive lateral and gravity load effects.



P_{u1} = factored applied gravity load
P_{u2} = factored self-weight of the wall (total)
e = eccentricity of applied gravity load
w_u = factored uniform lateral load

$$P_u = P_{u1} + \frac{P_{u2}}{2}$$

$$M_u = M_{ua} + P_u \Delta_u = \frac{M_{ua}}{1 - \dfrac{5P_u \ell_c^2}{(0.75)48E_c I_{cr}}}$$

$$M_{ua} = \frac{w_u \ell_c^2}{8} + \frac{P_{u1} e}{2}$$

$$M_u = \frac{w_u \ell_c^2}{8} + \frac{P_{u1} e}{2} + \left(P_{u1} + \frac{P_{u2}}{2}\right)\Delta_u$$

$$\Delta_u = \frac{5M_u \ell_c^2}{(0.75)48E_c I_{cr}}$$

Figure 21-5 Analysis of Wall According to 14.8

The design moment strength ϕM_n of the wall can be determined from the following equation:

$$\phi M_n = \phi A_{se} f_y \left(d - \frac{a}{2}\right) \qquad \textit{Eq. (2)}$$

where

$$a = \frac{A_{se} f_y}{0.85 f'_c \ell_w}$$

and ϕ is determined in accordance with 9.3.2.

In addition to satisfying the strength requirement of Eq. (14-3), the deflection requirement of 14.8.4 must also be satisfied. In particular, the maximum deflection Δ_s due to service loads, including P-Δ effects, shall not exceed $\ell_c/150$, where Δ_s is:

$$\Delta_s = \frac{5M\ell_c^2}{48E_c I_e} \qquad \textit{Eq. (14-9)}$$

where M = maximum unfactored moment due to service loads, including P-Δ effects

$$= \frac{M_{sa}}{1 - \frac{5P_s\ell_c^2}{48E_cI_e}} \qquad \textit{Eq. (14-10)}$$

and M_{sa} = maximum unfactored applied moment due to service loads, not including P-Δ effects

P_s = unfactored axial load at the design (mid-height) section including effects of self-weight

I_e = effective moment of inertia evaluated using the procedure of 9.5.2.3, substituting M for M_a.

It is important to note that Eq. (14-10) does not provide a closed form solution for M, since I_e is a function of M. Thus, an iterative process is required to determine Δ_s.

Example 21.3 illustrates the design of a nonprestressed precast wall panel by the alternated design method.

11.10 SPECIAL SHEAR PROVISIONS FOR WALLS

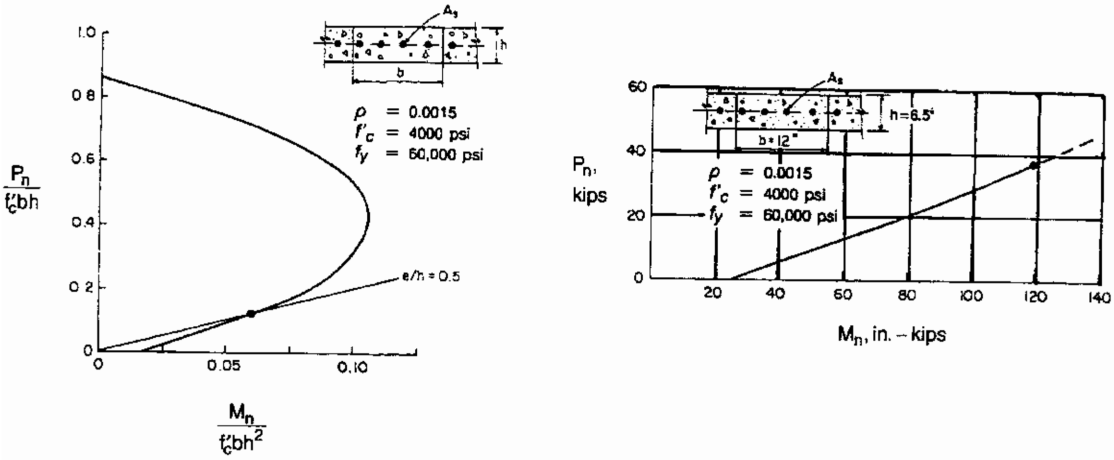
For most low-rise buildings, horizontal shear forces acting in the plane of walls are small, and can usually be neglected in design. Such in-plane forces, however, become an important design consideration in high-rise buildings. Design for shear shall be in accordance with the special provisions for walls in 11.10 (14.2.3). Example 21.4 illustrates in-plane shear design of walls, including design for flexure.

DESIGN SUMMARY

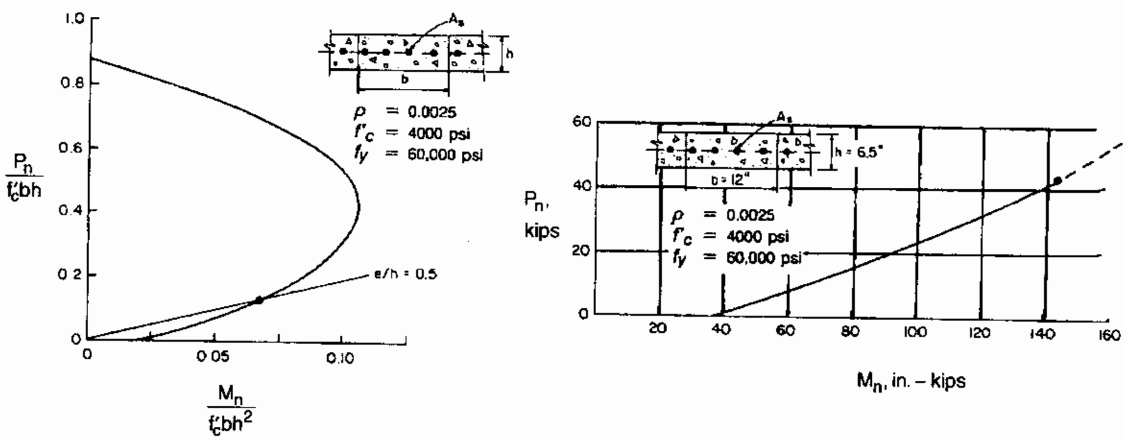
A trial procedure for wall design is suggested: first assume a wall thickness h and a reinforcement ratio ρ. Based on these assumptions, check the trial wall for the applied loading conditions.

It is not within the scope of Part 21 to include design aids for a broad range of wall and loading conditions. The intent is to present examples of various design options and aids. The designer can, with reasonable effort, produce design aids to fit the range of conditions usually encountered in practice. For example, strength interaction diagrams such as those plotted in Fig. 21-6(a) ($\rho = 0.0015$) and Fig. 21-6(b) ($\rho = 0.0025$) can be helpful design aids for evaluation of wall strength. The lower portions of the strength interaction diagrams are also shown for 6.5-in. thick walls. Design charts, such as the one shown in Fig. 21-7 can also be developed for specific walls. Figure 21-8 may be used to select wall reinforcement.

Prestressed walls are not covered specifically in Part 21. Prestressing of walls is advantageous for handling (precast panels) and for increased buckling resistance. For design of prestressed walls, the designer should consult Ref. 21.6.



(a) Reinforcement Ratio $\rho = 0.0015$



(b) Reinforcement Ratio $\rho = 0.0025$

Figure 21-6 Axial Load-Moment Interaction Diagram for Walls (f'_c = 4000 psi, f_y = 60 ksi)

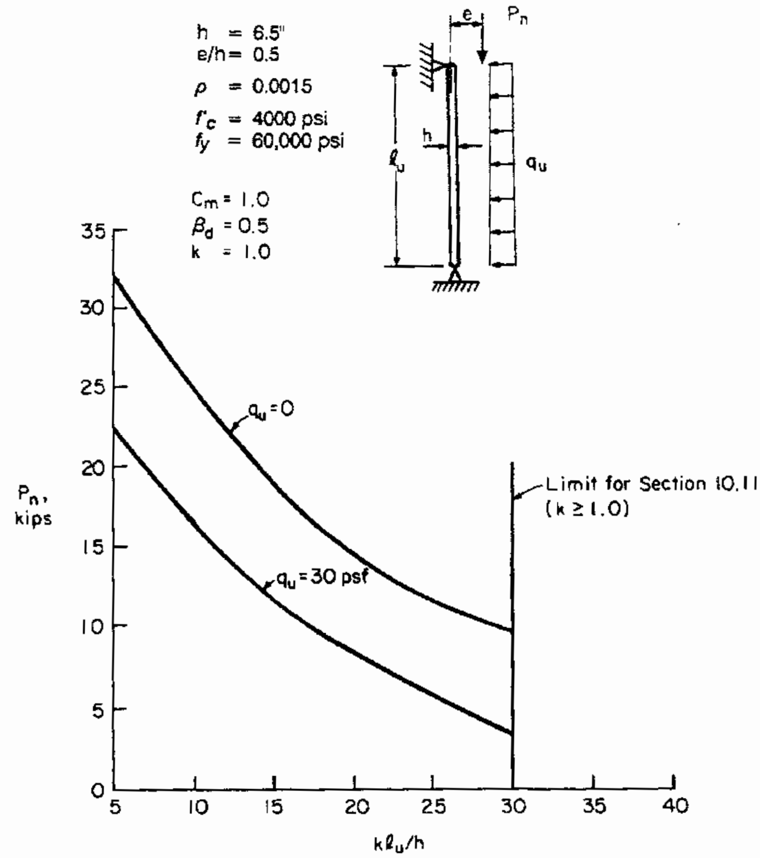


Figure 21-7 Design Chart for 6.5-in. Wall

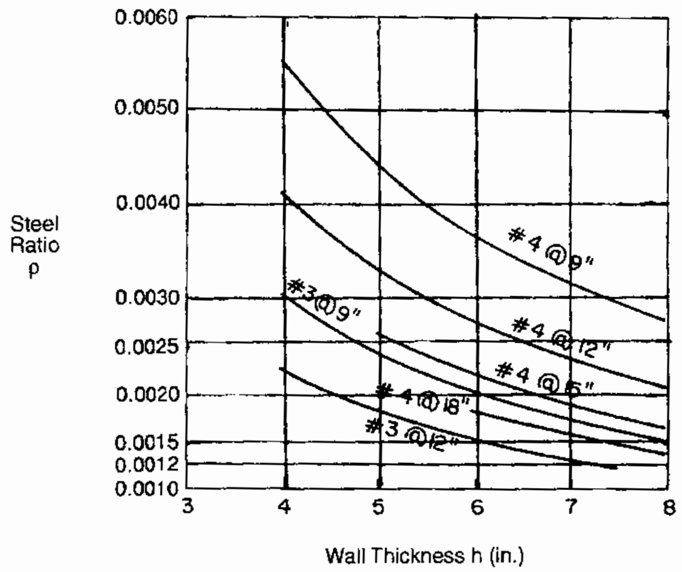


Figure 21-8 Design Aid for Wall Reinforcement

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21.3 Kripanaryanan, K.M., "Interesting Aspects of the Empirical Wall Design Equation," *ACI Journal*, Proceedings Vol. 74, No. 5, May 1977, pp. 204-207.

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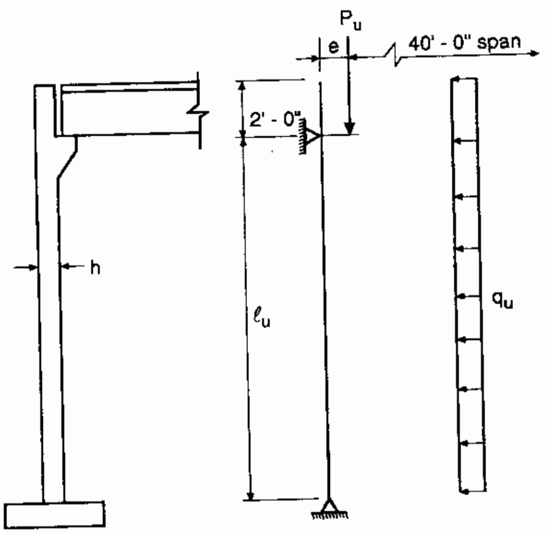
21.6 *PCI Design Handbook - Precast and Prestressed Concrete*, 5th Edition, Prestressed Concrete Institute, Chicago, IL, 1999.

21.7 Iyad M. Alsamsam and Mahmoud E. Kamara. *Simplified Design: Reinforced Concrete Buildings of Moderate Size and Height,* Portland Cement Association, EB104, 2004, pp 6-11.

21.8 Alex E. Cardenas, and Donald D. Magura, *Strength of High-Rise Shear Walls-Rectangular Cross Sections, Response of Multistory Concrete Structures to Lateral Forces,* SP-36, American Concrete Institute, Farmington Hills, MI, 1973, pp 119-150.

Example 21.1—Design of Tilt-up Wall Panel by Chapter 10 (14.4)

Design of the wall shown is required. The wall is restrained at the top edge, and the roof load is supported through 4 in. tee stems spaced at 4 ft on center.



Design data:

Roof dead load = 50 psf
Roof live load = 20 psf
Wind load = 20 psf
Unsupported length of wall ℓ_u = 16 ft
Effective length factor k = 1.0 (pinned-pinned end condition)
Concrete f'_c = 4,000 psi (w_c = 150 pcf)
Reinforcing steel f_y = 60,000 psi
Assume non-sway condition.

Calculations and Discussion	Code Reference

1. Trial wall selection

Try h = 6.5 in. with assumed e = 6.75 in.

Try a single layer of No. 4 @ 12 in. vertical reinforcement (A_s = 0.20 in.^{2}/ft) at centerline of wall

For a 1-ft wide design strip:

$$\rho_\ell = \frac{A_s}{bh} = \frac{0.20}{(12 \times 6.5)} = 0.0026 > 0.0012 \quad \text{O.K.}$$ *14.3.2 (a)*

2. Effective wall length for roof reaction *14.2.4*

Bearing width + 4 (wall thickness) = 4 + 4 (6.5) = 30 in. = 2.5 ft (governs)

Center-to-center distance between stems = 4 ft

3. Roof loading per foot width of wall

$$\text{Dead load} = \left[50 \times \left(\frac{4}{2.5}\right)\right] \times \frac{40}{2} = 1{,}600 \text{ plf}$$

$$\text{Live load} = \left[20 \times \left(\frac{4}{2.5}\right)\right] \times \frac{40}{2} = 640 \text{ plf}$$

$$\text{Wall dead load at mid-height} = \frac{6.5}{12} \times \left(\frac{16}{2} + 2\right) \times 150 = 813 \text{ plf}$$

4. Factored load combinations

Load comb. 1: $U = 1.2D + 0.5L_r$ — *Eq. (9-2)*

$P_u = 1.2\,(1.6 + 0.81) + 0.5\,(0.64) = 2.9 + 0.3 = 3.2$ kips

$M_u = 1.2\,(1.6 \times 6.75) + 0.5\,(0.64 \times 6.75) = 15.1$ in.-kips

$\beta_d = 2.9/3.2 = 0.91$

Load comb. 2: $U = 1.2D + 1.6L_r + 0.8W$ — *Eq. (9-3)*

$P_u = 1.2\,(1.6 + 0.81) + 1.6\,(0.64) + 0 = 2.9 + 1.0 = 3.9$ kips

$M_u = 1.2\,(1.6 \times 6.75) + 1.6\,(0.64 \times 6.75) + 0.8\,(0.02 \times 16^2 \times 12/8)$

$= 26.0$ in.-kips

$\beta_d = 2.9/3.9 = 0.74$

Load comb. 3: $U = 1.2D + 1.6W + 0.5L_r$ — *Eq. (9-4)*

$P_u = 1.2\,(1.6 + 0.81) + 0 + 0.5\,(0.64) = 3.2$ kips

$M_u = 1.2\,(1.6 \times 6.75) + 1.6\,(0.02 \times 16^2 \times 12/8) + 0.5\,(0.64 \times 6.75)$

$= 27.4$ in.-kips

$\beta_d = 2.9/3.2 = 0.91$

Load comb. 4: $U = 0.9D + 1.6W$ — *Eq. (9-6)*

$P_u = 0.9\,(1.6 + 0.81) + 0 = 2.2$ kips

$M_u = 0.9\,(1.6 \times 6.75) + 1.6\,(0.02 \times 16^2 \times 12/8)$

$= 22.0$ in.-kips

$\beta_d = 2.2/2.2 = 1.0$

5. Check wall slenderness

$$\frac{k\ell_u}{r} = \frac{1.0\,(16 \times 12)}{(0.3 \times 6.5)} = 98.5 < 100$$ *10.11.5*

where $r = 0.3h$ — *10.11.12*

Therefore, 10.11 may be used to account for slenderness effects.

6. Calculate magnified moments for non-sway case — 10.12

$M_c = \delta_{ns}M_2$ — Eq. (10-8)

$$\delta_{ns} = \frac{C_m}{1 - \left(\dfrac{P_u}{0.75P_c}\right)} \geq 1$$ — Eq. (10-9)

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2}$$ — Eq. (10-10)

$$EI = \frac{E_cI_g}{\beta}\left(0.5 - \frac{e}{h}\right) \geq 0.1\,\frac{E_cI_g}{\beta}$$ — Eq. (1)

$$\leq 0.4\,\frac{E_cI_g}{\beta}$$

$$\frac{e}{h} = \frac{6.75}{6.5} = 1.04 > 0.5$$

Thus, $EI = 0.1\left(\dfrac{E_cI_g}{\beta}\right)$

$E_c = 57{,}000\sqrt{4000} = 3.605 \times 10^6$ psi — 8.5.1

$$I_g = \frac{12 \times 6.5^3}{12} = 274.6 \text{ in.}^4$$

$$\beta = 0.9 + 0.5\,\beta^2_d - 12\rho \geq 1.0$$

$$= 0.9 + 0.5\,\beta^2_d - 12\,(0.0026)$$

$$= 0.869 + 0.5\,\beta^2_d \geq 1.0$$

$$EI = \frac{0.1 \times 3.605 \times 10^6 \times 274.6}{\beta} = \frac{99 \times 10^6}{\beta} \text{ lb-in.}^2$$

$$P_c = \frac{\pi^2 \times 99 \times 10^6}{\beta(16 \times 12)^2 \times 1000} = \frac{26.5}{\beta} \text{ kips}$$

$C_m = 1.0$ for members with transverse loads between supports 10.12.3.1

Determine magnified moment M_c for each load case.

Load Comb.	P_u (kips)	$M_2 = M_u$ (in.-kips)	β_d	β	EI (lb-in.^{2})	P_c (kips)	δ_{ns}	M_c (in.-kips)
1	3.2	15.1	0.91	1.28	77 x 10^6	20.7	1.26	19.0
2	3.9	26.0	0.74	1.14	87 x 10^6	23.2	1.29	33.5
3	3.2	27.4	0.91	1.28	77 x 10^6	20.7	1.26	34.5
4	2.2	22.0	1.00	1.37	72 x 10^6	19.3	1.18	26.0

7. Check design strength vs. required strength

Assume that the section is tension-controlled for each load combination, i.e., $\varepsilon_t \geq 0.005$ and $\phi = 0.90$. 10.3.4 9.3.2

The following table contains a summary of the strain compatibility analysis for each load combination, based on the assumption above:

Load Comb.	$P_n = P_u/\phi$ (kips)	a (in.)	c (in.)	ε_t (in./in.)
1	3.6	0.38	0.45	0.0187
2	4.3	0.40	0.47	0.0177
3	3.6	0.38	0.45	0.0187
4	2.4	0.35	0.42	0.0205

For example, the strain in the reinforcement ε_t is computed for load combination No. 2 as follows:

$$P_n = 0.85 f'_c ba - A_s f_y$$ 10.3.1 10.2.1

$$4.3 = (0.85)(4)(12)a - (0.2)(60) = 40.8a - 12$$

$$a = 0.40 \text{ in.}$$

$$c = a/\beta_1 = 0.4/0.85 = 0.47 \text{ in.}$$ 10.2.7.1 10.2.7.3

$$\varepsilon_t = \frac{0.003}{c}(d-c)$$ 10.2.2

$$= \frac{0.003}{0.47}(3.25-0.47)$$

$$= 0.0177 > 0.0050 \rightarrow \text{tension-controlled section}$$ 10.3.4

Note that the strain in the reinforcement for each of the load combinations is greater than 0.0050, so that the assumption of tension-controlled sections ($\phi = 0.90$) is correct.

For each load combination, the required nominal strength will be compared to the computed design strength. The results are tabulated below.

Load Comb.	Required Nominal Strength		Design Strength M_n (in.-kips)
	$P_n = P_u/\phi$ (kips)	$M_n = M_c/\phi$ (in.-kips)	
1	3.6	21.1	47.7
2	4.3	37.2	49.7
3	3.6	38.3	47.7
4	2.4	28.9	44.2

For example, the design strength M_n is computed for load combination No. 2 as follows:

$$M_n = 0.85\, f'_c\, ba\left(\frac{h}{2}-\frac{a}{2}\right) - A_s f_y\left(\frac{h}{2}-d_t\right)$$

$$= 0.85(4)(12)(0.40)\left(\frac{6.5}{2}-\frac{0.40}{2}\right) - 0.2(60)\left(\frac{6.5}{2}-3.25\right)$$

$$= 49.7 \text{ in.-kips}$$

The wall is adequate with the No. 4 @ 12 in. since the design strength is greater than the required nominal strength for all load combinations.

This conclusion can also be verified by utilizing pcaColumn program. Figure 21-4 shows the interaction diagram for the wall cross section with the applied factored loads

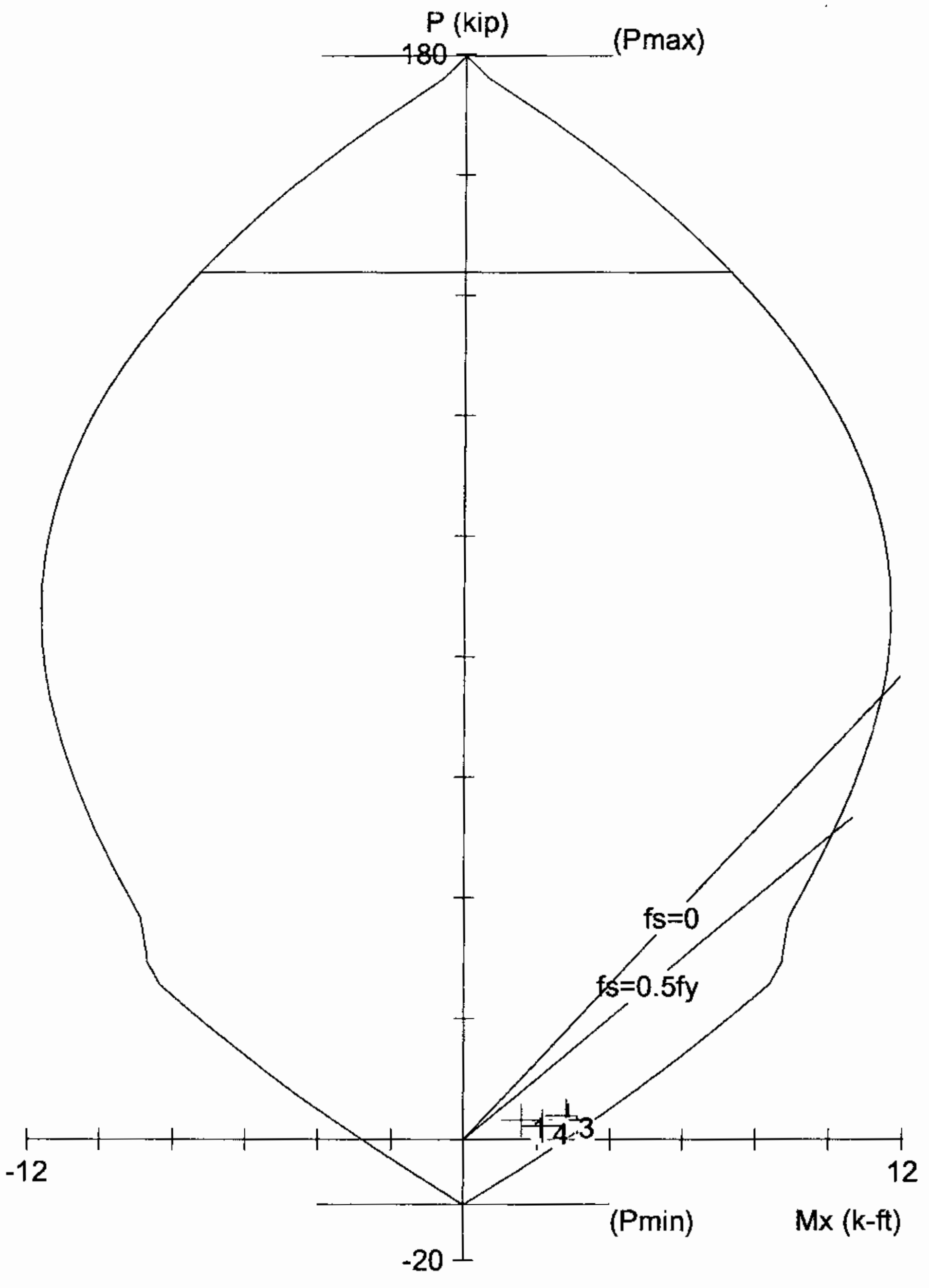
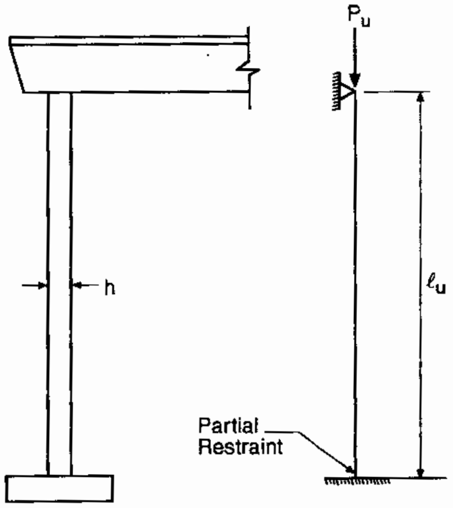


Figure 21-4 Interaction Diagram Generated Using pcaColumn Program

Example 21.2—Design of Bearing Wall by Empirical Design Method (14.5)

A concrete bearing wall supports a floor system of precast single tees spaced at 8 ft on centers. The stem of each tee section is 8 in. wide. The tees have full bearing on the wall. The height of the wall is 15 ft, and the wall is considered laterally restrained at the top.



Design Data:

Floor beam reactions: dead load = 28 kips
live load = 14 kips

$f'_c = 4000$ psi
$f_y = 60{,}000$ psi

Neglect weight of wall

Calculations and Discussion	Code Reference

The general design procedure is to select a trial wall thickness **h**, then check the trial wall for the applied loading conditions.

1. Select trial wall thickness h

$$h \geq \frac{\ell_u}{25} \text{ but not less than 4 in.}$$ *14.5.3.1*

$$\geq \frac{15 \times 12}{25} = 7.2 \text{ in.}$$

Try h = 7.5 in.

2. Calculate factored loading

$$P_u = 1.2D + 1.6L$$ *Eq. (9-2)*

$$= 1.2\,(28) + 1.6\,(14) = 33.6 + 22.4 = 56.0 \text{ kips}$$

3. Check bearing strength of concrete

Assume width of stem for bearing equal to 7 in., to allow for beveled bottom edges.

Loaded area $A_1 = 7 \times 7.5 = 52.5$ in.^{2}

Bearing capacity $= \phi(0.85f'_cA_1) = 0.65\,(0.85 \times 4 \times 52.5) = 116$ kips > 56.0 kips O.K. *10.17.1*

4. Calculate design strength of wall

Effective horizontal length of wall per tee reaction = $\begin{cases} 8 \times 12 = 96 \text{ in.} \\ 7 + 4(7.5) = 37 \text{ in.} \quad \text{(governs)} \end{cases}$ *14.2.4*

k = 0.8 *14.5.2*

$$\phi P_n = 0.55\phi f'_c A_g \left[1 - \left(\frac{k\ell_c}{32h}\right)^2\right]$$ *Eq. (14-1)*

$$= 0.55 \times 0.70 \times 4\,(37 \times 7.5)\left[1 - \left(\frac{0.8 \times 15 \times 12}{32 \times 7.5}\right)^2\right]$$

$= 273 \text{ kips} > 56 \text{ kips}$ O.K.

The 7.5-in. wall is adequate, with sufficient margin for possible effect of load eccentricity.

5. Determine single layer of reinforcement

Based on 1-ft width of wall and Grade 60 reinforcement (No. 5 and smaller):

Vertical $A_s = 0.0012 \times 12 \times 7.5 = 0.108 \text{ in.}^2/\text{ft}$ *14.3.2*

Horizontal $A_s = 0.0020 \times 12 \times 7.5 = 0.180 \text{ in.}^2/\text{ft}$ *14.3.3*

Spacing = $\begin{cases} 3h = 3 \times 7.5 = 22.5 \text{ in.} \\ 18 \text{ in.} \quad \text{(governs)} \end{cases}$ *14.3.5*

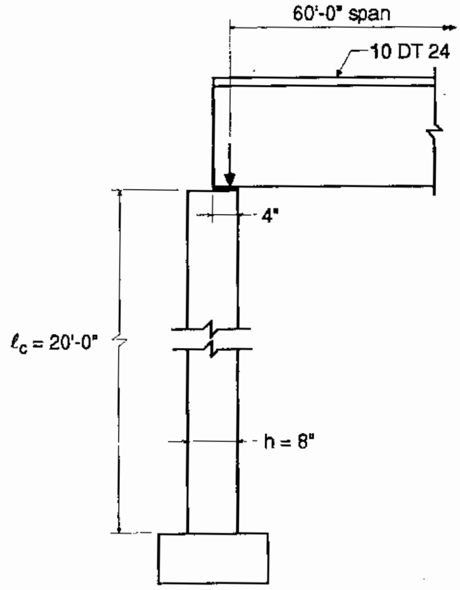
Vertical A_s: use No. 4 @ 18 in. on center ($A_s = 0.13 \text{ in.}^2/\text{ft}$)

Horizontal A_s: use No. 4 @ 12 in. on center ($A_s = 0.20 \text{ in.}^2/\text{ft}$)

Design aids such as the one in Fig. 21-8 may be used to select reinforcement directly.

Example 21.3—Design of Precast Panel by the Alternate Design Method (14.8)

Determine the required vertical reinforcement for the precast wall panel shown below. The roof loads are supported through the 3.75 in. webs of the 10DT24 which are spaced 5 ft on center.



Design data:

Weight of 10DT24 = 468 plf
Roof dead load = 20 psf
Roof live load = 30 psf
Wind load = 30 psf
Concrete $f_c' = 4000$ psi ($w_c = 150$ pcf)
Reinforcing steel $f_y = 60{,}000$ psi

Calculations and Discussion	Code Reference

1. Trial wall section

Try h = 8 in.
Try a single layer of No. 4 @ 9 in. vertical reinforcement ($A_s = 0.27$ in.^{2}/ft) at centerline of wall.

For a 1-ft wide design strip: ρ (gross) $= \dfrac{A_s}{\ell_w h} = \dfrac{0.27}{12 \times 8} = 0.0028 > 0.0012$ O.K. *14.3.2*

2. Distribution width of interior concentrated loads at mid-height of wall (see Fig. 21-4)

$$W + \frac{\ell_c}{2} = \frac{3.75}{12} + \frac{20}{2} = 10.3 \text{ ft}$$ *14.8.2.5*

S = 5.0 ft (governs)

3. Roof loading per foot width of wall

Dead load $= \left[\frac{468}{2} + (20 \times 5)\right]\left(\frac{60}{2}\right) = 10{,}020$ lbs/5 ft = 2,004 plf

Live load $= (30 \times 5)\left(\frac{60}{2}\right) = 4{,}500$ lbs/5 ft = 900 plf

Wall dead load $= \frac{8}{12} \times 20 \times 150 = 2{,}000$ plf

Eccentricity of the roof loads about the panel center line $= \frac{2}{3} \times 4 = 2.7$ in.

4. Factored load combinations at mid-height of wall (see Fig. 21-5)

a. Load comb. 1: $U = 1.2D + 0.5L_r$ — *Eq. (9-2)*

$$P_u = P_{u1} + \frac{P_{u2}}{2}$$

$$P_{u1} = (1.2 \times 2.0) + (0.5 \times 0.9) = 2.4 + 0.5 = 2.9 \text{ kips}$$

$$P_{u2} = 1.2 \times 2.0 = 2.4 \text{ kips}$$

$$P_u = 2.9 + \frac{2.4}{2} = 4.1 \text{ kips}$$

$$M_u = \frac{M_{ua}}{1 - \dfrac{5P_u\ell_c^2}{(0.75)\ 48E_cI_{cr}}}$$ *Eq. (14-6)*

$$M_{ua} = \frac{w_u\ell_c^2}{8} + \frac{P_{u1}e}{2} = 0 + \frac{2.9 \times 2.7}{2} = 3.9 \text{ in.-kips}$$

$$E_c = 57{,}000\sqrt{4000} = 3{,}605{,}000 \text{ psi}$$ *8.5.1*

$$I_{cr} = nA_{se}(d - c)^2 + \frac{\ell_w c^3}{3}$$ *Eq. (14-7)*

$$n = \frac{E_s}{E_c} = \frac{29{,}000}{3605} = 8.0$$

$$A_{se} = \frac{P_u + A_sf_y}{f_y} = \frac{4.1 + (0.27 \times 60)}{60} = 0.34 \text{ in.}^2\text{/ft}$$ *Eq. (14-8)*

$$a = \frac{A_{se}f_y}{0.85f'_c\ell_w} = \frac{0.34 \times 60}{0.85 \times 4 \times 12} = 0.50 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.50}{0.85} = 0.59 \text{ in.}$$

Therefore,

$$I_{cr} = 8.0 \times 0.34 \times (4 - 0.59)^2 + \frac{12 \times 0.59^3}{3} = 32.5 \text{ in.}^4$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right)d_t - 0.003$$

$$= \left(\frac{0.003}{0.59}\right)(4) - 0.003 = 0.0173 > 0.005$$

Therefore, section is tension-controlled — *10.3.4*

$\phi = 0.9$ — *9.3.2*

$$M_u = \frac{3.9}{1 - \dfrac{5 \times 4.1 \times (20 \times 12)^2}{0.75 \times 48 \times 3{,}605 \times 32.5}} = 5.4 \text{ in.-kips}$$ *Eq. (14-6)*

b. Load comb. 2: $U = 1.2D + 1.6L_r + 0.8W$ — *Eq. (9-3)*

$P_{u1} = (1.2 \times 2) + (1.6 \times 0.9) = 3.8$ kips

$P_{u2} = 1.2 \times 2.0 = 2.4$ kips

$$P_u = 3.8 + \frac{2.4}{2} = 5.0 \text{ kips}$$

$$M_{ua} = \frac{w_u\ell_c^2}{8} + \frac{P_{u1}e}{2} = \frac{0.8 \times 0.030 \times 20^2}{8} + \frac{3.8 \times (2.7/12)}{2}$$

$$= 1.2 + 0.4 = 1.6 \text{ ft-kips} = 19.2 \text{ in.-kips}$$

$$A_{se} = \frac{5.0 + (0.27 \times 60)}{60} = 0.35 \text{ in.}^2\text{/ft}$$ *Eq. (14-8)*

$$a = \frac{0.35 \times 60}{0.85 \times 4 \times 12} = 0.51 \text{ in.}$$

$c = \dfrac{0.51}{0.85} = 0.60$ in.

Therefore,

$I_{cr} = 8.0 \times 0.35 \times (4 - 0.60)^2 + \dfrac{12 \times 0.60^3}{3} = 33.2$ in.^{4} *Eq. (14-7)*

$\varepsilon_t = \left(\dfrac{0.003}{0.60}\right)(4) - 0.003 = 0.0170 > 0.005$

$\phi = 0.9$ *9.3.2*

$M_u = \dfrac{19.2}{1 - \dfrac{5 \times 5.0 \times (20 \times 12)^2}{0.75 \times 48 \times 3{,}605 \times 33.2}} = 28.8$ in.-kips *Eq. (14-6)*

c. Load comb. 3: $U = 1.2D + 1.6W + 0.5L_r$ *Eq. (9-4)*

$P_{u1} = (1.2 \times 2.0) + (0.5 \times 0.9) = 2.9$ kips

$P_{u2} = 1.2 \times 2.0 = 2.4$ kips

$P_u = 2.9 + \dfrac{2.4}{2} = 4.1$ kips

$M_{ua} = \dfrac{1.6 \times 0.03 \times 20^2}{8} + \dfrac{2.9 \times (2.7/12)}{2}$

$= 2.4 + 0.3 = 2.7$ ft-kips $= 32.4$ in.-kips

$A_{se} = \dfrac{4.1 + (0.27 \times 60)}{60} = 0.34$ in.^{2}/ft *Eq. (14-8)*

$a = \dfrac{0.34 \times 60}{0.85 \times 4 \times 12} = 0.5$ in.

$c = \dfrac{0.5}{0.85} = 0.59$ in.

Therefore,

$I_{cr} = 8 \times 0.34 \times (4 - 0.59)^2 + \dfrac{12 \times 0.59^3}{3} = 32.5$ in.^{4}

$\phi = 0.9$ as in load combination 1

$$M_u = \frac{32.4}{1 - \dfrac{5 \times 4.1 \times (20 \times 12)^2}{0.75 \times 48 \times 3605 \times 32.5}} = 45.0 \text{ in.-kips}$$

d. Load comb. 4: $U = 0.9D + 1.6W$ — *Eq. (9-6)*

$P_{u1} = 0.9 \times 2.0 = 1.8$ kips

$P_{u2} = 0.9 \times 2.0 = 1.8$ kips

$$P_u = 1.8 + \frac{1.8}{2} = 2.7 \text{ kips}$$

$$M_{ua} = \frac{1.6 \times 0.030 \times 20^2}{8} + \frac{1.8 \times (2.7/12)}{2} = 2.6 \text{ ft-kips} = 31.2 \text{ in.-kips}$$

$$A_{se} = \frac{2.7 + (0.27 \times 60)}{60} = 0.32 \text{ in.}^2/\text{ft}$$ *Eq. (14-8)*

$$a = \frac{0.32 \times 60}{0.85 \times 4 \times 12} = 0.47 \text{ in.}$$

$$c = \frac{0.47}{0.85} = 0.55 \text{ in.}$$

Therefore,

$$I_{cr} = 8.0 \times 0.32 \times (4 - 0.55)^2 + \frac{12 \times 0.55^3}{3} = 31.1 \text{ in.}^4$$ *Eq. (14-7)*

$$\varepsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003 = \left(\frac{0.003}{0.55}\right)(4) - 0.003 = 0.0188 > 0.005$$

$\phi = 0.9$ — *9.3.2*

$$M_u = \frac{31.2}{1 - \dfrac{5 \times 2.7 \times (20 \times 12)^2}{0.75 \times 48 \times 3605 \times 31.1}} = 38.7 \text{ in.-kips}$$ *Eq. (14-6)*

5. Check if section is tenson-controlled.

Assume section is tension-controlled $\phi = 0.9$ (Fig. R.9.3.2)

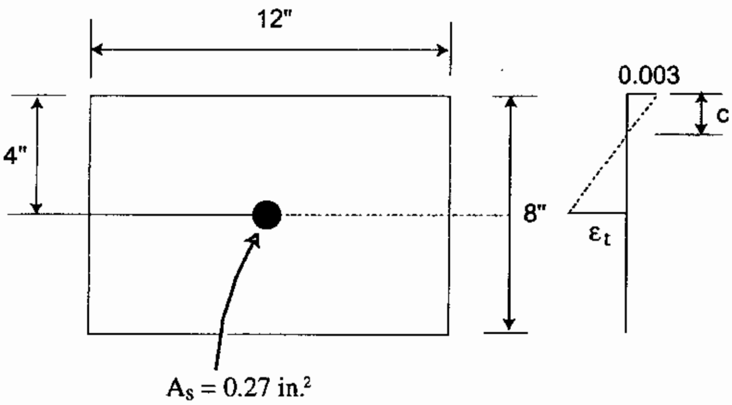
$$P_n = \frac{Pu}{\phi}$$

Lc1: U = 1.2D + 0.5 Lr
Pu = 4.1 kips

Lc2: U = 1.2D + 1.6Lr +0.8W
Pu = 5.0 kips (controls)

Lc3: U = 1.2D + 1.6Lr + 0.5W
Pu = 4.1 kips

Lc4: U = 0.9D + 1.6W
Pu = 2.7 kips



$$P_n = \frac{Pu}{\phi} = \frac{5.0}{0.9} = 5.56 \text{ kips}$$

$$a = \frac{Pu + A_s f_y}{0.85 f'_c b} = \frac{5.56 + 0.27 \times 60}{0.85 \times 4 \times 12} = \frac{21.76}{40.8} = 0.533 \text{ in.}$$

$$c = \frac{a}{0.85} = \frac{0.533}{0.85} = 0.627 \text{ in.}$$

$$\varepsilon_t = \frac{0.003}{c}(d - c) = \frac{0.003}{0.627}\left(\frac{8}{2} - 0.627\right)$$

$$= \frac{0.003}{0.627} \times 2.508$$

$$= 0.012 \geq 0.005$$

tension−controlledsection.

6. Determine M_{cr}

$$I_g = \frac{1}{12}\ell_w b^3 = \frac{1}{12} \times 12 \times 8^3 = 512 \text{ in.}^4$$

$$y_t = \frac{8}{2} = 4 \text{ in.}$$

$$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{4000} = 474.3 \text{ psi}$$ *Eq. (9-9)*

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474.3 \times 512}{4 \times 1000} = 60.7 \text{ in.-kips}$$

7. Check design moment strength ϕM_n

a. Load comb. 1

$$M_n = A_{se} f_y (d - \frac{a}{2}) = 0.34 \times 60 \times \left(4 - \frac{0.5}{2}\right) = 76.5 \text{ in.-kips}$$

$\phi M_n = 0.9 \times 76.5 = 68.9$ in.-kips $> M_u = 5.4$ in.-kips O.K. *14.8.3*
$> M_{cr} = 60.7$ in.-kips O.K. *14.8.2.4*

b. Load comb. 2

$$M_n = 0.35 \times 60 \times \left(4 - \frac{0.51}{2}\right) = 78.7 \text{ in.-kips}$$

$\phi M_n = 0.9 \times 78.7 = 70.8$ in.-kips $> M_u = 28.8$ in.-kips O.K. *14.8.3*
$> M_{cr} = 60.7$ in.-kips O.K. *14.8.2.4*

c. Load comb. 3

$$M_n = 0.34 \times 60 \times \left(4 - \frac{0.5}{2}\right) = 76.5 \text{ in.-kips}$$

$\phi M_n = 0.9 \times 76.5 = 68.9$ in.-kips $> M_u = 45.0$ in.-kips O.K.
$> M_{cr} = 60.7$ in.-kips O.K.

d. Load comb. 4

$$M_n = 0.32 \times 60 \times \left(4 - \frac{0.47}{2}\right) = 72.3 \text{ in.-kips}$$

$\phi M_n = 0.9 \times 72.3 = 65.1$ in.-kips $> M_u = 38.7$ in.-kips O.K. *14.8.3*
$> M_{cr} = 60.7$ in.-kips O.K. *14.8.2.4*

8. Check vertical stress at mid-height section

Load comb. 2 governs:

$$\frac{P_u}{A_g} = \frac{5000}{8 \times 12} = 52.1 \text{ psi} < 0.06 f'_c = 0.06 \times 4000 = 240 \text{ psi} \quad \text{O.K.}$$ *14.8.2.6*

9. Check mid-height deflection Δ_s

$$\Delta_s = \frac{5M\ell_c^2}{48E_cI_e}$$ *Eq. (14-9)*

$$M = \frac{M_{sa}}{1 - \dfrac{5P_s\ell_c^2}{48E_cI_e}}$$ *Eq. (14-10)*

Using Δ_s from Eq. (14-9), Eq. (14-10) can be rewritten as follows:

$M = M_{sa} + P_s\Delta_s$

$$M_{sa} = \frac{w\ell_c^2}{8} + \frac{P_{s1}e}{2} = \frac{0.030 \times 20^2}{8} + \frac{(2.0 + 0.9)(2.7/12)}{2} = 1.8 \text{ ft-kips} = 21.6 \text{ in.-kips}$$

$$P_s = P_{s1} + \frac{P_{s2}}{2} = (2.0 + 0.9) + \frac{2.0}{2} = 3.9 \text{ kips}$$

$$I_e = \left(\frac{M_{cr}}{M}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M}\right)^3\right] I_{cr}$$ *Eq. (9-8)*

Since I_e is a function of M, no closed form solution for Δ_s is possible. Determine Δ_s by iterative procedure.

Assume $\Delta_s = \dfrac{\ell_c}{150} = \dfrac{20 \times 12}{150} = 1.6$ in.
$M = 21.6 + (3.9 \times 1.6) = 27.8$ in.-kips

Since $M_{cr} = 60.7$ in.-kips > $M = 27.8$ in.-kips, $I_e = I_g = 512$ in.^{4}

$$M = \frac{21.6}{1 - \dfrac{5 \times 3.9 \times (20 \times 12)^2}{48 \times 3605 \times 512}} = 21.9 \text{ in.-kips}$$ *Eq. (14-10)*

$$\Delta_s = \frac{5 \times 21.9 \times (20 \times 12)^2}{48 \times 3605 \times 512} = 0.07 \text{ in.}$$ *Eq. (14-9)*

No further iterations are required since $I_e = I_g$.

Therefore,

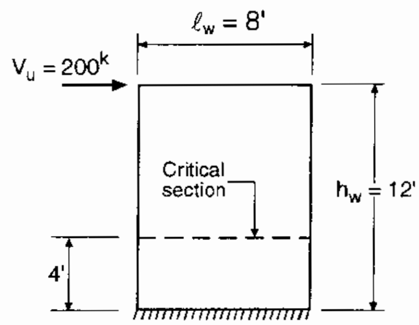
$\Delta_s = 0.07$ in. $< \dfrac{\ell_c}{150} = \dfrac{20 \times 12}{150} = 1.6$ in. O.K.

The wall is adequate with No. 4 @ 9 in. vertical reinforcement.

Example 21.4—Shear Design of Wall

Determine the shear and flexural reinforcement for the wall shown.

h = 8 in.
f_c' = 3000 psi
f_y = 60,000 psi



Calculations and Discussion	Code Reference

1. Check maximum shear strength permitted

$\phi V_n = \phi 10\sqrt{f_c'}hd$ *11.10.3*

where $d = 0.8\ell_w = 0.8 \times 8 \times 12 = 76.8$ in. *11.10.4*

$\phi V_n = 0.75 \times 10\sqrt{3000} \times 8 \times 76.8/1000 = 252.4 \text{ kips} > V_u = 200 \text{ kips}$ O.K.

2. Calculate shear strength provided by concrete V_c

Critical section for shear: *11.10.7*

$$\frac{\ell_w}{2} = \frac{8}{2} = 4 \text{ ft (governs)}$$

or

$$\frac{h_w}{2} = \frac{12}{2} = 6 \text{ ft}$$

$$V_c = 3.3\sqrt{f_c'}hd + \frac{N_u d}{4\ell_w}$$ *Eq. (11-29)*

$$= 3.3\sqrt{3000} \times 8 \times 76.8/1000 + 0 = 111 \text{ kips}$$

or

$$V_c = \left[0.6\sqrt{f'_c} + \frac{\ell_w\left(1.25\sqrt{f'_c} + \dfrac{0.2N_u}{\ell_w h}\right)}{\dfrac{M_u}{V_u} - \dfrac{\ell_w}{2}}\right]hd$$ *Eq. (11-30)*

$$= \left[0.6\sqrt{3000} + \frac{96\left(1.25\sqrt{3000} + 0\right)}{96 - 48}\right]\left(\frac{8 \times 76.8}{1000}\right) = 104 \text{ kips (governs)}$$

where $M_u = (12 - 4)\,V_u = 8V_u$ ft-kips $= 96V_u$ in.-kips

3. Determine required horizontal shear reinforcement

$V_u = 200 \text{ kips} > \phi V_c/2 = 0.75\,(104)/2 = 39.0 \text{ kips}$ *11.10.8*

Shear reinforcement must be provided in accordance with 11.10.9.

$V_u \le \phi V_n$ *Eq. (11-1)*

$\le \phi(V_c + V_s)$ *Eq. (11-2)*

$$\le \phi V_c + \frac{\phi A_v f_y d}{s_2}$$ *Eq. (11-31)*

$$\frac{A_v}{s_2} = \frac{(V_u - \phi V_c)}{\phi f_y d}$$

$$= \frac{[200 - (0.75 \times 104)]}{0.75 \times 60 \times 76.8} = 0.0353$$

For 2-No. 3: $s_2 = \dfrac{2 \times 0.11}{0.0353} = 6.2$ in.

2-No. 4: $s_2 = \dfrac{2 \times 0.20}{0.0353} = 11.3$ in.

2-No. 5: $s_2 = \dfrac{2 \times 0.31}{0.0353} = 17.6$ in.

Try 2-No. 4 @ 10 in.

$$\rho_h = \frac{A_v}{A_g} = \frac{2 \times 0.20}{8 \times 10} = 0.0050 > 0.0025 \quad \text{O.K.}$$ *11.10.9.2*

$$\text{Maximum spacing} = \begin{cases} \dfrac{\ell_w}{5} = \dfrac{8 \times 12}{5} = 19.2 \text{ in.} \\ 3h = 3 \times 8 = 24.0 \text{ in.} \\ 18.0 \text{ in. (governs)} \end{cases}$$ *11.10.9.3*

Use 2-No. 4 @ 10 in.

4. Determine vertical shear reinforcement

$$\rho_n = 0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)(\rho_h - 0.0025) \geq 0.0025$$ *Eq. (11-32)*

$$= 0.0025 + 0.5\ (2.5 - 1.5)\ (0.0050 - 0.0025)$$

$$= 0.0038$$

$$\text{Maximum spacing} = \begin{cases} \dfrac{\ell_w}{3} = \dfrac{8 \times 12}{3} = 32 \text{ in.} \\ 3h = 3 \times 8 = 24.0 \text{ in.} \\ 18.0 \text{ in. (governs)} \end{cases}$$ *11.10.9.5*

Use 2-No. 4 @ 13 in. ($\rho_n = 0.0038$)

5. Design for flexure

$M_u = V_u h_w = 200 \times 12 = 2{,}400$ ft-kips

Assume tension-controlled section ($\phi = 0.90$) *9.3.2*
with $d = 0.8\ell_w = 0.8 \times 96 = 76.8$ in. *11.10.4*
(Note: an exact value of d will be determined by a strain compatibility analysis below)

$$R_n = \frac{M_u}{\phi b d^2} = \frac{2400 \times 12{,}000}{0.9 \times 8 \times 76.8^2} = 678 \text{ psi}$$

$$\rho = \frac{0.85 f'_c}{fy}\left(1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right)$$

$$= \frac{0.85 \times 3}{60}\left(1 - \sqrt{1 - \frac{2 \times 678}{0.85 \times 3000}}\right) = 0.0134$$

$A_s = \rho bd = 0.0134 \times 8 \times 76.8 = 8.24$ in.^{2}

Try 9-No. 8 (A_s = 7.11 in.^{2}) at each end of wall, which provides less area of steel than that determined based on $d = 0.8\ell_w$.

Check moment strength of wall with 9-No. 8 bars using a strain compatibility analysis (see figure below for reinforcement layout).

From strain compatibility analysis (including No. 4 vertical bars):

c = 13.1 in.

d = 81.0 in.

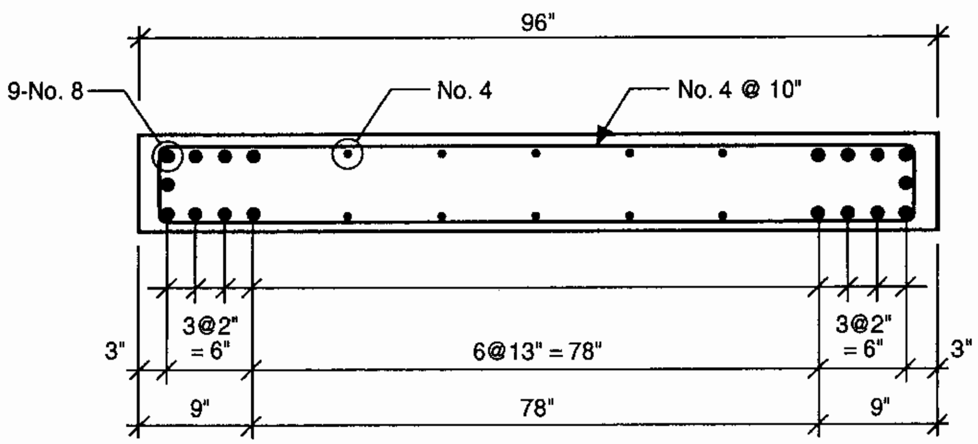
$\varepsilon_t = 0.0182 > 0.0050$

Therefore, section is tension-controlled as assumed and $\phi = 0.90$.

M_n = 3451 ft-kips

$\phi M_n = 0.9 \times 3451 = 3106$ ft-kips > 2400 ft-kips O.K.

Use 9-No. 8 bars each side (A_s = 7.11 in.^{2})



22

Footings

UPDATE FOR THE '05 CODE

Section 15.5.3 clarifies which proedure and code provisions apply for design of piles where the distance between the axis of the pile and the axis of the column is less then or equal to two times the distance between the top of the pile cap and the top of the pile.

GENERAL CONSIDERATIONS

Provisions of Chapter 15 apply primarily for design of footings supporting a single column (isolated footings) and do not provide specific design provisions for footings supporting more than one column (combined footings). The code states that combined footings shall be proportioned to resist the factored loads and induced reactions in accordance with the appropriate design requirements of the code. Detailed discussion of combined footing design is beyond the scope of Part 22. However, as a general design approach, combined footings may be designed as beams in the longitudinal direction and as an isolated footing in the transverse direction over a defined width on each side of the supported columns. Code references 15.1 and 15.2 are suggested for detailed design recommendations for combined footings.

15.2 LOADS AND REACTIONS

Footings must be designed to safely resist the effects of the applied factored axial loads, shears and moments. The size (base area) of a footing or the arrangement and number of piles is determined based on the allowable soil pressure or allowable pile capacity, respectively. The allowable soil or pile capacity is determined by principles of soil mechanics in accordance with general building codes. The following procedure is specified for footing design:

1. The footing size (plan dimensions) or the number and arrangement of piles is to be determined on the basis of unfactored (service) loads (dead, live, wind, earthquake, etc.) and the allowable soil pressure or pile capacity (15.2.2).

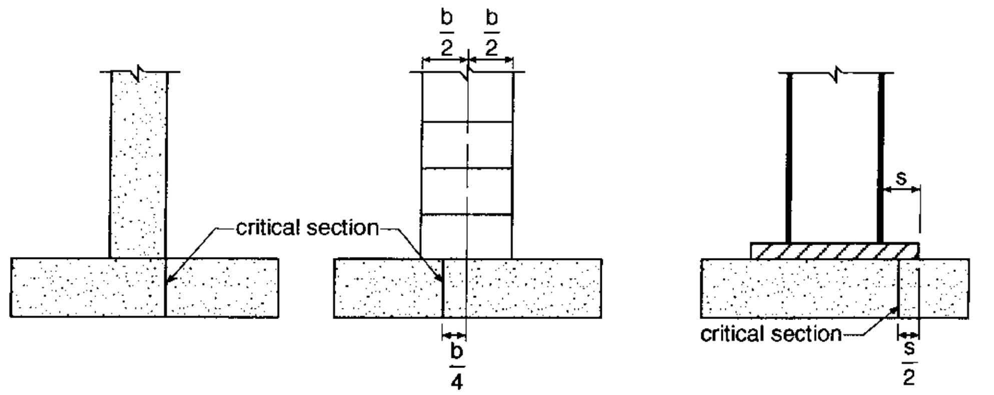
2. After having established the plan dimensions, the depth of the footing and the required amount of reinforcement are determined based on the appropriate design requirements of the code (15.2.1). The service pressures and the resulting shear and moments are multiplied by the appropriate load factors specified in 9.2 and are used to proportion the footing.

For purposes of analysis, an isolated footing may be assumed to be rigid, resulting in a uniform soil pressure for concentric loading, and a triangular or trapezoidal soil pressure distribution for eccentric loading (combined axial and bending effect). Only the computed bending moment that exists at the base of the column or pedestal is to be transferred to the footing. The minimum moment requirement for slenderness considerations in 10.12.3.2 need not be transferred to the footing (R15.2).

15.4 MOMENT IN FOOTINGS

At any section of a footing, the external moment due to the base pressure shall be determined by passing a vertical plane through the footing and computing the moment of the forces acting over the entire area of the footing on one side of the vertical plane. The maximum factored moment in an isolated footing is determined by passing a vertical plane through the footing at the critical sections shown in Fig. 22-1 (15.4.2). This moment is subsequently used to determine the required area of flexural reinforcement in that direction.

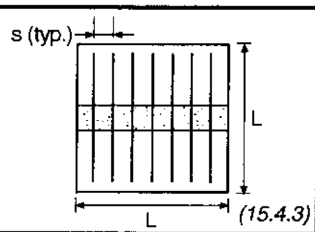
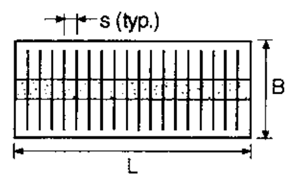
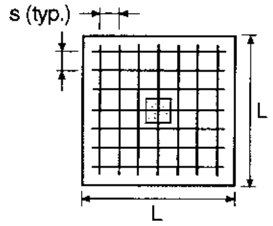
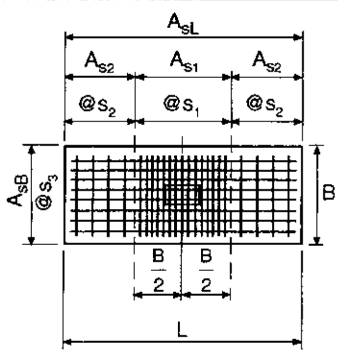
In one-way square or rectangular footings and two-way square footings, flexural reinforcement shall be distributed uniformly across the entire width of the footing (15.4.3). For two-way rectangular footings, the reinforcement must be distributed as shown in Table 22-1 (15.4.4).



(a) Concrete Column, Pedestal or Wall (b) Masonry Wall (c) Column with Steel Base Plate

Figure 22-1 Critical Location for Maximum Factored Moment in an Isolated Footing (15.4.2)

Table 22-1 Distribution of Flexural Reinforcement

Footing Type	Square Footing	Rectangular Footing
One-way	 *(15.4.3)*	 *(15.4.3)*
Two-way	 *(15.4.3)*	 $A_{s1} = \gamma_s A_{sL}$ $A_{s2} = \frac{(1-\gamma_s)A_{sL}}{2}$ $\beta = \frac{L}{B}$ $\gamma_s = \left(\frac{2}{\beta+1}\right)$ *(15.4.4)*

15.5 SHEAR IN FOOTINGS

Shear strength of a footing supported on soil or rock in the vicinity of the supported member (column or wall) must be determined for the more severe of the two conditions stated in 11.12. Both wide-beam action (11.12.1.1) and two-way action (11.12.1.2) must be checked to determine the required footing depth. Beam action assumes that the footing acts as a wide beam with a critical section across its entire width. If this condition is the more severe, design for shear proceeds in accordance with 11.1 through 11.5. Even though wide-beam action rarely controls the shear strength of footings, the designer must ensure that shear strength for beam action is not exceeded. Two-way action for the footing checks "punching" shear strength. The critical section for punching shear is a perimeter b_o around the supported member with the shear strength computed in accordance with 11.12.2.1. Tributary areas and corresponding critical sections for wide-beam action and two-way action for an isolated footing are illustrated in Fig. 22-2. Note that it is permissible to use a critical section with four straight sides for square or rectangular columns (11.12.1.3).

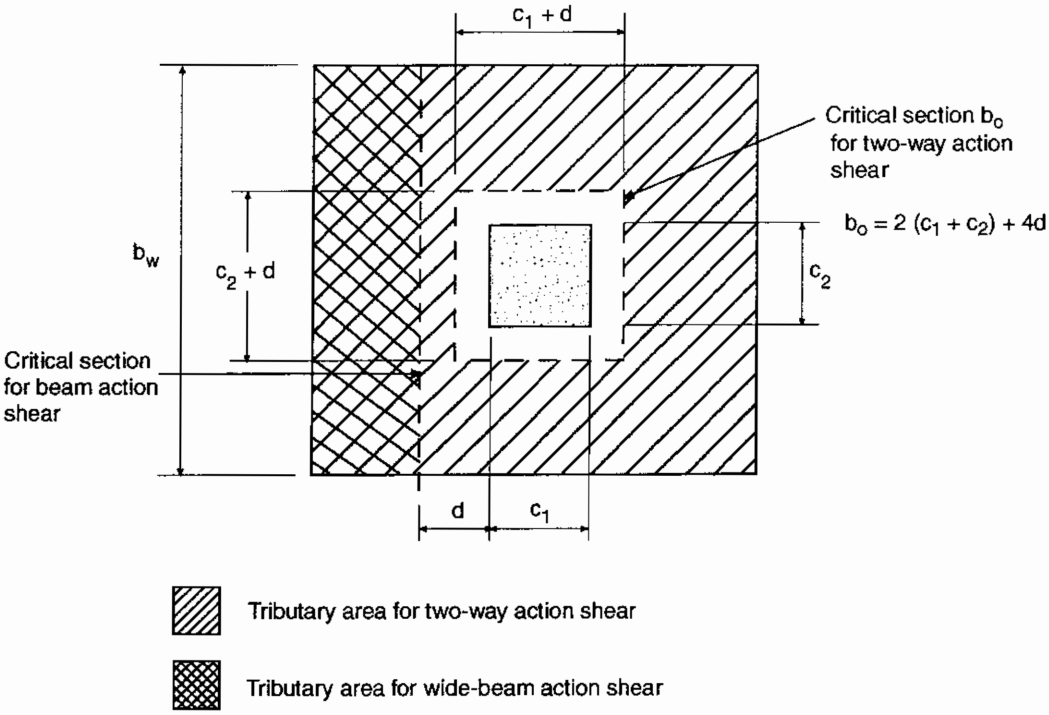


Figure 22-2 Tributary Areas and Critical Sections for Shear

In the design of a footing for two-way action, V_c is the smallest value obtained from Eqs. (11-33), (11-34), and (11-35). Eq. (11-35) established the upper limit of V_c at $4\sqrt{f'_c}\, b_o d$. Eq. (11-33) accounts for the effect of β, which is the ratio of the long side to the short side of the column, concentrated load, or reaction area. As β increases the concrete shear strength decreases (see Fig. 22-3). Eq. (11-34) was developed to account for the effect of b_o/d, and is based on tests that indicated shear strength decreases as b_o/d increases.

If the factored shear force V_u at the critical section exceeds the governing shear strength ϕV_c given by the minimum of Eqs. (11-33), (11-34), or (11-35), shear reinforcement must be provided. For shear reinforcement consisting of bars or wires and single- or multiple-leg stirrups, the shear strength may be increased to a maximum value of $6\sqrt{f'_c}\, b_o d$ (11.12.3.2), provided the footing has an effective depth d greater than or equal to 6 in., but not less than 16 times the shear reinforcement bar diameter (11.12.3). However, shear reinforcement must be designed to carry the shear in excess of $2\sqrt{f'_c}\, b_o d$ (11.12.3.1).

For footing design (without shear reinforcement), the shear strength equations may be summarized as follows:

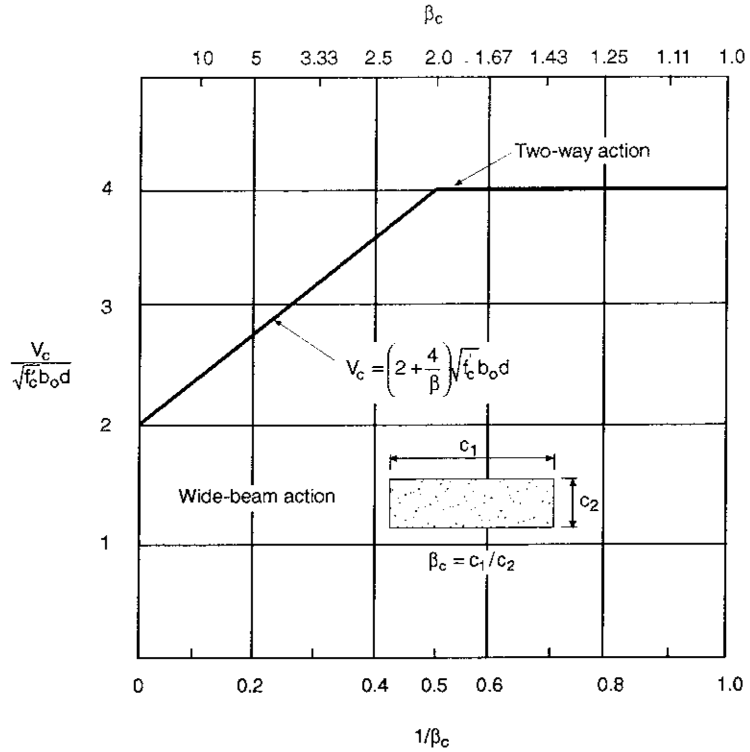


Figure 22-3 Shear Strength of Concrete in Footings

- Wide beam action

$$V_u \leq \phi V_n \qquad \textit{Eq. (11-1)}$$

$$\leq \phi\left(2\sqrt{f'_c}\, b_w d\right) \qquad \textit{Eq. (11-3)}$$

where b_w and V_u are computed for the critical section defined in 11.12.1.1 (see Fig. 22-2).

- Two-way action

$$V_u \leq \text{minimum of} \begin{cases} \left(2 + \frac{4}{\beta}\right)\sqrt{f'_c}\, b_o d & \textit{Eq. (11-33)} \\ \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c}\, b_o d & \textit{Eq. (11-34)} \\ 4\sqrt{f'_c}\, b_o d & \textit{Eq. (11-35)} \end{cases}$$

where

β = ratio of long side to short side of the column, concentrated load or reaction area

α_s = 40 for interior columns
= 30 for edge columns
= 20 for corner columns

b_o = perimeter of critical section shown in Fig. 22-2

15.8 TRANSFER OF FORCE AT BASE OF COLUMN, WALL, OR REINFORCED PEDESTAL

With the publication of ACI 318-83, 15.8 addressing transfer of force between a footing and supported member (column, wall, or pedestal) was revised to address both cast-in-place and precast construction. Section 15.8.1 gives general requirements applicable to both cast-in-place and precast construction. Sections 15.8.2 and 15.8.3 give additional rules for cast-in-place and precast construction, respectively. For force transfer between a footing and a precast column or wall, anchor bolts or mechanical connectors are specifically permitted by 15.8.3, with anchor bolts to be designed in accordance with Appendix D. (Prior to the '83 code, connection between a precast member and footing required either longitudinal bars or dowels crossing the interface, contrary to common practice.) Also note that walls are specifically addressed in 15.8 for force transfer to footings.

Section 15.8.3 contains requirements for the connection between precast columns and walls to supporting members. This section refers to 16.5.1.3 for minimum connection strength. Additionally, for precast columns with larger cross-sectional areas than required for loading, it is permitted to use a reduced effective area based on the cross-section required, but not less than one-half the total area when determining the nominal strength in tension.

The minimum tensile strength of a connection between a precast wall panel and its supporting member is required to have a minimum of two ties per panel with a minimum nominal tensile capacity of 10 kips per tie (16.5.1.3(b)).

All forces applied at the base of a column or wall (supported member) must be transferred to the footing (supporting member) by bearing on concrete and/or by reinforcement. Tensile forces must be resisted entirely by reinforcement. Bearing on concrete for both supported and supporting member must not exceed the concrete bearing strength permitted by 10.17 (see discussion on 10.17 in Part 6).

For a supported column, the bearing capacity ϕP_{nb} is

$$\phi P_{nb} = \phi(0.85 f'_c A_1) \qquad \textit{10.17.1}$$

where

f'_c = compressive strength of the column concrete

A_1 = loaded area (column area)

ϕ = 0.65 *9.3.2.4*

For a supporting footing,

$$\phi P_{nb} = \phi(0.85 f'_c A_1)\sqrt{\frac{A_2}{A_1}} \le 2\phi(0.85 f'_c A_1)$$

where

f'_c = compressive strength of the footing concrete

A_2 = area of the lower base of the largest frustrum of a pyramid, cone, or tapered wedge contained wholly within the footing and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal (see Fig. R10.17).

Example 22.4 illustrates the design for force transfer at the base of a column.

When bearing strength is exceeded, reinforcement must be provided to transfer the excess load. A minimum area of reinforcement must be provided across the interface of column or wall and footing, even where concrete bearing strength is not exceeded. With the force transfer provisions addressing both cast-in-place and precast construction, including force transfer between a wall and footing, the minimum reinforcement requirements are based on the type of supported member, as shown in Table 22-2.

Table 22-2 Minimum Reinforcement for Force Transfer Between Footing and Supported Member

	Cast-in-Place	Precast
Columns	$0.005A_g$ (15.8.2.1)	$\frac{200A_g}{f_y}$ (16.5.1.3 (a))
Walls	see 14.3.2 (15.8.2.2)	see 16.5.1.3(b) and (c)

For cast-in-place construction, reinforcement may consist of extended reinforcing bars or dowels. For precast construction, reinforcement may consist of anchor bolts or mechanical connectors. Reference 22.1 devotes an entire chapter on connection design for precast construction.

The shear-friction design method of 11.7.4 should be used for horizontal force transfer between columns and footings (15.8.1.4; see Example 22.6). Consideration of some of the lateral force being transferred by shear through a formed shear key is questionable. Considerable slip is required to develop a shear key. Shear keys, if provided, should be considered as an added mechanical factor of safety only, with no design shear force assigned to the shear key.

PLAIN CONCRETE PEDESTALS AND FOOTINGS

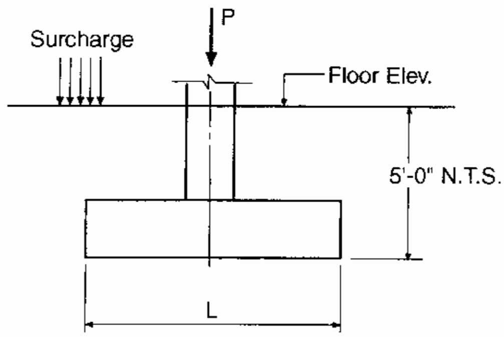
Plain concrete pedestals and footings are designed in accordance with Chapter 22. See Part 30 for an in-depth discussion and examples.

REFERENCE

22.1 *PCI Design Handbook—Precast and Prestressed Concrete*, MNL-120-04, 6th Edition, Precast/Prestressed Concrete Institute, Chicago, IL, 2004, 750 pp.

Example 22.1—Design for Base Area of Footing

Determine the base area A_f required for a square spread footing with the following design conditions:



Service dead load = 350 kips
Service live load = 275 kips
Service surcharge = 100 psf

Assume average weight of soil and concrete above footing base = 130 pcf

Allowable soil pressure at bottom of footing = 4.5 ksf

Column dimensions = 30 × 12 in.

Calculations and Discussion	Code Reference
1. Determination of base area:	
The base area of the footing is determined using service (unfactored) loads with the net permissible soil pressure.	
Weight of surcharge = 0.10 ksf	
Net allowable soil pressure = 4.5 - 0.75 = 3.75 ksf	
Required base area of footing:	*15.2.2*
$A_f = \dfrac{350 + 275}{3.75} = 167 \text{ ft}^2$	
Use a 13 × 13 ft square footing ($A_f = 169 \text{ ft}^2$)	
2. Factored loads and soil reaction:	
To proportion the footing for strength (depth and required reinforcement) factored loads are used.	*15.2.1*
$P_u = 1.2\,(350) + 1.6\,(275) = 860 \text{ kips}$	*Eq. (9-2)*
$q_s = \dfrac{P_u}{A_f} = \dfrac{860}{169} = 5.10 \text{ ksf}$	

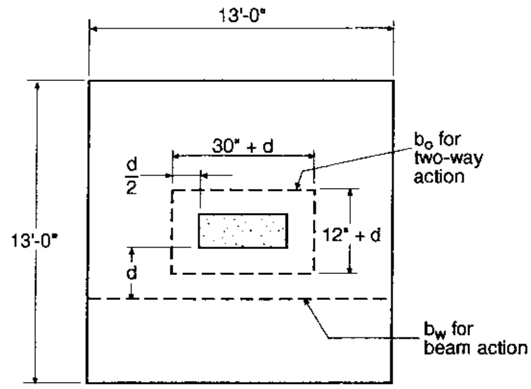
Example 22.2—Design for Depth of Footing

For the design conditions of Example 22.1, determine the overall thickness of footing required.

$f'_c = 3000$ psi

$P_u = 860$ kips

$q_s = 5.10$ ksf



Calculations and Discussion	Code Reference

Determine depth based on shear strength without shear reinforcement. Depth required for shear usually controls the footing thickness. Both wide-beam action and two-way action for strength computation need to be investigated to determine the controlling shear criteria for depth. *11.12*

Assume overall footing thickness = 33 in. and average effective thickness d = 28 in. = 2.33 ft

1. Wide-beam action:

$V_u = q_s \times$ tributary area

$b_w = 13$ ft = 156 in.

Tributary area = 13 (6.0 - 2.33) = 47.7 ft^2

$V_u = 5.10 \times 47.7 = 243$ kips

$$\phi V_n = \phi\left(2\sqrt{f'_c}\, b_w d\right)$$ *Eq. (11-3)*

$$= 0.75\left(2\sqrt{3000} \times 156 \times 28\right)/1000$$ *9.3.2.3*

$$= 359 \text{ kips} > V_u \quad \text{O.K.}$$

2. Two-way action:

$V_u = q_s \times$ tributary area

$$\text{Tributary area} = \left[(13 \times 13) - \frac{(30 + 28)(12 + 28)}{144}\right] = 152.9 \text{ ft}^2$$

$V_u = 5.10 \times 152.9 = 780$ kips

$$\frac{V_c}{\sqrt{f'_c}\,b_o d} = \text{minimum of} \begin{cases} 2 + \dfrac{4}{\beta} & \textit{Eq. (11-35)} \\ \dfrac{\alpha_s d}{b_o} + 2 & \textit{Eq. (11-36)} \\ 4 & \textit{Eq. (11-37)} \end{cases}$$

$b_o = 2\,(30 + 28) + 2\,(12 + 28) = 196$ in.

$\beta = \dfrac{30}{12} = 2.5$

$\dfrac{b_o}{d} = \dfrac{196}{28} = 7$

$\alpha_s = 40$ for interior columns

$$\frac{V_c}{\sqrt{f'_c}\,b_o d} = \begin{cases} 2 + \dfrac{4}{2.5} = 3.6 \quad \text{(governs)} \\ \dfrac{40}{7} + 2 = 7.7 \\ 4 \end{cases}$$

$\phi V_c = 0.75 \times 3.6\sqrt{3000} \times 196 \times 28/1000$

$= 812 \text{ kips} > V_u = 780 \text{ kips}$ O.K.

Example 22.3—Design for Footing Reinforcement

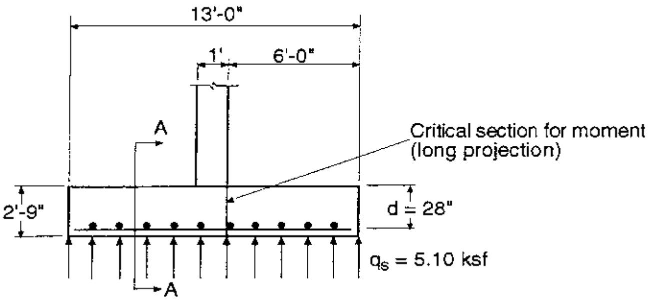
For the design conditions of Example 22.1, determine required footing reinforcement.

f'_c = 3000 psi

f_y = 60,000 psi

P_u = 860 kips

q_s = 5.10 ksf



Calculations and Discussion	Code Reference

1. Critical section for moment is at face of column *15.4.2*

   $M_u = 5.10 \times 13 \times 6^2/2 = 1193$ ft-kips

2. Compute required A_s assuming tension-controlled section ($\phi = 0.9$) *10.3.4, 9.3.2.1*

$$\text{Required } R_n = \frac{M_u}{\phi bd^2} = \frac{1193 \times 12 \times 1000}{0.9 \times 156 \times 28^2} = 130 \text{ psi}$$

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85f'_c}}\right)$$

$$= \frac{0.85 \times 3}{60}\left(1 - \sqrt{1 - \frac{2 \times 130}{0.85 \times 3000}}\right) = 0.0022$$

$$\rho \text{ (gross area)} = \frac{d}{h} \times 0.0022 = \frac{28}{33} \times 0.0022 = 0.0019$$

Check minimum A_s required for footings of uniform thickness; for Grade 60 reinforcement: *10.5.4*

ρ_{min} = 0.0018 < 0.0019 O.K. *7.12.2*

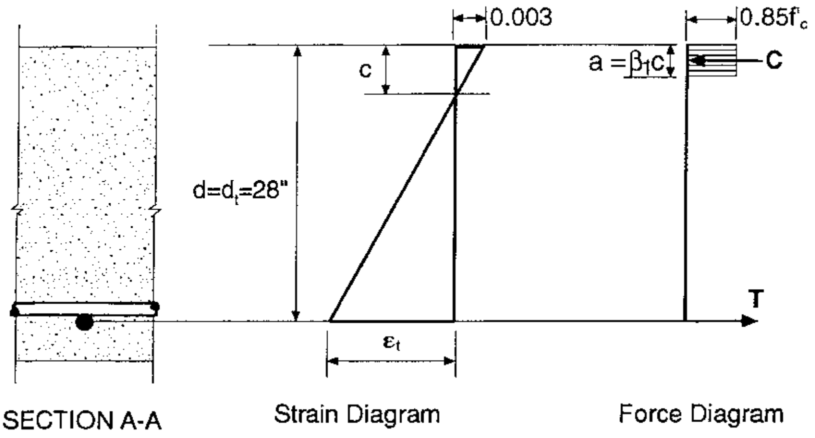
Required $A_s = \rho bd$

$A_s = 0.0022 \times 156 \times 28 = 9.60 \text{ in.}^2$

Try 13-No. 8 bars (A_s = 10.27 in.^{2}) each way

Note that a lesser amount of reinforcement is required in the perpendicular direction due to lesser M_u, but for ease of placement, the same uniformly distributed reinforcement will be used each way (see Table 22-1). Also note that d_t = 27 in. for perpendicular direction.

3. Check net tensile strain (ε_t)



$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$= \frac{10.27 \times 60}{0.85 \times 3 \times 156} = 1.55$$

$$c = \frac{a}{\beta_1} = \frac{1.55}{0.85} = 1.82$$

$$\frac{\varepsilon_t + 0.003}{d_t} = \frac{0.003}{c}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003$$

$$= \frac{0.003}{1.82} \times 28 - 0.003 = 0.043 > 0.004$$ *10.3.5*

Therefore, section is tension-controlled and initial assumption is valid, O.K.

Thus, use 13-No. 8 bars each way.

4. Check development of reinforcement. *15.6*

Critical section for development is the same as that for moment (at face of column). *15.6.3*

$$\ell_d = \left[\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \lambda}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right] d_b$$ *Eq. (12-1)*

Clear cover (bottom and side) = 3.0 in.

Center-to-center bar spacing $= \dfrac{156 - 2(3) - 2(0.5)}{12} = 12.4$ in.

$$c_b = \text{minimum of} \begin{cases} 3.0 + 0.5 = 3.5 \text{ in. (governs)} \\ \dfrac{12.4}{2} = 6.2 \text{ in.} \end{cases}$$ 12.2.4

$K_{tr} = 0$ (no transverse reinforcement)

$$\frac{c_b + K_{tr}}{d_b} = \frac{3.5 + 0}{1.0} = 3.5 > 2.5, \text{ use } 2.5$$ 12.2.3

$\psi_t = 1.0$ (less than 12 in. of concrete below bars) 12.2.4

$\psi_e = 1.0$ (uncoated reinforcement)

$\psi_t\psi_e = 1.0 < 1.7$

$\psi_s = 1.0$ (larger than No. 7 bars)

$\lambda = 1.0$ (normal weight concrete)

$$\ell_d = \left[\frac{3}{40}\,\frac{60{,}000}{\sqrt{3000}}\,\frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5}\right] \times 1.0 = 32.9 \text{ in.} > 12.0 \text{ in. O.K.}$$ 12.2.1

Since $\ell_d = 32.9$ in. is less than the available embedment length in the short direction

$\left(\dfrac{156}{2} - \dfrac{30}{2} - 3 = 60 \text{ in.}\right)$, the No. 8 bars can be fully developed.

Use 13-No. 8 each way.

Example 22.4—Design for Transfer of Force at Base of Column

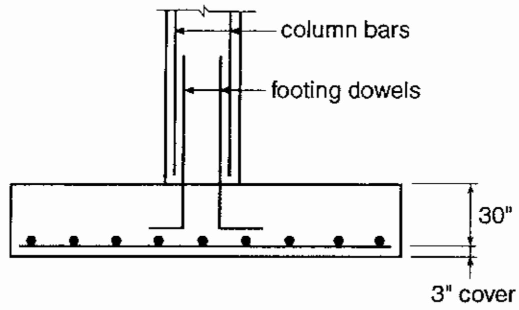
For the design conditions of Example 22.1, check force transfer at interface of column and footing.

f'_c (column) = 5000 psi

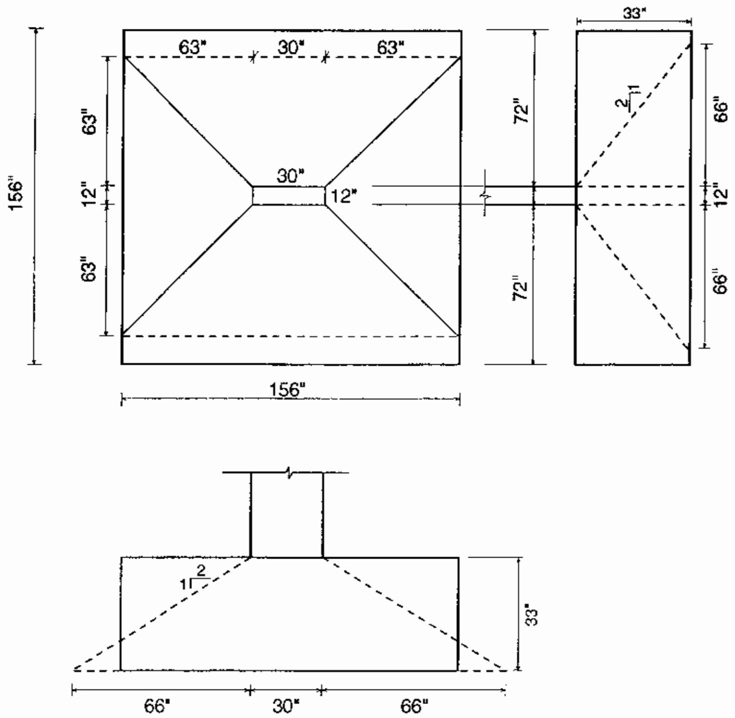
f'_c (footing) = 3000 psi

f_y = 60,000 psi

P_u = 860 kips



Calculations and Discussion	Code Reference
1. Bearing strength of column (f'_c = 5000 psi):	*15.8.1.1*
$\phi P_{nb} = \phi(0.85 f'_c A_1)$	*10.17.1*
$= 0.65\,(0.85 \times 5 \times 12 \times 30) = 995\text{ kips} > P_u = 860\text{ kips}$ O.K.	*9.3.2.4*
2. Bearing strength of footing (f'_c = 3000 psi):	*15.8.1.1*
The bearing strength of the footing is increased by a factor $\sqrt{A_2/A_1} \le 2$ due to the to the large footing area permitting a greater distribution of the column load.	*10.17.1*



A_1 is the column (loaded) area and A_2 is the plan area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal. For the 30 × 12 in. column supported on the 13 × 13 ft square footing, $A_2 = (66 + 12 + 66) \times (63 + 30 + 63)$.

$$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{144 \times 156}{30 \times 12}} = 7.9 > 2, \text{ use } 2$$

Note that bearing on the column concrete will always govern until the strength of the column concrete exceeds twice that of the footing concrete.

$$\phi P_{nb} = 2[\phi(0.85 f'_c A_1)]$$

$$= 2\,[0.65\,(0.85 \times 3 \times 12 \times 30)] = 1193 \text{ kips} > P_u = 860 \text{ kips} \quad \text{O.K.}$$

3. Required dowel bars between column and footing:

Even though bearing strength on the column and footing concrete is adequate to transfer the factored loads, a minimum area of reinforcement is required across the interface. *15.8.2.1*

$A_s\,(\text{min}) = 0.005\,(30 \times 12) = 1.80 \text{ in.}^2$

Provide 4-No. 7 bars as dowels ($A_s = 2.40$ in.^{2})

4. Development of dowel reinforcement in compression: *12.3.2*

In column:

$$\ell_{dc} = \left(\frac{0.02 f_y}{\sqrt{f'_c}}\right) d_b \geq (0.0003 f_y) d_b$$

For No. 7 bars:

$$\ell_{dc} = \left(\frac{0.02 \times 60{,}000}{\sqrt{5000}}\right) 0.875 = 14.9 \text{ in.}$$

$$\ell_{dc(min)} = 0.0003 \times 60{,}000 \times 0.875 = 15.8 \text{ in.} \quad \text{(governs)}$$

In footing:

$$\ell_{dc} = \left(\frac{0.02 \times 60{,}000}{\sqrt{3000}}\right) 0.875 = 19.2 \text{ in.} \quad \text{(governs)}$$

$$\ell_{dc(min)} = 0.0003 \times 60{,}000 \times 0.875 = 15.8 \text{ in.}$$

Available length for development in footing

= footing thickness - cover - 2 (footing bar diameter) - dowel bar diameter

= 33 - 3 - 2 (1.0) - 0.875 = 27.1 in. > 19.2 in.

Therefore, the dowels can be fully developed in the footing.

Example 22.5—Design for Transfer of Force by Reinforcement

For the design conditions given below, provide for transfer of force between the column and footing.

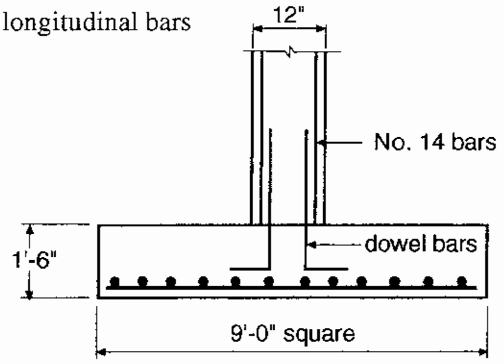
12 × 12 in. tied reinforced column with 4-No. 14 longitudinal bars

f'_c = 4000 psi (column and footing)

f_y = 60,000 psi

P_D = 200 kips

P_L = 100 kips



Calculations and Discussion	Code Reference

1. Factored load $P_u = (1.2 \times 200) + (1.6 \times 100) = 400$ kips — *Eq. (9-2)*

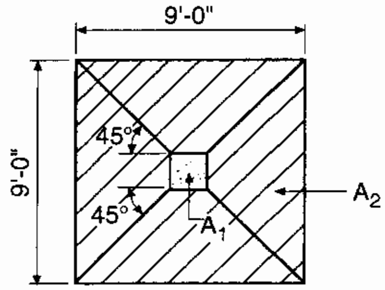
2. Bearing strength on column concrete: — *15.8.1.1*

$$\phi P_{nb} = \phi(0.85f'_cA_1) = 0.65(0.85 \times 4 \times 12 \times 12)$$ — *10.17.1*

$$= 318.2 \text{ kips} < P_u = 400 \text{ kips} \quad \text{N.G.}$$

The column load cannot be transferred by bearing on concrete alone. The excess load (400 - 318.2 = 81.8 kips) must be transferred by reinforcement. — *15.8.1.2*

3. Bearing strength on footing concrete: — *15.8.1.1*



$$\phi P_{nb} = \sqrt{\frac{A_2}{A_1}}\left[\phi(0.85f'_cA_1)\right]$$

$$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{9 \times 9}{1 \times 1}} = 9 > 2, \text{ use } 2$$

$$\phi P_{nb} = 2\,(318.2) = 636.4 \text{ kips} > 400 \text{ kips} \quad \text{O.K.}$$

4. Required area of dowel bars: — *15.8.1.2*

$$A_s\text{ (required)} = \frac{(P_u - \phi P_{nb})}{\phi f_y}$$

$$= \frac{81.8}{0.65 \times 60} = 2.10 \text{ in.}^2$$

A_s (min) = 0.005 (12 × 12) = 0.72 in.^{2} — *15.8.2.1*

Try 4-No. 8 bars (A_s = 3.16 in.^{2})

Example 22.5 (cont'd)	Calculations and Discussion	Code Reference

5. Development of dowel reinforcement

a. For development into the column, the No. 14 column bars may be lap spliced with the No. 8 footing dowels. The dowels must extend into the column a distance not less than the development length of the No. 14 column bars or the lap splice length of the No. 8 footing dowels, whichever is greater. *15.8.2.3*

For No. 14 bars:

$$\ell_{dc} = \left(\frac{0.02f_y}{\sqrt{f'_c}}\right) d_b = \left(\frac{0.02 \times 60{,}000}{\sqrt{4000}}\right) 1.693 = 32.1 \text{ in. (governs)}$$

$$\ell_{dc(min)} = \left(0.0003f_y\right) d_b = 0.0003 \times 60{,}000 \times 1.693 = 30.5 \text{ in.}$$

For No. 8 bars:

lap length $= 0.0005f_yd_b$ *12.16.1*

$= 0.0005 \times 60{,}000 \times 1.0 = 30$ in.

Development length of No. 14 bars governs.

The No. 8 dowel bars must extend not less than 33 in. into the column.

b. For development into the footing, the No. 8 dowels must extend a full development length. *15.8.2.3*

$$\ell_{dc} = \left(\frac{0.02f_y}{\sqrt{f'_c}}\right) d_b = \left(\frac{0.02 \times 60{,}000}{\sqrt{4000}}\right) \times 1.0 = 19.0 \text{ in. (governs)}$$

$$\ell_{dc(min)} = \left(0.0003f_y\right) d_b = 0.0003 \times 60{,}000 \times 1.0 = 18.0 \text{ in.}$$

This length may be reduced to account for excess reinforcement. *12.3.3(a)*

$$\frac{A_s \text{ (required)}}{A_s \text{ (provided)}} = \frac{2.10}{3.16} = 0.66$$

Required $\ell_{dc} = 19 \times 0.66 = 12.5$ in.

Available length for dowels development $\approx$ 18 - 5 = 13 in. > 12.5 in. required, O.K.

Note: In case the available development length is less than the required development length, either increase footing depth or use larger number of smaller size dowels. Also note that if the footing dowels are bent for placement on top of the footing reinforcement (as shown in the figure), the bent portion cannot be considered effective for developing the bars in compression (12.5.5).

Example 22.6—Design for Transfer of Horizontal Force at Base of Column

For the column and footing of Example 22.5, design for transfer of a horizontal factored force of 85 kips acting at the base of the column.

Design data:

Footing: size = 9×9 ft
thickness = 1ft-6 in.

Column: size = 12×12 in. (tied)
4-No. 14 longitudinal reinforcement

f'_c = 4000 psi (footing and column)

f_y = 60,000 psi

Calculations and Discussion	Code Reference
1. The shear-friction design method of 11.7 is applicable.	*15.8.1.4*
Check maximum shear transfer permitted:	*11.7.5*
$V_u \le \phi(0.2f'_cA_c)$ but not greater than $\phi(800A_c)$	
$\phi V_n = 0.75\,(0.2 \times 4 \times 12 \times 12) = 86.4$ kips	
$\phi(800A_c) = 0.75 \times 800 \times 12 \times 12/1000 = 86.4$ kips	
$V_u = 85$ kips $< \phi(0.2f'_cA_c)$ and $\phi(800A_c)$ O.K.	
The shear transfer of 85 kips is permitted at the base of 12×12 in. column.	
Strength requirement for shear:	
$V_u \le \phi V_n$	*Eq. (11-1)*
$V_n = V_u/\phi = A_{vf}f_y\mu$	*Eq. (11-25)*
Use $\mu = 0.6$ (concrete not intentionally roughened)	*11.7.4.3*
and $\phi = 0.75$ (shear)	
Required $A_{vf} = \dfrac{V_u}{\phi f_y \mu} = \dfrac{85}{0.75 \times 60 \times 0.6} = 3.15$ in.^{2}	*Eq. (11-25)*
A_s (provided) = 3.16 in.^{2} O.K.	
Therefore, use 4-No. 8 dowels ($A_s = 3.16$ in.^{2})	

If the 4-No. 8 dowels were not adequate for transfer of horizontal shear, the footing concrete in contact with the column concrete could be roughened to an amplitude of approximately 1/4 in. to take advantage of the higher coefficient of friction of 1.0:

$$\text{Required } A_{vf} = \frac{85}{0.75 \times 60 \times 1.0} = 1.89 \text{ in.}^2$$

2. Tensile development of No. 8 dowels, as required by 11.7.8

a. Within the column

$$\ell_d = \left[\frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s\lambda}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}\right]d_b$$ *Eq. (12-1)*

Clear cover to No. 8 bar ≈ 3.25 in.

Center-to-center bar spacing of No. 8 bars ≈ 4.5 in.

$$c_b = \text{minimum of}\begin{cases} 3.25 + 0.5 = 3.75 \text{ in.} \\ \dfrac{4.5}{2} = 2.25 \text{ in.} \quad \text{(governs)} \end{cases}$$ *12.2.4*

Assume $K_{tr} = 0$ (conservatively consider no transverse reinforcement)

$$\frac{c_b + K_{tr}}{d_b} = \frac{2.25 + 0}{1.0} = 2.25 < 2.5, \text{ use } 2.25$$ *12.2.3*

$\psi_t = 1.0$

$\psi_e = 1.0$

$\psi_t\psi_e = 1.0 < 1.7$

$\psi_s = 1.0$

$\lambda = 1.0$

$$\ell_d = \left(\frac{3}{40}\frac{60{,}000}{\sqrt{4000}}\frac{1.0 \times 1.0 \times 1.0}{2.25}\right) \times 1.0 = 31.6 \text{ in.}$$

Provide at least 32 in. of embedment into the column.

b. Within the footing

Use standard hooks at the ends of the No. 8 bars

Example 22.6 (cont'd)	Calculations and Discussion	Code Reference

$\ell_{dh} = \left(0.02\psi_e\lambda f_y / \sqrt{f'_c}\right)d_b$ *12.5.2*

$= \left(0.02 \times 1.0 \times 1.0 \times \dfrac{60{,}000}{\sqrt{4000}}\right) \times 1.0 = 19.0 \text{ in.}$

Modifications: *12.5.3*

cover normal to plane of 90° hook > 2.5 in.

cover on bar extension beyond hook ≥ 2 in.

$\ell_{dh} = 0.7 \times 19 = 13.3$ in. *12.5.3(a)*

Min. $\ell_{dh} = 8 \times d_b = 8$ in. < 13.3 in. *12.5.1*

Available development length =

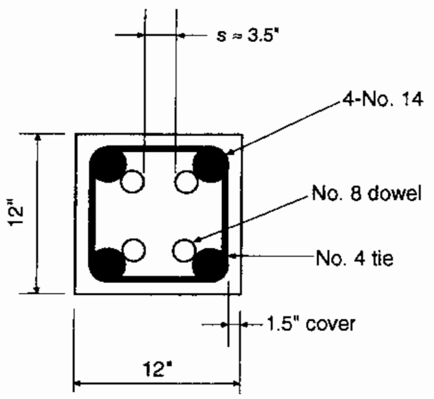
= 18 - 5 = 13 in. < 13.3 in. N.G.

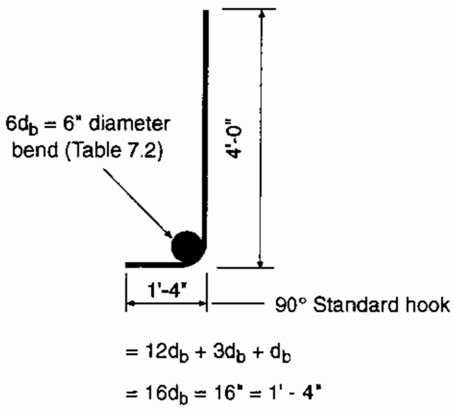
Increase footimg depth by 2 in. Total depth = 20 in.

Use 15 in. hook embedment into footing to secure dowels at the footing reinforcement.

Total length of No. 8 dowel = 32 + 15 = 47 in. Use 4 ft-0 in. long dowels.

Note: The top of the footing at the interface between column and footing must be clean and free of laitance before placement of the column concrete. *11.7.9*





No. 8 dowel detail

Example 22.7—Design for Depth of Footing on Piles

For the footing supported on the piles shown, determine the required thickness of the footing (pile cap).

Footing size = 8.5 × 8.5 ft

Column size = 16 × 16 in.

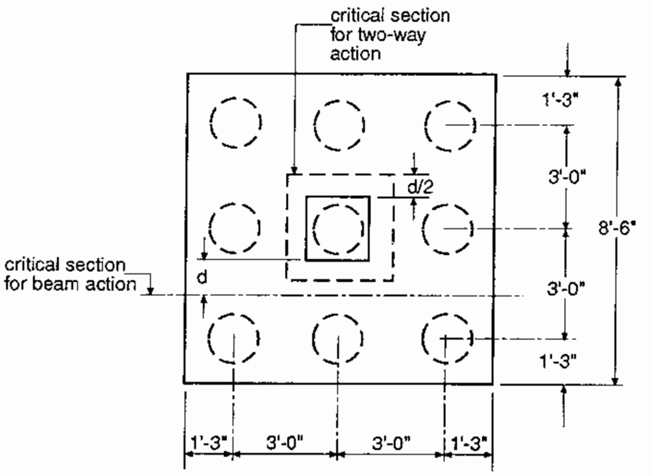
Pile diameter = 12 in.

f'_c = 4000 psi

Load per pile:

P_D = 20 kips

P_L = 10 kips



Calculations and Discussion	Code Reference

1. Depth required for shear usually controls footing thickness. Both wide-beam action and two-way action for the footing must be investigated. *11.12*

   Assume an overall footing thickness of 1 ft-9 in. with an average d ≈ 14 in. *15.7*

2. Factored pile loading:

   $P_u = 1.2\,(20) + 1.6\,(10) = 40$ kips *Eq. (9-2)*

3. Strength requirements for shear

   $V_u \le \phi V_n$ *Eq. (11-1)*

   a. Wide-beam action for footing: *11.12.1.1*

   3 piles fall within tributary area

   V_u (neglecting footing wt.) = 3 × 40 = 120 kips

   $\phi V_n = \phi\left(2\sqrt{f'_c}\,b_w d\right)$ *Eq. (11-3)*

   b_w = 8 ft-6 in. = 102 in.

   $\phi V_n = 0.75\left(2\sqrt{4000} \times 102 \times 14\right)/1000 = 135.4 \text{ kips} > V_u = 120 \text{ kips}$ O.K.

b. Two-way action: 11.12.1.2

8 piles fall within the tributary area

$V_u = 8 \times 40 = 320$ kips

$$\frac{V_c}{\sqrt{f'_c}b_od} = \text{smallest value of} \begin{cases} 2 + \dfrac{4}{\beta} & \text{Eq. (11-33)} \\ \dfrac{\alpha_s d}{b_o} + 2 & \text{Eq. (11-34)} \\ 4 & \text{Eq. (11-35)} \end{cases}$$

$$\beta = \frac{16}{16} = 1.0$$

$b_o = 4\,(16 + 14) = 120$ in.

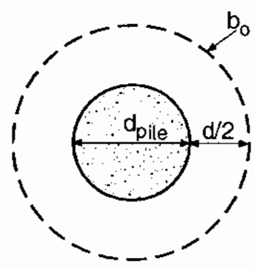
$\alpha_s = 40$ for interior columns

$$\frac{b_o}{d} = \frac{120}{14} = 8.6$$

$$\frac{V_c}{\sqrt{f'_c}b_od} = \begin{cases} 2 + \dfrac{4}{1} = 6 & \text{Eq. (11-33)} \\ \dfrac{40}{8.6} + 2 = 6.7 & \text{Eq. (11-34)} \\ 4 \quad \text{(governs)} & \text{Eq. (11-35)} \end{cases}$$

$$\phi V_c = 0.75 \times 4\sqrt{4000} \times 120 \times 14/1000$$

$$= 319 \text{ kips} \cong V_u = 320 \text{ kips} \quad \text{O.K.}$$



4. Check "punching" shear strength at corner piles. With piles spaced at 3 ft-0 in. on center, critical perimeters do not overlap.

$V_u = 40$ kips per pile

$$\frac{V_c}{\sqrt{f'_c}\,b_o d} = \text{minimum of}\begin{cases} 2 + \dfrac{4}{\beta} & \text{Eq. (11-33)} \\ \dfrac{\alpha_s d}{b_o} + 2 & \text{Eq. (11-34)} \\ 4 & \text{Eq. (11-35)} \end{cases}$$

$\beta = 1.0$ (square reaction area of equal area)

$b_o = \pi(12 + 14) = 81.7$ in.

$\alpha_s = 20$ (for corner columns) *11.12.2.1*

$$\frac{b_o}{d} = \frac{81.7}{14} = 5.8$$

$$\frac{V_c}{\sqrt{f'_c}\,b_o d} = \begin{cases} 2 + \dfrac{4}{1} = 6 \\ \dfrac{20}{5.8} + 2 = 5.4 \\ 4 \quad \text{(governs)} \end{cases}$$

$\phi V_c = 0.75 \times 4\sqrt{4000} \times 81.7 \times 14/1000 = 217 \text{ kips} > V_u = 40 \text{ kips}$ O.K.

23

Precast Concrete

GENERAL CONSIDERATIONS

Chapter 16 was completely rewritten for the 1995 code. Previous editions of Chapter 16 were largely performance oriented. The current chapter is more prescriptive, although the word "instructive" may be more appropriate, as the chapter provides much more guidance to the designer of structures which incorporate precast concrete. Not only does the chapter itself provide more requirements and guidelines, but the commentary contains some 25 references, as opposed to 4 in the 1989 code, thus encouraging the designer to make maximum use of the available literature.

The increase in instructive material is most notable in 16.5, Structural Integrity. Requirements for structural integrity were introduced in 7.13 of the 1989 code. For precast construction, this section required only that tension ties be provided in all three orthogonal directions (two horizontal and one vertical) and around the perimeter of the structure, without much further guidance. Reference 23.1 was given for precast bearing wall buildings. The recommendations given in that reference are now codified in 16.5.2. Section 16.5.1 applies primarily to precast structures other than bearing wall buildings and, as is the case with most of the rewritten Chapter 16, is largely a reflection of time-tested industry practice. Note that tilt-up concrete construction is a form of precast concrete. Reference 23.3 addresses all phases of design and construction of tilt-up concrete structures.

16.2 GENERAL

The code requires that precast members and connections be designed for "... loading and restraint conditions from initial fabrication to end use in the structure ..." Often, especially in the case of wall panels, conditions during handling are far more severe than those experienced during service. For this reason, and also because practices and details vary among manufacturers, precast concrete components are most often designed by specialty engineers employed by the manufacturer. Calculations, as well as shop drawings (16.2.4) are then submitted to the engineer-of-record for approval. This procedure is also usually followed in the design of connections. For more information on the relationship between the engineer-of-record and the specialty engineer, see Refs. 23.4 and 23.5.

As stated above, since 1995, the code encourages the use of other publications for the design of precast concrete structures. References 23.2, 23.6, and 23.7 are particularly useful to the designer. Of these, the most widely used is Ref. 23.7, the *PCI Design Handbook.*

Section 16.2.3 states that tolerances must be specified. This is usually done by reference to Industry documents[23.8, 23.9, 23.10], as noted in the commentary. Design of precast concrete members and connections is particularly sensitive to tolerances. Therefore, they should be specified in the contract documents together with required concrete strengths at different stages of construction [16.2.4(b)].

16.3 DISTRIBUTION OF FORCES AMONG MEMBERS

Section 16.3.1 covers distribution of forces perpendicular to the plane of members. Most of the referenced research relates to hollow-core slabs, and is also applicable to solid slabs that are connected by continuous grout keys. Members that are connected together by other means, such as weld plates on double tees, have through extensive use been found to be capable of distributing concentrated loads to adjacent members. It is common to assume for design purposes that up to 25% of a concentrated load can be transferred to each adjacent member; the connections should be designed accordingly. Since load transfer is dependent on compatible deflections, less distribution occurs nearer the support, as shown in Fig. 23-1(a) for hollow core slabs. Flanges of double tees are also designed for transverse load distribution over an effective width as illustrated in Fig. 23-1(b). Other types of decks may not necessarily follow the same pattern, because of different torsional resistance properties, but the same principles are applicable. A typical design is shown in Example 23.1.

Compatibility of the deflections of adjacent units is an important design consideration. For example, in the driving lane of a precast concrete double-tee parking deck, even if each member has adequate strength to carry a full wheel load, it is undesirable for each unit to deflect independently. It is common practice to use more closely spaced connections in those cases, to assure sharing of the load and to eliminate differential deflections between members.

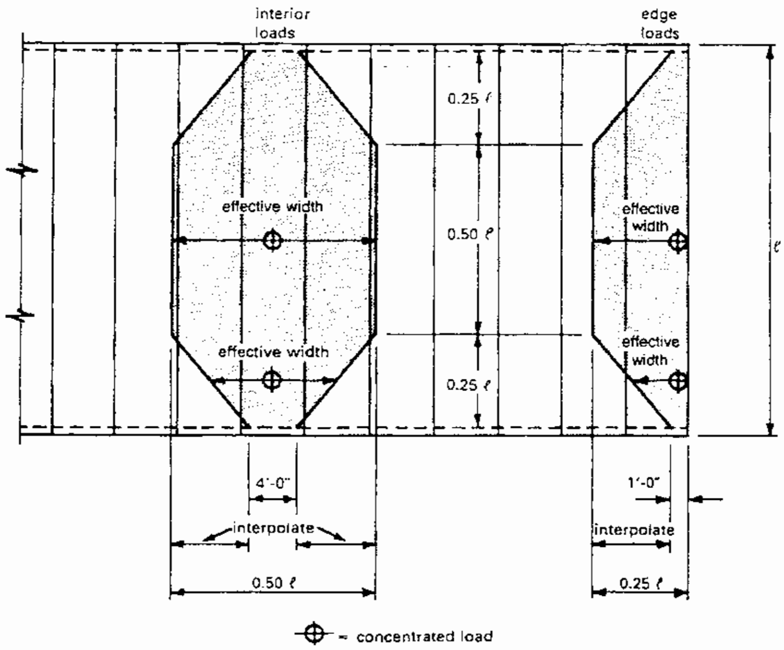
Section 16.3.2 covers distribution of in-plane forces. It requires a continuous load path for such forces. If these forces are tensile, they must be resisted by steel or steel reinforcement. Since these in-plane forces are usually caused by lateral loads such as those due to wind or earthquakes, and since such lateral loads can occur in any direction, nearly all of the continuous load path must be provided by steel or steel reinforcement. In that respect, the reinforcement and connections designed to meet the requirements of this section may also provide the continuous ties required by 16.5.

16.4 MEMBER DESIGN

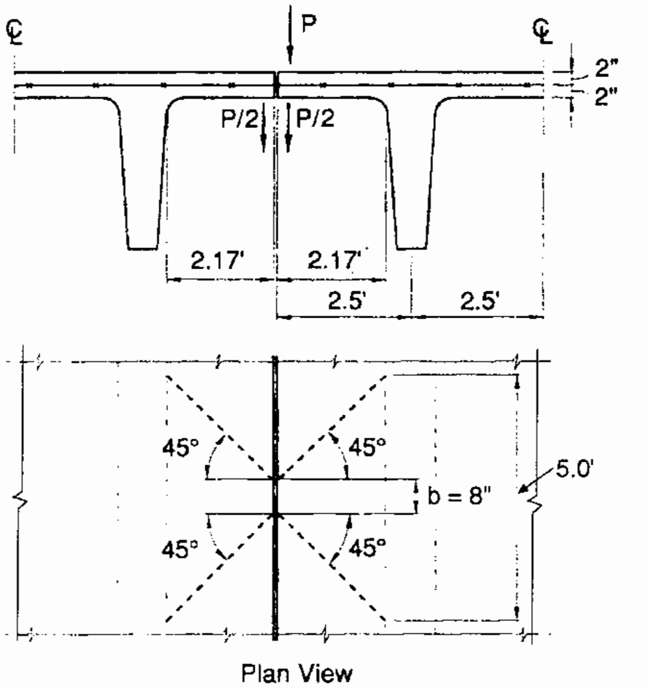
This section is primarily concerned with minimum reinforcement requirements for precast members. Section 16.4.1 waives transverse steel requirements in prestressed concrete members 12 ft or less in width, except where such steel is required for flexure. The commentary notes that an example of an exception to the waiver is the flanges of single and double tees. This section is intended primarily for hollow core and solid slabs where the transverse connection is typically grouted joints.

Section 16.4.2 reduces the minimum reinforcement in precast, nonprestressed walls, from that required for cast-in-place reinforced concrete walls in Chapter 14, to 0.001 times the gross cross-sectional area of the walls, in accordance with common industry practice. This is in recognition of the fact that much of the shrinkage of precast members occurs prior to attachment to the structure. Spacing of reinforcement in precast walls should not exceed 5 times the wall thickness or 30 in. for interior walls or 18 in. for exterior walls.

When wall panels are load-bearing, they are usually designed as compression members in accordance with Chapter 10. When they are not load-bearing (and often even when they are), the stresses during handling are usually critical. In those cases, it is common to place the lifting and dunnage points so that the stresses during handling do not exceed the modulus of rupture (with a safety factor), especially for architectural precast panels. If cracking is likely, crack control reinforcement in accordance with 10.6.4 is required.



(a) Hollow core slab



(b) Double tee

Figure 23-1 Assumed Load Distribution[23.7]

16.5 STRUCTURAL INTEGRITY

The provisions of 16.5.1.1 are intended to assure that there is a continuous load path from every precast member to the lateral load resisting system. The commentary gives several examples of how this may be accomplished. Section 16.5.1.2 is adopted from a similar requirement in the Uniform Building Code, as explained in the commentary.

Section 16.5.1.3 gives requirements for vertical tension ties. The requirement for columns in 16.5.1.3(a) applies not only to connections of columns to footings, but also to such connections as column splices. The 10,000 lb requirement for each of at least two ties per wall panel in 16.5.1.3(b) is from the *PCI Design Handbook*[23.7], and is the numerical equivalent of a common connection used in the precast concrete industry for many years. This is strictly an empirical value, and is intended to apply only to the dimensions of the hardware items in the connection, without including eccentricities in the design. Section 16.5.1.3(c) permits this connection to be into a reinforced concrete floor slab, as is common with tilt-up construction.

Section 16.5.2 in essence codifies some of the recommendations of Ref. 23.11, which gives numerical values for tension ties in bearing wall buildings. This report is based on a series of tests conducted at the Portland Cement Association's laboratories[23.12] in the late 1970s.

16.6 CONNECTION AND BEARING DESIGN

Section 16.6.1 lists the several ways that precast members can be connected, and then allows design by analysis or test. Special mention is made of 11.7, Shear Friction, as this is a commonly used analysis/design tool. See Part 16 of this document. Examples on application of shear friction design procedure are also given in *PCI Design Handbook*[23.7] and *Design and Typical Details of Connections for Precast and Prestressed Concrete.* [23.6]

Section 16.6.2 describes several important considerations when designing for bearing of precast elements. The minimum bearing lengths of 16.6.2.2(a) are particularly important. It should be emphasized that these are minimum values, and that the structure should be detailed with significantly longer bearing lengths, to allow for tolerance in placement. Section 16.6.2.3 makes it clear that positive moment reinforcement need not extend out of the end of the member, provided it goes at least to the center of the bearing.

16.7 ITEMS EMBEDDED AFTER CONCRETE PLACEMENT

In precasting plants, it has long been common practice to place certain embedded items in the concrete after it has been cast. This practice is recognized in this section and constitutes an exception to the provisions of 7.5.1. Conditions to embed items in the concrete while it is in a plastic state are: (1) embedded item is not required to be hooked or tied to reinforcement within the concrete, (2) embedded item is secured in its position until concrete hardens, and (3) concrete is properly consolidated around each embedment.

16.8 MARKING AND IDENTIFICATION

The purpose of identification marks on precast members is to facilitate construction and avoid placing errors. Each precast member should be marked to show date of manufacture and should be identified according to placing drawings.

16.9 HANDLING

This section re-emphasizes the general requirement of 16.2.1. Handling stresses and deformations must be considered during the design of precast concrete members. Erection steps and hardware required for each step must be shown on contract or erection drawings.

16.10 STRENGTH EVALUATION OF PRECAST CONSTRUCTION

It is always desirable, and certainly safer and more economical, to test a suspect precast concrete member before it is integrated into the structure. This new section describes how Chapter 20 provisions can be applied to this case. The test loads specified in 20.3.2 must be adjusted to simulate the load portion carried by the suspect member when it is in the final, composite mode. The acceptance criteria of 20.5 apply to the isolated precast member.

REFERENCES

23.1 PCI Committee on Building Code and PCI Technical Activities Committee, "Proposed Design Requirements for Precast Concrete," *PCI Journal*, V. 31, No. 6, November-December 1986, pp. 32-47.

23.2 ACI Committee 550, "Design Recommendations for Precast Concrete Structures," (ACI 550R-96, Reapproved 2001). American Concrete Institute, Farmington Hills, MI, 1996. Also in ACI Manual of Concrete Practice, Part 6.

23.3 ACI Committee 551, "Tilt-Up Concrete Structures," (ACI 551R-92, Reapproved 2003), American Concrete Institute, Farmington Hills, MI, 1992. Also in *ACI Manual of Concrete Practice*, Part 6.

23.4 The Case Task Group on Specialty Engineering, "National Practice Guidelines for Specialty Structural Engineers," Council of American Structural Engineers, Washington, DC, 1994, 11 pp.

23.5 ACI Committee on Responsibility in Concrete Construction, "Guidelines for Authorities and Responsibilities in Concrete Design and Construction," *Concrete International*, Vol. 17, No. 9, September 1995, pp. 66-69.

23.6 "Design and Typical Details of Connections for Precast and Prestressed Concrete," MNL-123-88, 2nd Edition, Precast/Prestressed Concrete Institute, Chicago, 1988, 270 pp.

23.7 "PCI Design Handbook—Precast and Prestressed Concrete," MNL-120-04, 6th Edition, Precast/Prestressed Concrete Institute, Chicago, 2004, 750 pp.

23.8 "Manual for Quality Control for Plants and Production of Precast and Prestressed Concrete Products," MNL-116-99, 4th Edition, Precast/Prestressed Concrete Institute, Chicago, 1999, 340 pp.

23.9 "Manual for Quality Control for Plants and Production of Architectural Precast Concrete," MNL-117-96, 3rd Edition, Precast/Prestressed Concrete Institute, Chicago, 1996, 226 pp.

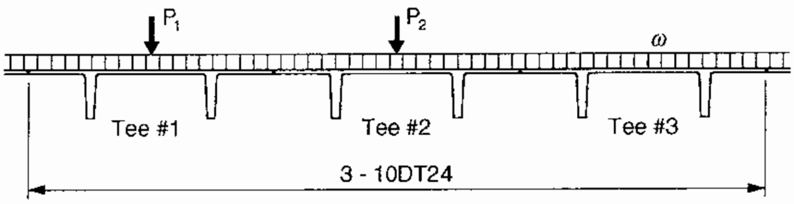
23.10 PCI Committee on Tolerances, "Tolerances for Precast and Prestressed Concrete," *PCI Journal*, V. 30, No. 1, January-February 1985, pp. 26-112.

23.11 PCI Committee on Precast Concrete Bearing Wall Buildings, "Considerations for the Design of Precast Concrete Bearing Wall Buildings to Withstand Abnormal Loads," *PCI Journal*, V. 21, No. 2, March-April 1976, pp. 18-51.

23.12 "Design and Construction of Large-Panel Concrete Structures," six reports, EB100D, 1976-1980, 762 pp.; three studies, EB102D, 1980, 300 pp.; Portland Cement Association, Skokie, IL.

Example 23.1—Load Distribution in Double Tees

Required: Compute the factored moments and shears for each of three double tees of the following roof:



Given:
Double tees = 10DT24 (self weight = 468 plf), h = 24 in.
Span = 60 ft
Superimposed DL = 15 psf, LL = 30 psf
Concentrated dead load on Tee #1, P_1 = 20 kips @ 3 ft from left support
Concentrated dead load on Tee #2, P_2 = 20 kips @ midspan

Calculations and Discussion	Code Reference

1. Assume:

   Concentrated dead load P_1 cannot be distributed to adjacent tees since it is near the support.

   Concentrated dead load P_2 is distributed, with 25 percent to adjacent tees and 50 percent to the tee supporting the load, i.e.

   0.25 (20 kips) = 5 kips to Tee #1
   0.50 (20 kips) = 10 kips to Tee #2
   0.25 (20 kips) = 5 kips to Tee #3

2. Factored uniform dead and live loads, for each tee

   DL = 468 + 15 (10 ft width) = 0.618 kip/ft

   LL = 30 (10 ft width) = 0.30 kip/ft

   w_u = 1.2D + 1.6L *Eq. (9-2)*

   = 1.2 (0.618) + 1.6 (0.30) = 1.222 kip/ft

3. Factored moments and shears for Tee #1

   Factored concentrated dead load next to support = 1.2 (20) = 24 kips

   Factored concentrated dead load at midspan = 1.2 (5) = 6 kips

   w_u = 1.222 kip/ft

Reaction at left support $= \dfrac{57}{60}(24) + \dfrac{6}{2} + \dfrac{1.222(60)}{2} = 62.46$ kips

For prestressed concrete members, design for shear at distance h/2 *11.1.3.2*

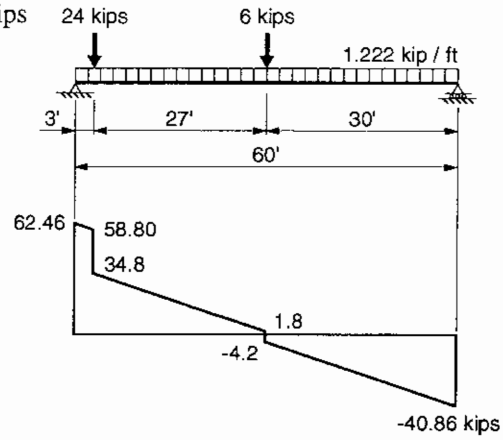
V_u (left) = 62.46 - 1.222 = 61.24 kips

Reaction at right support $= \dfrac{3}{60}(24) + \dfrac{6}{2} + \dfrac{1.222(60)}{2} = 40.86$ kips

At distance h/2, V_u (right) = 40.86 - 1.222 = 39.64 kips

Maximum moment is at midspan

M_u (max) = 40.86 (30) - 1.222 (30) (15) = 676 ft-kips



Factored loads on Tee #1

4. Factored moments and shears for Tee #2

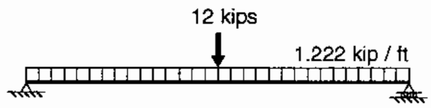
$$M_u = \frac{w_u \ell^2}{8} + \frac{P\ell}{4}$$

$$= \frac{1.222(60)^2}{8} + \frac{1.2(10)(60)}{4} = 730 \text{ ft-kips}$$

$$\text{Maximum reaction} = \frac{w_u \ell}{2} + \frac{P}{2}$$

$$= \frac{1.222(60)}{2} + \frac{1.2(10)}{2} = 42.66 \text{ kips}$$

At distance h/2, V_u = 42.66 - 1.222 = 41.44 kips



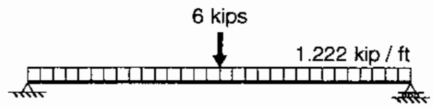
Factored loads on Tee #2

5. Factored moments and shears for Tee #3

$$M_u = \frac{1.222(60)^2}{8} + \frac{1.2(5)(60)}{4} = 640 \text{ ft-kips}$$

$$\text{Maximum reaction} = \frac{1.222(60)}{2} + \frac{1.2(5)}{2} = 39.66 \text{ kips}$$

At distance h/2, V_u = 39.66 - 1.222 = 38.44 kips



Factored loads on Tee #3

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24

Prestressed Concrete — Flexure

UPDATE FOR THE '05 CODE

Significant changes to the provisions for prestressed concrete design for flexure are:

- Construction joint location limitation of 6.4.4 previously applied to both reinforced and prestressed concrete, they are now waived for prestressed concrete.

- A definition for "Transfer length" was added in 2.2, and was illustrated schematically in Figure R9.3.2.7(a) and R9.3.2.7(b) for pretensioned bonded and debonded strands, respectively. Strength reduction factor" ϕ applicable within transfer and development lengths has been revised (9.3.2.7) to eliminate a discontinuity in computed flexural strength.

- The permissible flexural tensile strength in two-way prestressed slabs of $f_t \leq 7.5\sqrt{f_c'}$ introduced in the 2002 code has been changed to $f_t \leq 6\sqrt{f_c'}$ as prescribed in previous codes (18.3.3) prior to 2002.

- The limiting depth of 36 in. for required skin reinforcement has been changed from the effective depth to the overall member height h (18.4.4.4). The crack control provisions for skin reinforcement of 10.6.7 have been simplified and made consistent with those required for flexural tension reinforcement in 10.6.4.

- An unnecessary sentence describing redistribution of negative moments in R18.10.3 was removed from the commentary to eliminate potential confusion.

GENERAL CONSIDERATIONS

In prestressed members, compressive stresses are introduced into the concrete to reduce tensile stresses resulting from applied loads including the self weight of the member (dead load). Prestressing steel, such as strands, bars, or wires, is used to impart compressive stresses to the concrete. Pretensioning is a method of prestressing in which the tendons are tensioned before concrete is placed and the prestressing force is primarily transferred to the concrete through bond. Post-tensioning is a method of prestressing in which the tendons are tensioned after the concrete has hardened and the prestressing force is primarily transferred to the concrete through the end anchorages.

The act of prestressing a member introduces "prestressing loads" to the member. The induced prestressing loads, acting in conjunction with externally applied loads, must provide serviceability and strength to the member beginning immediately after prestress force transfer and continuing throughout the life of the member. Prestressed structures must be analyzed taking into account prestressing loads, service loads, temperature, creep, shrinkage and the structural properties of all materials involved.

The code states that all provisions of the code apply to prestressed concrete, unless they are in conflict with Chapter 18 or are specifically excluded. The exclusions, listed in 18.1.3, are necessary because some empirical or simplified analytical methods employed elsewhere in the code may not adequately account for the effects of prestressing forces.

Deflections of prestressed members calculated according to 9.5.4 should not exceed the values listed in Table 9.5(b). According to 9.5.4, prestressed concrete members, like any other concrete members, should be designed to have adequate stiffness to prevent deformations which may adversely affect the strength or serviceability of the structure.

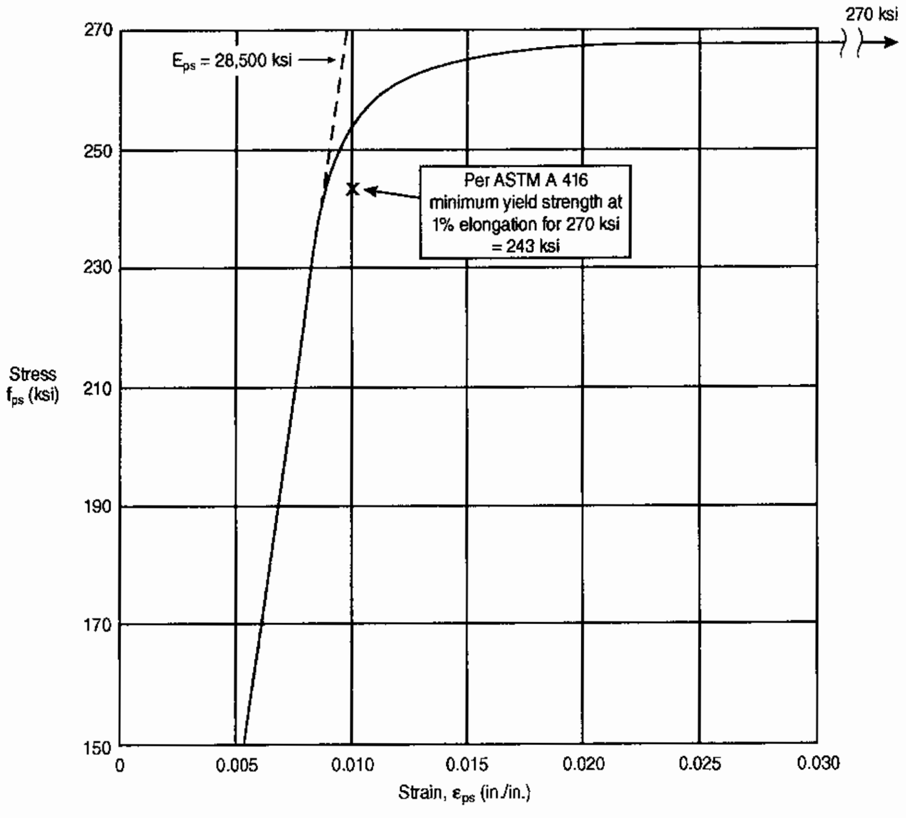
PRESTRESSING MATERIALS

The most commonly used prestressing material in the United States is Grade 270 ksi low-relaxation, seven-wire strand, defined by ASTM A 416. The most common size is 1/2-in., although there is increasing use of 0.6-in. strand, especially for post-tensioning. The properties of these strands are as follows:

Nominal Diameter, in.	1/2	0.6
Area, sq. in.	0.153	0.217
Breaking stress f_{pu}, ksi	270	270
Breaking strength, kips	41.3	58.6
Jacking stress, ksi = 0.75 f_{pu}	202.5	202.5

Virtually identical metric strands are used in metric countries.

The Prestressed Concrete Institute's *PCI Design Handbook*, 6th edition, Ref. 24.1, gives a standard stress-strain curve for this material, as shown in Fig. 24-1. This curve is approximated by the two expressions given below the figure.



The above curve can be approximated by the following equations: $\varepsilon_{ps} \le 0.0086$: $f_{ps} = 28{,}500\ \varepsilon_{ps}$ (ksi)

$\varepsilon_{ps} > 0.0086$: $f_{ps} = 270 - \dfrac{0.04}{\varepsilon_{ps} - 0.007}$ (ksi)

Figure 24-1 Stress-Strain Curve for Grade 270, Low Relaxation Strand[24.1]

NOTATION AND TERMINOLOGY

The following symbols are used in 18.4.4, which deals with serviceability requirements for cracked prestressed flexural members.

Δf_{ps} = stress in prestressing steel at service loads less decompression stress, psi. See Fig. 24-2

f_{dc} = decompression stress. Stress in the prestressing steel when stress is zero in the concrete at the same level as the centroid of the tendons, psi. See Fig. 24-2

s = center-to-center spacing of flexural tension steel near the extreme tension face, in. Where there is only one bar or tendon near the extreme tension face, s is the width of extreme tension face

Note, $f_{dc} = f_{se} + f_c \times E_{ps}/E_c$ where f_c is the concrete stress at level of steel under dead load and prestress. f_{dc} may be conservatively taken as f_{se}.

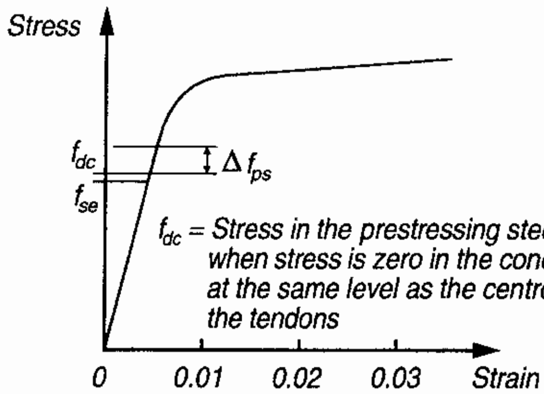


Figure 24-2 Decompression Stress f_{dc}

The following definitions found in 2.2 are consistently used in Chapter 18 and throughout the code. They reflect industry terminology.See Fig. 24-2

Prestressing steel — High-strength steel element, such as wire, bar, or strand, or a bundle of such elements, used to impart prestress forces to concrete.

Tendon — In pretensioned applications, the tendon is the prestressing steel. In post-tensioned applications, the tendon is a complete assembly consisting of anchorages, prestressing steel, and sheathing with coating for unbonded applications or ducts with grout for bonded applications.

Bonded tendon — Tendon in which prestressing steel is bonded to concrete either directly or through grouting.

Unbonded tendon — Tendon in which the prestressing steel is prevented from bonding to the concrete and is free to move relative to the concrete. The prestressing force is permanently transferred to the concrete at the tendon ends by the anchorages only.

Duct — A conduit (plain or corrugated) to accommodate prestressing steel for post-tensioned installation. Requirements for post-tensioning ducts are given in 18.17.

Sheathing — A material encasing prestressing steel to prevent bonding of the prestressing steel with the surrounding concrete, to provide corrosion protection, and to contain the corrosion inhibiting coating.

18.2 GENERAL

The code specifies strength and serviceability requirements for all concrete members, prestressed or nonprestressed. This section requires that, for prestressed members, both strength and behavior at service conditions must be checked. All load stages that may be critical during the life of the structure, beginning with the transfer of the prestressing force to the member and including handling and transportation, must be considered.

This section also calls attention to several structural issues specific to prestressed concrete structures that must be considered in design:

18.2.3...Stress concentrations. See 18.13 for requirements for post-tensioned anchorages.

18.2.4...Compatibility of deformation with adjoining construction. An example of the effect of prestressing on adjoining parts of a structure is the need to include moments caused by axial shortening of prestressed floors in the design of the columns which support the floors.

18.2.5...Buckling of prestressed members. This section addresses the possibility of buckling of any part of a member where prestressing tendons are not in contact with the concrete. This can occur when prestressing steel is in an oversize duct, and with external prestressing described in 18.22.

18.2.6...Section properties. The code requires that the area of open post-tensioning ducts be deducted from section properties prior to bonding of prestressing tendons. For pretensioned members and post-tensioned members after grouting, the commentary allows the use of gross section properties, or effective section properties that may include the transformed area of bonded tendons and nonprestressed reinforcement.

18.3 DESIGN ASSUMPTIONS

In applying fundamental structural principles (equilibrium, stress-strain relations, and geometric compatibility) to prestressed structures, certain simplifying assumptions can be made. For computation of strength (18.3.1), the basic assumptions given for nonprestressed members in 10.2 apply, except that 10.2.4 applies only to nonprestressed reinforcement. For investigation of service load conditions, the "elastic theory" (referring to the linear variation of stress with strain) may be used. Where concrete is cracked, the concrete resists no tension. For analysis at service load conditions, the moduli of elasticity for concrete and nonprestressed reinforcement are given in 8.5. The modulus of elasticity for prestressed reinforcement is not given but can generally be taken as described in Fig. 24-1.

Section 18.3.3 defines three classes of prestressed flexural members, as follows:

Uncracked	Class U:	$f_t \leq 7.5\sqrt{f'_c}$
Transition	Class T:	$7.5\sqrt{f'_c} < f_t \leq 12\sqrt{f'_c}$
Cracked	Class C:	$f_t > 12\sqrt{f'_c}$

Table 24-1 summarizes the applicable requirements for these three classes of prestressed flexural members and, for comparison, for nonprestressed flexural members as well.

Class U and Class T members correspond to those designed by 18.4.2(c) and 18.4.2(d), respectively, of ACI 318-99 and earlier editions of the code. In ACI 318-99, 18.4.2(d) required deflections to be checked by a cracked section analysis if tensile stresses exceeded $6\sqrt{f'_c}$, but the section was not assumed to be cracked unless the stress exceeded $7.5\sqrt{f'_c}$. This inconsistency was eliminated in 2002 Code by setting the dividing tensile stress between Classes U and T at $7.5\sqrt{f'_c}$.

Class C permits design using any combination of prestressing steel and reinforcement. It "fills the gap" between prestressed and nonprestressed concrete. For Class C members, a cracked section analysis or stresses is required by 18.3.4; whereas, for Class T members, an approximate cracked section analysis is required for deflection only. Unfortunately, a cracked section stress analysis for combined flexure and axial load (from the prestress) is complex. Reference 24.2 gives one method of accomplishing this.

Section 18.3.3 requires that prestressed two-way slab systems be designed as Class U with $f_t \le 6\sqrt{f'_c}$.

Table 24-1 Serviceability Design Requirements

	Prestressed			
	Class U	Class T	Class C	Nonprestressed
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 18.3.4	Gross section 18.3.4	Cracked section 18.3.4	No requirement
Allowable stress at transfer	18.4.1	18.4.1	18.4.1	No requirement
Allowable compressive stress based on uncracked section properties	18.4.2	18.4.2	No requirement	No requirement
Tensile stress at service loads 18.3.3	$\le 7.5\sqrt{f'_c}$	$7.5\sqrt{f'_c} < f_t \le 12\sqrt{f'_c}$	No requirement	No requirement
Deflection calculation basis	9.5.4.1 Gross section	9.5.4.2 Cracked section, bilinear	9.5.4.2 Cracked section, bilinear	9.5.2, 9.5.3 Effective moment of inertia
Crack control	No requirement	No requirement	10.6.4 Modified by 18.4.4.1	10.6.4
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis	$M/(A_s$ x lever arm), or $0.6f_y$
Side skin reinforcement	No requirement	No requirement	10.6.7	10.6.7

18.4 SERVICEABILITY REQUIREMENTS — FLEXURAL MEMBERS

Both concrete and prestressing tendon stresses are limited to ensure satisfactory behavior immediately after transfer of prestress and at service loads. The code provides different permissible stresses for conditions immediately after prestress transfer (before time-dependent losses) and for conditions at service loads (after all prestress losses have occurred).

For conditions immediately after prestress transfer, the code allows: extreme fiber compressive stress of $0.60f'_{ci}$; extreme fiber tensile stress of $3\sqrt{f'_{ci}}$, except $6\sqrt{f'_{ci}}$ is permitted at the ends of simply supported members. Where tensile stress exceeds the permissible values, bonded nonprestressed reinforcement shall be provided to resist the total tensile force assuming an uncracked section.

The permissible compressive stress due to prestress plus total service loads is limited to $0.60f'_c$. A permissible stress equal to $0.45f'_c$ has been added for the condition of prestress plus sustained loads. It should be noted that the "sustained loads" mentioned in 18.4.2(a) include any portion of the live load that will be sustained for a sufficient period to cause significant time-dependent deflections.

Concrete tensile stress limitations for Class U and T at service loads apply to the "precompressed" tensile zone which is that portion of the member cross-section in which flexural tension occurs under dead and live loads.

For Class C prestressed members, crack control is accomplished through a steel spacing requirement based on that of 10.6.4 and Eq. (10-4) for nonprestressed concrete. Eq. (10-4) is modified by 18.4.4. The maximum spacing between tendons is reduced to 2/3 of that permitted for bars, to account for lesser bond, compared to deformed bars. The quantity Δf_{ps}, the stress in the prestressing steel at service loads less the decompression stress f_{dc} is the stress in the prestressing steel when the stress is zero in the concrete at the same level as the centroid of the tendons. The code permits f_{dc} to be conservatively taken as the effective prestress f_{se}. The following shows Eq. (10-4), and as modified by 18.4.4.

Eq. (10-4) in 10.6.4:

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \le 12\left(\frac{40{,}000}{f_s}\right)$$

As modified by 18.4.4:

$$s = \frac{2}{3}\left[15\left(\frac{40{,}000}{\Delta f_{ps}}\right) - 2.5c_c\right]$$

The quantity of Δf_{ps} shall not exceed 36,000 psi. If Δf_{ps} is not greater than 20,000 psi, the above spacing limits need not apply.

The 2/3 modifier is to account for bond characteristics of strands, which are less effective than those of deformed bars.When both reinforcement and bonded tendons are used to meet the spacing requirement, the spacing between a bar and a tendon shall not exceed 5/6 of that given by Eq. (10-4).

Where h of a beam of a Class C exceeds 36 in., skin reinforcement consisting of reinforcement or bonded tendons shall be provided as required by 10.6.7.

18.5 PERMISSIBLE STRESSES IN PRESTRESSING STEEL

The permissible tensile stresses in all types of prestressing steel, in terms of the specified minimum tensile strength f_{pu}, are summarized as follows:

a. Due to tendon jacking force: .... $0.94f_{py}$ but not greater than $0.80f_{pu}$
   low-relaxation wire and strands ($f_{py} = 0.90f_{pu}$) .... $0.80f_{pu}$
   stress-relieved wire and strands, and plain bars (ASTM A722) ($f_{py} = 0.85f_{pu}$) .... $0.80f_{pu}$
   deformed bars (ASTM A722) ($f_{py} = 0.80f_{pu}$) .... $0.75f_{pu}$

b. Immediately after prestress transfer: .... $0.82f_{py}$ but not greater than $0.74f_{pu}$
   low-relaxation wire and strands ($f_{py} = 0.90f_{pu}$) .... $0.74f_{pu}$
   stress-relieved wire and strands, and plain bars ($f_{py} = 0.85f_{pu}$) .... $0.70f_{pu}$
   deformed bars ($f_{py} = 0.80f_{pu}$) .... $0.66f_{pu}$

c. Post-tensioning tendons, at anchorages and couplers, immediately after tendon anchorage .... $0.70f_{pu}$

Note that the permissible stresses given in 18.5.1(a) and (b) apply to both pretensioned and post-tensioned tendons. Pretensioned tendons are often jacked to 75 percent of f_{pu}. This will result in a stress below 0.74 f_{pu} after transfer.

18.6 LOSS OF PRESTRESS

A significant factor which must be considered in design of prestressed members is the loss of prestress due to various causes. These losses can dramatically affect the behavior of a member at service loads. Although calculation procedures and certain values of creep strain, friction factors, etc., may be recommended, they are at best only an estimate. For the design of members whose behavior (deflection in particular) is sensitive to prestress losses, the engineer should establish through tests the time-dependent properties of materials to be used in the analysis/design of the structure. Refined analyses should then be performed to estimate the prestress losses. Specific provisions for computing friction loss in post-tensioning tendons are provided in 18.6.2. Allowance for other types of prestress losses are discussed in Ref. 24.1. Note that the designer is required to show on the design drawings the magnitude and location of prestressing forces as required by 1.2.1(g).

ESTIMATING PRESTRESS LOSSES

Lump sum values of prestress losses that were widely used as a design estimate of prestress losses prior to the '83 code edition (35,000 psi for pretensioning and 25,000 psi for post-tensioning) are now considered obsolete. Also, the lump sum values may not be adequate for some design conditions.

Reference 24.3 offers guidance to compute prestress losses and it is adaptable to computer programs. It allows step-by-step computation of losses which is necessary for rational analysis of deformations. The method is too tedious for hand calculations.

Reference 24.4 presents a reasonably accurate and easy procedure for estimating prestress losses due to various causes for pretensioned and post-tensioned members with bonded and unbonded tendons. The procedure, which is intended for practical design applications under normal design conditions, is summarized below. The simple equations enable the designer to estimate the prestress loss from each source rather than using a lump sum value. The reader is referred to Ref. 24.4 for an in-depth discussion of the procedure, including sample computations for typical prestressed concrete beams. Quantities used in loss computations are defined in the summary of notation which follows this section.

COMPUTATION OF LOSSES

Elastic Shortening of Concrete (ES)

For members with bonded tendons:

$$ES = K_{es}E_s \frac{f_{cir}}{E_{ci}} \tag{1}$$

where K_{es} = 1.0 for pretensioned members

K_{es} = 0.5 for post-tensioned members where tendons are tensioned in sequential order to the same tension. With other post-tensioning procedures, the value for K_{es} may vary from 0 to 0.5.

$$f_{cir} = K_{cir}f_{cpi} - f_g \tag{2}$$

where K_{cir} = 1.0 for post-tensioned members

K_{cir} = 0.9 for pretensioned members.

For members with unbonded tendons:

$$ES = K_{es}E_s \frac{f_{cpa}}{E_{ci}} \tag{1a}$$

Creep of Concrete (CR)

For members with bonded tendons:

$$CR = K_{cr} \frac{E_s}{E_c} (f_{cir} - f_{cds}) \tag{3}$$

where $K_{cr} = 2.0$ for pretensioned members

$K_{cr} = 1.6$ for post-tensioned members

For members made of sand lightweight concrete the foregoing values of K_{cr} should be reduced by 20 percent.

For members with unbonded tendons:

$$CR = K_{cr} \frac{E_s}{E_c} f_{cpa} \tag{3a}$$

Shrinkage of Concrete (SH)

$$SH = 8.2 \times 10^{-6} K_{sh} E_s \left(1 - 0.06 \frac{V}{S}\right)(100 - RH) \tag{4}$$

where $K_{sh} = 1.0$ for pretensioned members

K_{sh} is taken from Table 24-2 for post-tensioned members.

Table 24-2 Values of K_{sh} for Post-Tensioned Members

Time, days*	1	3	5	7	10	20	30	60
K_{sh}	0.92	0.85	0.80	0.77	0.73	0.64	0.58	0.45

**Time after end of moist curing to application of prestress*

Relaxation of Tendons (RE)

$$RE = [K_{re} - J(SH + CR + ES)]\, C \tag{5}$$

where the values of K_{re}, J, and C are taken from Tables 24-3 and 24-4.

Table 24-3 Values of K_{re} and J

Type of Tendon	Kre (psi)	J
270 Grade stress-relieved strand or wire	20,000	0.15
250 Grade stress-relieved strand or wire	18,500	0.14
240 or 235 Grade stress-relieved wire	17,600	0.13
270 Grade low-relaxation strand	5000	0.040
250 Grade low-relaxation wire	4630	0.037
240 or 235 Grade low-relaxation wire	4400	0.035
145 or 160 Grade stress-relieved bar	6000	0.05

Table 24-4 Values of C

f_{pi}/f_{pu}	Stressed relieved strand or wire	Stress-relieved bar or low relaxation strand or wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

Friction

Computation of friction losses is covered in 18.6.2. When the tendon is tensioned, the friction losses computed can be checked with reasonable accuracy by comparing the measured tendon elongation and the prestressing force applied by the tensioning jack.

SUMMARY OF NOTATION

A_c = area of gross concrete section at the cross-section considered
A_{ps} = total area of prestressing steel
C = a factor used in Eq. (5), see Table 24-4
CR = stress loss due to creep of concrete
e = eccentricity of center of gravity of prestressing steel with respect to center of gravity of concrete at the cross-section considered
E_c = modulus of elasticity of concrete at 28 days
E_{ci} = modulus of elasticity of concrete at time prestress is applied
E_s = modulus of elasticity of prestressing steel. Usually 28,000,000 psi
ES = stress loss due to elastic shortening of concrete
f_{cds} = stress in concrete at center of gravity of prestressing steel due to all superimposed permanent dead loads that are applied to the member after it has been prestressed
f_{cir} = net compressive stress in concrete at center of gravity of prestressing steel immediately after the prestress has been applied to the concrete. See Eq. (2).
f_{cpa} = average compressive stress in the concrete along the member length at the center of gravity of the

prestressing steel immediately after the prestress has been applied to the concrete

f_{cpi} = stress in concrete at center of gravity of prestressing steel due to P_{pi}

f_g = stress in concrete at center of gravity of prestressing steel due to weight of structure at time prestress is applied

f_{pi} = stress in prestressing steel due to P_{pi}, $= P_{pi}/A_{ps}$

f_{pu} = specified tensile strength of prestressing steel, psi

I_c = moment of inertia of gross concrete section at the cross-section considered

J = a factor used in Eq. (5), see Table 24-3

K_{cir} = a factor used in Eq. (2)

K_{cr} = a factor used in Eq. (3)

K_{es} = a factor used in Eqs. (1) and (1a)

K_{re} = a factor used in Eq. (5), see Table 24-3

M_d = bending moment due to dead weight of member being prestressed and to any other permanent loads in place at time of prestressing

M_{ds} = bending moment due to all superimposed permanent dead loads that are applied to the member after it has been prestressed

P_{pi} = prestressing force in tendons at critical location on span after reduction for losses due to friction and seating loss at anchorages but before reduction for ES, CR, SH, and RE

RE = stress loss due to relaxation of prestressing steel

RH = average relative humidity surrounding the concrete member (see Fig. 24-3)

SH = stress loss due to shrinkage of concrete

V/S = volume to surface ratio, usually taken as gross cross-sectional area of concrete member divided by its perimeter

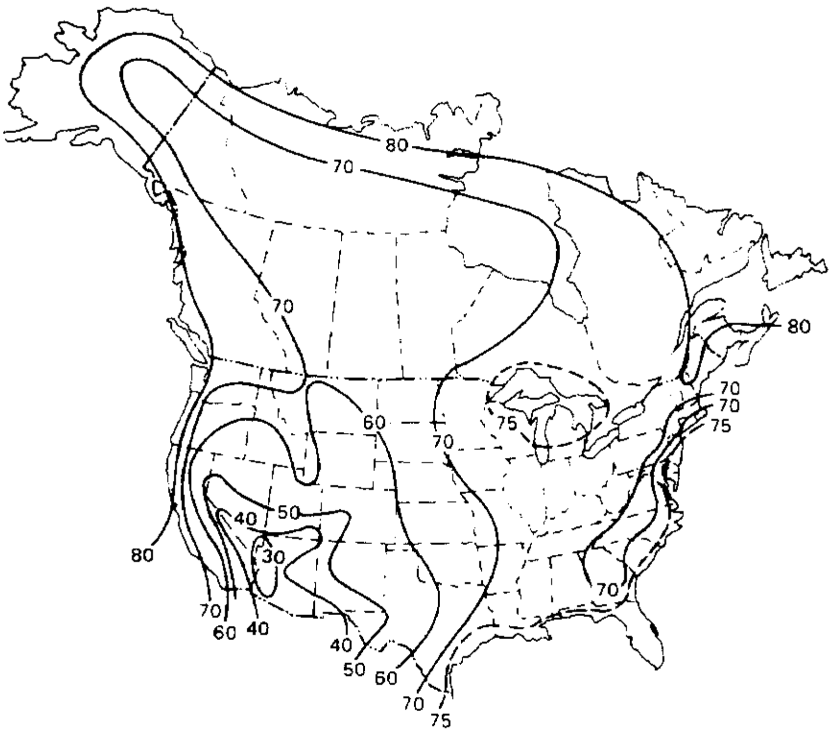


Figure 24-3 Annual Average Ambient Relative Humidity

18.7 FLEXURAL STRENGTH

The flexural strength of prestressed members can be calculated using the same assumptions as for nonprestressed members. Prestressing steel, however, does not have a well defined yield point as does mild reinforcement. As a prestressed cross-section reaches its flexural strength (defined by a maximum compressive concrete strain of 0.003), stress in the prestressed reinforcement at nominal strength, f_{ps}, will vary depending on the amount of prestressing. The value of f_{ps} can be obtained using the conditions of equilibrium, stress-strain relations, and strain compatibility (Design Example 24-4 illustrates the procedure). However, the analysis is quite cumbersome, especially in the case of unbonded tendon. For bonded prestressing, the compatibility of strains can be considered at an individual section, while for unbonded tendon, compatibility relations can be written only at the anchorage points and will depend on the entire cable profile and member loading. To avoid such lengthy calculations, the code allows f_{ps} to be obtained by the approximate Eqs. (18-3), (18-4), and (18-5).

For members with bonded prestressing steel, an approximate value of f_{ps} given by Eq. (18-3) may be used for flexural members reinforced with a combination of prestressed and nonprestressed reinforcement (partially prestressed members), taking into account effects of any nonprestressed tension reinforcement (ω), any compression reinforcement (ω'), the concrete compressive strength f'_c, rectangular stress block factor β_1, and an appropriate factor for type of prestressing material used $\left(\gamma_p\right)$. For a fully prestressed member (without nonprestressed tension or compression reinforcement), Eq. (18-3) reduces to:

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f'_c}\right)$$

where γ_p = 0.55 for deformed bars $\left(f_{py}/f_{pu} \geq 0.80\right)$

= 0.40 for stress-relieved wire and strands, and plain bars $\left(f_{py}/f_{pu} \geq 0.85\right)$

= 0.28 for low-relaxation wire and strands $\left(f_{py}/f_{pu} \geq 0.90\right)$

and β_1, as defined in 10.2.7.3,

$$\begin{aligned}
\beta_1 &= 0.85 \text{ for } f'_c \leq 4000 \text{ psi} \\
&= 0.80 \text{ for } f'_c = 5000 \text{ psi} \\
&= 0.75 \text{ for } f'_c = 6000 \text{ psi} \\
&= 0.70 \text{ for } f'_c = 7000 \text{ psi} \\
&= 0.65 \text{ for } f'_c \geq 8000 \text{ psi}
\end{aligned}$$

Eq. (18-3) can be written in nondimensional form as follows:

$$\omega_p = \omega_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\omega_{pu}\right) \qquad (6)$$

where

$$\omega_p = \frac{A_{ps}f_{ps}}{bd_pf'_c} \qquad (7)$$

$$\omega_{pu} = \frac{A_{ps}f_{pu}}{bd_pf'_c} \qquad (8)$$

The moment strength of a prestressed member with bonded tendons may be computed using Eq. (18-3) only when all of the prestressed reinforcement is located in the tension zone. When part of the prestressed reinforcement is located in the compression zone of a cross-section, Eq. (18-3), involving d_p, is not valid. Flexural strength for such a condition must be computed by a general analysis based on strain compatibility and equilibrium, using the stress-strain properties of the prestressing steel and the assumptions given in 10.2.

For members with unbonded prestressing steel, an approximate value of f_{ps} given by Eqs. (18-4) and (18-5) may be used. Eq. (18-5) applies to members with high span-to-depth ratios (> 35), such as post-tensioned one-way slabs, flat plates and flat slabs.

With the value of f_{ps} known, the nominal moment strength of a rectangular section, or a flanged section where the stress block is within the compression flange, can be calculated as follows:

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) = A_{ps}f_{ps}\left(d_p - 0.59\,\frac{A_{ps}f_{ps}}{bf'_c}\right) \tag{9}$$

where a = the depth of the equivalent rectangular stress block $= \dfrac{A_{ps}f_{ps}}{0.85bf'_c}$ (10)

or in nondimensional terms:

$$R_n = \omega_p\,(1 - 0.59\omega_p) \tag{11}$$

where $R_n = \dfrac{M_n}{b(d_p)^2 f'_c}$ (12)

18.8 LIMITS FOR REINFORCEMENT OF FLEXURAL MEMBERS

Prestressed concrete sections are classifed as tension-controlled, transition, or compression-controlled based on net tensile strain. These classifications are defined in 10.3.3 and 10.3.4, with appropriate ϕ-factors in 9.3.2. These requirements are the same as those for nonprestressed concrete.

Figure 24-3 shows the relationship between the coefficient of resistance $\phi M_n/(bd^2)$ and the reinforcement ratio ρ_p for prestressed flexural members. Grade 270 ksi prestressing steel has a useful strength 4.5 times that of Grade 60 reinforcement. Compare Fig. 24-4 to Fig. 7-3. Higher concrete strengths are normally used with prestressed concrete, so Fig. 24-4 shows curves for f'_c from 5000 to 8000 psi; whereas, Fig. 7-3 shows curves for f'_c from 3000 to 6000 psi. The curves for f'_c of 5000 and 6000 psi are almost identical in the two figures.

In both figures, the curves have a break point corresponding to a net tensile strain of 0.005. Beyond that point, the reduction in ϕ in the transition region almost cancels the benefit of increased reinforcement index. For both nonprestressed and prestressed concrete, the best design is to stay in the tension-controlled region, using compression reinforcement, if necessary, to maintain the net tensile strain, ε_t at 0.005 or more.

As in previous ACI 318 codes, there is no absolute limit on the reinforcement index for prestressed members. But it will always be advantageous to design the tension-controlled region at critical sections, as there is little or no gain in design strength in the transition region.

Critical parameters at the tension-controlled limit may be tabulated. The effective prestress f_{se} will normally be at least 0.6 f_{pu}, or 162 ksi, if a jacking stress of 0.75 f_{pu} is used. This amounts to a 20 percent loss. The total steel strain when $\varepsilon_t = 0.005$ is equal to $162/28{,}500 + 0.005 = 0.01068$. Using the stress-strain curve shown in Fig. 24-1, $f_{ps} = 270 - 0.04/(0.01068-0.007) = 259$ ksi. A section will be tension-controlled when d_t is taken equal to d_p.

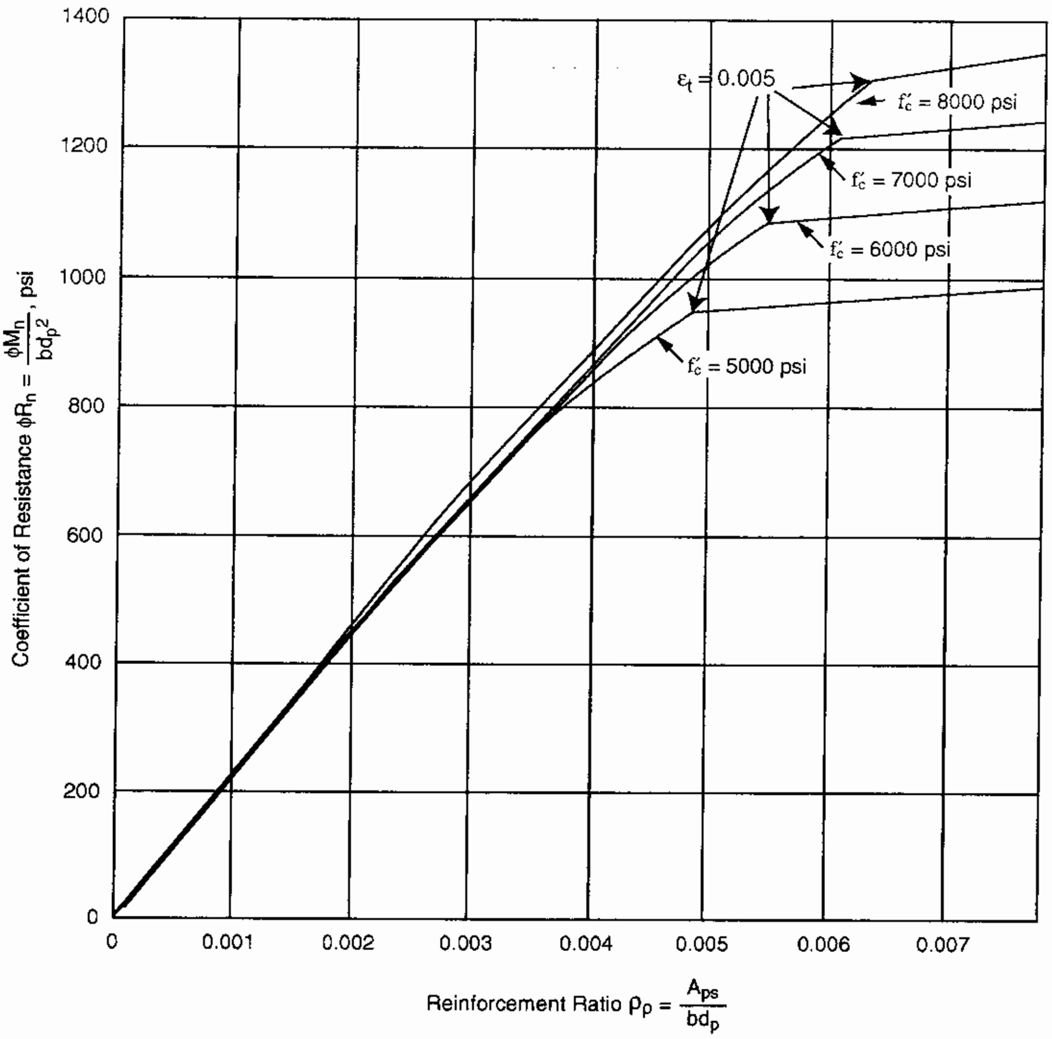


Figure 24-4 Design Strength Curves (ϕR_n vs. ρ_p) for Type 270k Low Relaxation Strand

Table 24-5 shows design parameters for prestressed sections at the tension-controlled strain limit, indicated by the added subscript t. The rows for R_{nt}, ϕ_{nt}, and ω_{pt} are identical to those in Table 6-1. The row for ω_{put} shows values slightly higher than ω_{pt}, because ω_{put} is based on f_{pu} of 270 ksi; whereas, ω_{put} is based on f_{ps} of 259 ksi. The final row for ρ_{pt} shows values much smaller than for ρ_t in Table 6-1, because of the much higher strength of the prestressing strand.

The following is a short-cut procedure for finding the flexural strength of sections in which the Grade 270 ksi low-relaxation prestressing steel can reasonably be assumed to be in one layer with $d_p = d_t$, and with $f_{se} \geq 162$ ksi.

1. Assume section is at tension-controlled limit, and $f_{ps} = 259$ ksi.

2. Compute steel tension T and equal compressive force C.

3. Find depth of stress block a and depth to neutral axis c.

4. Is $c/d_p \leq 0.375$? If so, proceed. If not, add compression steel to make $c/d_p \leq 0.375$.

5. Compute provided design strength $\phi M_n = 0.9(T)(d\text{-}a/2)$.

Table 24-5 Design Parameters at Strain Limit of 0.005 for Tension-Controlled Sections

	$f'_c = 3000$ $\beta_1 = 0.85$	$f'_c = 4000$ $\beta_1 = 0.85$	$f'_c = 5000$ $\beta_1 = 0.80$	$f'_c = 6000$ $\beta_1 = 0.75$	$f'_c = 8000$ $\beta_1 = 0.65$	$f'_c = 10{,}000$ $\beta_1 = 0.65$
R_{nt}	683	911	1084	1233	1455	1819
ϕR_{nt}	615	820	975	1109	1310	1637
ω_{pt}	0.2709	0.2709	0.2550	0.2391	0.2072	0.2072
ω_{put}	0.2823	0.2823	0.2657	0.2491	0.2159	0.2159
ρ_{pt}	0.00314	0.00418	0.00492	0.00554	0.00640	0.00800

For $f_{se} \geq 162$ ksi in low-relaxation Grade 270 ksi strand

6. If provided $\phi M_n \geq$ required, stop. Section is adequate. If not proceed.
7. If the deficiency in provided ϕM_n is more than 4 percent, steel must be added. If deficiency is less than 4 percent, strain compatibility may be used in an attempt to find a higher f_{ps} in order to justify adequacy of the section.

Section 18.8.2 requires the total amount of prestressed and nonprestressed reinforcement of flexural members to be adequate to develop a design moment strength at least equal to 1.2 times the cracking moment strength ($\phi M_n = 1.2 M_{cr}$), where M_{cr} is computed by elastic theory using a modulus of rupture equal to $7.5\sqrt{f'_c}$. The provisions of 18.8.2 are analogous to 10.5 for nonprestressed members. They are intended as a precaution against abrupt flexural failure resulting from rupture of the prestressing tendons immediately after cracking. The provision ensures that cracking will occur before flexural strength is reached, and by a large enough margin so that significant deflection will occur to warn that the ultimate capacity is being approached. The typical bonded prestressed member will have a fairly large margin between cracking strength and flexural strength, but the designer must be certain by checking it.

The cracking moment M_{cr} for a prestressed member is determined by summing all the moments that will cause a stress in the bottom fiber equal to the modulus of rupture f_r. Referring to Fig. 24-5 for an unshored prestressed composite member taking compression as negative and tension as positive:

$$-\left(\frac{P_{se}}{A_c}\right) - \left(\frac{P_{se}e}{S_b}\right) + \left(\frac{M_d}{S_b}\right) + \left(\frac{M_a}{S_c}\right) = +f_r$$

Solving for $M_a = \left(f_r + \frac{P_{se}}{A_c} + \frac{P_{se}e}{S_b}\right) S_c - M_d\left(\frac{S_c}{S_b}\right)$

Since $M_{cr} = M_d + M_a$

$$M_{cr} = \left(f_r + \frac{P_{se}}{A_c} + \frac{P_{se}e}{S_b}\right) S_c - M_d\left(\frac{S_c}{S_b} - 1\right) \qquad (13)$$

For a prestressed member alone (without composite slab), $S_c = S_b$. Therefore, M_{cr} reduces to

$$M_{cr} = \left(f_r + \frac{P_{se}}{A_c}\right) S_b + P_{se}e \qquad (14)$$

Examples 24.6 and 24.7 illustrate computation of the cracking moment strength of prestressed members.

Note that an exception in 18.8.2 waives the $1.2M_{cr}$ requirement for (a) two-way, unbonded post-tensioned slabs, and (b) flexural members with shear and flexural strength at least twice that required by 9-2.

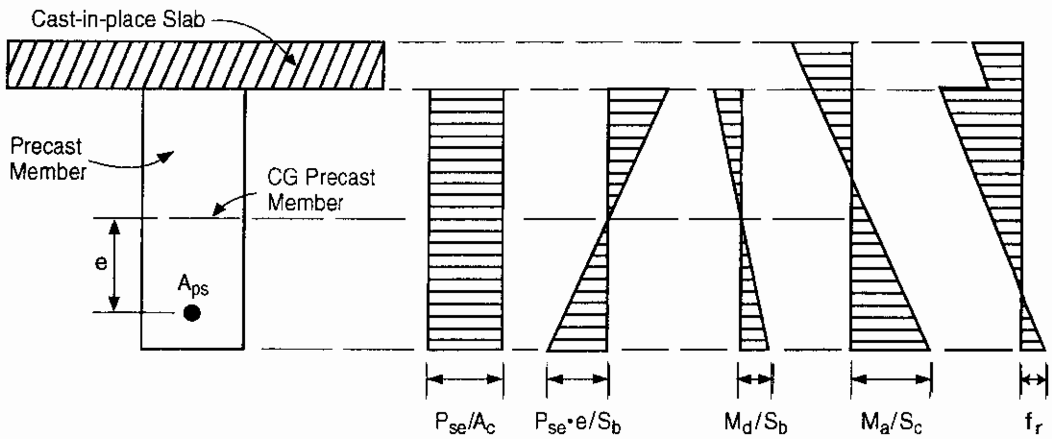
For flexural strength: $\phi M_n \geq 2M_u \geq 2\,(1.2M_d + 1.6M_\ell)$

For shear strength: $\phi V_n \geq 2V_u \geq 2\,(1.2V_d + 1.6V_\ell)$

The $1.2M_{cr}$ provision often requires excessive reinforcement for certain prestressed flexural members especially for short span hollow-core members. The exception is intended to limit the amount of additional reinforcement required to amounts that provide for ductility, and is comparable in concept to those for nonprestressed members in 10.5.3.

Introduced in the 1999 edition of the code, the waiver of the $1.2M_{cr}$ provision for two-way, unbonded post-tensioned slabs brings the code in line with current practices which have been shown to be technically sound and safe (Ref. 24.5).

Section 18.8.3 prescribes a qualitative requirement stating that some bonded reinforcement or tendons must be placed as close to the tension face as is practicable.



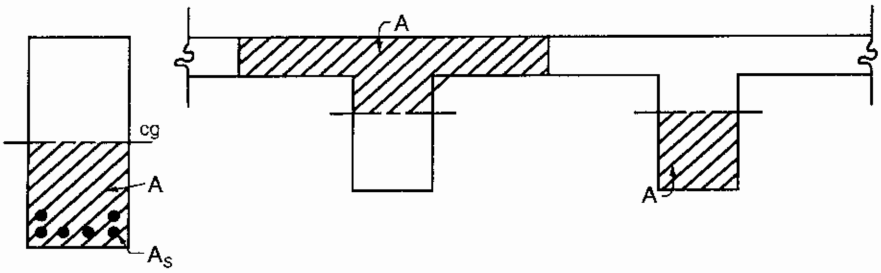
A_{ps} = area of prestressed reinforcement in tensile zone
A_c = area of precast member
S_b = section modulus for bottom of precast member
S_c = section modulus for bottom of composite member
P_{se} = effective prestress force
e = eccentricity of prestress force
M_d = dead load moment of composite member
M_a = additional moment to cause a stress in bottom fiber equal to modulus of rupture f_r

Figure 24-5 Stress Conditions for Evaluating Cracking Moment Strength

18.9 MINIMUM BONDED REINFORCEMENT

A minimum amount of bonded reinforcement is desirable in members with unbonded tendons. Reference to R18.9 is suggested.

For all flexural members with unbonded prestressing tendons, except two-way solid slabs, a minimum area of bonded reinforcement computed by Eq. (18-6) must be uniformly distributed over the precompressed tensile zone as close as practicable to the extreme tension fiber. Figure 26-6 illustrates application of Eq. (18-6).

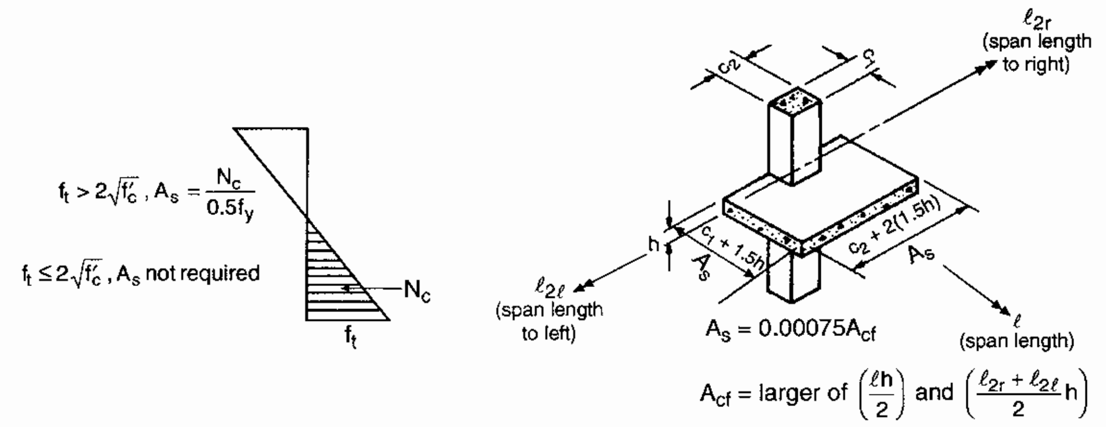


$A_s = 0.004A$

Figure 24-6 Bonded Reinforcement for Flexural Members

For solid slabs, the special provisions of 18.9.3 apply. Depending on the tensile stress in the concrete at service loads, the requirements for positive moment areas of solid slabs are illustrated in Fig. 24-7(a). Formerly, 18.9.3 applied only to flat plates. Starting with ACI 318-02, it also applies to two-way flat slab systems with drop panels.

The requirement for minimum area of bonded reinforcement in two-way flat plates at column supports was revised in the 1999 code edition to reflect the intent of the original research recommendations (Ref. 24.5). This revision increases the minimum reinforcement requirement over interior columns for rectangular panels in one direction, and, for square panels, doubles the minimum reinforcement requirement over exterior columns normal to the slab edge. Figure 24-7(b) illustrates the minimum bonded reinforcement requirements for the negative moment areas at column supports. The bonded reinforcement must be located within the width $c_2 + 2\,(1.5h)$ as shown, with a minimum of four bars spaced at not more than 12 in. Similarly, minimum bonded reinforcement should be provided parallel to slab edge.



(a) Positive Moment Areas

(b) Negative Moment Areas

Figure 24-7 Bonded Reinforcement for Flat Plates

18.10.4 Redistribution of Negative Moments in Continuous Prestressed Flexural Members

The special provisions for moment redistribution in 8.4, apply equally to prestressed and nonprestressed continuous flexural members. See Part 8 for details.

18.11 COMPRESSION MEMBERS — COMBINED FLEXURE AND AXIAL LOADS

Provisions of the code for calculating the strength of prestressed members are the same as for members without prestressing. Additional considerations include (1) accounting for prestressing strains, and (2) using an appropriate stress-strain relation for the prestressing tendons. Example 24.7 illustrates the calculation procedure.

For compression members with an average concrete stress due to prestressing of less than 225 psi, minimum nonprestressed reinforcement must be provided (18.11.2.1). For compression members with an average concrete stress due to prestressing equal to or greater than 225 psi, 18.11.2.2 requires that all prestressing tendons be enclosed by spirals or lateral ties, except for walls.

REFERENCES

24.1 "PCI Design Handbook – Precast and Prestressed Concrete," MNL 120-04 6th Edition, Precast/Prestressed Concrete Institute, Chicago, 2004, 750 pp.

24.2 Mast, R. F., "Analysis of Cracked Prestressed Sections: A Practical Approach," *PCI Journal,* Vol. 43, No. 4, July-August 1975, pp. 43-75.

24.3 PCI Committee on Prestress Losses, "Recommendations for Estimating Prestress Losses," *PCI Journal,* Vol. 20, No. 4, July-August 1975, pp. 43-75.

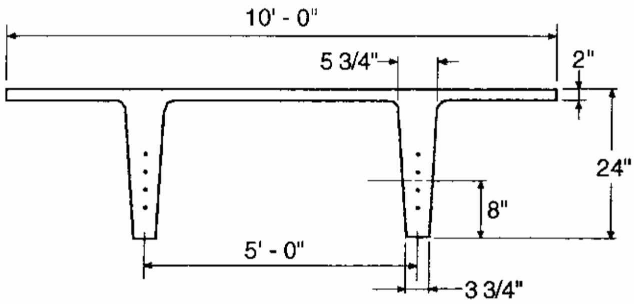
24.4 Zia, Paul, et al., "Estimating Prestress Losses," *Concrete International: Design and Construction,* Vol. 1, No. 6, June 1979, pp. 32-38.

24.5 ACI 423.3R-96 Report, ""Recommendations for Concrete Members Prestressed with Unbonded Tendons," American Concrete Institute, Farmington Hills, Michigan.

Example 24.1—Estimating Prestress Losses

For the simply supported double-tee shown below, estimate loss of prestress using the procedures of Ref. 24.4, as outlined earlier under "Computation of Losses." Assume the unit is manufactured in Green Bay, WI.

live load = 40 psf
roof load = 20 psf
dead load = 47 psf = 468 plf
span = 48 ft
f'_{ci} = 3500 psi
f'_c = 5000 psi
8 - 0.5 in. diameter low-relaxation strands
A_{ps} = 8 (0.153 in.^{2}) = 1.224 in.^{2}
e = 9.77 in. (all strands straight)
f_{pu} = 270,000 psi
f_{py} = $0.90f_{pu}$
jacking stress = $0.74f_{pu}$ = 200 ksi



Assume the following for loss computations:
E_{ci} = 3590 ksi
E_c = 4290 ksi
E_s = 28,500 ksi

Section Properties
A_c = 449 in.^{2}
I_c = 22,469 in.^{4}
y_b = 17.77 in.
y_t = 6.23 in.
V/S = 1.35 in.

Calculations and Discussion	Code Reference

1. Elastic Shortening of Concrete (ES); using Eq. (1)

$$ES = K_{es}E_s \frac{f_{cir}}{E_{ci}} = 1.0\,(28{,}500)\,\frac{0.725}{3590} = 5.8 \text{ ksi}$$

where

K_{es} = 1.0 for pretensioned members

$$f_{cir} = K_{cir}f_{cpi} - f_g$$

$$= K_{cir}\left(\frac{P_{pi}}{A_c} + \frac{P_{pi}e^2}{I_c}\right) - \frac{M_d e}{I_c}$$

$$= 0.9\left(\frac{245}{449} + \frac{245 \times 9.77^2}{22{,}469}\right) - \frac{1617 \times 9.77}{22{,}469} = 0.725 \text{ ksi}$$

K_{cir} = 0.9 for pretensioned members

$P_{pi} = 0.74 f_{pu} A_{ps} = 0.74\ (270)\ (1.224) = 245$ kips

$$M_d = 0.468 \times 48^2 \times \frac{12}{8} = 1617 \text{ in.-kips (dead load of unit)}$$

2. Creep of Concrete (CR); using Eq. (3)

$$CR = K_{cr} \frac{E_s}{E_c} (f_{cir} - f_{cds}) = 2.0 \times \frac{28{,}500}{4290} (0.725 - 0.30) = 5.6 \text{ ksi}$$

$$\text{where } f_{cds} = M_{ds} \frac{e}{I_c} = 691 \times \frac{9.77}{22{,}469} = 0.30 \text{ ksi}$$

$$M_{ds} = 0.02 \times 10 \times 48^2 \times \frac{12}{8} = 691 \text{ in.-kips (roof load only)}$$

and $K_{cr} = 2.0$ for pretensioned members.

3. Shrinkage of Concrete (SH); using Eq. (4)

$$SH = 8.2 \times 10^{-6} K_{sh} E_s \left(1 - 0.06 \frac{V}{S}\right) (100 - RH)$$

$$= 8.2 \times 10^{-6} \times 1.0 \times 28{,}500\ (1 - 0.06 \times 1.35)\ (100 - 75) = 5.4 \text{ ksi}$$

RH = average relative humidity surrounding the concrete member from Fig. 24-3. For Green Bay, Wisconsin, RH = 75%

and $K_{sh} = 1.0$ for pretensioned members.

4. Relaxation of Tendon Stress (RE); using Eq. (5)

$$RE = [K_{re} - J\ (SH + CR + ES)]\ C$$

$$= [5 - 0.04\ (5.4 + 5.6 + 5.8)]\ 0.95 = 4.1 \text{ ksi}$$

where, for 270 Grade low-relaxation strand:

$K_{re} = 5$ ksi (Table 24-3)

$J = 0.040$ (Table 24-3)

$$C = 0.95 \text{ (Table 24-4 for } \frac{f_{pi}}{f_{pu}} = 0.74)$$

5. Total allowance for loss of prestress

$ES + CR + SH + RE = 5.8 + 5.6 + 5.4 + 4.1 = 20.9$ ksi *18.6.1*

6. Stress, f_p, and force, P_p, immediately after transfer.

   Assume that one-fourth of relaxation loss occurs prior to release.

   $f_p = 0.74f_{pu} - (ES + 1/4\ RE)$

   $= 0.74\ (270) - [5.8 + 1/4\ (4.1)] = 193.0$ ksi

   $P_p = f_pA_{ps} = 193.0 \times 1.224 = 236$ kips

7. Effective prestress stress f_{se} and effective prestress force P_e after all losses

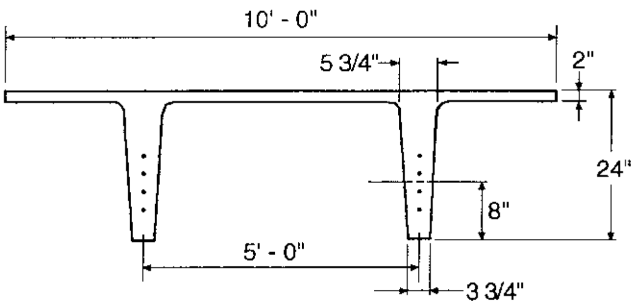
   $f_{se} = 0.74f_{pu}$ - allowance for all prestress losses

   $= 0.74\ (270) - 20.9 = 179$ ksi

   $P_e = f_{se}A_{ps} = 179 \times 1.224 = 219$ kips

Example 24.2—Investigation of Stresses at Prestress Transfer and at Service Load

For the simply supported double-tee considered in Example 24.1, check all permissible concrete stresses immediately after prestress transfer and at service load assuming the unit is used for roof framing. Use losses computed in Example 24.1.



live load = 40 psf
roof load = 20 psf
dead load = 47 psf = 468 plf
span = 48 ft
f'_{ci} = 3500 psi
f'_c = 5000 psi
8 - 0.5 in. diameter low-relaxation strands
$A_{ps} = 8 (0.153 \text{ in.}^2) = 1.224 \text{ in.}^2$
e = 9.77 in. (all strands straight)
f_{pu} = 270,000 psi
$f_{py} = 0.90f_{pu}$
jacking stress = $0.74f_{pu}$ = 200 ksi
stress after transfer = 193 ksi
force after transfer = $P_p = 1.224 \times 193 = 236$ kips

Section Properties
$A_c = 449 \text{ in.}^2$
$I_c = 22{,}469 \text{ in.}^4$
y_b = 17.77 in.
y_t = 6.23 in.
V/S = 1.35 in.

Calculations and Discussion	Code Reference
1. Calculate permissible stresses in concrete.	18.4
At prestress transfer (before time-dependent losses):	18.4.1
Compression: $0.60f'_{ci} = 0.60(3500) = 2100$ psi	
Tension: $6\sqrt{f'_{ci}} = 355$ psi (at ends of simply supported members; otherwise $3\sqrt{f'_{ci}}$)	
At service load (after allowance for all prestress losses):	18.4.2
Compression: $0.45f'_c = 2250$ psi - Due to sustained loads Compression: $0.60f'_c = 3000$ psi - Due to total loads	
Tension: $12\sqrt{f'_c} = 849$ psi	18.3.3(b)
2. Calculate service load moments at midspan:	

Example 24.2 (cont'd)	Calculations and Discussion	Code Reference

$$M_d = \frac{w_d \ell^2}{8} = \frac{0.468 \times 48^2}{8} = 134.8 \text{ ft-kips (beam dead load)}$$

$$M_{ds} = \frac{w_{ds} \ell^2}{8} = \frac{0.02 \times 10 \times 48^2}{8} = 57.6 \text{ ft-kips (roof dead load)}$$

$$M_{sus} = M_d + M_{ds} = 134.8 + 57.6 = 192.4 \text{ ft-kips (sustained load)}$$

$$M_\ell = \frac{w_\ell \ell^2}{8} = \frac{0.04 \times 10 \times 48^2}{8} = 115.2 \text{ ft-kips (live load)}$$

$$M_{tot} = M_d + M_{ds} + M_\ell = 134.8 + 57.6 + 115.2 = 307.6 \text{ ft-kips (total load)}$$

3. Calculate service load moments at transfer point

Assume transfer point located at $50d_b = 25$ in. from end of beam. Assume distance from end of beam to center of support is 4 in. Therefore, x = 25 - 4 = 21 in. = 1.75 ft. *11.4.3*

$$M_d = \frac{w_d x}{2}(\ell - x) = \frac{0.468 \times 1.75}{2}(48 - 1.75) = 18.9 \text{ ft-kips (beam dead load)}$$

Additional moment calculations at this location are unnecessary because conditions immediately after release govern at this location.

4. Calculate extreme fiber stresses by "linear elastic theory" which leads to the following well known formulas:

$$f_t = \frac{P}{A} - \frac{Pey_t}{I} + \frac{My_t}{I}$$

$$f_b = \frac{P}{A} + \frac{Pey_b}{I} - \frac{My_b}{I}$$

where, from Example 24.1

$P = P_p = 236$ kips (immediately after transfer)

$P = P_e = 219$ kips (at service load)

Table 24-4 Stresses in Concrete Immediately after Prestress Transfer (psi)

	At Assumed Transfer Point		At Mid Span	
	Top	Bottom	Top	Bottom
P_p/A P_pey/I $M_d y/I$	+526 -639 +63	+526 +1824 -180	+526 -639 +448	+526 +1824 -1279
Total	-50 (O.K.)	+2170 (say O.K.)	+335 (O.K.)	+1071 (O.K.)
Permissible	-355	+2100	+2100	+2100

Compression (+)
Tension (-)

Table 24-5 Stresses in Concrete at Service Loads (psi)

	At Midspan – Sustained Loads		At Midspan – Total Loads	
	Top	Bottom	Top	Bottom
P_e/A P_eey/I My/I	+488 -594 +640	+488 +1695 -1826	+488 -594 +1023	+488 +1695 -2919
Total	+534 (O.K.)	+357 (O.K.)	+917 (O.K.)	-736 (O.K.)
Permissible	+2250	+2250	+3000	-849

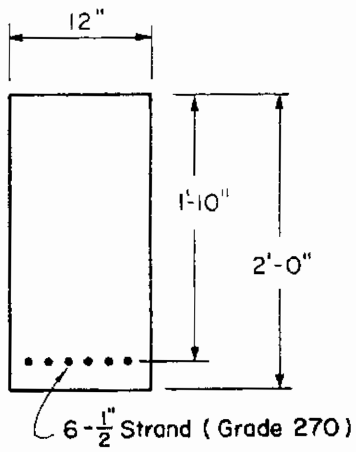
Compression (+)
Tension (-)

Example 24.3—Flexural Strength of Prestressed Member Using Approximate Value for f_{ps}

Calculate the nominal moment strength of the prestressed member shown.

f'_c = 5000 psi

f_{pu} = 270,000 psi (low-relaxation strands; $f_{py} = 0.90f_{pu}$)



Calculations and Discussion	Code Reference

1. Calculate stress in prestressed reinforcement at nominal strength using approximate value for f_{ps}. For a fully prestressed member, Eq. (18-3) reduces to:

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f'_c}\right)$$ *Eq. (18-3)*

$$= 270\left(1 - \frac{0.28}{0.80} \times 0.00348 \times \frac{270}{5}\right) = 252 \text{ ksi}$$

where

$$\gamma_p = 0.28 \text{ for } \frac{f_{py}}{f_{pu}} = 0.90 \text{ for low-relaxation strand}$$

β_1 = 0.80 for f'_c = 5000 psi *10.2.7.3*

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{6 \times 0.153}{12 \times 22} = 0.00348$$

2. Calculate nominal moment strength from Eqs. (9) and (10) of Part 24

Compute the depth of the compression block:

$$a = \frac{A_{ps}f_{ps}}{0.85bf'_c} = \frac{0.918 \times 252}{0.85 \times 12 \times 5} = 4.54 \text{ in.} \qquad \text{Eq. (10)}$$

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) \qquad \text{Eq. (9)}$$

$$M_n = 0.918 \times 252\left(22 - \frac{4.54}{2}\right) = 4565 \text{ in-kips} = 380 \text{ ft-kips}$$

3. Check to see if tension controlled *10.3.4*

$$c/d_p = (a/\beta_1)/d_p = \left(\frac{4.54}{0.80}\right)/22$$

$c/d_p = 0.258 < 0.375$ *R9.3.2.2*

Tension controlled $\phi = 0.9$

Example 24.4—Flexural Strength of Prestressed Member Based on Strain Compatibility

The rectangular beam section shown below is reinforced with a combination of prestressed and nonprestressed strands. Calculate the nominal moment strength using the strain compatibility (moment-curvature) method.

f'_c = 5000 psi

f_{pu} = 270,000 psi (low-relaxation strand; $f_{py} = 0.9f_{pu}$)

E_{ps} = 28,500 ksi

jacking stress = $0.74f_{pu}$

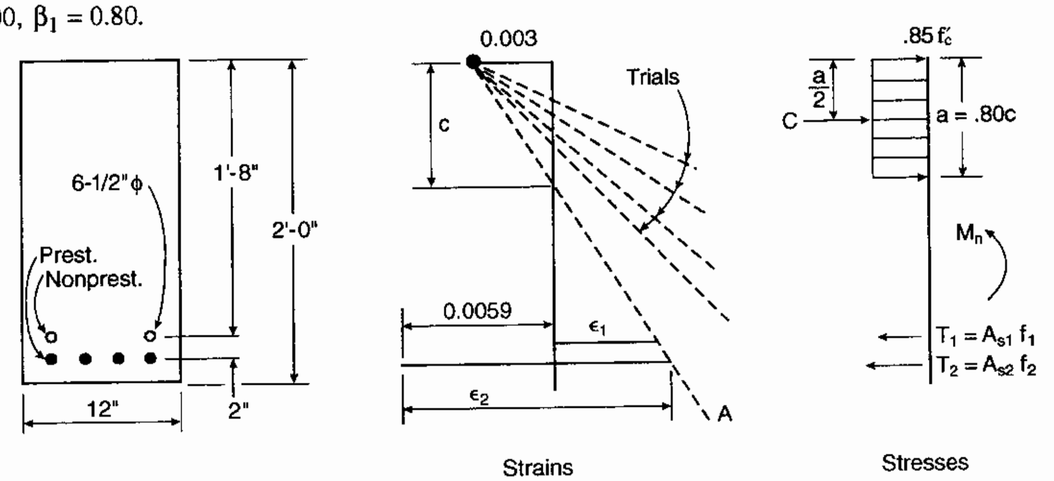
losses = 31.7 ksi (calculated by method of Ref. 24.4. See 18.6 — Loss of Prestress for procedure.)

Calculations and Discussion	Code Reference

1. Calculate effective strain in prestressing steel.

$$\varepsilon = (0.74f_{pu} - \text{losses})/E_{ps} = (0.74 \times 270 - 31.7)/28{,}500 = 0.0059$$

2. Draw strain diagram at nominal moment strength, defined by the maximum concrete compressive strain of 0.003 and an assumed distance to the neutral axis, c. For f'_c = 5000, $\beta_1 = 0.80$. *18.3.1*



Strains

Stresses

3. Obtain equilibrium of horizontal forces.

The "strain line" drawn above from point 0 must be located to obtain equilibrium of horizontal forces:

$$C = T_1 + T_2$$

To compute T_1 and T_2, strains ε_1 and ε_2 are used with the stress-strain relation for the strand to determine the corresponding stresses f_1 and f_2. Equilibrium is obtained using the following iterative procedure:

a. assume c (location of neutral axis)

b. compute ε_1 and ε_2

c. obtain f_1 and f_2 from the equations at the bottom of Fig. 24-1.

d. compute $a = \beta_1 c$

e. compute $C = 0.85 f_c' ab$

f. compute T_1 and T_2

g. check equilibrium using $C = T_1 + T_2$

h. if $C < T_1 + T_2$, increase c, or vice versa and return to step b of this procedure. Repeat until satisfactory convergence is achieved.

Estimate a neutral axis location for first trial. Estimate stressed strand at 260 ksi, unstressed strand at 200 ksi.

$T = \Sigma A_{ps} f_s = 0.306\ (200) + 0.612\ (260) = 220 \text{ kips} = C$

$a = C/(0.85\ f_c' b) = 220/(0.85 \times 5 \times 12) = 4.32$ in.

$c = a/\beta_1 = 4.32/0.80 = 5.4$ in. Use c = 5.4 in. for first try

The following table summarizes the iterations required to solve this problem:

Trial No.	c in.	ε_1	ε_2	f_1 ksi	f_2 ksi	a in.	C kips	T_1 kips	T_2 kips	$T_1 + T_2$ kips
1	5.4	0.0081	0.0151	231	265	4.32	220	71	162	233
2 O.K.	5.6	0.0077	0.0147	220	265	4.48	228.5	67	162	229

4. Calculate nominal moment strength.

Using C = 228.5 kips, $T_1 = 67$ kips and $T_2 = 162$ kips, the nominal moment strength can be calculated as follows by taking moments about T_2:

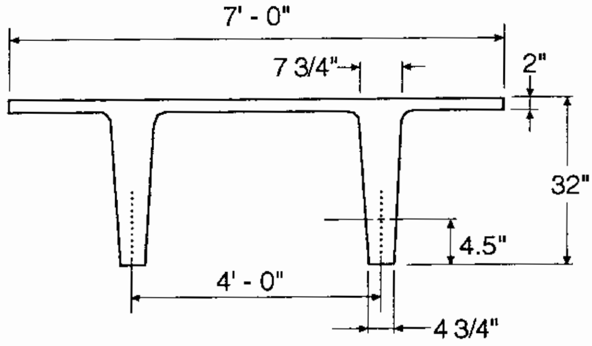
$$M_n = \{[(d_2 - a/2) \times C] - [(d_2 - d_1) \times T_1]\}/12$$

$$= \{[(22 - (4.48/2) \times 228.5] - [(22 - 20) \times 67]\}/12 = 365 \text{ ft-kips}$$

Example 24.5—Tension-Controlled Limit for Prestressed Flexural Member

For the double tee section shown below, check limits for the prestressed reinforcement provided.

f'_c = 5000 psi
22 - 0.5 in. diameter low-relaxation strands
A_{ps} = 22 (0.153 in.^{2}) = 3.366 in.^{2}
f_{pu} = 270,000 psi
f_{py} = 0.90f_{pu}



Calculations and Discussion	Code Reference

Example No. 24.5.1

1. Calculate stress in prestressed reinforcement at nominal strength using Eqs. (6) and (8).

$$\omega_{pu} = \frac{A_{ps}f_{pu}}{bd_pf'_c} = \frac{3.366 \times 270}{84 \times 27.5 \times 5} = 0.079$$

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\omega_{pu}\right) = 270\left(1 - \frac{0.28}{0.8} \times 0.079\right) = 263 \text{ ksi}$$ *Eq. (18-3)*

where

γ_p = 0.28 for low-relaxation strands

β_1 = 0.80 for f'_c = 5000 psi *10.2.7.3*

2. Calculate required depth of concrete stress block.

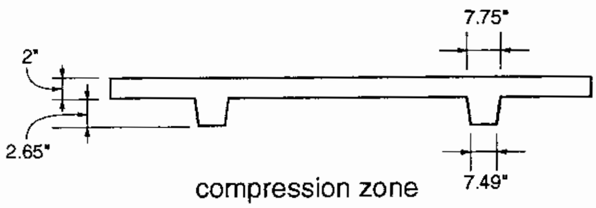
$$a = \frac{A_{ps}f_{ps}}{0.85bf'_c} = \frac{3.366 \times 263}{0.85 \times 84 \times 5} = 2.48 \text{ in.} > h_f = 2 \text{ in.}$$

3. Calculate area of compression zone.

$$A_c = \frac{A_{ps}f_{ps}}{0.85f'_c} = \frac{3.366 \times 263}{0.85 \times 5} = 208.3 \text{ in.}^2$$

4. Find depth a of stress block, and c.

$$A = 2 \times 84 = 168 \text{ in.}^2$$
$$+ 2 \times 2.65 \times \frac{7.75 + 7.49}{2} = 40.4$$
$$208.4 \text{ in.}^2$$



compression zone

a = 4.65 in.
c = a/β_1 = 4.65/0.8 = 5.81 in.

5. Check to see if tension-controlled — *10.3.4*

   $c/d_t = 5.81/30.0 = 0.19 < 0.375$ — *R9.3.2.2*

   (By definition,dimension "d_t" should be measured to the bottom strand)

   Section is tension-controlled

Note: In Step 1, Eq. 18-3 was used to find f_{ps}. But, with the stress block in the web, the value of ω_{pu} used in Eq. (18-5) was not correct, although the error is small in this case. A strain compatibility analysis gives c = 6.01 in. and f_{ps} = 266 ksi.

Example No. 24.5.2

Check the limits of reinforcement using a 3 in. thick flange on the member in Example 24.5.1. The overall depth remains 32 in.

1. f_{ps} = 263 ksi *No change from Example 24.5.1*

2. a = 2.48 *No change from Example 24.5.1, Step 2*

   $< h_f = 3$ in.

   Since the stress block is entirely within the flange, the section acts effectively as a rectangular section.

3. Check c/d_p ratio

   $c = a/\beta_1 = 2.48/0.8 = 3.10$ in.

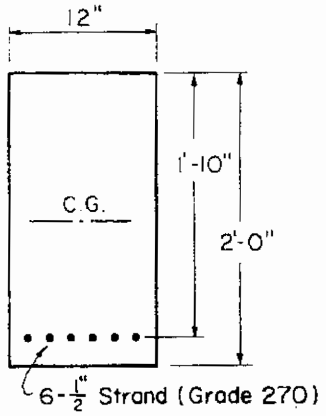
   $c/d_t = 3.10/30.0 = 0.10 < 0.375$ — *R9.3.2.2*

   Section is tension controlled.

Example 24.6—Cracking Moment Strength and Minimum Reinforcement Limit for Non-composite Prestressed Member

For the non-composite prestressed member of Example 24.3, calculate the cracking moment strength and compare it with the design moment strength to check the minimum reinforcement limit.

f'_c = 5000 psi
f_{pu} = 270,000 psi
jacking stress = $0.70f_{pu}$
Assume 20% losses



Calculations and Discussion	Code Reference

1. Calculate cracking moment strength using Eq. (14) developed in Part 24.

$$M_{cr} = \left(f_r + \frac{P_{se}}{A_c}\right)S_b + (P_{se} \times e) \qquad \text{Eq. (14)}$$

$f_r = 7.5\sqrt{f'_c} = 530$ psi *Eq. (9-10)*

Assuming 20% losses:

$$P_{se} = 0.8 \times [6 \times 0.153 \times (0.7 \times 270)] = 139 \text{ kips}$$

$$S_b = \frac{bh^2}{6} = \frac{12 \times 24^2}{6} = 1152 \text{ in.}^3$$

$$A_c = bh = 12 \times 24 = 288 \text{ in.}^2$$

$$e = 12 - 2 = 10 \text{ in.}$$

$$M_{cr} = \left(0.530 + \frac{139}{288}\right)1152 + (139 \times 10) = 2557 \text{ in.-kips} = 213 \text{ ft-kips}$$

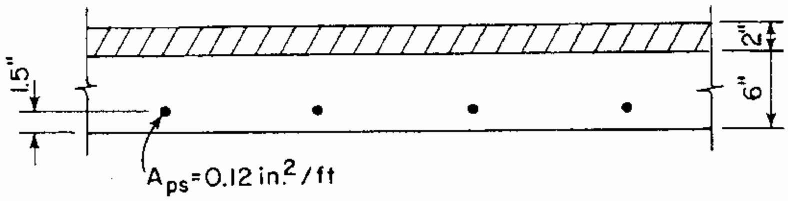
Note that cracking moment strength needs to be determined for checking minimum reinforcement per 18.8.3.

2. Section 18.8.3 requires that the total reinforcement (prestressed and nonprestressed) must be adequate to develop a design moment strength at least equal to 1.2 times the cracking moment strength. From Example 24.3, M_n = 380 ft-kips.

Example 24.6 (cont'd)	Calculations and Discussion	Code Reference
$\phi M_n \geq 1.2M_{cr}$		*18.8.3*
$0.9\ (380) > 1.2\ (213)$		
$342 > 256$ O.K.		

Example 24.7—Cracking Moment Strength and Minimum Reinforcement Limit for Composite Prestressed Member

For the 6 in. precast solid flat slab with 2 in. composite topping, calculate the cracking moment strength. The slab is supported on bearing walls with 15 ft span.



Section properties per foot of width:

A_c = 72 in.^{2} (precast slab)
S_b = 72 in.^{3} (precast slab)
S_c = 132.7 in.^{3} (composite section)

f'_c = 5000 psi (all-lightweight concrete, w_c = 125 pcf)
f_{pu} = 250,000 psi (stress-relieved strand)
jacking stress = $0.70f_{pu}$
Assume 25% losses

Calculations and Discussion	Code Reference

1. Calculate cracking moment strength using Eq. (13) developed for unshored composite members. All calculations are based on one foot width of slab. *18.8.3*

$$M_{cr} = \left(f_r + \frac{P_{se}}{A_c} + \frac{P_{se}e}{S_b}\right)S_c - M_d\left(\frac{S_c}{S_b} - 1\right) \quad (13)$$

$f_r = 0.75\left(7.5\sqrt{5000}\right) = 398$ psi reduced for all-lightweight concrete *9.5.2.3*

Assuming 25% losses:

$P_{se} = 0.75\,(0.12 \times 0.7 \times 250) = 15.75$ kips

$e = 3 - 1.5 = 1.5$ in.

$w_d = (6 + 2)/12 \times 125 = 83$ psf $= 0.083$ ksf (weight of precast slab + composite topping)

$$M_d = \frac{w_d\ell^2}{8} = \frac{0.083 \times 15^2}{8} = 2.33 \text{ ft-kips} = 28.0 \text{ in.-kips}$$

$$M_{cr} = \left[\left(0.398 + \frac{15.75}{72} + \frac{15.75 \times 1.5}{72}\right)132.7\right] - \left[28.0\left(\frac{132.7}{72} - 1\right)\right]$$

$= 125.4 - 23.6 = 101.8$ in.-kips

2. Calculate design moment strength and compare with cracking moment strength. All calculations based on one foot width of slab.

$A_{ps} = 0.12 \text{ in.}^2, \quad d_p = 8.0 - 1.5 = 6.5 \text{ in.}$

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{0.12}{12 \times 6.5} = 0.00154$$

With no additional tension or compression reinforcement, Eq. (18-3) reduces to:

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f_c'}\right) = 250\left(1 - \frac{0.4}{0.8} \times 0.00154 \times \frac{250}{5}\right) = 240.4 \text{ ksi}$$

$$a = \frac{A_{ps}f_{ps}}{0.85f_c'b} = \frac{0.12 \times 240.4}{0.85 \times 5 \times 12} = 0.57 \text{ in.}$$

$M_n = A_{ps}f_{ps}(d_p - a/2) = 0.12 \times 240.4\,(6.5 - 0.57/2) = 179.3$ in.-kips

$\phi M_n = 0.9\,(179.3) = 161.4$ in.-kips

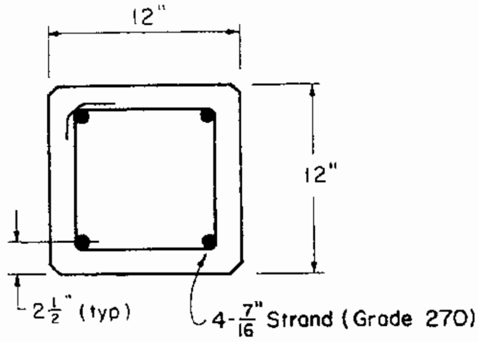
$\phi M_n \geq 1.2\,(M_{cr})$ *18.8.3*

$161.4 > 1.2\,(101.8) = 122.2$ O.K.

Example 24.8—Prestressed Compression Member

For the short column shown, calculate the nominal strength M_n for a nominal axial load P_n = 30 kips.

Calculate design strength.



f_c' = 5000 psi
f_{pu} = 270,000 psi (low-relaxation strand)
jacking stress = $0.70f_{pu}$
Assume 10% losses

Calculations and Discussion	Code Reference

Eq. 18-3 should not be used when prestressing steel is in the compression zone. The same "strain compatibility" procedure used for flexure must be used here. The only difference is that for columns the load P_n must be included in the equilibrium of axial forces.

1. Calculate effective prestress.

$$f_{se} = 0.9 \times 0.7f_{pu} = 0.9 \times 0.7 \times 270 = 170 \text{ ksi}$$

$$P_e = A_{ps}f_{se} = 4 \times 0.115 \times 170 = 78.2 \text{ kips}$$

2. Calculate average prestress on column section.

$$f_{pc} = \frac{P_e}{A_g} = \frac{78.2}{12^2} = 0.54 \text{ ksi}$$

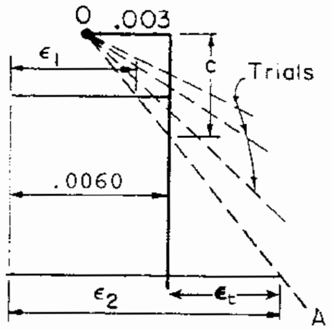
Minimum reinforcement as per 10.9.1 not required because f_{pc} = 0.54 ksi > 0.225 ksi. *18.11.2.1*

Since f_{pc} = 0.54 ksi > 0.225 ksi, lateral ties satisfying the requirements of 18.11.2.2 must enclose all prestressing tendons.

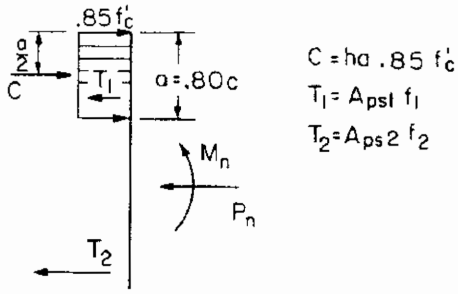
3. Calculate effective strain in prestressing steel.

$$\varepsilon = \frac{f_{se}}{E_p} = \frac{170}{28{,}500} = 0.0060$$

4. Draw strain diagram at nominal moment strength, defined by the maximum concrete compressive strain of 0.003 and an assumed distance to the neutral axis, c. For f_c' = 5000 psi, β_1 = 0.80.



Strains



Stresses

5. Obtain equilibrium of axial forces. The strain line OA drawn above, must be such that equilibrium of axial forces exists.

   $C = T_1 + T_2 + P_n$

   This can be done by trial-and-error as outlined in Example 24.4. Assuming different values of c, the following trial table is obtained:

Trial No.	c in.	ε_1	ε_2	f_1* ksi	f_2* ksi	a in.	C kips	T_1 kips	T_2 kips	$T_1 + T_2 + P_n$ kips
1	3.0	0.0055	0.0125	157	263	2.40	122.4	36.1	60.4	126.5
2	3.2	0.0053	0.0119	152	261	2.56	130.6	35.0	60.2	135.2
3 O.K.	3.1	0.0054	0.0122	154	262	2.48	126.5	35.5	60.3	125.8

**From equation in Fig. 24-1.*

6. Calculate nominal moment strength.

   Using C = 126.5 kips (from the sum of the other forces), P_n = 30 kips, T_1 = 35.5 kips, and T_2 = 60.3 kips, the moment strength can be calculated as follows by taking moments about P_n, located at the centroid of the section:

   $M_n = \{[(h/2 - a/2) \times C] - [(h/2 - 2.5) \times T_1] + [(h/2 - 2.5) \times T_2]\}/12$

   $= [(4.76 \times 126.5) - (3.5 \times 35.5) + (3.5 \times 60.3)]/12 = 57.4$ ft-kips

7. Calculate design strength

   $\varepsilon_t = \varepsilon_2 - 0.0060 = 0.0122 - 0.0060 = 0.0062 > 0.005$

   Section is tension-controlled $\phi = 0.9$ *10.3.4*

   $\phi P_n = 0.9 \times 30 = 27$ kips *9.3.2.1*

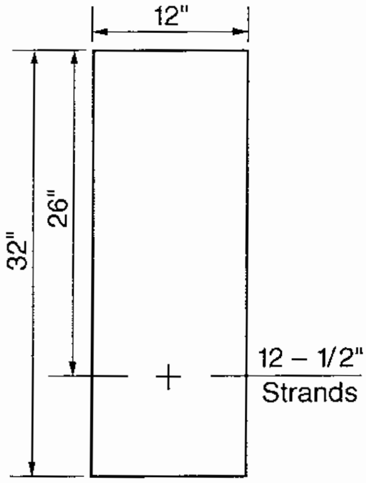
   $\phi M_n = 0.9 \times 57.4 = 51.7$ ft-kips

Example 24.9—Cracked Section Design When Tension Exceeds $12\sqrt{f'_c}$

Do the serviceability analysis for the beam shown.

$f'_c = 6000$ psi
depth $d_p = 26$ in.
effective prestress $f_{se} = 150$ ksi
decompression stress $f_{dc} = 162$ ksi
span = 40 ft

	w k/ft	Midspan moments in.-k
Self-weight	0.413	992
Additional dead load	1.000	2400
Live load	1.250	3000
Sum	2.663	6392



Calculations and Discussion	Code Reference

1. Check tension at service loads, based on gross section.

$P = A_{ps}f_{se} = 1.836 \times 150 = \quad 275.4$ kip

$P/A = 275.4/384 \quad = \quad 0.717$ ksi

$Pe/S = 275.4 \times 10/2048 \quad 1.345$

$S = bh^2/6 = 12(32)^2/6 = 2.048$ in.^{3}

$\Sigma M/S = 6392/2048 \quad -3.121$

-1.059 ksi tension

$12\sqrt{f'_c} = 12\sqrt{f'_c} = 930$ psi $= 0.930$ ksi

Tension exceeds $12\sqrt{f'_c}$. Design as a Class C member — *18.3.3(c)*

2. A cracked section stress analysis is required — *18.3.4*

Cracked transformed section properties, similar to those used for working stress analysis of ordinary (nonprestressed) reinforced concrete will be used. The area of steel elements is replaced by a "transformed" area of concrete equal to n times the actual steel area, where n is the ratio of the modulus of elasticity of steel to that of concrete.

The modular ratio $n = E_{ps}/E_c = 28{,}500/4415 = 6.455$

where $E_c = 57{,}000\sqrt{f'_c} = 57{,}000\sqrt{6000} = 4415$ ksi — *8.5.1*

The transformed steel area A_t is:

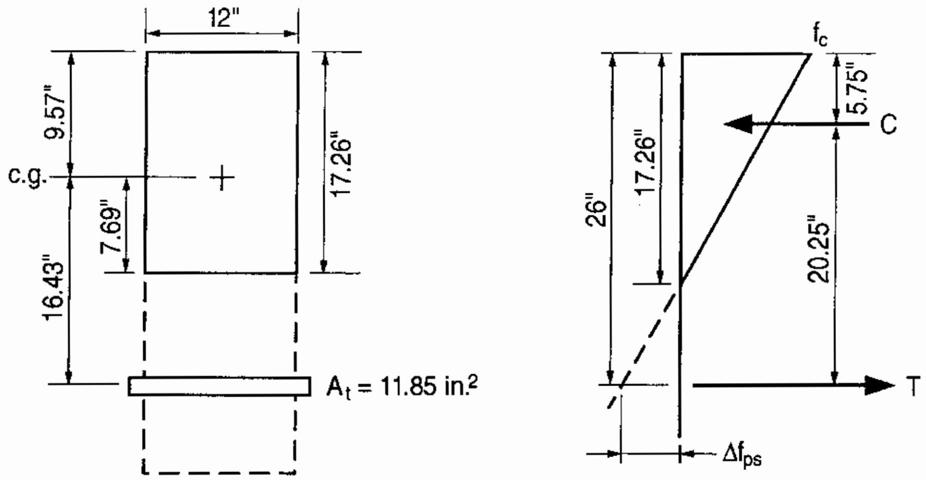
$A_t = nA_{ps} = 6.455 \times 1.836 = 11.85$ in.^{2}

The force P_{dc} at decompression (when the stress in the concrete at the same level as the prestressing steel is zero) is:

$$P_{dc} = A_{ps}f_{dc} = 1.836 \times 162 = 297.4 \text{ kips}$$

3. The stress analysis of a cracked section with axial load (from the prestress) requires, at best, the solution of a cubic equation. A more general approach is to find a neutral axis location that satisfies horizontal force equilibrium and produces the given bending moment. Reference 24.2 gives one way to accomplish this. It is too lengthy to be presented in detail here.

   The results give a neutral axis depth c of 17.26 in., with a concrete stress f_c of 3.048 ksi and a transformed steel stress $\Delta f_{ps}/n$ of 1.545 ksi. The actual Δf_{ps} is $1.545 \times 6.455 = 9.97$ ksi.



4. The transformed section properties are

   $A = 219$ in.^{2}

   $I = 8524$ in.^{4}

   $y_t = 9.57$ in.

5. Equilibrium may be checked manually,

   $C = f_c bc/2 = 3.048\,(12)(17.26)/2 \qquad = 315.7\text{k}$

   C acts at top kern of compression zone

   $= d_c/3$ for rectangular area

   $17.26/3 = 5.75$ in.

   $T = P_{dc} + \Delta f_{ps}\,(A_{ps}) = 297.4 + 9.97\,(1.836) \qquad = 315.7\text{k} = C$ Check

   $M = C$ or $T \times$ lever arm

   $= 315.7 \times 20.25 \qquad = 6392$ in.-kips Check

Example 24.9 (cont'd)	Calculations and Discussion	Code Reference

6. Check limits on Δf_{ps}

Δf_{ps} is less than code limit of 36 ksi O.K. — *18.4.3.3*

Δf_{ps} is less than 20 ksi, so the spacing requirements of 18.4.4.1 and 18.4.4.2 need not be applied. — *18.4.3.3*

7. Check deflection

Live load deflection calculations based on a cracked section analysis are required for Class C members. — *9.5.4.2*

Use the "bilinear moment-deflection relationship," as described in Ref. 24.1 — *9.5.4.2*

8. Find cracking moment M_{cr}, using P_{dc}

$P/A + Pe/S + M_{cr}/S = f_r$

modulus of rupture $f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{6000} = 581$ psi — *9.5.2.3*

$297/384 + 297 \times 10/2048 + 0.581 = M_{cr}/2048$

$M_{cr} = 5750$ in.-kips

$M_d = \underline{3392}$

2358 = live load moment applied to gross section

balance of M_ℓ 642 = live load moment applied to cracked section

9. Compute deflections before and after cracking

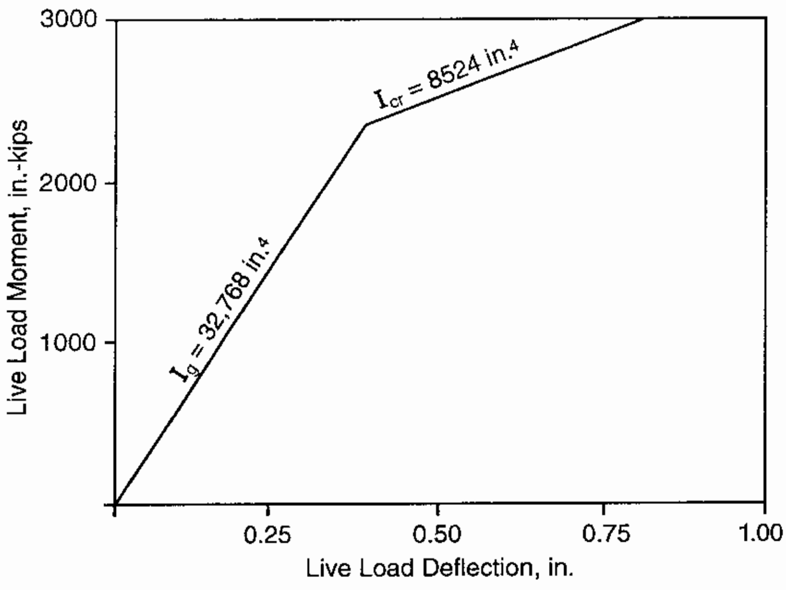
$$\Delta_L = \frac{5}{48}\,\frac{2358\,L^2}{EI_g} + \frac{5}{48}\,\frac{642\,L^2}{EI_{cr}}$$

$$= \frac{5}{48}\,\frac{2358 \times 480^2}{4415 \times 32768} + \frac{5}{48}\,\frac{642 \times 480^2}{4415 \times 8524}$$

$\Delta_L = 0.39 + 0.41 = 0.80$ in.

Δ_L is < (L/360 = 480/365 = 1.33 in.) O.K. — *9.5.2.6*, *Table 9.5(b)*

The live load deflection is shown graphically below.


3000
2000
1000
Live Load Moment, in.-kips
0.25
0.50
0.75
1.00
Live Load Deflection, in.
I_{cr} = 8524 in.^{4}
I_g = 32,768 in.^{4}

Blank

25

Prestressed Concrete Shear

UPDATE FOR THE '05 CODE

New Section 11.4.1 was added to clarify that the distance from extreme compression fiber to centroid of prestressed and nonprestressed longitudinal tension reinforcement need not be taken less than 80 percent of the overall height of member. Commentary was added in R11.5.6.1 to flag that results of tests on pretensioned concrete hollow core units with overall height greater than 12.5 in. have shown that web shear cracking strengths in end regions can be less than strength V_{cw} computed by Eq. (11-12).

GENERAL CONSIDERATIONS

The basic equations for shear design of prestressed concrete, Eqs. (11-10), (11-11), and (11-12), were introduced in the 1963 code. Although well founded on test results, they have been found difficult to apply in practice. A simplified Eq. (11-9) was introduced in the 1971 code.

In order to understand Eqs. (11-10) and (11-12), it is best to review the principles on which ACI shear design is based. These principles are <u>empirical</u>, based on a large number of tests.

- The shear resisted by concrete and the shear resisted by stirrups are additive.
- The shear resisted by the concrete after shear cracks form is at least equal to the shear existing in the concrete at the location of the shear crack at the time the shear crack forms.

How does one compute the shear resisted by the concrete at the time a shear crack forms? There are two possibilities.

1. Web shear. A diagonal shear crack originates in the web, near the neutral axis, caused by principal tension in the web.
2. Flexure-shear. A crack starts as a flexural crack on the tension face of a flexural member. It then extends up into the web, and develops into a diagonal shear crack. This can happen at a much lower principal tensile stress than that causing a web shear crack, because of the tensile stress concentration at the tip of the crack.

Web Shear

The apparent tensile strength of concrete in direct tension is about $4\sqrt{f'_c}$. When the principal tension at the center of gravity of the cross section reaches $4\sqrt{f'_c}$, a web shear crack will occur. Section 11.4.3.2 states "...V_{cw} shall be computed as the shear...that results in a principal tensile stress of $4\sqrt{f'_c}$ ..."

The compression from the prestress helps to reduce the principal tension. The computation of principal tension due to combined shear and compression can be somewhat tedious. The code gives a simplified procedure.

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p \qquad \textit{Eq. (11-12)}$$

The term V_p in Eq. (11-12) is the vertical component of the tension in the prestressing tendons. This is additive for web shear strength (but not for flexure-shear strength).

A comparison to test results is shown below.

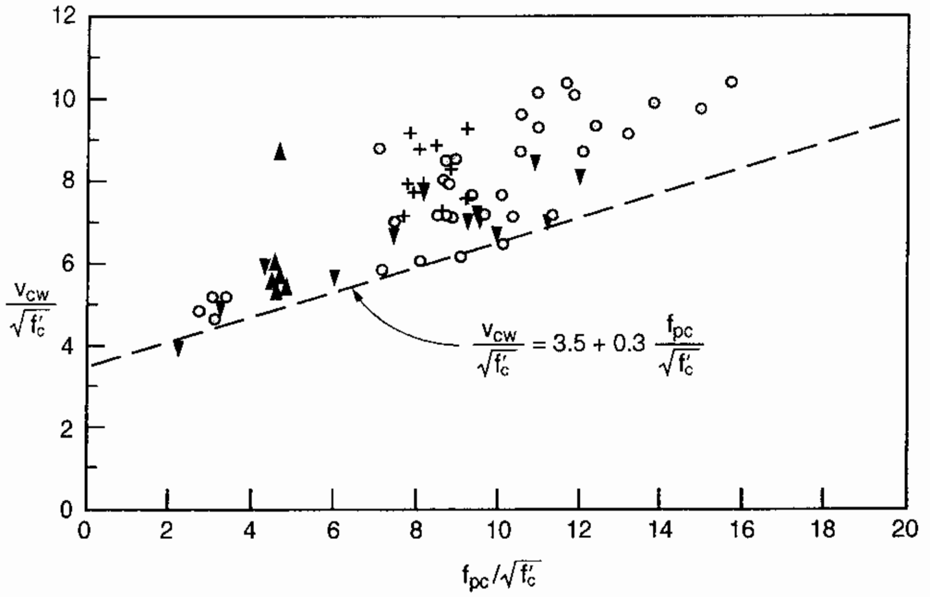


Figure 25-1 Diagonal Cracking in Regions not Previously Cracked

The compression from prestressing increases the shear strength by 30 percent of the P/A level, f_{pc}, of compression.

For nonprestressed beams, the principal tension at the center of gravity of the section is equal to the shear. Why does Eq. (11-3) for shear in nonprestressed members permit only $2\sqrt{f'_c}$ shear resisted by the concrete? Because shear strength is reduced by flexural cracking. In nonprestressed beams, shear is almost always influenced by flexural tension. But, prestressing reduces the flexural cracking.

Flexure-Shear in Prestressed Concrete

In prestressed beams, flexural cracking is delayed by the prestress – usually until loaded beyond service load. It is worthwhile to account for the beneficial effects of prestressing.

In the 1950s, it was thought that draping strands would increase shear strength, by the vertical component V_p of the prestressing force. Tests showed just the opposite. Why? Because draping the strands reduces the flexural cracking strength in the shear span.

The tests were done with concentrated loads; whereas, the dead load of the beam was a uniform load. For this reason, when the shear design method was developed from the test results, the dead load and test load shears were treated separately.

Flexure-Shear

Equation (11-10) is the equation for shear resistance provided by the concrete, as governed by flexural cracks that develop into shear cracks. The shear strength of the concrete at a given cross section is taken equal to the shear at the section at the time a flexural crack occurs, plus a small increment of shear which transforms the flexural crack into an inclined crack. Equation (10-10) may be expressed in words as follows.

V_{ci} = shear existing at the time of flexural cracking plus an added increment to convert it into a shear crack. The added increment is $0.6b_w d_p \sqrt{f'_c}$.

The shear existing at the time of flexural cracking is the dead load shear V_d plus the added shear $V_i M_{cre}/M_{max}$.

What is the origin of the term $V_i M_{cre}/M_{max}$?

The term V_i is the factored ultimate shear at the section, less the dead load shear.

The term M_{cre} is the <u>added</u> moment (over and above stresses due to prestress and dead load) causing $6\sqrt{f'_c}$ tension in the extreme fiber.

The added moment M_{cre} is calculated by finding the bottom fiber stress f_{pe} due to prestress, subtracting the bottom fiber stress f_d due to dead loads, adding $6\sqrt{f'_c}$ tension, and multiplying the result by the section modulus for the section resisting live loads. This is Eq. (11-11) of the code.

$$M_{cr} = (I/y_t)(6\sqrt{f'_c} + f_{pe} - f_d) \qquad \textit{Eq. (11-11)}$$

Note: In the above discussion, "bottom" means "tension side" for continuous members.

The term M_{max} is the factored ultimate moment of the section, less the dead load moment.

To better understand the meaning of these terms and their use in Eq. (11-10), refer to Fig. 25-2.

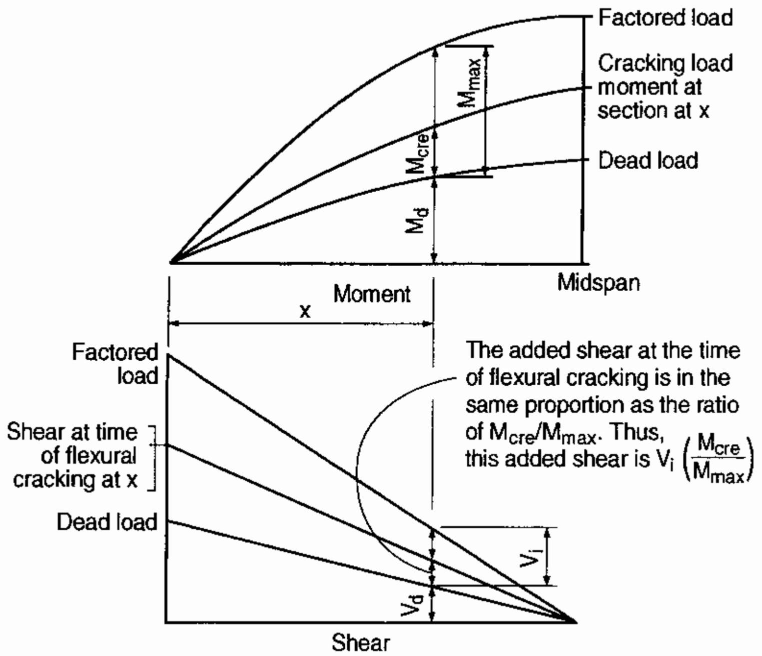


Figure 25-2 Origin of ($V_i M_{cre}/M_{max}$) Term in Eq. (11-10)

The quantity $V_i M_{cre}/M_{max}$ is the shear due to an <u>added</u> load (over and above the dead load) which causes the tensile stress in the extreme fiber to reach $6\sqrt{f'_c}$. The added load is applied to the composite section (if composite).

After a flexural crack forms, a small amount of additional shear is needed to transform the crack into a shear crack. This is determined empirically, as shown in Fig. 25-3.

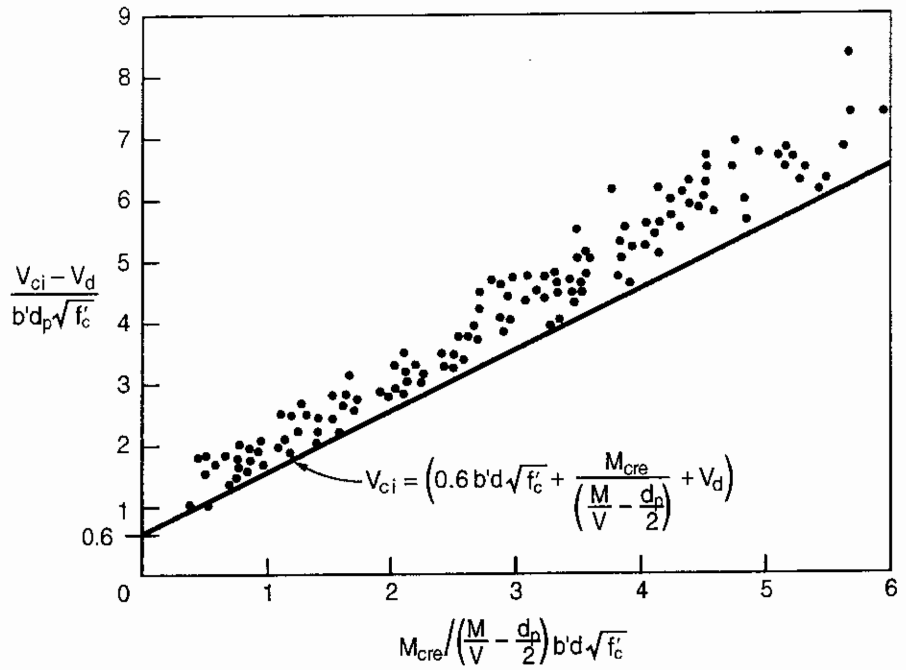


Figure 25-3 Diagonal Cracking in those Regions of Beams Previously Cracked in Flexure

The intercept at 0.6 produces the first term in Eq. (11-10), $0.6b_w d_p\sqrt{f'_c}$.

Note: The quantity "$-d_p/2$" shown in the expressions of Fig. 25-3 was later dropped, as a conservative simplication.

The notation used in Eqs. (11-10) and (11-11) is follows.

M_{cre} = moment causing flexural cracking at section due to externally applied loads

M_{max} = maximum factored moment at section due to externally applied loads

V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}.

Unfortunately, the subscripts are confusing.

M_{cre} is <u>not</u> the total cracking moment. It is <u>not</u> the same as M_{cr} that is used to check for minimum reinforcement in Example 24.6.

M_{max} is not the total factored moment. It is the total factored moment less the dead load moment.

It would seem that V_i and M_{max} should have the same subscript, because they both relate to the differences between the same two loadings.

To make matters worse, the term "externally applied loads" is ambiguous. Apparently, dead load is not regarded as "externally applied," perhaps because the weight comes from the "internal" mass of the member. But, R11.4.3 says that superimposed dead load on a composite section should be considered an externally applied load. The commentary explains a good reason for this, but the confusion still exists.

The shear strength must be checked at various locations along the shear span, a process that is tedious. For manual shear calculations, the simplified process described in 11.4.2 is adequate for most cases.

11.1 SHEAR STRENGTH FOR PRESTRESSED MEMBERS

The basic requirement for shear design of prestressed concrete members is the same as for reinforced concrete members: the design shear strength ϕV_n must be greater than the factored shear force V_u at all sections (11.1).

$$\phi V_n \geq V_u \qquad \textit{Eq. (11-1)}$$

For both reinforced and prestressed concrete members, the nominal shear strength V_n is the sum of two components: the nominal shear strength provided by concrete V_c and the nominal shear strength provided by shear reinforcement V_s.

$$V_n = V_c + V_s \qquad \textit{Eq. (11-2)}$$

Therefore,

$$\phi V_c + \phi V_s \geq V_u$$

The nominal shear strength provided by concrete V_c is assumed to be equal to the shear existing at the time an inclined crack forms in the concrete.

Beginning with the 1977 code, shear design provisions have been presented in terms of shear forces V_n, V_c, and V_s, to better clarify application of the material strength reduction factor ϕ for shear design. In force format, the ϕ factor is directly applied to the material strengths, i.e., ϕV_c and ϕV_s.

11.1.2 Concrete Strength

Section 11.1.2 restricts the concrete strength that can be used in computing the concrete contribution because of a lack of shear test data for high strength concrete. The limit does not allow $\sqrt{f'_c}$ to be greater than 100 psi, which corresponds to f'_c = 10,000 psi. Note, the limit is expressed in terms of $\sqrt{f'_c}$, as it denotes diagonal tension. The limit can be exceeded if minimum shear reinforcement is provided as specified in 11.1.2.1.

11.1.3 Location for Computing Maximum Factored Shear

Section 11.1.3 allows the maximum factored shear V_u to be computed at a distance from the face of the support when all of the following conditions are satisfied:

a. the support reaction, in the direction of the applied shear, introduces compression into the end regions of the member,

b. loads are applied at or near the top of the member, and

c. no concentrated load occurs between the face of the support and the critical section.

For prestressed concrete sections, 11.1.3.2 states that the critical section for computing the maximum factored shear V_u is located at a distance of h/2 from the face of the support. This differs from the provisions for reinforced (nonprestressed) concrete members, in which the critical section is located at d from the face of the support. For more details concerning maximum factored shear force at supports, see Part 12.

11.2 LIGHTWEIGHT CONCRETE

The adjustments to shear strength for lightweight concrete given in 11.2 apply equally to prestressed and nonprestressed concrete members.

11.4 SHEAR STRENGTH PROVIDED BY CONCRETE FOR PRESTRESSED MEMBERS

Section 11.4 provides two approaches to determining the nominal shear strength provided by concrete V_c. A simplified approach is presented in 11.4.2 with a more detailed approach presented in 11.4.3. In both cases, the shear strength provided by concrete is assumed to be equal to the shear existing at the time an inclined crack forms in the concrete.

11.4.1 NOTATION

For prestressed members, the depth d used in shear calculations is defined as follows.

d = distance from extreme compression fiber to centroid of prestressed and nonprestressed longitudinal tension reinforcement, if any, but need not be less than **0.80*h***.

11.4.2 Simplified Method

The use of this simplified method is limited to prestressed members with an effective prestress force not less than 40 percent of the tensile strength of the flexural reinforcement, which may consist of only prestressed reinforcement or a combination of prestressed and conventional reinforcement.

$$V_c = \left(0.6\sqrt{f'_c} + 700\,\frac{V_u d_p}{M_u}\right) b_w d \qquad \textit{Eq. (11-9)}$$

but need not be less than $2\sqrt{f'_c}\,b_w d$.

V_c must not exceed $5\sqrt{f'_c}\,b_w d$ or V_{cw} (11.4.3.2) computed considering the effects of transfer length (11.4.4) and debonding (11.4.5) which apply in regions near the ends of pretensioned members.

It should be noted that for the term $V_u d_p/M_u$ in Eq. (11-9), d_p must be taken as the actual distance from the extreme compression fiber to the centroid of the prestressed reinforcement rather than the 0.8h allowed elsewhere in the code.

The shear strength must be checked at various locations along the shear span. The commentary notes that for simply supported members subjected to uniform loads, the quantity of $V_u d_p/M_u$ may be expressed as:

$$\frac{V_u d_p}{M_u} = \frac{d_p(\ell - 2x)}{x(\ell - x)}$$

Figure 25-4, useful for a graphical solution, is also given in the commentary.

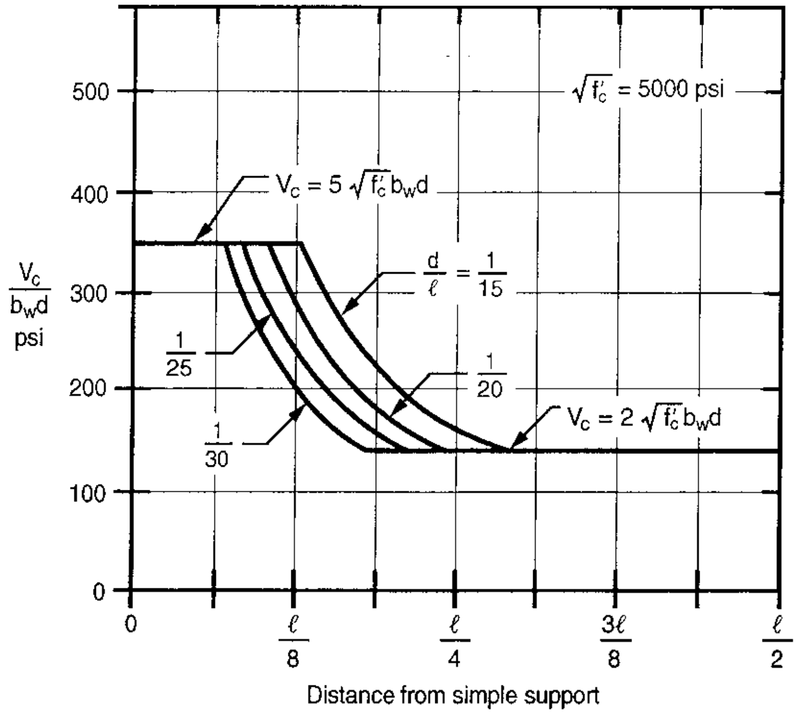


Figure 25-4 Application of Eq. (11-9) to Uniformly Loaded Prestressed Members (Fig. R11.4.2

The use of this figure is illustrated in Example 25-2. Additional figures for graphical solutions of shear strength are given in Ref. 25.1.

11.4.3 Detailed Method

The origin of this method is discussed under General Considerations, at the beginning of Part 25.

Two types of inclined cracking have been observed in prestressed concrete members: flexure-shear cracking and web-shear cracking. Since the nominal shear strength from concrete is assumed to be equal to the shear causing inclined cracking of the concrete, the detailed method provides equations to determine the nominal shear strength for both types of cracking.

The two types of inclined cracking are illustrated in Fig. 25-5 which is found in R11.4.3. The nominal shear strength provided by concrete V_c is taken as the lesser shear causing the two types of cracking, which are discussed below. The detailed expressions for V_c in 11.4.3 may be difficult to apply without design aids, and should be used only when the simplified expression for V_c in 11.4.2 is not adequate.

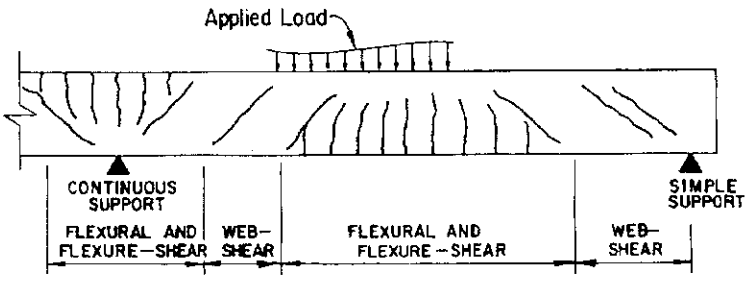


Figure 25-5 Types of Cracking in Concrete Beams (Fig. R11.4.3)

11.4.3.1 Flexure-Shear Cracking, V_{ci}—Flexure-shear cracking occurs when flexural cracks, which are initially vertical, become inclined under the influence of shear. The shear at which this occurs can be taken as

$$V_{ci} = 0.6\sqrt{f'_c}\, b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}} \qquad \textit{Eq. (11-10)}$$

Note that V_{ci} need not be taken less than $1.7\sqrt{f'_c}\, b_w d$.

The added moment M_{cre} to cause flexural cracking is computed using the equation

$$M_{cre} = \left(\frac{I}{y_t}\right)\left(6\sqrt{f'_c} + f_{pe} - f_d\right) \qquad \textit{Eq. (11-11)}$$

where f_{pe} is the compressive stress in concrete due to effective prestress force only (after allowance for all prestress losses) at the extreme fiber of the section where tensile stress is caused by externally applied loads.

V_{ci} usually governs for members subject to uniform loading. The total nominal shear strength V_{ci} is assumed to be the sum of three parts:

1. the shear force required to transform a flexural crack into an inclined crack — $0.6\sqrt{f'_c}\, b_w d_p$;
2. the unfactored dead load shear force — V_d; and
3. the portion of the remaining factored shear force that will cause a flexural crack to initially occur — $V_i M_{cre}/M_{max}$.

For non-composite members, V_d is the shear force caused by the unfactored dead load. For composite members, V_d is computed using the unfactored self weight plus unfactored superimposed dead load.

The load combination used to determine V_i and M_{max} is the one that causes maximum moment at the section under consideration. The value V_i is the factored shear force resulting from the externally applied loads occurring simultaneously with M_{max}. For composite members, V_i may be determined by subtracting V_d from the shear force resulting from the total factored loads, V_u. Similarly, $M_{max} = M_n - M_d$. When calculating the cracking moment M_{cre}, the load used to determine f_d is the same unfactored load used to compute V_d.

11.4.3.2 Web-Shear Cracking, V_{cw} — Web-shear cracking occurs when the principal diagonal tension in the web exceeds the tensile strength of the concrete. This shear is approximately equal to

$$V_{cw} = \left(3.5\sqrt{f'_c} + 0.3 f_{pc}\right) b_w d_p + V_p \qquad \textit{Eq. (11-12)}$$

where f_{pc} is the compressive stress in concrete (after allowance for all prestress losses) at the centroid of the cross-section resisting externally applied loads or at the junction of the web and flange when the centroid lies within the flange.

V_p is the vertical component of the effective prestress force, which is present only when strands are draped or deflected.

The expression for web shear strength V_{cw} usually governs for heavily prestressed beams with thin webs, especially when the beam is subject to large concentrated loads near simple supports. Eq. (11-12) predicts the shear strength at first web-shear cracking.

An alternate method for determining the web shear strength V_{cw} is to compute the shear force corresponding to dead load plus live load that results in a principal tensile stress of $4\sqrt{f'_c}$ at the centroidal axis of the member, or at the interface of web and flange when the centroidal axis is located in the flange. This alternate method may be advantageous when designing members where shear is critical. Note the limitation on V_{cw} in the end regions of pretensioned members as provided in 11.4.4 and 11.4.5.

11.4.4, 11.4.5 Special Considerations for Pretensioned Members

Section 11.4.4 applies to situations where the critical section located at h/2 from the face of the support is within the transfer length of the prestressing tendons. This means that the full effective prestress force is not available for contributing to the shear strength. A reduced value of effective prestress force must be used using linear interpolation between no stress in the tendons at the end of the member to full effective prestress at the transfer length from the end of the member, which is taken to be 50 diameters (d_b) for strand and $100d_b$ for a single wire.

Section 11.4.5 is provided to ensure that the effect on shear strength of reduced prestress is properly taken into account when bonding of some of the tendons is intentionally prevented (debonding) near the ends of a pretensioned member, as permitted by 12.9.3.

11.5 SHEAR STRENGTH PROVIDED BY SHEAR REINFORCEMENT FOR PRESTRESSED MEMBERS

The design of shear reinforcement for prestressed members is the same as for reinforced nonprestressed concrete members discussed in Part 12, except that V_c is computed differently (as discussed above) and another minimum shear reinforcement requirement applies (11.5.6.4). Therefore, see Part 12 for a complete discussion of design of shear reinforcement.

11.5.6.1 The code permits a slightly wider spacing of (3/4)h (instead of d/2) for prestressed members, because the shear crack inclination is flatter in prestressed members.

As permitted by 11.5.6.2, shear reinforcement may be omitted in any member if shown by physical tests that the required strength can be developed without shear reinforcement. Section 11.5.6.2 clarifies conditions for appropriate tests. Also, commentary discussion gives further guidance on appropriate tests to meet the intent of 11.5.6.2. The commentary also calls attention to the need for sufficient stirrups in all thin-web, post-tensioned members to support the tendons in the design profile, and to provide reinforcement for tensile stresses in the webs resulting from local deviations of the tendons from the design tendon profile.

11.5.6.4 Minimum Reinforcement for Prestressed Members—For prestressed members, minimum shear reinforcement is computed as the smaller of Eqs. (11-13) and (11-14). However, Eq. (11-13) will generally give a higher minimum than Eq. (11-14). Note that Eq. (11-14) may not be used for members with an effective prestress force less than 40 percent of the tensile strength of the prestressing reinforcement.

REFERENCE

25.1 "PCI Design Handbook – Precast and Prestressed Concrete," MNL 120-04 6th Edition, Precast/Prestressed Concrete Institute, Chicago, 2004, 750 pp.

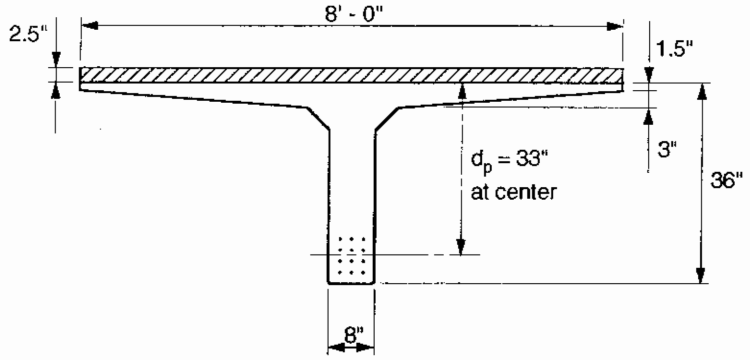
Example 25.1—Design for Shear (11.4.1)

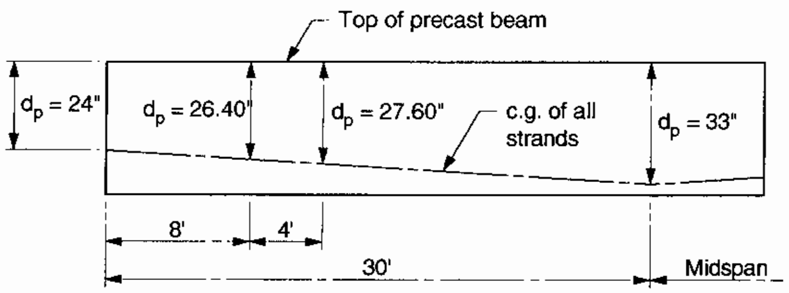
For the prestressed single tee shown, determine shear requirements using V_c by Eq. (11-9).

Precast concrete: f'_c = 5000 psi (sand lightweight, w_c = 120 pcf)
Topping concrete: f'_c = 4000 psi (normal weight, w_c = 150 pcf)
Prestressing steel: Twelve 1/2-in. dia. 270 ksi strands (single depression at midspan)
Span = 60 ft (simple)
Dead load = 725 lb/ft (includes topping)
Live load = 720 lb/ft
f_{se} (after all losses) = 150 ksi

Precast Section:
A = 570 in.^{2}
I = 68,917 in.^{4}
y_b = 26.01 in.
y_t = 9.99 in.

Composite Section:
y_{bc} = 29.27 in.





Strand Profile in Precast Girder

Calculations and Discussion	Code Reference

1. Determine factored shear force V_u at various locations along the span. The results are shown in Fig. 25-6.

2. Determine shear strength provided by concrete V_c using Eq. (11-9). The effective prestress f_{se} is greater than 40 percent of f_{pu} (150 ksi > 0.40 × 270 = 108 ksi). Note that the value of d need not be taken less than 0.8h for shear strength computations. Typical computations using Eq. (11-9) for a section 8 ft from support are as follows, assuming the shear is entirely resisted by the web of the precast section: *11.4.1* *11.0*

$$w_u = 1.2\,(0.725) + 1.6\,(0.720) = 2.022 \text{ kips/ft}$$

$$V_u = \left[\left(\frac{60}{2}\right) - 8\right]2.022 = 44.5 \text{ kips}$$

$M_u = (30 \times 2.022 \times 8) - (2.022 \times 8 \times 4) = 421$ ft-kips

For the non-composite section, at 8 ft from support, determine distance d to centroid of tendons.

d = 26.40 in. (see strand profile)

For composite section, d = 26.4 + 2.5 = 28.9 in. < 0.8h = 30.8 in. use d = 30.8 in.

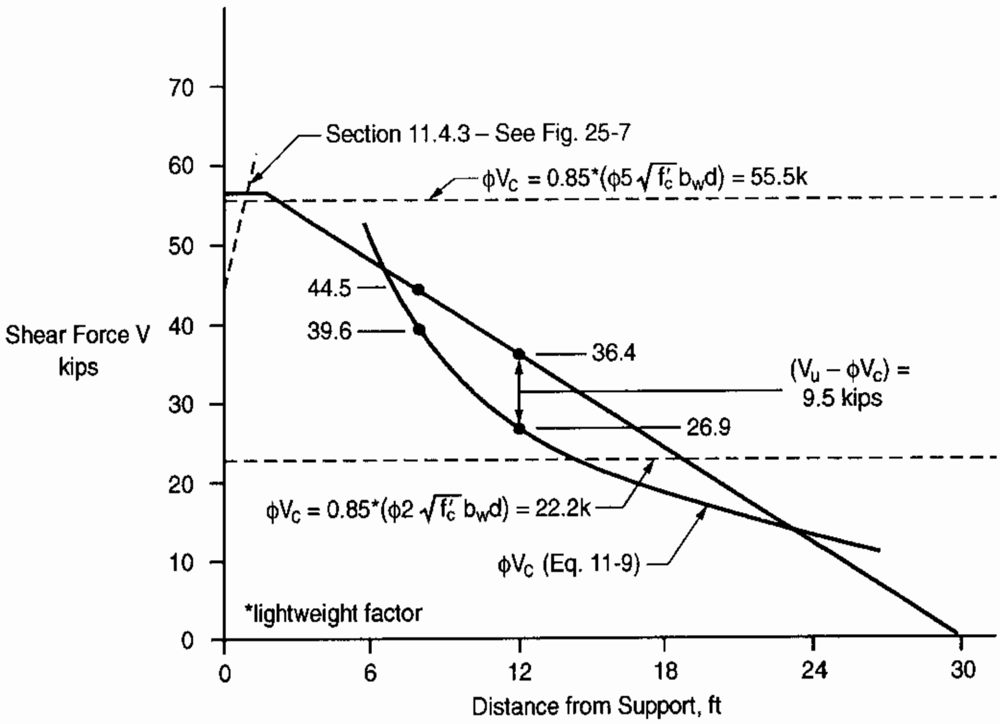


Figure 25-6 Shear Force Variation Along Member

$$V_c = \left(0.6\sqrt{f_c'} + 700\,\frac{V_u d_p}{M_u}\right) b_w d$$ *Eq. (11-9)*

but not less than $2\sqrt{f_c'}\, b_w d$ *11.4.2*

nor greater than $5\sqrt{f_c'}\, b_w d$ *11.4.2*

Since the precast section utilizes sand lightweight concrete, all $\sqrt{f_c'}$ terms must be reduced by the factor 0.85. *11.2.1.2*

Note: Total effective depth, d_p = 28.9 in., must be used in $V_u d_p/M_u$ term rather than 0.8h which is used elsewhere. *11.4.1*

$$V_c = \left(0.6 \times 0.85\sqrt{5000} + 700 \times 44.5 \times 28.90/\left(421 \times 12\right)\right)8 \times 30.8$$

$$= (36 + 178)\ 8 \times 30.8 = 52.8 \text{ kips} \quad \text{(governs)}$$

$$\geq 2 \times 0.85\sqrt{5000} \times 8 \times 30.8 = 29.6 \text{ kips}$$

$$\leq 5 \times 0.85\sqrt{5000} \times 8 \times 30.8 = 74.0 \text{ kips}$$

$$\phi V_c = 0.75 \times 52.8 = 39.6 \quad \text{(see Fig. 25-6)}$$ *11.2.1.2*

Note: For members simply supported and subject to uniform loading, $V_u d_p/M_u$ in. Eq. (11-9) becomes a simple function of d/ℓ, where ℓ is the span length,

$$V_c = \left[0.6\sqrt{f'_c} + 700 d_p \frac{(\ell - 2x)}{x(\ell - x)}\right] b_w d$$ *Eq. (11-9)*

where x is the distance from the support to the section being investigated. At 8 ft from the support,

$$V_c = \left[0.6 \times 0.85\sqrt{5000} + 700 \times 28.90 \frac{(60 - 16)}{8\ (60 - 8)\ 12}\right] 8 \times 30.8 = 52.8 \text{ kips}$$

3. In the end regions of pretensioned members, the shear strength provided by concrete V_c may be limited by the provisions of 11.4.4. For this design, 11.4.4 does not apply because the section at h/2 is farther out into the span than the bond transfer length (see Fig. 25-7). The following will, however, illustrate typical calculations to satisfy 11.4.4. Compute V_c at face of support, 10 in. from end of member.

Bond transfer length for 1/2-in. diameter strand = 50 (0.5) = 25 in. *11.4.3*

Prestress force at 10 in. location: $P_{se} = (10/25)\ 150 \times 0.153 \times 12 = 110.2$ kips

Vertical component of prestress force at 10 in. location:

$$\text{slope} = \frac{(d_{CL} - d_{end})}{\frac{\ell}{2}} = \frac{(33 - 24)}{30 \times 12} = 0.025$$

$$V_p \approx P \times \text{slope} = (110.2)\ (0.025) = 2.8 \text{ kips}$$

For composite section, d = 28.90 in., use 0.8h = 30.8 in. *11.4.2.3*

M_d (unfactored weight of precast unit + topping) = 214.4 in.-kips

Distance of composite section centroid above the centroid of precast unit,

$c = y_{bc} - y_b = 29.27 - 26.01 = 3.26$ in.

Tendon eccentricity, $e = d_{end} + 10 \text{ in.} \times \text{slope} - y_t = 24 + 10 \times 0.025 - 9.99$

$= 14.26$ in. below the centroid of the precast section

$$f_{pc} \text{ (see notation definition)} = \frac{P}{A_g} - (Pe)\frac{c}{I_g} + M_d\frac{c}{I_g}$$

$$= \frac{110.2}{570} - 110.2\,(14.26)\left(\frac{3.26}{68{,}917}\right) + 214.4\left(\frac{3.26}{68{,}917}\right) = 129 \text{ psi}$$

where A_g and I_g are for the precast section alone.

$$V_{cw} = \left(3.5\sqrt{f'_c} + 0.3f_{pc}\right) b_w d_p + V_p$$ *Eq. (11-12)*

$$= \left[\left(3.5 \times 0.85\sqrt{5000} + 0.3 \times 129\right) 8 \times 28.9\right] + 2800 = 60.4 \text{ kips}$$

$\phi V_{cw} = 0.75 \times 60.4 = 45.3$ kips

The results of this analysis are shown graphically in Fig. 25-7.

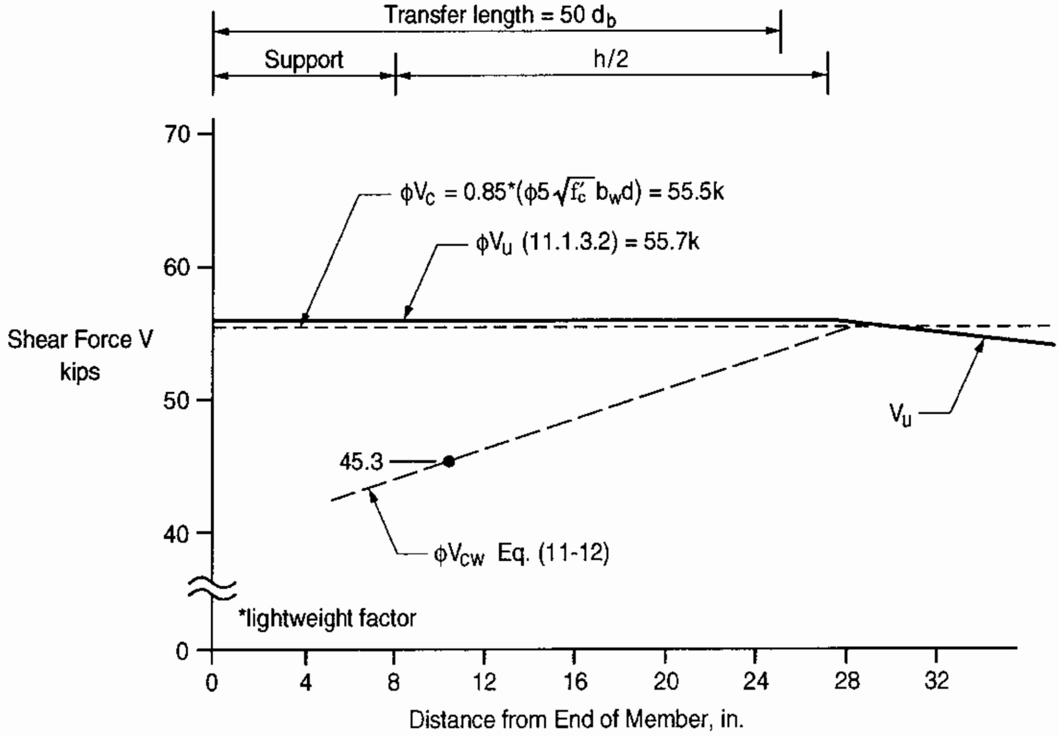


Figure 25-7 Shear Force Variation at End of Member

Example 25.1 (cont'd)	Calculations and Discussion	Code Reference

4. Compare factored shear V_u with shear strength provided by concrete ϕV_c. Where $V_u > \phi V_c$, shear reinforcement must be provided to carry the excess. Minimum shear reinforcement requirement should also be checked.

Shear reinforcement required at 12 ft from support is calculated as follows:

$d = 30.10$ in. (use in $V_u d_p/M_u$ term)

$M_u = 30 \times 2.24 \times 12 - 2.24 \times 12 \times 6 = 645$ ft-kips

$$V_u = \left[\left(\frac{60}{2}\right) - 12\right] 2.022 = 36.4 \text{ kips}$$

$$V_c = \left[(0.6 \times 0.85\sqrt{5000}) + 700 \times 40.3 \times 30.10/(645 \times 12)\right] 8 \times 30.8 = 35.9 \text{ kips}$$

$$\phi V_c = 0.75 \times 35.9 = 26.9 \text{ kips}$$

$$A_v = \frac{(V_u - \phi V_c)\,s}{\phi f_y d} = \frac{(36.4 - 26.9)\,12}{0.75 \times 60 \times 30.8} = 0.082 \text{ in.}^2/\text{ft}$$

Check minimum required by 11.5.6.3 and 11.5.6.4.

$$A_v(\min) = 0.75\sqrt{f_c'}\,\frac{b_w s}{f_y} = 0.75\sqrt{5000}\left(\frac{8 \times 12}{60{,}000}\right) = 0.085 \text{ in.}^2/\text{ft} \qquad \textit{Eq. (11-13)}$$

but not less than $50\frac{b_w s}{f_y}$ (not controlling for $f_c' > 4444$ psi)

$$A_v(\min) = \frac{A_{ps}}{80}\,\frac{f_{pu}}{f_{yt}}\,\frac{s}{d}\sqrt{\frac{d}{b_w}} \qquad \textit{Eq. (11-14)}$$

$$= \frac{1.84}{80} \times \frac{270}{60} \times \frac{12}{30.8}\sqrt{\frac{30.8}{8}} = 0.079 \text{ in.}^2/\text{ft}$$

The lesser A_v (min) from Eqs. (11-13) and (11-14) may be used

The required A_v is very slightly above minimum A_v

Maximum stirrup spacing = (3/4)d = (3/4) × 30.8 = 23.1 in.

Use No. 3 stirrups @ 18 in. for entire member length. ($A_v = 0.147$ in.^{2}/ft)

Example 25.2—Shear Design Using Fig. 25-4

Determine the shear reinforcement for the beam of Example 24.9

$f'_c = 6000$ psi

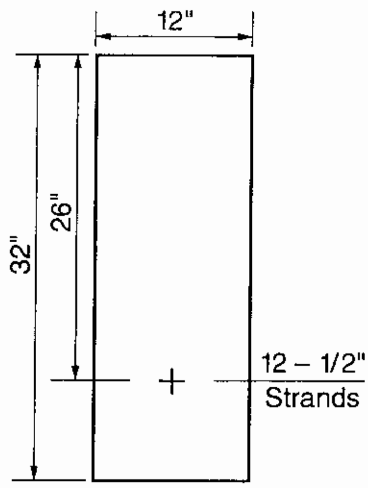
depth $d_p = 26$ in.

effective prestress $f_{se} = 150$ ksi

decompression stress $f_{dc} = 162$ ksi

span = 40 ft.

	w k/ft	Midspan moments in.-k
Self-weight	0.413	992
Additional dead load	1.000	2400
Live load	1.250	3000
Sum	2.663	6392



Calculations and Discussion	Code Reference

1. Calculate factored shear at support

$$V_u = 1.2D + 1.6L = [1.2(0.413 + 1.000) + 1.6(1.250)] \times \frac{40}{2}$$

$= 73.9$ kips

2. Prepare to use Fig. 25-4

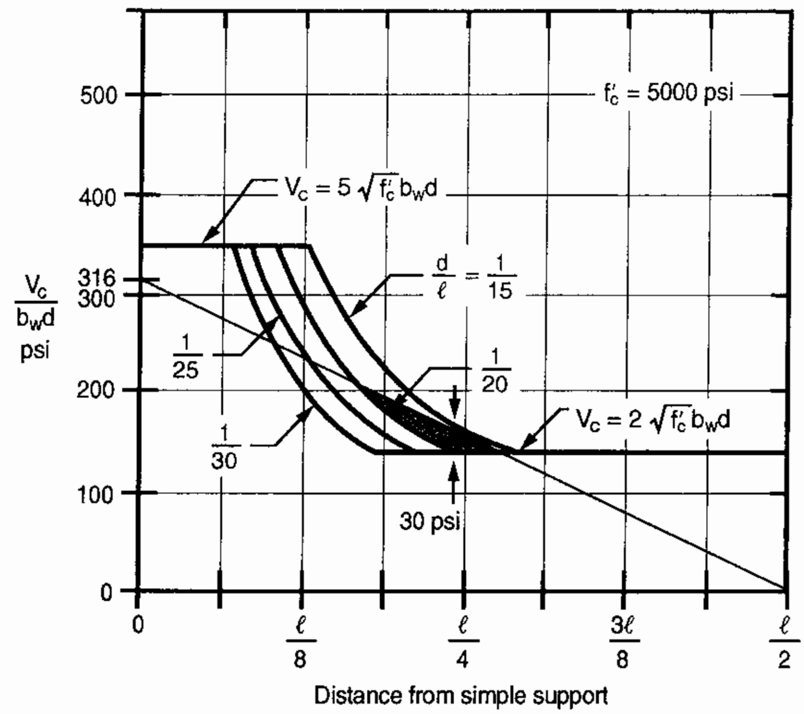
Note: Figure 25-4 is for $f'_c = 5000$ psi. Its use for $f'_c = 6000$ psi will be about 10 percent conservative.

$d/\ell = 26/480 = 1/18.5$

Use curve for $\ell/d = 1/20$

$$\frac{V_u}{\phi b_w d} = \frac{73.9}{0.75 \times 12 \times 26} = 0.316 \text{ ksi} = 316 \text{ psi}$$

3. Draw line for required nominal shear strength on Fig. 25-4, and find V_s required



The area where shear reinforcement is required is shaded. The maximum nominal shear stress to be resisted by shear reinforcement is 29 psi.

$V_s = 0.03 \text{ ksi} \times b \times d = 0.030 \times 12 \times 26 = 9.4 \text{ kips}$

$$A_v = \frac{V_s s}{f_{yt} d} = \frac{9.4 \times 12}{60 \times 26} = 0.07 \text{ in.}^2\text{/ft}$$ *Eq. (11-15)*

4. Check minimum reinforcement.

$$A_v = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_{yt}}, \text{ but not less than}$$ *Eq. (11-13)*

$$50\frac{b_w s}{f_{yt}}$$

$0.75\sqrt{6000} = 58.1$ controls

$A_v = 58.1 \times 12 \times 12/60{,}000 = 0.14 \text{ in.}^2\text{/ft}$

$$A_v = \frac{A_{ps} f_{pu} s}{80 f_{yt} d}\sqrt{\frac{d}{b_w}}$$ *Eq. (11-14)*

$$A_v = \frac{1.836 \times 270 \times 12}{80 \times 60 \times 26}\sqrt{\frac{26}{12}} = 0.07 \text{ in.}^2\text{/ft}$$

The lesser of A_v by Eqs. (11-13) and (11-14) may be used, but not less than A_v required.

5. Select stirrups

$A_v = 0.07$ in.²/ft

Maximum s = (3/4)d $\leq$ 24 in. *11.5.5.1*

s = (3/4)(26) = 19.5 in.

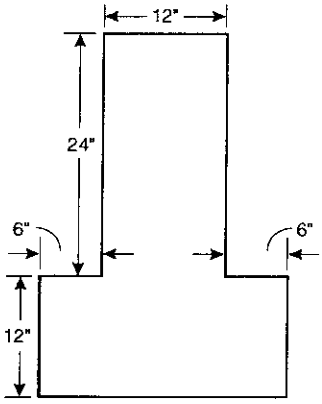
Use twin No. 3 @ 18 in.

$A_v = 0.22/1.5 = 0.15$ in.²/ft O.K.

This is required where V_u exceeds $\phi V_c/2$ *11.5.6.1*
Most designers would provide it for the full length of the member.

Example 25.3—Shear Design Using 11.4.2

For the simple span pretensioned ledger beam shown, determine shear requirements using V_c by Eqs. (11-10) and (11-12).



$A = 576 \text{ in.}^2$ $\quad w_d = 5.486$ kips/ft

$I = 63{,}936 \text{ in.}^4$ $\quad w_\ell = 5.00$ kips/ft

$h = 36$ in.

$y_b = 15$ in.

$f'_c = 6$ ksi

$\ell = 24$ feet

16 1/2 in. Grade 270 ksi strands, $P = 396.6$ kips

$e_{end} = e_{msp}$ (midspan) $= 10$ in.

Calculations and Discussion	Code Reference
A systematized procedure is needed, to expedite the calculations.	
1. Determine midspan moments and end shears	
$M_d = w_d\ell^2/8 = 5.486 \times 24^2/8 = 395 \text{ ft-kips} = 4740 \text{ in.-kips}$	
$M_\ell = w_\ell\ell^2/8 = 5.00 \times 24^2/8 = 360 \text{ ft-kips} = 4320 \text{ in.-kips}$	
$M_u = 1.2\,M_d + 1.6\,M_\ell = 1.2 \times 4740 + 1.6 \times 4320 = 12{,}600 \text{ in.-kips}$	*Eq. (9-2)*
$M_{max} = M_u - M_d = 12{,}600 - 4740 = 7860 \text{ in.-kips}$	*11.0*
$V_d = w_d\ell/2 = 5.486 \times 24/2 = 65.8 \text{ kips}$	
$V_\ell - w_\ell\ell/2 = 5 \times 24/2 = 60.0 \text{ kips}$	
$V_u = 1.2\,V_d + 1.6\,V_\ell = 1.2 \times 65.8 + 1.6 \times 60 = 175.0 \text{ kips}$	*Eq. (9-2)*
$V_i = V_u - V_d = 175 - 65.8 = 109.2 \text{ kips}$	*11.0*
2. Define factors for converting midspan moments and end shears to moments and shears at a distance x/ℓ from support, for $x/\ell = 0.3$.	
V factor $= 1-2(x/\ell) = 1-2(0.3) = 0.4$	
M factor $= 4(x/\ell - (x/\ell)^2) = 4 \times (0.3-0.3^2) = 0.84$	

3. Compute V_3, the third term in Eq. (11-10)

$P/A = 396.6/576 = 0.689$

$Pe/S_b = 396.6 \times 10/4262 = 0.930$

$-M_d/S_b = 0.84\ M_d\ (msp)/S_b = -0.934$

$+6\sqrt{f'_c} = 6\sqrt{6000} = 465 = 0.465$

$\overline{\quad 1.150 \text{ ksi}}$

$M_{cre} = S_b\,(1.150 \text{ ksi}) = 4900 \text{ in.-kips}$ — *Eq. (11-11)*

$V_i = 0.4V_i\ (end) = 0.4 \times 109.2 = 43.7 \text{ kips}$

$M_{max} = 0.84\ M_{max}\ (msp) = 0.84 \times 7860 = 6602 \text{ in.-kips}$

$$V_3 = \frac{V_i M_{cre}}{M_{max}} = \frac{43.7 \times 4900}{6602} = 32.4 \text{ kips}$$ *Eq. (11-10)*

4. Compute the remaining terms V_1 and V_2 in Eq. (11-10), and solve for V_{ci} — *Eq. (11-10)*

$d = 31$, but not less than $0.8d = 28.8$. Use $d = 31$ — *11.4.1*

$V_1 = 0.6 b_w d_p \sqrt{f'_c} = 0.6 \times 12 \times 31\sqrt{6000} = 17.3\text{k}$

$V_2 = V_d = 0.4(V_d\text{end}) = 0.4 \times 65.8 = 26.3 \text{ kips}$

$V_{ci} = V_1 + V_2 + V_3 = 17.3 + 26.3 + 32.4 = 76.0 \text{ kips}$ — *Eq. (11-10)*

5. Compute V_u, and find V_s to be resisted by stirrups
$V_u = 0.4\ V_u\ (end) = 0.4 \times 175.0 = 70 \text{ kips}$

ϕ for shear $= 0.75$ — *9.3.2.3*

$V_s = V_n - V_c = V_u/\phi - V_c = 70/0.75 - 76 = 17.3 \text{ kips}$ — *Eq. (11-2)*

6. Find required stirrups

$$A_v = \frac{V_s s}{f_{yt} d} = \frac{17.3 \times 12}{60 \times 31} = 0.11 \text{ in.}^2\text{/ft}$$

Minimum requirements

$A_v = 0.75\sqrt{f'_c}\ b_w s/f_{yt}$ when $f'_c > 4444$ psi — *Eq. (11-13)*

$= 0.75\sqrt{6000} \times 12 \times 12/60{,}000 = 0.14 \text{ in.}^2$

$$A_v = \frac{A_{ps} f_{pu} s}{80\ f_{yt} d}\sqrt{\frac{d}{b_w}} = \frac{2.448 \times 270 \times 12}{80 \times 60 \times 31}\sqrt{\frac{31}{12}} = 0.086 \text{ in.}^2$$ *Eq. (11-14)*

The minimum need only be the lesser of that required by Eqs. (11-13) or (11-14). — *11.5.6.4*

So, the required A_v of 0.09 in.^{2}/ft controls

Maximum spacing = (3/4)d = (3/4)31 = 23.25 in.

Say, twin No. 3 at 18 in., $A_v = 2 \times 0.11/1.5 = 0.15$ in.²/ft.

7. Compute required shear reinforcement at support

Because the ledger beam is loaded on the ledges, not "near the top," shear must be checked at the support, not at h/2 from the support for prestressed members. *11.1.3*

At the support, the prestress force P is assumed to be zero, for simplicity.

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3\,f_{pc})\,b_w d_p + V_p$$ *Eq. (11-12)*

$$= (3.5\sqrt{6000}) \times 12 \times 31 = 100.9 \text{ kips}$$

$$V_s = V_n - V_c = V_u/\phi - V_c = 175/0.75 - 100.9$$ *Eq. (11-12)*

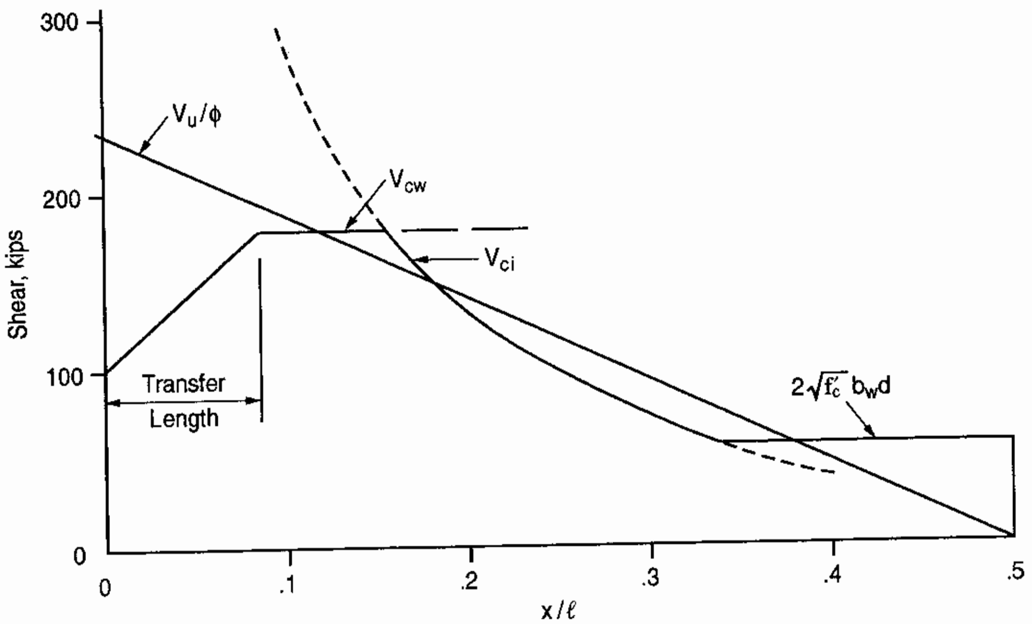
$$V_s = 132.4\text{k}$$

$$A_v = \frac{V_s s}{f_{yt} d} = \frac{132.4 \times 12}{60 \times 31} = 0.85 \text{ in.}^2$$ *Eq. (11-15)*

Say twin No. 4 at 4 in., A_v = .40/.33 = 1.20 in.²/ft near end.

Referring to Step 6, this is above minimum requirements.

8. Repeat the processes described above for various sections along the shear span (not shown). The results are shown below.



Note: Minimum V_c of $2\sqrt{f'_c}\,b_w d$ permitted by 11.4.2 was used.

9. Notes:

   1. A spreadsheet can be set up, with each column containing data for various values of x/ℓ and the shear and moment factors in Step 2.

   2. For members with draped tendons, additional factors for varying eccentricity, depth, and tendon slope (for computing V_p) need to be added in Step 2.

   3. For composite members, the portions of dead load applied before and after composite behavior is obtained need to be separated. The dead load applied after the beam becomes composite should not be included in V_d and M_d terms. See R11.4.3.

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26

Prestressed Slab Systems

UPDATE FOR THE '05 CODE

Section 6.4.4, which requires that construction joints in slabs be located in the middle third of the span, is excluded from application to prestressed concrete. Most construction joints in continuous prestressed concrete slabs are located close to the quarter point of the span where the tendon profile is near mid-depth of the member.

Section 18.3.3 defines two-way prestressed slab systems as Class U and reduces the maximum permissible flexural tensile stress f_t from $7.5\sqrt{f'_c}$ to $6\sqrt{f'_c}$.

INTRODUCTION

Six code sections are particularly significant with respect to analysis and design of prestressed slab systems:

Section 11.12.2—Shear strength of prestressed slabs

Section 11.12.6—Shear strength of prestressed slabs with moment transfer

Section 18.3.3—Permissible flexural tensile stresses

Section 18.4.2—Permissible flexural compressive stresses

Section 18.7.2—Determination of f_{ps} for calculation of flexural strength

Section 18.12—Prestressed slab systems

Discussion of each of these code sections is presented below, followed by Example 26.1 of a post-tensioned flat plate. The design example illustrates application of the above code sections as well as general applicability of the code to analysis and design of post-tensioned flat plates.

11.12.2 Shear Strength

Section 11.12.2 contains specific provisions for calculation of shear strength in two-way prestressed concrete systems. At columns of two-way prestressed slabs (and footings) utilizing unbonded tendons and meeting the bonded reinforcement requirements of 18.9.3, the shear strength V_n must not be taken greater than the shear strength V_c computed in accordance with 11.12.2.1 or 11.12.2.2, unless shear reinforcement is provided in accordance with 11.12.3 or 11.12.4. Section 11.12.2.2 gives the following value of the shear strength V_c at columns of two-way prestressed slabs:

$$V_c = \left(\beta_p\sqrt{f'_c} + 0.3f_{pc}\right) b_o d + V_p \qquad \textit{Eq. (11-36)}$$

Equation (11-36) includes the term β_p which is the smaller of 3.5 and $(\alpha_s d/b_o + 1.5)$. The term $\alpha_s d / b_o$ is to account for a decrease in shear strength affected by the perimeter area aspect ratio of the column, where α_s is to be taken as 40 for interior columns, 30 for edge columns, and 20 for corner columns. f_{pc} is the average value of

f_{pc} for the two directions, and V_p is the vertical component of all effective prestress forces crossing the critical section. If the shear strength is computed by Eq. (11-36), the following must be satisfied; otherwise, 11.12.2.1 for nonprestressed slabs applies:

a. no portion of the column cross-section shall be closer to a discontinuous edge than 4 times the slab thickness,

b. f'_c in Eq. (11-36) shall not be taken greater than 5000 psi, and

c. f_{pc} in each direction shall not be less than 125 psi, nor be taken greater than 500 psi.

In accordance with the above limitations, shear strength Eqs. (11-33), (11-34), and (11-35) for nonprestressed slabs are applicable to columns closer to the discontinuous edge than 4 times the slab thickness. The shear strength V_c is the lesser of the values given by these three equations. For usual design conditions (slab thicknesses and column sizes), the controlling shear strength at edge columns will be $4\sqrt{f'_c}b_od$.

11.12.6 Shear Strength with Moment Transfer

For moment transfer calculations, the controlling shear stress at columns of two-way prestressed slabs with bonded reinforcement in accordance with 18.9.3 is governed by Eq. (11-36), which could be expressed as a shear stress for use in Eq. (11-40) as follows:

$$v_c = \beta_p\sqrt{f'_c} + 0.3f_{pc} + \frac{V_p}{b_od} \qquad \textit{Eq. (11-36)}$$

If the permissible shear stress is computed by Eq. (11-36), the following must be satisfied:

a. no portion of the column cross-section shall be closer to a discontinuous edge than 4 times the slab thickness,

b. f'_c in Eq. (11-36) shall not be taken greater than 5000 psi, and

c. f_{pc} in each direction shall not be less than 125 psi, nor be taken greater than 500 psi.

For edge columns under moment transfer conditions, the controlling shear stress will be the same as that permitted for nonprestressed slabs. For usual design conditions, the governing shear stress at edge columns will be $4\sqrt{f'_c}$.

18.3.3 Permissible Flexural Tensile Stresses

This section requires that prestressed two-way slab systems be designed as Class U (Uncracked) members but with the permissible flexural tensile stress limited to $6\sqrt{f'_c}$.

18.4.2 Permissible Flexural Compressive Stresses

In 1995, Section 18.4.2 increased the permissible concrete service load flexural compressive stress under total load from $0.45f'_c$ to $0.60f'_c$, but imposed a new limit of $0.45f'_c$ for sustained load. This involves some judgment on the part of designers in determining the appropriate sustained load.

18.7.2 f_{ps} for Unbonded Tendons

In prestressed elements with unbonded tendons having a span/depth ratio greater than 35, the stress in the prestressed reinforcement at nominal strength is given by:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p} \qquad \textit{Eq. (18-5)}$$

but not greater than f_{py}, nor (f_{se} + 30,000).

Nearly all prestressed one-way slabs and flat plates will have span/depth ratios greater than 35. Equation (18-5) provides values of f_{ps} which are generally 15,000 to 20,000 psi lower than the values of f_{ps} given by Eq. (18-4) which was derived primarily from results of beam tests. These lower values of f_{ps} are more compatible with values of f_{ps} obtained in more recent tests of prestressed one-way slabs and flat plates. Application of Eq. (18-5) is illustrated in Example 26.1.

18.12 SLAB SYSTEMS

Section 18.12 provides analysis and design procedures for two-way prestressed slab systems, including the following requirements:

1. Use of the Equivalent Frame Method of 13.7 (excluding 13.7.7.4 and 13.7.7.5), or more detailed analysis procedures, is required for determination of factored moments and shears in prestressed slab systems. According to References 26.1 and 26.4, for two-way prestressed slabs, the equivalent frame slab-beam strips would not be divided into column and middle strips as for a typical nonprestressed two-way slab, but would be designed as a total beam strip.

2. Spacing of tendons or groups of tendons in one direction shall not exceed 8 times the slab thickness nor 5 ft. Spacing of tendons shall also provide a minimum average prestress, after allowance for all prestress losses, of 125 psi on the slab section tributary to the tendon or tendon group. Special consideration must be given to tendon spacing in slabs with concentrated loads.

3. A minimum of two tendons shall be provided in each direction through the critical shear section over columns. This provision, in conjunction with the limits on tendon spacing outlined in Item 2 above, provides specific guidance for distributing tendons in prestressed flat plates in accordance with the "banded" pattern illustrated in Fig. 26-1. This method of tendon installation is widely used and greatly simplifies detailing and installation procedures.

Calculation of equivalent frame properties is illustrated in Example 26.1. Tendon distribution is also discussed in this example.

References 26.1 and 26.4 illustrate application of ACI 318 requirements for design of one-way and two-way post-tensioned slabs, including detailed design examples.

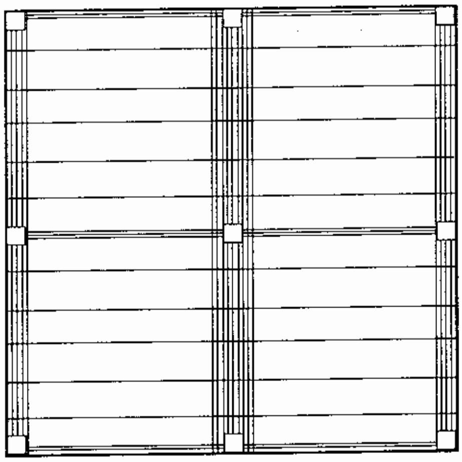

Figure 26-1 Banded Tendon Distribution

REFERENCES

26.1 *Design of Post-Tensioned Slabs Using Unbonded Tendons*, Post-Tensioning Institute, 3rd. ed., Phoenix, AZ, 2004.

26.2 *Continuity in Concrete Building Frames*, Portland Cement Association, Skokie, IL, 1986.

26.3 *Estimating Prestress Losses,* Zia, P., Preston, H. K., Scott, N. L., and Workman, E. B., Concrete International : Design and Construction, V. 1, No. 6, June 1979, pp. 32-38.

26.4 *Design Fundamentals of Post-Tensioned Concrete Floors,* Aalami, B. O., and Bommer, A., Post-Tensioning Institute, Phoenix, AZ, 1999.

Example 26.1—Two-Way Prestressed Slab System

Design a typical transverse equivalent frame strip of the prestressed flat plate with partial plan and section shown in Figure 26-2.

f'_c = 4000 psi; w = 150 pcf (slab and columns)

f_y = 60,000 psi

f_{pu} = 270,000 psi

Live load = 40 psf
Partition load = 15 psf

Reduce live load in accordance with general building code. For this example live load is reduced in accordance with IBC 2003, Section 1607.9.2.

Required minimum concrete cover to tendons 1.5 in. from the bottom of the slab in end spans, 0.75 in. top and bottom elsewhere.

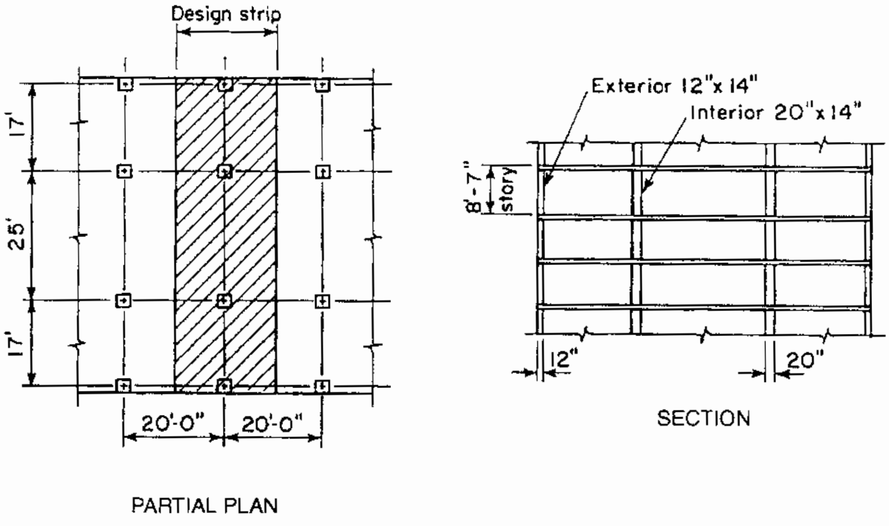

Figure 26-2 Equivalent Frame

Calculations and Discussion	Code Reference

1. Slab Thickness

For two-way prestressed slabs, a span/depth ratio of 45 typically results in overall economy and provides satisfactory structural performance.[26.1]

Slab thickness:

Longitudinal span: $20 \times 12/45 = 5.3$ in.
Transverse span: $25 \times 12/45 = 6.7$ in.

Use 6-1/2 in. slab.

Slab weight = 81 psf
Partition load = 15 psf
Total dead load = 81 + 15 = 96 psf

Span 2:
Reduced live load (IBC 1607.9.2)
Live load = $40[1 - 0.08(500 - 150)/100] = 29$ psf
Factored dead load = $1.2 \times 96 = 115$ psf
Factored live load = $1.6 \times 29 = 47$ psf

Total load = 125 psf, unfactored
= 162 psf, factored

Spans 1 and 3:
Reduced live load (IBC 1607.9.2)
Live load = 40[1 – 0.08(340 – 150)/100] = 34 psf
Factored dead load = 1.2 × 96 = 115 psf
Factored live load = 1.6 × 34 = 55 psf
Total load = 130 psf, unfactored
= 170 psf, factored

2. Design Procedure

Assume a set of loads to be balanced by parabolic tendons. Analyze an equivalent frame subjected to the net downward loads according to 13.7. Check flexural stresses at critical sections, and revise load balancing tendon forces as required to obtain permissible flexural stresses according to 18.3.3 and 18.4.

When final forces are determined, obtain frame moments for factored dead and live loads. Calculate secondary moments induced in the frame by post-tensioning forces, and combine with factored load moments to obtain design factored moments. Provide minimum bonded reinforcement in accordance with 18.9.

Check design flexural strength and increase nonprestressed reinforcement if required by strength criteria. Investigate shear strength, including shear due to vertical load and due to moment transfer, and compare total to permissible values calculated in accordance with 11.12.2.

3. Load Balancing

Arbitrarily assume the tendons will balance 80% of the slab weight (0.8 × 0.081 = 0.065 ksf) in the controlling span (Span 2), with a parabolic tendon profile of maximum permissible sag, for the initial estimate of the required prestress force F_e:

Maximum tendon sag in Span 2 = 6.5 – 1 – 1 = 4.5 in.

$$F_e = \frac{w_{bal}L^2}{8a} = \frac{0.8(0.081)(25)^2(12)}{8(4.5)} = 13.5 \text{ kips / ft}$$

Assume 1/2 in. diameter (cross-sectional area = 0.153 in.^{2}), 270 ksi seven-wire low relaxation strand tendons with 14 ksi long-term losses (Reference 26.3). Effective force per tendon is 0.153 [(0.7 × 270) – 14] = 26.8 kips, where the tensile stress in the tendons immediately after tendon anchorage = $0.70f_{pu}$. *18.5.1(c)*

For a 20-ft bay, 20 × 13.5/26.8 = 10.1 tendons.

Use 10-1/2 in. diameter tendons/bay

$F_e = 10 \times 26.8/20 = 13.4$ kips/ft

$f_{pc} = F_e/A = 13.4/(6.5 \times 12) = 0.172$ ksi

Actual balanced load in Span 2:

$$w_{bal} = \frac{8F_e a}{L^2} = \frac{8(13.4)(4.5)}{12 \times 25^2} = 0.064 \text{ ksf}$$

Adjust tendon profile in Spans 1 and 3 to balance same load as in Span 2:

$$a = \frac{w_{bal}L^2}{8F_e} = \frac{0.064(17)^2(12)}{8(13.4)} = 2.1 \text{ in.}$$

Midspan cgs = (3.25 + 5.5)/2 – 2.1 = 2.275 in say 2.25 in.
Actual sag in Spans 1 and 3 = (3.25 + 5.5)/2 – 2.25 = 2.125 in.
Actual balanced load in Spans 1 and 3 =

$$w_{bal} = \frac{8(13.4)(2.125)}{17^2(12)} = 0.066 \text{ ksf}$$

4. Tendon Profile

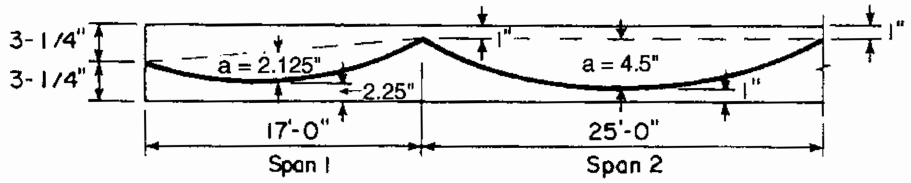

Figure 26-3 Tendon Profile

Net load causing bending:
Span 2:
$w_{net} = 0.125 - 0.064 = 0.061$ ksf
Spans 1 and 3:
$w_{net} = 0.130 - 0.066 = 0.064$ ksf

5. Equivalent Frame Properties — *13.7*

   a. Column stiffness. — *13.7.4*

Column stiffness, including effects of "infinite" stiffness within the slab-column joint (rigid connection), may be calculated by classical methods or by simplified methods which are in close agreement. The following approximate stiffness K_c will give results within five percent of "exact" values.[26.1]

$K_c = 4EI/(\ell - 2h)$

where ℓ = center-to-center column height and h = slab thickness.

For exterior columns (14 × 12 in.):

$I = 14 \times 12^3/12 = 2016 \text{ in.}^4$

$E_{col}/E_{slab} = 1.0$

$K_c = (4 \times 1.0 \times 2016)/[103 - (2 \times 6.5)] = 90 \text{ in.}^3$

$\Sigma K_c = 2 \times 90 = 180 \text{ in.}^3$ (joint total)

Stiffness of torsional members is calculated as follows: — *13.7.5*

$C = (1 - 0.63\, x/y)\, x^3y/3$ — *13.0*

$= [1 - (0.63 \times 6.5/12)]\,(6.5^3 \times 12)/3 = 724 \text{ in.}^4$

$$K_t = \frac{9CE_{cs}}{\ell_2\,(1 - c_2/\ell_2)^3}$$ — *R13.7.5*

$$= \frac{9 \times 724 \times 1.0}{(20 \times 12)\,(1 - 1.17/20)^3} = 32.5 \text{ in.}^3$$

$\Sigma K_t = 2 \times 32.5 = 65 \text{ in.}^3$ (joint total)

Exterior equivalent column stiffness (see ACI 318R-89, R13.7.4):

$1/K_{ec} = 1/\Sigma K_t + 1/\Sigma K_c$

$K_{ec} = (1/65 + 1/180)^{-1} = 48 \text{ in.}^3$

For interior columns (14 × 20 in.):

$I = 14 \times 20^3/12 = 9333 \text{ in.}^4$

$K_c = (4 \times 1.0 \times 9333)/[103 - (2 \times 6.5)] = 415 \text{ in.}^3$

$\Sigma K_c = 2 \times 415 = 830 \text{ in.}^3$ (joint total)

$C = [1 - (0.63 \times 6.5/20)] (6.5^3 \times 20)/3 = 1456 \text{ in.}^4$

$$K_t = \frac{9 \times 1456 \times 1.0}{240\,(1 - 1.17/20)^3} = 65 \text{ in.}^3$$

$\Sigma K_t = 2 \times 65 = 130 \text{ in.}^3$ (joint total)

$K_{ec} = (1/130 + 1/830)^{-1} = 112 \text{ in.}^3$

b. Slab-beam stiffness. *13.7.3*

Slab stiffness, including effects of infinite stiffness within slab-column joint, can be calculated by the following approximate expression.[26.1]

$K_s = 4EI/(\ell_1 - c_1/2)$

where ℓ_1 = length of span in direction of analysis measured center-to-center of supports and c_1 = column dimension in direction of ℓ_1.

At exterior column:

$K_s = (4 \times 1.0 \times 20 \times 6.5^3)/[(17 \times 12) - 12/2] = 111 \text{ in.}^3$

At interior column (spans 1 & 3):

$K_s = (4 \times 1.0 \times 20 \times 6.5^3)/[(17 \times 12) - 20/2] = 113 \text{ in.}^3$

At interior column (span 2):

$K_s = (4 \times 1.0 \times 20 \times 6.5^3)/[(25 \times 12) - 20/2] = 76 \text{ in.}^3$

c. Distribution factors for analysis by moment distribution.

Slab distribution factors:

At exterior joints $= 111/(111 + 48) = 0.70$

At interior joints for spans 1 and 3 $= 113/(113 + 76 + 112) = 0.37$

At interior joints for span 2 $= 76/301 = 0.25$

6. Moment Distribution—Net Loads

Since the nonprismatic section causes only very small effects on fixed-end moments and carryover factors, fixed-end moments will be calculated from FEM $= wL^2/12$ and carryover factors will be taken as COF = 1/2.

For Spans 1 and 3, net load FEM $= 0.064 \times 17^2/12 = 1.54$ ft-kips

For Span 2 net load FEM $= 0.061 \times 25^2/12 = 3.18$ ft-kips

Note that since live load is less than three-quarters dead load, patterned or "skipped" live load is not required. Maximum factored moments are based upon full live load on all spans simultaneously. *(13.7.6.2)*

Table 26-1 Moment Distribution—Net Loads
(all moments are in ft-kips)

DF	0.70	0.37	0.25
FEM	-1.54	-1.54	-3.18
Distribution	+1.08	-0.61	+0.41
Carry-over	+0.31	-0.54	-0.21
Distribution	-0.22	+0.12	-0.08
Final	-0.37	-2.57	-3.06

7. Check Net Stresses (tension positive, compression negative)

a. At interior face of interior column:

Moment at column face = centerline moment + $Vc_1/3$ (see Ref. 26.2):

$$-M_{max} = -3.06 + \frac{1}{3}\left(\frac{0.061 \times 25}{2}\right)\left(\frac{20}{12}\right)$$

$$= -2.64 \text{ ft-kips}$$

$S = bh^2/6 = 12 \times 6.5^2/6 = 84.5 \text{ in.}^3$

$$f_{t,b} = -f_{pc} \pm \frac{M_{net}}{S_{t,b}} = -0.172 \pm \frac{12 \times 2.64}{84.5} = 0.172 \pm 0.375 = +0.203, -0.547 \text{ ksi}$$

Allowable Tension = $6\sqrt{4000} = 0.379$ ksi *18.3.3*
At top 0.203 ksi applied < 0.379 allowable OK

Allowable compression under total load = $0.60f'_c = 0.6 \times 4000 = 2.4$ ksi *18.4.2(b)*
At bottom 0.547 ksi applied < 2.4 ksi allowable OK

Allowable compression under sustained load = $0.45 \times 4000 = 1.8$ ksi *18.4.2(a)*
0.547 ksi applied under total load < 1.8 ksi allowable under sustained load OK
(regardless of value of sustained load).

b. At midspan of Span 2:

$+M_{max} = (0.061 \times 25^2/8) - 3.18 = +1.59$ ft-kips

$$f_{t,b} = -f_{pc} \mp \frac{M_{net}}{S_{t,b}} = -0.172 \mp \frac{12 \times 1.59}{84.5} = -0.172 \mp 0.226 = -0.398, +0.054 \text{ ksi}$$

Compression at top 0.398 < 1.8 ksi allowable sustained load < 2.4 ksi allowable total load O.K. Tension at bottom 0.054 ksi applied < 0.379 ksi allowable O.K.

When the tensile stress exceeds $2\sqrt{f'_c}$ in positive moment areas, the total tensile force N_c must be carried by bonded reinforcement. For this slab, $2\sqrt{4000} = 0.126 \text{ ksi} > 0.054 \text{ ksi}$. Therefore, positive moment bonded reinforcement is not required. When it is, the calculation for the required amount of bonded reinforcement is done as follows (refer to Figure 26-4). *18.9.3.2*

$$y = \frac{f_t}{f_t + f_c}(h) \text{ in.}$$

$$N_c = \frac{12(y)(f_t)}{2} \text{ kips/ft}$$

$$A_s = \frac{N_c}{0.5f_y} \text{ in.}^2\text{/ft}$$

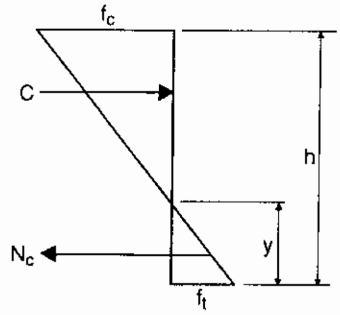

Figure 26-4 Stress Distribution

Determine minimum bar lengths for this reinforcement in accordance with *18.9.4* (Note that conformance to *Chapter 12* is also required.)

Calculate deflections under total loads using usual elastic methods and gross concrete section properties *(9.5.4)*. Limit ***computed*** deflections to those specified in Table *9.5(b)*.

This completes the service load portion of the design.

8. Flexural Strength

a. Calculation of design moments.

Design moments for statically indeterminate post-tensioned members are determined by combining frame moments due to factored dead and live loads with secondary moments induced into the frame by the tendons. The load balancing approach directly includes both primary and secondary effects, so that for service conditions only "net loads" need be considered.

At design flexural strength, the balanced load moments are used to determine secondary moments by subtracting the primary moment, which is simply $F_e \times e$, at each support. For multistory buildings where typical vertical load design is combined with varying moments due to lateral loading, an efficient design approach would be to analyze the equivalent frame under each case of dead, live, balanced, and lateral loads, and combine the cases for each design condition with appropriate load factors. For this example, the balanced load moments are determined by moment distribution as follows:

For spans 1 and 3, balanced load FEM $= 0.066 \times 17^2/12 = 1.59$ ft-kips

For span 2, balanced load FEM $= 0.064 \times 25^2/12 = 3.33$ ft-kips

Table 26-2 Moment Distribution—Balanced Loads (all moments are in ft-kips)

DF	0.70	0.37	0.25
FEM	+1.59	+1.59	+3.33
Distribution	-1.11	+0.64	-0.44
Carry-over	-0.32	+0.56	+0.22
Distribution	+0.22	-0.13	+0.09
Final	+0.38	+2.66	+3.20

Since the balanced load moment includes both primary (M_1) and secondary (M_2) moments, secondary moments can be found from the following relationship:

$M_{bal} = M_1 + M_2$, or $M_2 = M_{bal} - M_1$

The primary moment M_1 equals $F_e \times e$ at any point ("e" is the distance between the cgs and the cgc, the "eccentricity" of the prestress force).

Thus, the secondary moments are:

At an exterior column:

$M_2 = 0.38 - (13.4 \times 0/12) = 0.38$ ft-kips

At an interior column:

Spans 1 and 3,

$M_2 = 2.66 - 13.4\,(3.25 - 1.0)/12 = 0.15$ ft-kips

Span 2,

$M_2 = 3.20 - (13.4 \times 2.25)/12 = 0.69$ ft-kips

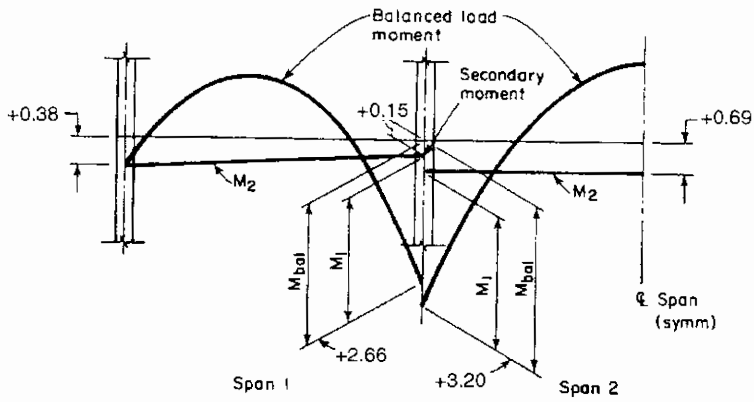

Figure 26-5 Moment Diagram

Factored load moments:
Spans 1 and 3: w_u = 170 psf
Span 2: w_u = 162 psf

For spans 1 and 3, factored load FEM = 0.170 × $17^2/12$ = 4.09 ft-kips

For span 2, factored load FEM = 0.162 × $25^2/12$ = 8.44 ft-kips

Table 26-3 Moment Distribution—Factored Loads (all moments are in ft-kips)

DF	0.70	0.37	0.25
FEM	-4.09	-4.09	-8.44
Distribution	+2.86	-1.61	+1.09
Carry-over	+0.81	-1.43	-0.55
Distribution	-0.57	+0.33	-0.22
Final	-0.99	-6.80	-8.12

Combine the factored load and secondary moments to obtain the total negative design moments. The results are given in Table 26-4.

Table 26-4 Design Moments at Face of Column (all moments are in ft-kips)

	Span 1		Span 2
Factored load moments	-0.99	-6.80	-8.12
Secondary moments	+0.38	+0.15	+0.69
Moments at column centerline	-0.61	-6.65	-7.43
Moment reduction to face of column, $Vc_1/3$	+0.48	+0.80	+1.13
Design moments at face of column	-0.13	-5.85	-6.30

Calculate total positive design moments at interior of span:

For span 1,

$V_{ext} = (0.170 \times 17/2) - (6.65 - 0.61)/17$

$= 1.45 - 0.36 = 1.09$ kips/ft

$V_{int} = 1.45 + 0.36 = 1.81$ kips/ft

Distance x to location of zero shear and maximum positive moment from centerline of exterior column:

$x = 1.09/0.170 \approx 6.42$ ft

End span positive moment $= (0.5 \times 1.09 \times 6.42) - 0.61 = 2.89$ ft-kips/ft (including M_2)

For span 2,

$V = 0.162 \times 25/2 = 2.03$ kips/ft

Interior span positive moment $= -7.43 + (0.5 \times 2.03 \times 12.5) = 5.26$ ft-kips/ft (including M_2)

b. Calculation of flexural strength.

Check slab at interior support. Section 18.9.3.3 requires a minimum amount of bonded reinforcement in negative moment areas at column supports regardless of service load stress levels. More than the minimum may be required for flexural strength. The minimum amount is to help ensure flexural continuity and ductility, and to control cracking due to overload, temperature, or shrinkage.

$A_s = 0.00075A_{cf}$ *Eq. (18-8)*

where

A_{cf} = larger cross-sectional area of the slab-beam strips of the two orthogonal equivalent frames intersecting at a column of a two-way slab.

$$A_s = 0.00075 \times 6.5 \times \left(\frac{17 + 25}{2}\right) \times 12 = 1.23 \text{ in.}^2$$

Try 6-No. 4 bars. Space bars at 6 in. on center, so that they are within the column width plus 1.5 times slab thickness on either side of column. *18.9.3.3*

Bar length $= [2 \times (25 - 20/12)/6] + 20/12 = 9$ ft-5 in. *18.9.4.2*

For average one-foot strip:

$A_s = 6 \times 0.20/20 = 0.06 \text{ in.}^2/\text{ft}$

Initial check of flexural strength will be made considering this reinforcement.

Calculate stress in tendons at nominal strength:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p}$$ *Eq. (18-5)*

With 10 tendons in 20 ft bay:

$\rho_p = A_{ps}/bd_p = 10 \times 0.153/(20 \times 12 \times 5.5) = 0.00116$

$f_{se} = (0.7 \times 270) - 14 = 175 \text{ ksi}$ *18.5.1, 18.6, Reference 3*

$f_{ps} = 175 + 10 + 4/(300 \times 0.00116) = 175 + 10 + 12 = 197 \text{ ksi}$

f_{ps} shall not be taken greater than $f_{py} = 0.85f_{pu} = 230 \text{ ksi} > 197$
or $f_{se} + 30 = 205 \text{ ksi} > 197$ OK *18.7.2(c)*

$A_{ps}f_{ps} = 10 \times 0.153 \times 197/20 = 15.1 \text{ kips/ft}$

$A_sf_y = 0.06 \times 60 = 3.6 \text{ kips/ft}$

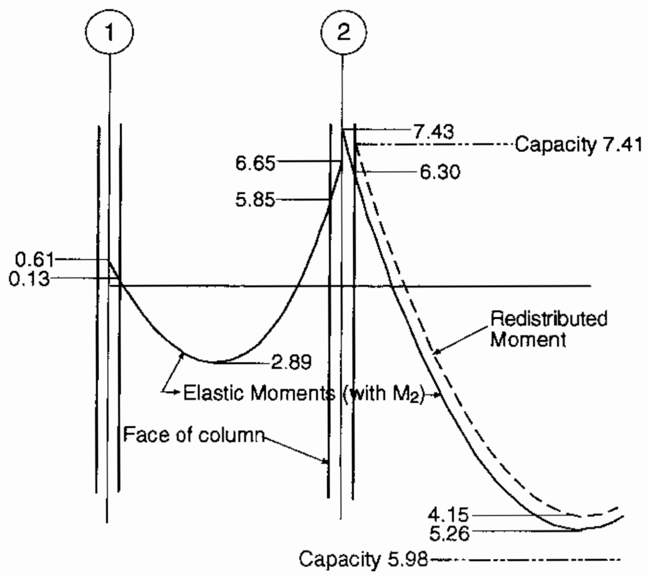

Figure 26-6 Moments in ft-kips

$$a = \frac{A_{ps}f_{ps} + A_s f_y}{0.85 f'_c b} = \frac{15.1 + 3.6}{0.85 \times 4 \times 12} = 0.46 \text{ in.}$$

$$c = a/\beta_1 = 0.46/0.85 = 0.54 \text{ in.}$$

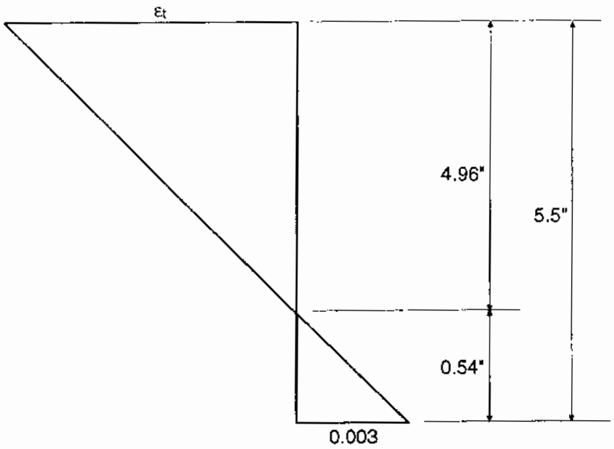

Figure 26-7 Strain Diagram at Interior Support

$\varepsilon_t = (5.5 - 0.54) \times 0.003/0.54 = 0.028$ therefore tension controlled, $\phi = 0.9$ — *9.3.2, 10.3.4*

Since the bars and tendons are in the same layer:

$$\left(d - \frac{a}{2}\right) = \left(5.5 - \frac{0.46}{2}\right)/12 = 0.44 \text{ ft}$$

$\phi M_n = 0.9 \times (15.1 + 3.6) \times 0.44 = 7.41$ ft-kips/ft > 6.30 ft-kips/ft OK. — *9.3.2.1*

Since there is excess negative moment capacity available, use moment redistribution to increase the negative moment and minimize the positive moment demand in Span 2. Note that the actual inelastic moment redistribution occurs at the positive moment section of Span 2.

Permissible change in negative moment = $1000\varepsilon_t = 1000(0.028) = 28\% > 20\%$ max — *18.10.4.1, 8.4*

Available increase in negative moment = $0.2 \times 6.30 = 1.26$ ft-kips/ft

Actual increase in negative moment = Minimum capacity – Elastic Negative Moment
= 7.41 – 6.30 = 1.11 ft-kips/ft < 1.26 available O.K.

Minimum design positive moment in Span 2 = 5.26 − 1.11 = 4.15 ft-kips/ft

Capacity at midspan of Span 2 (no bonded reinforcement required):

$A_{ps}f_{ps} = 15.1$ kips/ft

$$a = \frac{15.1}{0.85 \times 4 \times 12} = 0.37 \text{ in.}$$

$$\frac{c}{d_t} = \frac{\frac{0.37}{0.85}}{5.5} = 0.079 < 0.375, \text{ therefore tension controlled.}$$ 9.3.2.2 10.3.4

$$\left(d - \frac{a}{2}\right) = \frac{5.5 - \frac{0.37}{2}}{12} = 0.44 \text{ ft}$$

At center of span,

$\phi M_n = 0.9 \times (15.1) \times 0.44 = 5.98$ ft-kips/ft > 4.15 OK at midspan

Check positive moment capacity in Span 1:

$$\left(d - \frac{a}{2}\right) = \frac{(6.5 - 2.25) - \frac{0.37}{2}}{12} = 0.39 \text{ ft}$$

$$\frac{c}{d_t} = \frac{\frac{0.37}{0.85}}{4.25} = 0.102 < 0.375, \text{ therefore, tension controlled}$$ 9.3.2.2 10.3.4

$\phi M_n = 0.9 \times (15.1) \times 0.39 = 5.30$ ft-kips/ft > 2.89 OK at midspan

Exterior columns:

A_s minimum = 0.00075 × 20 × 12 × 6.5 = 1.17 in² use 6-#4 bars

A_s = 6 × 0.2/20 = 0.06 in²/ft

A_sf_y = 0.06 × 60 = 3.6 kips/ft

ρ_p = 10 × 0.153/(12 × 20 × 3.25) = 0.00196

f_{ps} = 175 + 10 + 4/(300 × 0.00196) = 192 ksi

$A_sf_{ps} = 10 \times 0.153 \times 192/20 = 14.7$ kips/ft

$$a = \frac{14.7+3.6}{0.85 \times 4 \times 12} = 0.45 \text{ in}$$

$\varepsilon_t = (5.5\text{-}0.53) \times 0.003/0.53 = 0.028$, therefore, tension controlled, $\phi = 0.9$ *9.3.2* *10.3.4*

Tendons:

$$\left(d - \frac{a}{2}\right) = \frac{(3.25) - \frac{0.45}{2}}{12} = 0.25 \text{ ft}$$

Rebar:

$$\left(d - \frac{a}{2}\right) = \frac{(5.5) - \frac{0.45}{2}}{12} = 0.44 \text{ ft}$$

$\phi M_n = 0.9 \times [(14.7 \times 0.25) + (3.6 \times 0.44)] = 4.73$ ft-kips/ft > 0.13 OK

This completes the design for flexural strength.

9. Shear and Moment Transfer Strength at Exterior Column *11.12.6* *13.5.3*

   a. Shear and moment transferred at exterior column.

      $V_u = (0.170 \times 17/2) - (6.65 - 0.61)/17 = 1.09$ kips/ft

      Assume building enclosure is masonry and glass, weighing 0.40 kips/ft.

      Total slab shear at exterior column:

      $V_u = [(1.2 \times 0.40) + 1.09] \times 20 = 31.4$ kips

      Transfer moment $= 20\,(0.61) = 12.2$ ft-kips
      (factored moment at exterior column centerline $= 0.61$ ft-kips/ft)

   b. Combined shear stress at inside face of critical transfer section.

      For shear strength equations, see Part 16.

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u c}{J}$$ *R11.12.6.2*

where (referring to Table 16-2: edge column-bending perpendicular to edge)

$d \approx 0.8 \times 6.5 = 5.2$ in.

$c_1 = 12$ in.

$c_2 = 14$ in.

$b_1 = c_1 + d/2 = 14.6$ in.

$b_2 = c_2 + d = 19.2$ in.

$$c = \frac{b_1^2}{(2b_1 + b_2)} = 4.40 \text{ in.}$$

$A_c = (2b_1 + b_2)\,d = 252$ in.^{2}

$J/c = [2b_1 d\,(b_1 + 2b_2) + d^3\,(2b_1 + b_2)/b_1]/6 = 1419$ in^3

$$\gamma_v = 1 - \gamma_f$$ *Eq. (11-39)*

$$= 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} = 0.37$$ *13.5.3.2*

$$v_u = \frac{31400}{252} + \frac{0.37 \times 12.2 \times 12000}{1419} = 163 \text{ psi}$$

c. Permissible shear stress (for members without shear reinforcement). *11.12.6.2*

$\phi v_n = \phi V_c/(b_o d)$ *Eq. (11-20)*
where V_c is defined in 11.12.2.1 or 11.12.2.2

For edge columns:

$\phi v_n = \phi 4\sqrt{f_c'} = 0.85 \times 4\sqrt{4000} = 215$ psi > 163 O.K. *11.12.2.1*

d. Check moment transfer strength. *13.5.3*

Although the transfer moment is small, for illustrative purposes, check the moment strength of the effective slab width (width of column plus 1.5 times the slab thickness on each side) for moment transfer. Assume that of the 10 tendons required for the 20 ft bay width, 3 tendons are anchored within the column and are bundled together across *13.5.3.2*

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28

Strength Evaluation of Existing Structures

INTRODUCTION

Chapter 20 was revised in 1995 to flag the need to monitor during load tests not only deflections, but also cracks related to shear and/or bond, along with spalling and crushing of the concrete. In cases involving deterioration of the structure, acceptance of a building should be based on a load test. Further, the acceptance should include a time limit. Periodic inspections and strength reevaluations should be specified depending on the nature of the deterioration. When structure dimensions, size and location of reinforcement, and material properties are known, higher strength reduction factors were introduced in ACI 318-95 for analytical evaluations of the strength of existing structures.

Strength evaluation of an existing structure requires experience and sound engineering judgment. Chapter 20 provides guidance for investigating the safety of a structure when:

1. Materials of a building are considered to be deficient in quality.
2. There is evidence indicating faulty construction.
3. A building has deteriorated.
4. A building will be used for a new function.
5. A building or a portion of it does not appear to satisfy the requirements of the code.

The provisions of Chapter 20 should not be used for approval of special systems of design and construction. Approval of such systems is covered in 1.4.

References 28.1 and 28.2 published by the Concrete Reinforcing Steel Institute (CRSI) are suggested additional guides for strength evaluation of existing structures. Information about reinforcing steel found in old reinforced concrete structures is given in CRSI Engineering Data Report Number 11.[28.3]

20.1 STRENGTH EVALUATION - GENERAL

Strength evaluation of structures can be performed analytically or experimentally. Applicability of the analytical procedure depends on whether the source of deficiency is critical to the structure's strength under: (1) flexural and/or axial load, or (2) shear and/or bond. The behavior and strengths of structural concrete under flexural and/or axial load strengths can be accurately predicted based on Navier's hypothesis of "plane section before loading remains plane after loading." On the other hand, available theories and models are not as reliable to predict the shear and bond behavior and strengths of structural concrete. Code provisions for one- and two-way shear, and for bond are semi-empirical. Shear and bond failures can be brittle.
Analytical strength evaluations suffice for acceptance of buildings if two conditions are met (20.1.2). First, the source of deficiency should be critical to flexural, axial load, or combined flexural and axial load strengths. It cannot be critical to shear or bond strengths. Second, it should be possible to establish the actual building dimensions, size and location of reinforcement, and material properties. If both conditions are not met, strength evaluations should be determined by a load test as prescribed in 20.3. If causes of concern relate to flexure or axial load, but it is not possible or feasible to determine material properties, a physical test may be appropriate.

Analytical evaluations of shear strength are not precluded if they are "well understood." If shear or bond strength is critical to the safety concerns, physical test may be the most efficient solution. Wherever possible and appropriate, it is desirable to support the results of the load tests by analysis (R20.1.3).

If the safety concerns are due to deterioration, strength evaluation may be through a load test. If the building satisfies the acceptance criteria of 20.5, the building should be allowed to remain in service for a specified period of time as a function of the nature of the deterioration. Periodic reevaluations of the building should be conducted.

20.2 DETERMINATION OF REQUIRED DIMENSIONS AND MATERIAL PROPERTIES

If strength evaluation of a building is performed through analysis, actual dimensions, location and size of reinforcement, and material properties should be established. Measurements should be taken at critical sections where calculated stress would reach a maximum value. When shop drawings are available, spot checks should be made to confirm location and size of reinforcing bars shown on the drawings. Nondestructive testing techniques are available to determine location and size of reinforcement, and estimate the strength of concrete. Unless they are already known, actual properties of reinforcing steel or prestressing tendons should be determined from samples extracted from the structure.

An analytical strength evaluation requires the use of the load factors of 9.2 and the strength reduction factors of 20.2.5. One of the purposes of the strength reduction factors ϕ given in R9.3.1 is "to allow for the probability of understrength members due to variations in material strengths and dimensions." When actual member dimensions, size and location of reinforcement, and concrete and reinforcing steel properties are measured, Chapter 20 specifies higher strength reduction factors. A comparison of the strength reduction factors of 20.2.5 to those of 9.3 is given in Table 28-1. The ratios of strength reduction factors of Chapter 20 to those of Chapter 9 are listed in the last column of the table. For analytical evaluation of columns and bearing on concrete, strength reduction factors ϕ of 20.2.5 are about 20 percent higher than those of 9.3. For flexure in beams and axial tension, the increase is 11 percent, while for shear and torsion it is 6 percent.

An increase in strength reduction factors, as specified in Chapter 20, results in an increase in computed member strengths. Nominal axial compressive strength of columns is in great part a function of the product of the column cross sectional area and the concrete compressive strength. As concrete compressive strength is subject to large variability, the strength reduction factors of Chapter 9 are lower for axial compression than for flexure. Because the actual concrete compressive strength is measured for strength evaluation of existing structures (20.1.2), a higher increase in strength reduction factor ϕ is specified for columns in 20.2.5.

Table 28-1 Comparison of Strength Reduction Factors

	Strength reduction factor		
	Ch. 20	Ch. 9	Ch. 20/Ch. 9
Tension-controlled sections, as defined in 10.3.4	1.00	0.90	1.11
Compression-controlled sections, as defined in 10.3.3			
Members with spiral reinforcement conforming to 10.9.3	0.85	0.75	1.21
Other reinforced members	0.80	0.70	1.23
Shear and torsion	0.80	0.75	1.07
Bearing on concrete	0.80	0.65	1.23

20.3 LOAD TEST PROCEDURE

The number and arrangement of spans or panels loaded should be selected to maximize the deflection and stresses in the critical regions of the structural elements of which strength is in doubt (20.3.1). If adjoining elements are expected to contribute to the load carrying capacity, magnitude of the test load or placement should be adjusted to compensate for this contribution. As in earlier editions of the code, the total test load is specified at 0.85 (1.4D + 1.7L), where D is the sum of dead loads or related internal moments and forces, and L is defined as the live loads or related internal moments and forces. The total test load includes the dead load already in place (20.3.2). The portion of the structure being load tested should be at least 56 days old, unless all concerned parties agree to conduct the test at an earlier age (20.3.3).

Note, starting with ACI 318-02, the load factors and strength reduction factors were revised in 9.2 and 9.3, respectively. In spite of these changes, the test load intensity prescribed in 20.3.2 has remained unchanged. Committee 318 felt that it was appropriate to maintain the same factored test load intensity, for designs complying with the new load factors and strength reduction factors of Chapter 9.

20.4 LOADING CRITERIA

Loading criteria are specified in 20.4. Initial values of all response measurements (deflection, strain, crack width, etc.) should be read and recorded not more than one hour before load application. When simulating uniformly distributed loads, arching of the applied loads must be avoided. Figure 28-1 illustrates arching action.[28.1] Sufficient gap should be provided between loading stacks so as to prevent contact, and hence arching, after member deflection, while assuring stability of the test loads.

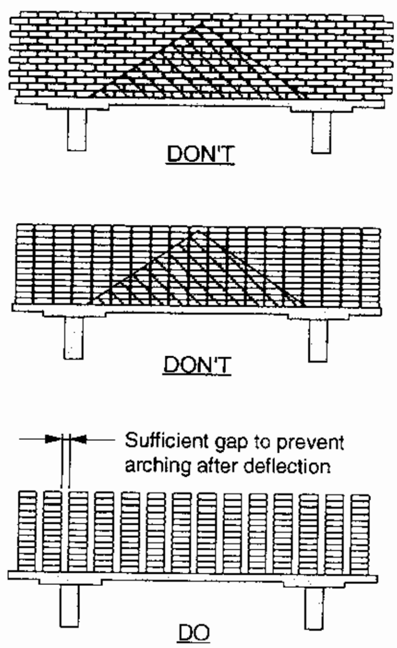

Figure 28-1 Arching Effect Shifts Applied Load to Ends of Span

Test load should be applied in not less than four approximately equal increments. A set of test load and response measurements is to be recorded after each load increment and after the total load has been applied for at least 24 hours. A set of final response measurements is to be recorded 24 hours after the test load is removed.

20.5 ACCEPTANCE CRITERIA

Evidence of failure includes spalling or crushing of concrete (20.5.1), excessive deflections (20.5.2), shear cracks (20.5.3 and 20.5.4), and bond cracks (20.5.5). No simple rules can be developed for application to all types of structures and conditions. However, in members without transverse reinforcement, projection of diagonal (inclined) cracks on an axis parallel to the longitudinal axis of the member should be monitored. If the projection of any diagonal crack is longer than the member depth at mid length of the crack, the member may be deficient in shear. If sufficient damage has occurred so that the structure is considered to have failed that test, retesting is not permitted since it is considered that damaged members should not be put into service even at a lower rating.

Deflection criteria must satisfy the following conditions (20.5.2):

1. When maximum deflection exceeds $\ell_t^2/(20{,}000h)$, the percentage recovery must be at least 75 percent after 24 hours, where

   h = overall thickness of member, in.

   ℓ_t = span of member under load test, in. (The shorter span for two-way slab systems.) Span is the smaller of (a) distance between centers of supports, and (b) clear distance between supports plus thickness h of member. Span for a cantilever must be taken as twice the distance from support to cantilever end, in.

2. When maximum deflection is less than $\ell_t^2/(20{,}000h)$, recovery requirement is waived. Figures 28-2 and 28-3 illustrate application of the limiting deflection criteria to the first load test. Figure 28-2 illustrates the limiting deflection versus member thickness for a sample span of 20 ft. Figure 28-3 depicts the limiting deflection versus span for a member 8 in. thick.

3. Members failing to meet the 75 percent recovery criterion may be retested.

4. Before retesting, 72 hours must have elapsed after load removal. On retest, the recovery must be 80 percent.

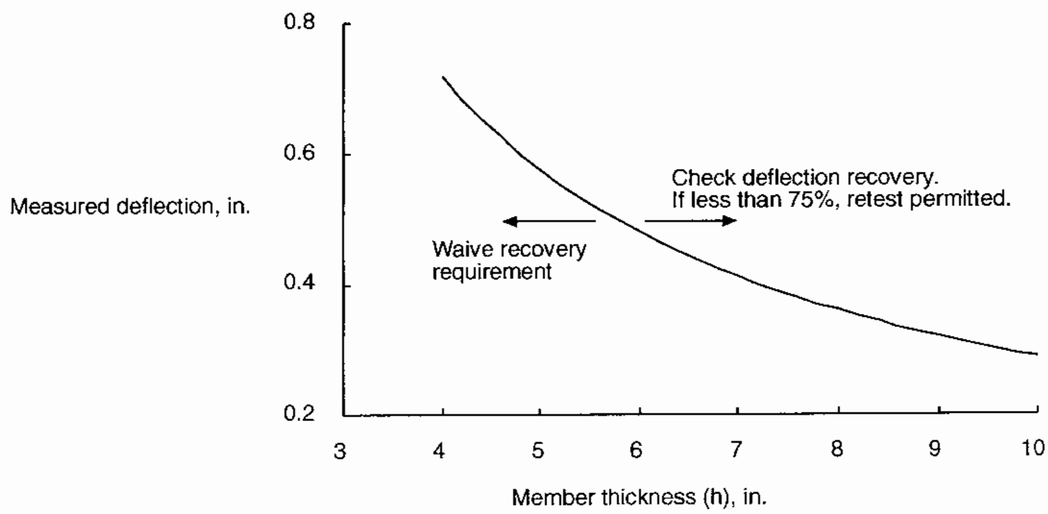

Figure 28-2 Load Testing Acceptance Criteria for Members with Span Length ℓ_t = 20 ft

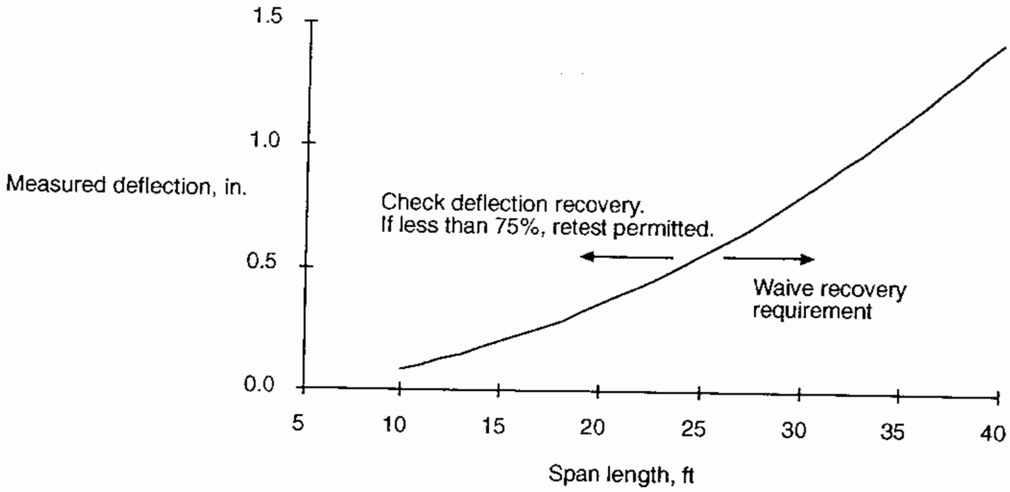

Figure 28-3 Load Testing Criteria for Members with Overall Thickness h = 8 in.

20.6 PROVISION FOR LOWER LOAD RATING

If analytical strength evaluations (20.1.2) indicate that a structure is inadequate, if the deflections of 20.5.2 are exceeded, or if cracks criteria of 20.5.3 are not met, the structure can be used for a lower load rating, if approved by the building official.

20.7 SAFETY

During load testing, shoring normally must be provided under the loaded members to assure safety. The shoring must not interfere with the test procedure or affect the test results. At no time during the load test should the deformed structure touch or bear against the shoring.

REFERENCES

28.1 "Applications of ACI 318 Load Test Requirements," Structural Bulletin No. 16, Concrete Reinforcing Steel Institute, Schaumburg, IL, November 1987.

28.2 "Proper Load Tests Protect the Public," Engineering Data Report Number 27, Concrete Reinforcing Steel Institute, Schaumburg, IL.

28.3 "Evaluation of Reinforcing Steel in Old Reinforced Concrete Structures," Engineering Data Report Number 48, Concrete Reinforcing Steel Institute, Schaumburg, IL, 2001, 4 pp.

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29

Special Provisions for Seismic Design

UPDATE FOR THE '05 CODE

A new term, design story drift ratio, has been defined as the relative difference of design displacements between the top and the bottom of a story, divided by the story height.

Sections 9.4 and 10.9.3 have been modified to allow the use of spiral reinforcement with specified yield strength of up to 100 ksi. A provision added to 21.2.5 specifically prohibits such use in members resisting earthquake-induced forces in structures assigned to Seismic Design Category D, E, or F.

Section 21.5.4 modifies the development length requirements of Chapter 12 for longitudinal beam bars terminating at exterior beam-column joints of structures assigned to high seismic design categories. But then 21.7.2.3 of ACI 318-02 required that all continuous reinforcement in structural walls must be anchored or spliced in accordance with the provisions for reinforcement in tension in 21.5.4. Section 21.9.5.4 of ACI 318-02 further required that all continuous reinforcement in diaphragms, trusses, ties, chords, and collector elements to be anchored or spliced in accordance with the provisions for reinforcement in tension as specified in 21.5.4. Sections 21.7.2.3 and 21.9.5.4 were very confusing to the user, because 21.5.4 is really not applicable to situations covered by those sections. This problem existed with ACI 318 editions prior to 2002 as well.

In a very significant and beneficial change, the requirements of 21.7.2.3 have been modified to remove the reference to beam-column joints in 21.5.4. All reference now is directly to the provisions of Chapter 12. The requirement that mechanical splices of reinforcement conform to 21.2.6, and welded splices to 21.2.7, has now been placed in 21.7.2.3. Consequently, 21.7.6.4(f) of 21.7.6.6 of ACI 318-02 have been deleted.

In a companion change, 21.9.5.4 now requires that all continuous reinforcement in diaphragms, trusses, struts, ties, chords, and collector elements be developed or spliced for f_y in tension.

Structural truss elements, struts, ties, diaphragm chords, and collector elements with compressive stresses exceeding $0.2 f'_c$ at any section are required to be specially confined by 21.9.5.3. The special transverse reinforcement may be discontinued at a section where the calculated compressive stress is less than $0.15 f'_c$. Stresses are calculated for factored forces using a linear elastic model and gross-section properties of the elements considered. In recent seismic codes and standards, collector elements of diaphragms are required to be designed for forces amplified by a factor, to account for the overstrength in the vertical elements of the seismic-force-resisting system. The amplification factor ranges between 2 and 3 for concrete structures, depending upon the document selected and on the type of seismic system. To account for this amplification factor, 21.9.5.3 now additionally states that where design forces have been amplified to account for the overstrength of the vertical elements of the seismic-force-resisting system, the limits of $0.2 f'_c$ and $0.15 f'_c$ shall be increased to $0.5 f'_c$ and $0.4 f'_c$, respectively.

In a very significant change, provisions for shear reinforcement at slab-column joints have been added in a new section 21.11.5, to reduce the likelihood of punching shear failure in two-way slabs without beams. A prescribed amount and detailing of shear reinforcement is required unless either 21.11.5(a) or (b) is satisfied.

BACKGROUND

Special provisions for earthquake resistance were first introduced into the 1971 edition of the ACI code in Appendix A, and were included with minor revisions in ACI 318-77. The original provisions of Appendix A were intended to apply only to reinforced concrete structures located in regions of high seismicity, and designed with a substantial reduction in total lateral seismic forces (as compared with the elastic response forces), in anticipation of inelastic structural behavior. Also, several changes were incorporated into the main body of the 1971 code specifically to improve toughness, in order to increase the resistance of concrete structures to earthquakes or other catastrophic loads. While Appendix A was meant for application to lateral force-resisting frames and walls in regions of high seismicity, the main body of the code was supposed to be sufficient for regions where there is a probability of only moderate or light earthquake damage.

The special provisions of Appendix A were extensively revised for the 1983 code, to reflect current knowledge and practice of the design and detailing of monolithic reinforced concrete structures for earthquake resistance. Appendix A to ACI 318-83 for the first time included special detailing for frames in zones of moderate seismic hazard.

The special provisions for earthquake resistance formed Chapter 21 of the 1989 code edition. This move from an appendix into the main body of the code was made for reasons discussed in Part 1 of this publication.

For buildings located in regions of low seismic risk, or for structures assigned to low seismic performance or design categories, no special design or detailing is required; the general requirements of Chapters 1 through 18 and 22 of the code apply. Concrete structures proportioned by Chapters 1 through 18 and 22 of the code are expected to provide a level of toughness adequate for structures subject to low seismic risk.

For buildings located in regions of moderate seismic hazard, or for structures assigned to intermediate seismic performance or design categories, moment frames proportioned to resist earthquake effects require some additional reinforcement details. The additional detailing requirements apply only to frames (beams, columns, and slabs) to which earthquake-induced forces have been assigned in design. There are no additional requirements for structural walls provided to resist the effects of wind and earthquakes, or for structural components that are not designed to be part of the lateral force-resisting system a of building located in regions of moderate seismic hazard, except for the connection requirements in 21.13 for intermediate precast structural walls. Structural walls proportioned by the non-seismic provisions of the code are considered to have sufficient toughness at drift levels anticipated in structures subject to moderate seismic risk.

For buildings located in regions of high seismic hazard, or for structures assigned to high seismic performance or design categories, where damage to construction has a high probability of occurrence, all structural components, irrespective of whether they are included in the lateral force-resisting system, must satisfy the requirements of 21.2 through 21.10. The special proportioning and detailing provisions of Chapter 21 are intended to produce a concrete structure with adequate toughness to respond inelastically under severe earthquake excitation.

GENERAL CONSIDERATIONS

Economical earthquake-resistant design should aim at providing appropriate dynamic characteristics in structures so that acceptable response levels would result under the design earthquake. The structural properties that can be modified to achieve the desired results are the magnitude and distribution of stiffness and mass and the relative strengths of the structural members.

In some structures, such as slender free-standing towers or smoke stacks, which depend for their stability on the stiffness of the single element making up the structure, or in nuclear containment buildings where a more-than-usual conservatism in design is required, yielding of the principal elements in the structure cannot be tolerated. In such cases, the design needs to be based on an essentially elastic response to moderate-to-strong earthquakes, with the critical stresses limited to the range below yield.

In most buildings, particularly those consisting of frames and other multiply-redundant systems, however, economy is achieved by allowing yielding to take place in some members under moderate-to-strong earthquake motion.

The performance criteria implicit in most earthquake code provisions require that a structure be able to:

1. Resist earthquakes of minor intensity without damage; a structure would be expected to resist such frequent but minor shocks within its elastic range of stresses.

2. Resist moderate earthquakes with negligible structural damage and some nonstructural damage; with proper design and construction, it is expected that structural damage due to the majority of earthquakes will be repairable.

3. Resist major catastrophic earthquakes without collapse; some structural and nonstructural damage is expected.

The above performance criteria allow only for the effects of a typical ground shaking. The effects of landslides, subsidence or active faulting in the immediate vicinity of the structure, which may accompany an earthquake, are not considered.

While no clear quantitative definition of the above earthquake intensity ranges has been given, their use implies the consideration not only of the actual intensity level but also of their associated probability of occurrence with reference to the expected life of a structure.

The principal concern in earthquake-resistant design is the provision of adequate strength and toughness to assure life safety under the most intense earthquake expected at a site during the life of a structure. Observations of building behavior in recent earthquakes, however, have made engineers increasingly aware of the need to ensure that buildings that house facilities essential to post-earthquake operations—such as hospitals, power plants, fire stations and communication centers—not only survive without collapse, but remain operational after an earthquake. This means that such buildings should suffer a minimum amount of damage. Thus, damage control is at times added to life safety as a second design consideration.

Often, damage control becomes desirable from a purely economic point of view. The extra cost of preventing severe damage to the nonstructural components of a building, such as partitions, glazing, ceiling, elevators and other mechanical systems, may be justified by the savings realized in replacement costs and from continued use of a building after a strong earthquake.

The principal steps involved in the earthquake-resistant design of a typical concrete structure according to building code provisions are as follows:

1. Determination of seismic zone, or seismic performance, or design category
   Seismic design category combines the seismic hazard at the site of the structure, the occupancy of the structure, and the soil characteristics at the site of the structure. It's a relatively new concept, for an understanding of which, the reader may consult Ref. 29.1. Seismic performance category is a function only of the seismic hazard at the site of the structure and the occupancy of the structure. Seismic zone considers only the seismic hazard at the site of the structure.

2. Determination of design earthquake forces
   a. calculation of base shear corresponding to computed or estimated fundamental period of vibration of the structure (a preliminary design of the structure is assumed here)
   b. distribution of the base shear over the height of the building

3. Analysis of the structure under the (static) lateral earthquake forces calculated in step 1, as well as under gravity and wind loads, to obtain member design forces.

4. Designing members and joints for the critical combinations of gravity and lateral (wind or seismic) loads.

5. Detailing members for ductile behavior in accordance with the seismic zone, or the seismic performance or design category of the structure.

It is important to note that some buildings are required to be designed by a dynamic, rather than a static, lateral force procedure when one or more criteria of the static procedure are not satisfied.

In the International Building Code (IBC)[29.2], as well as in the model codes that preceeded it, the design base shear represents the total horizontal seismic force that may be assumed acting parallel to the axis of the structure considered. The force in the other horizontal direction is usually assumed to act non-concurrently. Depending on the building and the seismic zone or seismic performance or design category, the seismic forces may need to be applied in the direction that produces the most critical load effect. The requirement that orthogonal effects be considered in the proportioning of a structural element may be satisfied by designing the element for 100 percent of the prescribed seismic forces in one direction plus 30 percent of the prescribed forces in the perpendicular direction. The combination requiring the greater component strength must be used for design. The vertical component of the earthquake ground motion is included in the load combinations involving earthquake forces that are prescribed in the IBC. Special provisions are also required for structural elements that are susceptible to vertical earthquake forces (cantilever beams and slabs; prestressed members).

The code-specified design lateral forces have a general distribution that is compatible with the typical envelope of maximum horizontal shears indicated by elastic dynamic analyses for regular structures. However, the code forces are substantially smaller than those that would be developed in a structure subjected to the anticipated earthquake intensity, if the structure were to respond elastically to such ground excitation. Thus, buildings designed under the present codes would be expected to undergo fairly large deformations when subjected to a major earthquake. These large deformations will be accompanied by yielding in many members of the structure, which is the intent of the codes. The reduced code-specified forces must be coupled with additional requirements for the design and detailing of members and their connections in order to ensure sufficient deformation capacity in the inelastic range.

The capacity of a structure to deform in a ductile manner (i.e., to deform beyond the yield limit without significant loss of strength), allows such a structure to dissipate a major portion of the energy from an earthquake without collapse. Laboratory tests have demonstrated that cast-in-place and precast concrete members and their connections, designed and detailed by the present codes, do possess the necessary ductility to allow a structure to respond inelastically to earthquakes of major intensity without significant loss of strength.

21.2 GENERAL REQUIREMENTS

21.2.1 Scope

Sections 21.2.1.2 through 21.2.1.4 contain the required detailing requirements based on the structural framing system, seismic hazard level at the site, level of energy dissipation planned in the structural design, and the occupancy of the building.

Traditionally, seismic risk levels have been classified as low, moderate, and high. The seismic risk level, or the seismic performance or design category of a building is regulated by the legally adopted building code of the region or is determined by a local authority (1.1.8.3). Table R1.1.8.3 contains a summary of the seismic risk levels, seismic performance categories (SPC), and seismic design categories (SDC) specified in the IBC, the three prior model building codes now called *legacy* codes, as well as other resource documents (see R21.2.1).

The provisions of Chapters 1 through 18 and Chapter 22 of ACI 318 apply to structures in regions of low seismic hazard or to structures assigned to low seismic performance or design categories (21.2.1.2). The design and detailing requirements of these chapters are intended to provide adequate toughness for structures in these regions or assigned to these categories. Ordinary moment frames (cast-in-place or precast) and ordinary structural walls are the structural systems that can be utilized. It is important to note that the requirements of Chapter 21 apply when the design seismic forces are computed using provisions for intermediate or special concrete systems.

In regions of moderate seismic hazard or for structures assigned to satisfy intermediate seismic performance or design categories, intermediate or special moment frames, or ordinary, intermediate, or special structural walls shall be used (21.2.1.3). Provisions for intermediate moment frames and intermediate precast structural walls are contained in 21.12 and 21.13, respectively.

Special moment frames (cast-in-place or precast), special structural walls (cast-in-place or precast), and diaphragms and trusses complying with 21.2.2 through 21.2.8 and 21.3 through 21.10 shall be used in regions of high seismic hazard or for structures assigned to satisfy high seismic performance or design categories (21.2.1.4). Members not proportioned to resist earthquake forces shall comply with 21.11. The provisions of 21.2.2 through 21.2.8 and 21.3 through 21.11 have been developed to provide adequate toughness should the design earthquake occur.

The requirements of Chapter 21 as they apply to various structural components are summarized in Table R21.2.1.

21.2.2 Analysis and Proportioning of Structural Members

The interaction of all structural and nonstructural components affecting linear and nonlinear structural response are to be considered in the analysis (21.2.2.1). Consequences of failure of structural and nonstructural components not forming part of the lateral force-resisting system shall also be considered (21.2.2.2). The intent of 21.2.2.1 and 21.2.2.2 is to draw attention to the influence of nonstructural components on structural response and to hazards from falling objects.

Section 21.2.2.3 alerts the designer to the fact that the base of the structure as defined in analysis may not necessarily correspond to the foundation or ground level. It requires that structural members below base, which transmit forces resulting from earthquake effects to the foundation, shall also comply with the requirements of Chapter 21.

Even though some element(s) of a structure may not be considered part of the lateral force-resisting system, the effect on all elements due to the design displacements must be considered (21.2.2.4).

21.2.3 Strength Reduction Factors

The strength reduction factors of 9.3.2 are not based on the observed behavior of cast-in-place or precast concrete members under load or displacement cycles simulating earthquake effects. Some of those factors have been modified in 9.3.4 in view of the effects on strength due to large displacement reversals into the inelastic range of response.

Section 9.3.4(a) refers to members such as low-rise walls or portions of walls between openings, which are proportioned such as to make it impractical to raise their nominal shear strength above the shear corresponding to nominal flexural strength for the pertinent loading conditions.

21.2.4, 21.2.5 Limitations on Materials

A minimum specified concrete compressive strength f'_c of 3,000 psi and a maximum specified reinforcement yield strength f_y of 60,000 psi are mandated. These limits are imposed as reasonable bounds on the variation of material properties, particularly with respect to their unfavorable effects on the sectional ductilities of members in which they are used. A decrease in the concrete strength and an increase in the yield strength of the tensile reinforcement tend to decrease the ultimate curvature and hence the sectional ductility of a member subjected to flexure.

The statement in 21.2.1.1, referencing 1.1.1, helps to clarify that no maximum specified compressive strength applies. Limitations on the compressive strength of lightweight aggregate concrete is discussed below.

There is evidence suggesting that lightweight concrete ranging in strength up to 12,500 psi can attain adequate ultimate strain capacities. Testing to examine the behavior of high-strength, lightweight concrete under high-intensity, cyclic shear loads, including a critical study of bond characteristics, has not been extensive in the past. However, there are test data showing that properly designed lightweight concrete columns, with concrete strength ranging up to 6,200 psi, maintained ductility and strength when subjected to large inelastic deformations from load reversals. Committee 318 feels that a limit of 5,000 psi on the strength of lightweight concrete is advisable, pending further testing of high-strength lightweight concrete members under reversed cyclic loading. Note that lightweight concrete with a higher design compressive strength is allowed if it can be demonstrated by experimental evidence that structural members made with that lightweight concrete possess strength and toughness equal to or exceeding those of comparable members made with normal weight concrete of the same strength.

Chapter 21 requires that reinforcement for resisting flexure and axial forces in frame members and wall boundary elements be ASTM A 706 Grade 60 low-alloy steel, which is intended for applications where welding or bending, or both, are important. However, ASTM A 615 billet steel bars of Grade 40 or 60 may be used in these members if the following two conditions are satisfied:

$$\text{actual } f_y \leq \text{specified } f_y + 18{,}000 \text{ psi}$$

$$\frac{\text{actual tensile strength}}{\text{actual } f_y} \geq 1.25$$

The first requirement helps to limit the magnitude of the actual shears that can develop in a flexural member above that computed on the basis of the specified yield strength of the reinforcement when plastic hinges form at the ends of a beam. Note that retests shall not exceed this value by more than an additional 3,000 psi. The second requirement is intended to ensure steel with a sufficiently long yield plateau.

In the "strong column-weak beam" frame intended by the code, the relationship between the moment strengths of columns and beams may be upset if the beams turn out to have much greater moment strengths than intended. Thus, the substitution of Grade 60 steel of the same area for specified Grade 40 steel in beams can be detrimental. The shear strength of beams and columns, which is generally based on the condition of plastic hinges forming at the ends of the members, may become inadequate if the moment strengths of member ends would be greater than intended as a result of the steel having a substantially greater yield strength than specified.

Sections 9.4 and 10.9.3 have been modified to allow the use of spiral reinforcement with specified yield strength of up to 100 ksi. A sentence added to 21.2.5 specifically prohibits such use in members resisting earthquake-induced forces in structures assigned to Seismic Design Category D, E, or F. This is largely the result of some misgiving that high-strength spiral reinforcement may be less ductile than conventional mild reinforcement and that spiral failure has in fact been observed in earthquakes. There are fairly convincing arguments, however, against such specific prohibitions. Spiral failure, primarily observed in bridge columns, have invariably been the result of insufficient spiral reinforcement, rather than the lack of ductility of the spiral reinforcement. Also, prestressing steel, which is the only high-strength steel available on this market, is at least as ductile as welded wire reinforcement which is allowed to be used as transverse reinforcement.

21.2.6 Mechanical Splices

Section 21.2.6 contains provisions for mechanical splices. According to 21.2.6.1, a Type 1 mechanical splice shall conform to 12.14.3.2, i.e., the splice shall develop in tension or compression at least 125 percent of the specified yield strength f_y of the reinforcing bar. A Type 2 mechanical splice shall also conform to 12.14.3.2 and shall develop the specified tensile strength of the spliced bar.

During an earthquake, the tensile stresses in the reinforcement may approach the tensile strength of the reinforcement as the structure undergoes inelastic deformations. Thus, Type 2 mechanical splices can be used at any location in a member (21.2.6.2). The locations of Type 1 mechanical splices are restricted since the tensile stresses in the reinforcement in yielding regions of the member can exceed the strength requirements of 12.14.3.2. Consequently, Type 1 mechanical splices are not permitted within a distance equal to twice the member depth from the face of the column or beam or from sections where yielding of the reinforcement is likely to occur due to inelastic lateral displacements (21.2.6.2).

21.2.7 Welded Splices

The requirements for welded splices are in 21.2.7. Welded splices shall conform to the provisions of 12.14.3.4, i.e., the splice shall develop at least 125 percent of the specified yield strength f_y of the reinforcing bar (21.2.7.1). Similar to Type 1 mechanical splices, welded splices are not permitted within a distance equal to twice the member depth from the face of the column or beam or from sections where yielding of the reinforcement is likely to occur due to inelastic lateral displacements; in yielding regions of the member, the tensile stresses in the reinforcement can exceed the strength requirements of 12.14.3.4 (21.2.7.1).

According to 21.2.7.2, welding of stirrups, ties, inserts or other similar elements to longitudinal reinforcement that is required by design is not permitted. Welding of crossing reinforcing bars can lead to local embrittlement of the steel. If such welding will facilitate fabrication or field installation, it must be done only on bars added expressly for construction. Note that this provision does not apply to bars that are welded with welding operations under continuous competent control, as is the case in the manufacture of welded wire reinforcement.

21.2.8 Anchoring to Concrete

The requirements in this section pertain to anchors resisting earthquake-induced forces in structures located in regions of moderate or high seismic hazard. The design of such anchors must conform to the additional requirements of D.3.3 of Appendix D. See Part 34 for additional information.

21.3 FLEXURAL MEMBERS OF SPECIAL MOMENT FRAMES

The left-hand column of Table 29-1 contains the requirements for flexural members of special moment frames (as noted above, special moment frames, which can be cast-in-place or precast, are required in regions of high seismic hazard or for structures assigned to satisfy high seismic performance or design categories). These requirements typically apply to beams of frames and other flexural members with negligible axial loads (21.3.1). Special precast moment frames must also satisfy the provisions of 21.6, which are discussed below. For comparison purposes, Table 29-1 also contains the corresponding requirements for flexural members of intermediate and ordinary cast-in-place moment frames. See Chapter 16 and Part 23 for additional information on precast systems.

21.3.1 Scope

Flexural members of special moment frames must meet the general requirements of 21.3.1.1 through 21.3.1.4. These limitations have been guided by experimental evidence and observations of reinforced concrete frames that have performed well in past earthquakes. Members must have sufficient ductility and provide efficient moment transfer to the supporting columns. Note that columns subjected to bending and having a factored axial load $P_u \leq A_g f'_c/10$ may be designed as flexural members, where A_g is the gross area of the section.

Table 29-1 Flexural Members of Frames

	Special Moment Frames	Intermediate and Ordinary CIP Moment Frames
General	Flexural frame members shall satisfy the following conditions: • Factored axial compressive force ≤ $A_g f_c' / 10$ • Clear span ≥ 4 × effective depth • Width to depth ratio ≥ 0.3 • Width ≥ 10 in. • Width ≤ width of supporting member + distances on each side of the supporting member not exceeding three-fourths of the depth of the flexural member **21.3.1**	Intermediate — Factored axial compressive force ≤ $A_g f_c' / 10$. **21.12.2** Ordinary — No similar requirements.
Flexural Requirements	Minimum reinforcement shall not be less than $\frac{3\sqrt{f_c'}\,b_w d}{f_y}$ and $\frac{200\,b_w d}{f_y}$ at any section, top and bottom, unless provisions in 10.5.3 are satisfied. **21.3.2.1**	Same requirement, except as provided in 10.5.2, 10.5.3, and 10.5.4, although minimum reinforcement need only be provided at sections where tensile reinforcement is required by analysis. **10.5**
	The reinforcement ratio (ρ) shall not exceed 0.025. **21.3.2.1**	The net tensile strain ε_t at nominal strength shall not be less than 0.004. **10.3.5**
	At least two bars shall be provided continuously at both top and bottom of section. **21.3.2.1**	Provide minimum structural integrity reinforcement. **7.13**
	Positive moment strength at joint face ≥ 1/2 negative moment strength at that face of the joint. **21.3.2.2**	Intermediate — Positive moment strength at joint face ≥ 1/3 negative moment strength at that face of the joint. **21.12.4.1** Ordinary — No similar requirement.
	Neither the negative nor the positive moment strength at any section along the member shall be less than 1/4 the maximum moment strength provided at the face of either joint. **21.3.2.2**	Intermediate — Same requirement, except it is needed to provide only 1/5 of the maximum moment strength at the face of either joint at every section along the member. **21.12.4.1** Ordinary — No similar requirement.

— continued on next page —

A_g = gross area of section

b_w = width of web

d = effective depth of section

f_c' = specified compressive strength of concrete

f_y = specified yield strength of reinforcement

Table 29-1 Flexural Members of Frames (cont'd)

	Special Moment Frames	Intermediate and Ordinary CIP Moment Frames
Splices	Lap splices of flexural reinforcement are permitted only if hoop or spiral reinforcement is provided over the lap length. Hoop and spiral reinforcment spacing shall not exceed $d/4$ or 4 in. Mechanical splices shall conform to 21.2.6 and welded splices shall conform to 21.2.7. **21.3.2.3, 21.3.2.4**	There is no requirement that splices be enclosed in hoops.
Splices	Lap splices are not to be used: • Within joints. • Within a distance of twice the member depth from the face of the joint. • At locations where analysis indicates flexural yielding caused by inelastic lateral displacements of the frame. **21.3.2.3**	No similar requirement.
Transverse Reinforcement	Hoops are required over a length equal to twice the member depth from the face of the supporting member toward midspan at both ends of the flexural member. **21.3.3.1**	Intermediate — Same requirement. **21.12.4.2** Ordinary — No similar requirement.
Transverse Reinforcement	Hoops are required over lengths equal to twice the member depth on both sides of a section where flexural yielding may occur in connection with inelastic lateral displacements of the frame. **21.3.3.1**	Reinforcement for flexural members subject to stress reversals shall consist of closed stirrups extending around flexural reinforcement. Also, provide minimum structural integrity reinforcement. **7.11.2, 7.13**
Transverse Reinforcement	Where hoops are required, the spacing shall not exceed: $d/4$ 8 × diameter of smallest longitudinal bar 24 × diameter of hoop bars 12 in. The first hoop shall be located not more than 2 in. from the face of the supporting member. **21.3.3.2**	Intermediate — Same requirement. **21.12.4.2** Ordinary — No similar requirement.
Transverse Reinforcement	Where hoops are required, longitudinal bars on the perimeter shall have lateral support conforming to 7.10.5.3. **21.3.3.3**	No similar requirement.
Transverse Reinforcement	Where hoops are not required, stirrups with seismic hooks at both ends shall be spaced at a distance not more than $d/2$ throughout the length of the member. **21.3.3.4**	Intermediate — Similar requirement except that seismic hooks are not required. **21.12.4.3**
Transverse Reinforcement	Transverse reinforcment must also be proportioned to resist the entire design shear force, neglecting the contribution of concrete to shear strength, if certain conditions are met. **21.3.4**	Intermediate — Transverse reinforcement must also be proportioned to resist the design shear force. **21.12.3** Ordinary — Provide sufficient transverse reinforcement for shear and torsion. **11.5, 11.6**

21.3.2 Flexural Reinforcement

The reinforcement requirements for flexural members of special moment frames are shown in Fig. 29-1. To allow for the possibility of the positive moment at the end of a beam due to earthquake-induced lateral displacements exceeding the negative moment due to gravity loads, 21.3.2.2 requires a minimum positive moment strength at the ends of the beam equal to at least 50 percent of the corresponding negative moment strength. The minimum moment strength at any section of the beam is based on the moment strength at the faces of the supports. These requirements ensure strength and ductility under large lateral displacements. The limiting ratio of 0.025 is based primarily on considerations of steel congestion and also on limiting shear stresses in beams of typical proportions. The requirement that at least two bars be continuous at both the top and the bottom of the beam is for construction purposes.

The flexural requirements for flexural members of intermediate moment frames are similar to those shown in Fig. 29-1 (see Table 29-1).

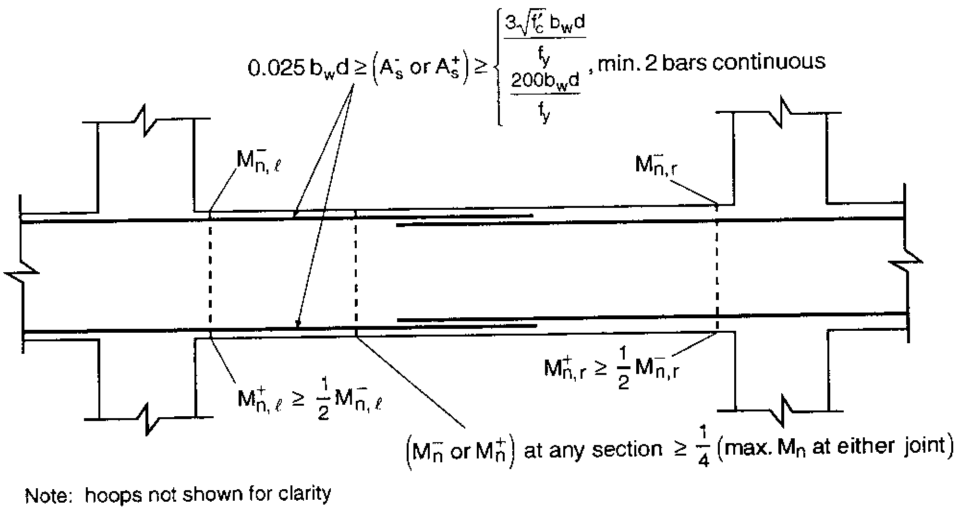
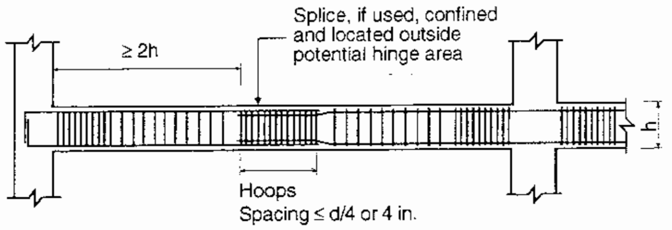


Figure 29-1 Reinforcement Requirements for Flexural Members of Special Moment Frames

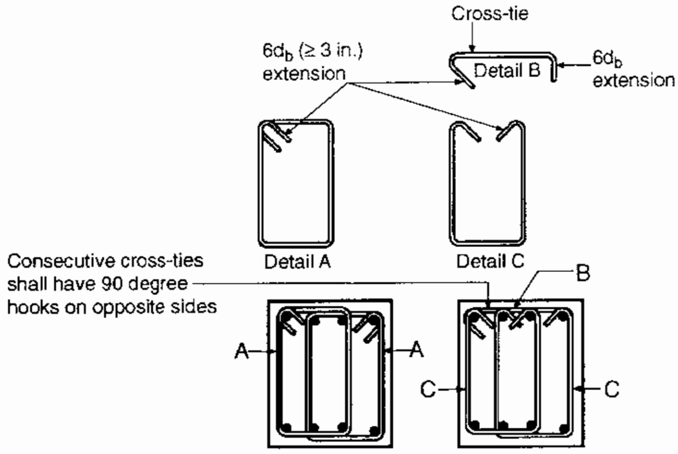
Lap splices of flexural reinforcement must be placed at locations away from potential hinge areas subjected to stress reversals under cyclic loading (see Fig. 29-2). Where lap splices are used, they should be designed as tension lap splices and must be properly confined. Mechanical splices and welded splices must conform to 21.2.6 and 21.2.7, respectively.

21.3.3 Transverse Reinforcement

Adequate confinement is required at the ends of flexural members, where plastic hinges are likely to form, in order to ensure sufficient ductility of the members under reversible loads. Transverse reinforcement is also required at these locations to assist the concrete in resisting shear and to maintain lateral support for the reinforcing bars. For flexural members of special moment frames, the transverse reinforcement for confinement must consist of hoops as shown in Fig. 29-3. Hoops must be used for confinement in flexural members of intermediate moment frames as well (21.12.4.2). Shear strength requirements for flexural members are given in 21.3.4 for special moment frames and 21.12.3 for intermediate moment frames.

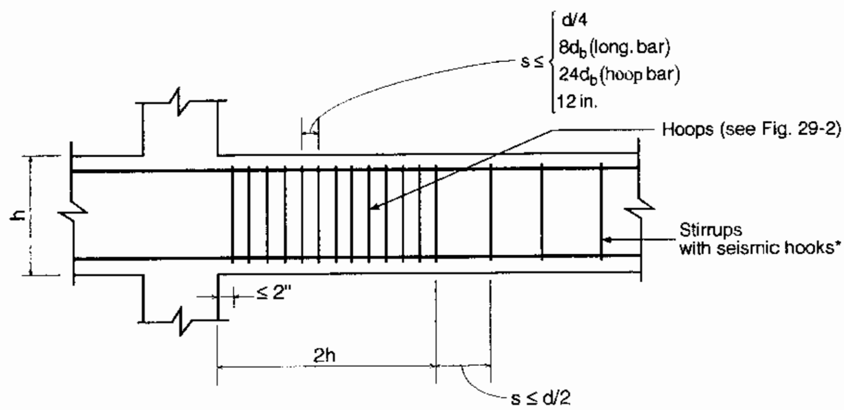


Typical Splice Details



Single- and Two-Piece Hoops

Figure 29-2 Splices and Hoop Reinforcement for Flexural Members of Special Moment Frames



*Seismic hooks are not required for intermediate moment frames

Figure 29-3 Transverse Reinforcement for Flexural Members of Special and Intermediate Moment Frames

21.3.4 Shear Strength Requirements

Typically, larger forces than those prescribed by the governing building code are induced in structural members during an earthquake. Designing for shear forces from a combined gravity and lateral load analysis using the code-prescribed load combinations is not conservative, since in reality the reinforcement may be stressed beyond its yield strength, resulting in larger than anticipated shear forces. Adequate shear reinforcement must be provided so as to preclude shear failure prior to the development of plastic hinges at the ends of the beam. Thus, a flexural member of a special moment frame must be designed for the shear forces associated with probable moment strengths M_{pr} acting at the ends and the factored tributary gravity load along its span (21.3.4.1). The probable moment strength M_{pr} is associated with plastic hinging in the flexural member, and is defined as the strength of the beam with the stress in the reinforcing steel equal to $1.25f_y$ and a strength reduction factor of 1.0:

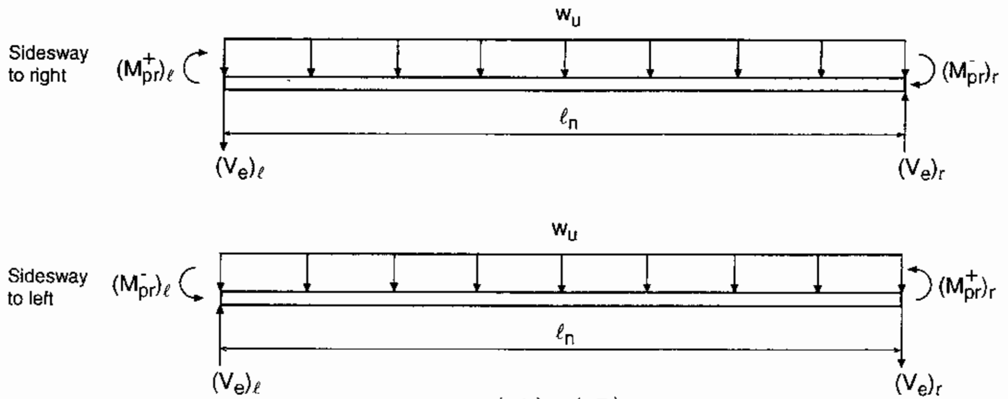
$$M_{pr} = A_s(1.25f_y)\left(d - \frac{a}{2}\right) \text{ (rectangular section with tension reinforcement only)}$$

where $a = \dfrac{A_s(1.25f_y)}{0.85f'_c b}$

Note that sidesway to the right and to the left must both be considered to obtain the maximum shear force (see Fig. 29-4). The use of $1.25f_y$ for the stress in the reinforcing steel reflects the possibility that the actual yield strength may be in excess of the specified value and the likelihood that the deformation in the tensile reinforcement will be in the strain-hardening range. By taking $1.25f_y$ as the stress in the reinforcement and 1.0 as the strength reduction factor, the chance of shear failure preceding flexural yielding is reduced.

In determining the required shear reinforcement over the lengths identified in 21.3.3.1, the contribution of the shear strength of the concrete V_c is taken as zero if the shear force from seismic loading is one-half or more of the required shear strength and the factored axial compressive force including earthquake effects is less than $A_g f'_c/20$ (21.3.4.2). The purpose of this requirement is to provide adequate shear reinforcement to increase the probability of flexural failure. Note that the strength reduction factor ϕ to be used is 0.75 or 0.85, depending on whether Chapter 9 or Appendix C load combinations are used (see 9.3.2.3 or C.3.2.3).

Shear reinforcement shall be in the form of hoops over the lengths specified in 21.3.3.1 (21.3.3.5); at or near regions of flexural yielding, spalling of the concrete shell is very likely to occur. Details of hoop reinforcement are given in 21.3.3.6 (see Fig. 29-2). Where hoops are not required, stirrups with seismic hooks at both ends may be used (21.3.3.4, 21.3.3.5). A minimum amount of transverse reinforcement is required throughout the entire length of flexural members to safeguard against any loading cases that were unaccounted for in design.



$$\text{Maximum } V_e = \frac{(M^{\pm}_{pr})_\ell + (M^{\mp}_{pr})_r}{\ell_n} + \frac{w_u \ell_n}{2}$$

For intermediate moment frames, use nominal moment strength (M_n).

Figure 29-4 Design Shear Forces for Flexural Members of Special Moment Frames

The transverse reinforcement provided within the lengths specified in 21.3.3.1 shall satisfy the requirement for confinement or shear, whichever governs.

A similar analysis is required for frame members of intermediate moment frames except that the nominal moment strength M_n of the member is used instead of the probable moment strength (21.12.3). Also to be found in 21.12.3 is an alternate procedure where the earthquake effects are doubled in lieu of using the nominal moment strength.

21.4 SPECIAL MOMENT FRAME MEMBERS SUBJECTED TO BENDING AND AXIAL LOAD

The left hand column of Table 29-2 contains the requirements for special moment frame members subjected to combined bending and axial loads. These requirements would typically apply to columns of frames and other flexural members that carry a factored axial load $P_u > A_g f_c'/10$. For comparison purposes, Table 29-2 also contains the corresponding requirements for intermediate and ordinary cast-in-place moment frame members subject to combined bending and axial loads.

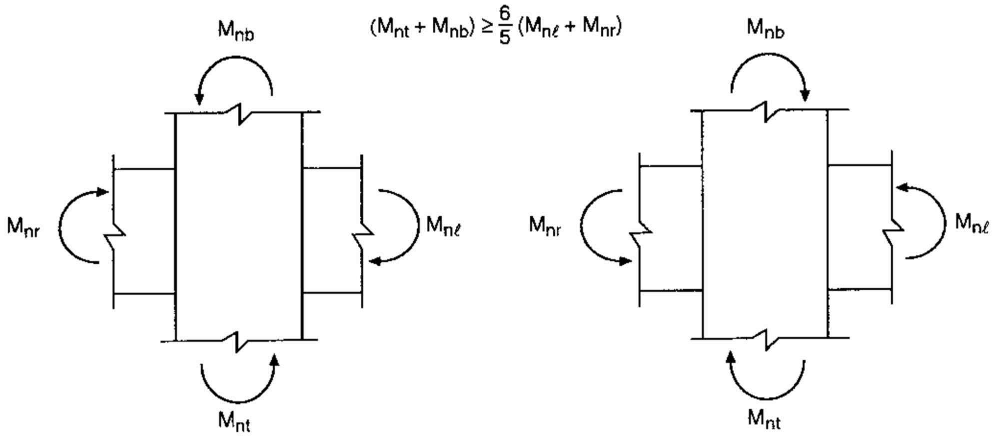
21.4.1 Scope

Section 21.4.1 is intended primarily for columns of special moment frames. Frame members other than columns that do not satisfy 21.3.1 are proportioned and detailed according to 21.4. The geometric constraints are largely reflective of prior practice. Unlike in the case of flexural members, a column-like member violating the dimensional limitations of 21.4.1 need not be excluded from the lateral force-resisting system, if it is designed as a wall in accordance with 21.7.

21.4.2 Minimum Flexural Strength of Columns

Columns must be provided with sufficient strength so that they will not yield prior to the beam at a beam-column joint. Lateral sway caused by column hinging may result in excessive damage. Yielding of the columns prior to the beams could also result in total collapse of the structure. For these reasons, columns are designed with 20% higher flexural strength as compared to beams meeting at the same joint, as shown in Fig. 29-5. In 21.4.2.2, nominal strengths of the columns and girder are calculated at the joint faces, and those strengths are used in Eq. (21-1). The column flexural strength is calculated for the factored axial load, consistent with the direction of the lateral forces considered, resulting in the lowest flexural strength.

$$(M_{nt} + M_{nb}) \geq \frac{6}{5}(M_{n\ell} + M_{nr})$$



Subscripts ℓ, r, t, and b stand for left support, right support, top of column, and bottom of column, respectively.

Figure 29-5 "Strong Column-Weak Beam" Frame Requirements for Special Moment Frames

When computing the nominal flexural strength of girders in T-beam construction, slab reinforcement within an effective slab width defined in 8.10 shall be considered as contributing to the flexural strength if the slab reinforcement is developed at the critical section for flexure. Research has shown that using the effective flange width in 8.10 gives reasonable estimates of the negative bending strength of girders at interior joints subjected to interstory displacements approaching 2% of the story height.

Table 29-2 Frame Members Subjected to Bending and Axial Loads

	Special Moment Frames	Intermediate and Ordinary CIP Moment Frames
General	Frame members under this classification must meet the following requirements: • Factored axial compressive force > $A_g f'_c$ / 10. • Shortest cross-sectional dimension ≥ 12 in. • Ratio of shortest cross-sectional dimension to perpendicular dimension ≥ 0.4. **21.4.1**	Intermediate — Factored axial compressive force > $A_g f'_c$ / 10. **21.12.2** Ordinary — No similar requirements.
Flexural Requirements	The flexural strengths of columns shall satisfy the following: $\Sigma M_c \geq (6/5)\, \Sigma M_g$ where ΣM_c = sum of moments at the faces of the joint, corresponding to the nominal flexural strengths of the columns. ΣM_g = sum of moments at the faces of the joint, corresponding to the nominal flexural strengths of the girders. In T-beam costruction, slab reinforcement within an effective slab width defined in 8.10 shall be considered as contributing to flexural strength. If this requirement is not satisfied, the lateral strength and stiffness of the column shall not be considered when determining the strength and stiffness of the structure, and the column shall conform to 21.11; also, the column must have transverse reinforcement over its full height as specified in 21.4.4.1 through 21.4.4.3. **21.4.2**	No similar requirements.
	The reinforcement ratio (ρ_g) shall not be less than 0.01 and shall not exceed 0.06. **21.4.3.1**	The reinforcement ratio (ρ_g) shall not be less than 0.01 and shall not exceed 0.08. For a compression member with a cross section larger than required by considerations of loading, the reinforcement ratio (ρ_g)can be reduced below 0.01, but never below 0.005 (10.8.4). **10.9**
Splices	Mechanical splices shall conform to 21.2.6 and welded splices shall conform to 21.2.7. Lap splices are permitted only within the center half of the member length, must be tension lap splices, and shall be enclosed within transverse reinforcement conforming to 21.4.4.2 and 21.4.4.3. **21.4.3.2**	There is no restriction on the location of splices which are typically located just above the floor for ease of construction.
Transverse Reinforcement	The transverse reinforcement requirements discussed in the following five items on the next page need only be provided over a length (ℓ_0) from each joint face and on both sides of any section where flexural yielding is likely to occur. The length (ℓ_0) shall not be less than: depth of member 1/6 clear span 18 in. **21.4.4.4**	Intermediate — The length (ℓ_0) is the same as for special moment frames. **21.12.5.2** Ordinary — No similar requirements.

— continued on next page —

A_{ch} = cross-sectional area of member measured out-to-out of transverse reinforcement
A_g = gross area of section
f'_c = specified compressive strength of concrete
f_{yt} = specified yield stress of transverse reinforcement
b_c = cross-sectional dimension of column core measured center-to-center of outer legs of the transverse reinforcement comprising area A_{sh}
h_x = maximum horizontal spacing of hoop or crosstie legs on all faces of the column
s = spacing of transverse reinforcement
s_o = longitudinal spacing of transverse reinforcement within the length ℓ_0.

Table 29-2 Frame Members Subjected to Bending and Axial Loads (cont'd)

	Special Moment Frame	Intermediate and Ordinary CIP Moment Frames
Transverse Reinforcement (continued)	Ratio of spiral reinforcement (ρ_s) shall not be less than the value given by: $$\rho_s = 0.12\frac{f'_c}{f_{yt}} \geq 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f'_c}{f_{yt}}$$ **21.4.4.1**	Ratio of spiral reinforcement (ρ_s) shall not be less than the value given by: $$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f'_c}{f_{yt}}$$ and shall conform to the provisions in 7.10.4. **10.9.3**
	Total cross-sectional area of rectangular hoop reinforcement for confinement (A_{sh}) shall not be less than that given by the following two equations: $$A_{sh} = 0.3\left(sb_c f'_c / f_{yt}\right)\left[\left(A_g / A_{ch}\right) - 1\right]$$ $$A_{sh} = 0.09\left(sb_c f'_c / f_{yt}\right)$$ **21.4.4.1**	Transverse reinforcement must be provided to satisfy both shear and lateral support requirements for longitudinal bars. **7.10.5, 11.1**
	If the thickness of the concrete outside the confining transverse reinforcement exceeds 4 in., additional transverse reinforcement shall be provided at a spacing ≤ 12 in. Concrete cover on the additional reinforcement shall not exceed 4 in. **21.4.4.1**	No similar requirements.
	Transverse reinforcement shall be spaced at distances not exceeding 1/4 minimum member dimension, 6 × longitudinal bar diameter, 4 in. ≤ $s_o = 4 + [(14 - h_x)/3] \leq 6$ in. **21.4.4.2**	Intermediate—Maximum spacing s_o is 8 × smallest longitudinal bar diameter, 24 × hoop bar diameter, 1/2 smallest cross-sectional dimension, or 12 in. First hoop to be located no further than $s_o/2$ from the joint face. **21.12.5.2** Ordinary — No similar requirement.
	Cross ties or legs of overlapping hoops shall not be spaced more than 14 in. on center in the direction perpendicular to the longitudinal axis of the member. Vertical bars shall not be farther than 6 in. clear from a laterally supported bar. **21.4.4.3, 7.10.5.3**	Vertical bars shall not be farther than 6 in. clear from a laterally supported bar. **7.10.5.3**
	Where the transverse reinforcement as discussed above is no longer required, the remainder of the column shall contain spiral or hoop reinforcement spaced at distances not to exceed 6 × longitudinal bar diameter 6 in. **21.4.4.6**	Intermediate — Outside the length ℓ_o, spacing of transverse reinforcement shall conform to 7.10 and 11.5.4.1. **21.12.5.4** Ordinary — Transverse reinforcement to conform to 7.10 and 11.5.4.1.
	Transverse reinforcement must also be proportioned to resist the design shear force (V_e). **21.4.5**	Intermediate — Transverse reinforcement must also be proportioned to resist the design shear forces specified in 21.12.3. Ordinary — Provide sufficient transverse reinforcement for shear. **11.5.4, 11.5.6**
	Columns supporting reactions from discontinued stiff members, such as walls, shall have transverse reinforcement as specified in 21.4.4.1 through 21.4.4.3 over their full height, if the factored axial compressive force, including earthquake effects, exceeds ($A_g f'_c$ / 10). This transverse reinforcement shall extend into the discontinued member for at least the development length of the largest longitudinal reinforcement in the column in accordance with 21.5.4. If the column terminates on a footing or mat, the transverse reinforcement shall extend at least 12 in. into the footing or mat. **21.4.4.5**	No similar requirement.

If Eq. (21-1) is not satisfied at a joint, columns supporting reactions from that joint are to be provided with transverse reinforcement as specified in 21.4.4 over their full height (21.4.2.3), and shall be ignored in determining the calculated strength and stiffness of the structure (21.4.2.1). These columns must also conform to the provisions for frame members not proportioned to resist earthquake motions as given in 21.11 (21.4.2.1).

No similar provisions are included for intermediate or ordinary moment frames.

21.4.3 Longitudinal Reinforcement

The maximum allowable reinforcement ratio is reduced from 8% to 6% for columns in special moment frames (21.4.3.1). This lower ratio prevents congestion of steel, which reduces the chance of improperly placed concrete. It also prevents the development of large shear stresses in the columns. Typically, providing a reinforcement ratio larger than about 3% is not practical or economical.

Mechanical splices shall conform to 21.2.6 and welded splices shall conform to 21.2.7 (21.4.3.2). When lap splices are used, they are permitted only within the center half of the member length and are to be designed as tension lap splices (see Fig. 29-6). Transverse reinforcement conforming to 21.4.4.2 and 21.4.4.3 is required along the length of the lap splice.

There are no restrictions on the location of lap splices in intermediate or ordinary moment frames.

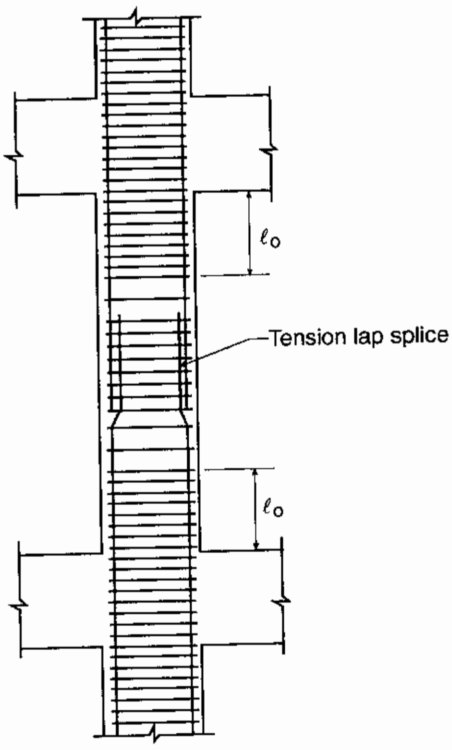
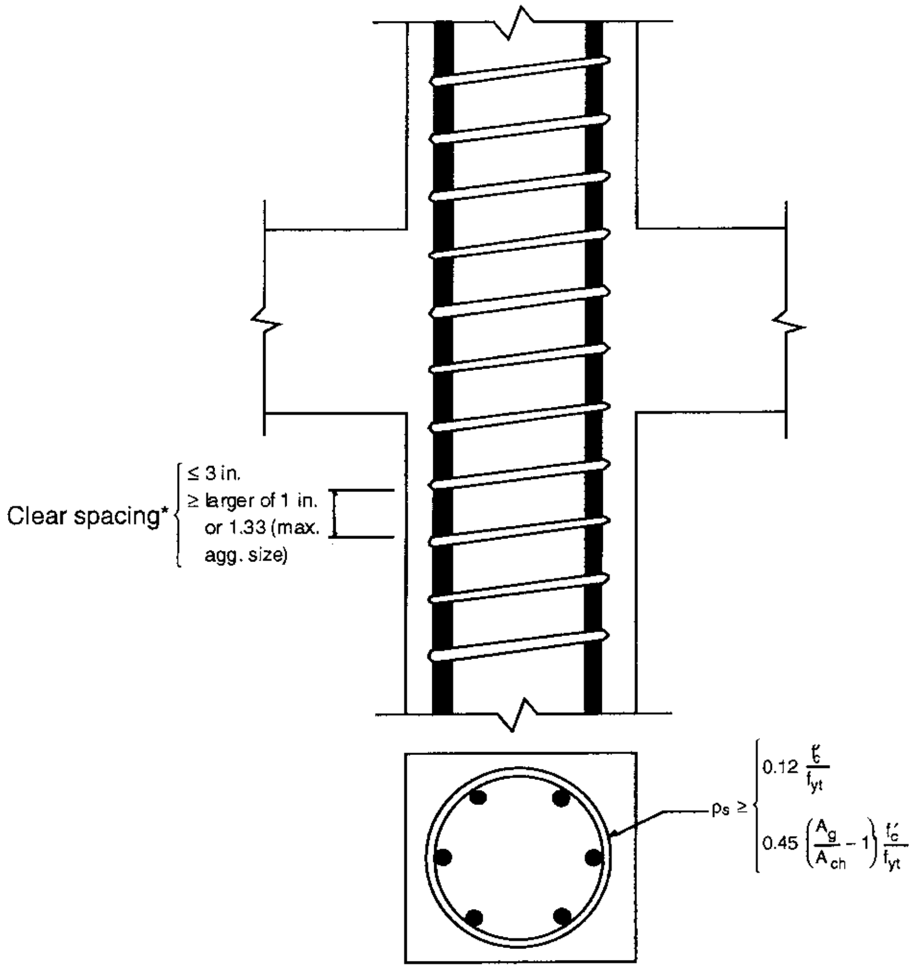


Figure 29-6 Typical Lap Splice Details for Columns in Special Moment Frames

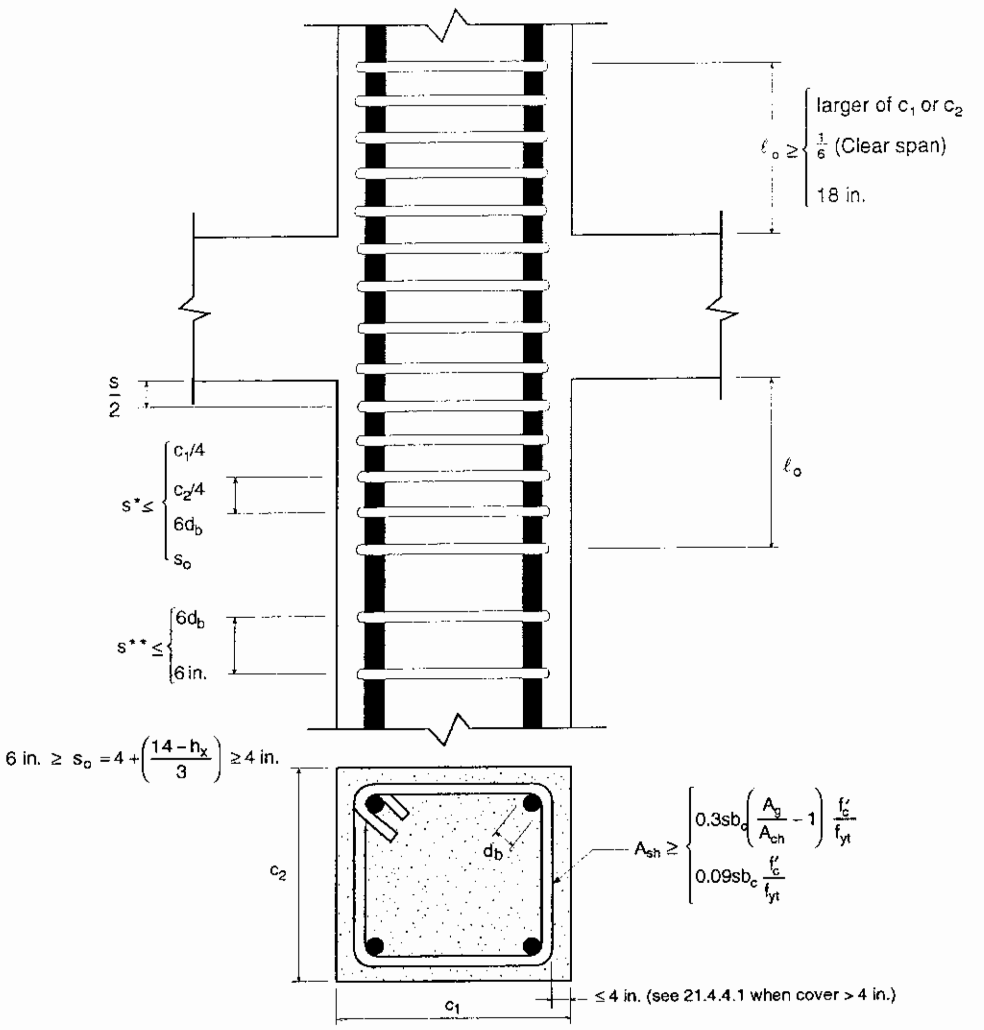
21.4.4 Transverse Reinforcement

Column ends require adequate confinement to ensure column ductility in the event of hinge formation. They also require adequate shear reinforcement in order to prevent shear failure prior to the development of flexural yielding at the column ends. The correct amount, spacing, and location of the transverse reinforcement must be provided so that both the confinement and the shear strength requirements are satisfied. For special moment frames, the transverse reinforcement must be spiral or circular hoop reinforcement or rectangular hoop reinforcement, as shown in Fig. 29-7. Spiral reinforcement is generally the most efficient form of confinement reinforcement; however, the extension of the spirals into the beam-column joint may cause some construction difficulties.



*Clear spacing for spiral reinforcement. Circular hoops to be spaced per 21.4.4.2.

Figure 29-7 Confinement Requirements at Column Ends
(a) spiral or circular hoop reinforcement



*For intermediate moment frames, s ≤ 8 × smallest long. bar diameter; 24 × transverse bar diameter; $\frac{1}{2}$ × smaller of c_1 or c_2; 12 in.

**For intermediate moment frames, s shall conform to 7.10 and 11.5.4.1

†Confinement requirements for special moment frames need not be satsified for intermediate moment frames

Figure 29-7 Confinement Requirements at Column Ends
(b) rectangular hoop reinforcement

Figure 29-8 shows an example of transverse reinforcement provided by one hoop and three crossties. 90-degree hooks are not as effective as 135-degree hooks. Confinement will be sufficient if crosstie ends with 90-degree hooks are alternated.

The requirements of 21.4.4.2 and 21.4.4.3 must be satisfied for the configuration of rectangular hoop reinforcement. The requirement that spacing not exceed one-quarter of the minimum member dimension is to obtain adequate concrete confinement. Restraining longitudinal reinforcement buckling after spalling is the rationale behind the spacing being limited to 6 bar diameters. Section 21.4.4.2 permits the 4 in. spacing for confinement to be relaxed to a maximum of 6 in. if the horizontal spacing of crossties or legs of overlapping hoops is limited to 8 in.

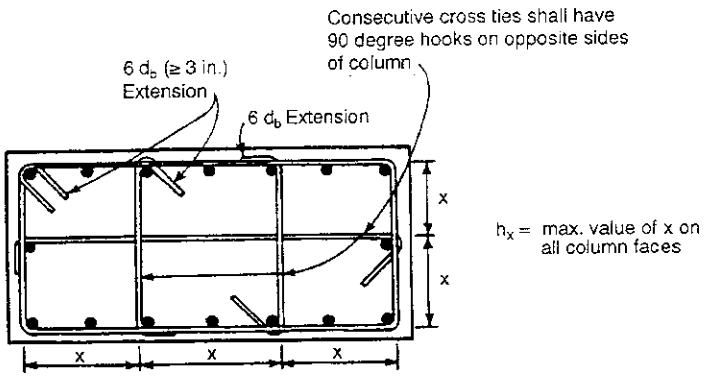


Figure 29-8 Transverse Reinforcement in Columns

Additional transverse reinforcement at a maximum spacing of 12 in. is required when concrete thickness outside the confining transverse reinforcement exceeds 4 in. This additional reinforcement will help reduce the risk of portions of the shell falling away from the column. The required amount of such reinforcement is not specifically indicated; the 1997 UBC [29.3] specifies a minimum amount equal to that required for columns that are not part of the lateral force-resisting system.

For columns supporting discontinued stiff members (such as walls) as shown in Fig. 29-9, transverse reinforcement in compliance with 21.4.4.1 through 21.4.4.3 needs to be provided over the full height of the column and must be extended at least the development length of the largest longitudinal column bars into the discontinued member (wall). The transverse reinforcement must also extend at least 12 in. into the footing or mat, if the column terminates on a footing or mat.

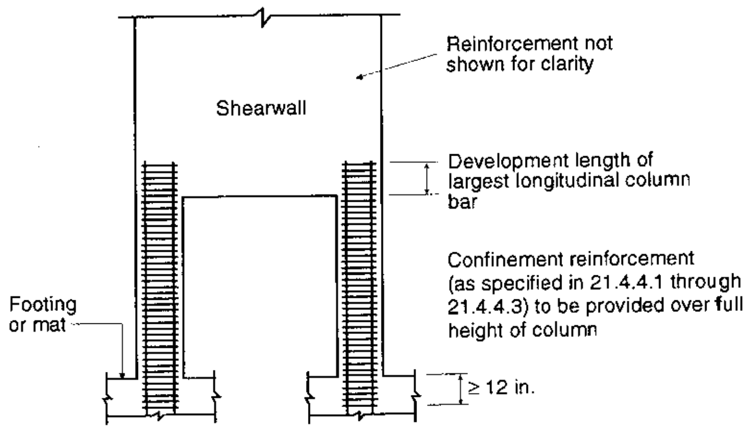


Figure 29-9 Columns Supporting Discontinued Stiff Members

As indicated in Fig. 29-7, there are transverse reinforcement requirements for columns of intermediate moment frames in 21.12.5.

21.4.5 Shear Strength Requirements

In addition to satisfying confinement requirements, the transverse reinforcement in columns must resist the maximum shear forces associated with the formation of plastic hinges in the frame (21.4.5.1). Although the provisions of 21.4.2 are intended to have most of the inelastic deformation occur in the beams, the provisions of 21.4.5.1 recognize that hinging can occur in the column. Thus, as in the case of beams, the shear reinforcement in the columns is based on the probable moment strengths M_{pr} that can be developed at the ends of the column.

The probable moment strength is to be the maximum consistent with the range of factored axial loads on the column; sidesway to the right and to the left must both be considered (see Fig. 29- 10). It is obviously conservative to use the probable moment strength corresponding to the balanced point.

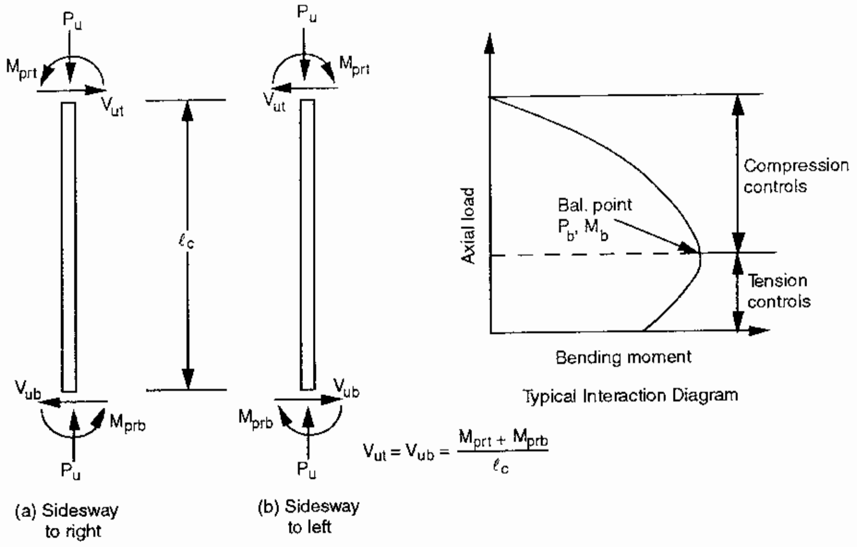


Figure 29-10 Loading Cases for Design of Shear Reinforcement in Columns of Special Moment Frames

Section 21.4.5.1 points out that the column shear forces need not exceed those determined from joint strengths based on the probable moment strengths of the beams framing into the joint. When beams frame on opposite sides of a joint, the combined probable moment strength may be taken as the sum of the negative probable moment strength of the beam on one side of the joint and the positive probable moment strength of the beam on the other side. The combined probable moment strength of the beams is then distributed appropriately to the columns above and below the joint, and the shear forces in the column are computed based on this distributed moment. It is important to note that in no case is the shear force in the column to be taken less than the factored shear force determined from analysis of the structure under the code-prescribed seismic forces (21.4.5.1).

Provisions for proportioning the transverse reinforcement are contained in 21.4.5.2. As in the case of beams, the strength reduction factor ϕ to be used wth the Chapter 9 load combinations is 0.75 (see 9.3.4 and 9.3.2.3).

The shear forces in intermediate frame members subjected to combined bending and axial force are determined in the same manner as for flexural members of intermediate moment frames, i.e., nominal moment strengths at member ends are used to compute the shear forces (21.12.3).

21.5 JOINTS OF SPECIAL MOMENT FRAMES

The overall integrity of a structure is dependent on the behavior of the beam-column joint. Degradation of the joint can result in large lateral deformations which can cause excessive damage or even failure. The left-hand column of Table 29-3 contains the requirements for joints of special moment frames. For intermediate and ordinary cast-in-place frames, the beam-column joints do not require the special design and detailing requirements as for special moment frames. It may be prudent, however, to apply the same line of thinking to intermediate frame joints as to special moment frame joints.

Slippage of the longitudinal reinforcement in a beam-column joint can lead to an increase in the joint rotation. Longitudinal bars must be continued through the joint or must be properly developed for tension (21.5.4) and compression (Chapter 12) in the confined column core. The minimum column size requirement of 21.5.1.4 reduces the possibility of failure from loss of bond during load reversals that take the steel beyond its yield point.

21.5.2 Transverse Reinforcement

The transverse reinforcement in a beam-column joint is intended to provide adequate confinement of the concrete to ensure its ductile behavior and to allow it to maintain its vertical load-carrying capacity even after spalling of the outer shell.

Table 29-3 Joints of Frames

	Special Moment Frames	Intermediate and Ordinary CIP Moment Frames
Longitudinal Beam Reinforcement	Beam longitudinal reinforcement terminated in a column shall be extended to the far face of the confined column core and anchored in tension according to 21.5.4 and in compression according to Chapter 12. **21.5.1.3**	No similar requirement.
	Where longitudinal beam reinforcement extends through a joint, the column dimension parallel to the beam reinforcement shall not be less than 20 times the diameter of the largest longitudinal bar for normal weight concrete. For lightweight aggregate concrete, this dimension shall be not less than 26 times the bar diameter. **21.5.1.4**	No similar requirements.
Transverse Reinforcement	The transverse hoop reinforcement required for column ends (21.4.4) shall be provided within the joint, unless the joint is confined by structural members as specified in 21.5.2.2. If members frame into all four sides of the joint and the member width at the column face is at least 3/4 the column width, the transverse reinforcement can be reduced to 50% of the requirements of 21.4.4.1 within the depth of the shallowest member. The spacing required in 21.4.4.2 shall not exceed 6 in. at these locations. **21.5.2.1, 21.5.2.2**	No similar requirement.

f'_c = specified compressive strength of concrete

f_y = specified yield strength of reinforcement

Table 29-3 Joints of Frames (cont'd)

	Special Moment Frames	Intermediate and Ordinary CIP Moment Frames
Shear Strength	The nominal shear strength of the joint shall not exceed the forces specified below for normal-weight aggregate concrete. For joints confined on all four faces .... $20\sqrt{f'_c}\ A_j$ For joints confined on three faces or on two opposite faces .... $15\sqrt{f'_c}\ A_j$ For other joints: .... $12\sqrt{f'_c}\ A_j$ where: A_j = effective cross-sectional area within a joint in a plane parallel to the plane of the reinforcement generating shear in the joint. The joint depth shall be the overall depth of the column. Where a beam frames into a support of larger width, the effective width of the joint shall not exceed the smaller of: 1. Beam width plus the joint depth. 2. Twice the smaller perpendicular distance from the longitudinal axis of the beam to the column side. A joint is considered to be confined if confining members frame into all faces of the joint. A member that frames into a face is considered to provide confinement at the joint if at least 3/4 of the face of the joint is covered by the framing member. **21.5.3**	Although it is not required, it may be prudent to check the shear strength of the joint in intermediate moment frames. The force in the longitudinal beam reinforcement may be taken as $1.0f_y$ rather than the $1.25f_y$ required for special moment frames.
	In determining shear forces in the joints, forces in the longitudinal beam reinforcement at the joint face shall be calculated by assuming that the stress in the flexural tensile reinforcement is $1.25f_y$. **21.5.1.1**	No similar requirement.
	For lightweight aggregate concrete, the nominal shear strength of the joint shall not exceed 3/4 of the limits given in 21.5.3.1. **21.5.3.2**	No similar requirement.

Minimum confinement reinforcement of the same amount required for potential hinging regions in columns, as specified in 21.4.4, must be provided within a beam-column joint around the column reinforcement, unless the joint is confined by structural members as specified in 21.5.2.2.

For joints confined on all four faces, a 50% reduction in the amount of confinement reinforcement is allowed. A member that frames into a face is considered to provide confinement to the joint if at least three-quarters of the face of the joint is covered by the framing member. The code further allows that where a 50% reduction in the amount of confinement reinforcement is permissible, the spacing specified in 21.4.4.2(b) may be increased to 6 in. (21.5.2.2).

The minimum amount of confinement reinforcement, as noted above, must be provided through the joint regardless of the magnitude of the calculated shear force in the joint. The 50% reduction in the amount of confinement reinforcement allowed for joints having horizontal members framing into all four sides recognizes the beneficial effect provided by these members in resisting the bursting pressures that can be generated within the joint.

21.5.3 Shear Strength

The most significant factor in determining the shear strength of a beam-column joint is the effective area A_j of the joint, as shown in Fig. 29-11. For joints that are confined by beams on all four faces, the shear strength of the joint is equal to $20\sqrt{f'_c}\,A_j$. If the joint is confined only on three faces, or on two opposite faces, the strength must be reduced by 25% to $15\sqrt{f'_c}\,A_j$. For other cases, the shear strength is equal to $12\sqrt{f'_c}\,A_j$. It is important to note that the shear strength is a function of the concrete strength and the cross-sectional area only. Test results show that the shear strength of the joint is not altered significantly with changes in transverse reinforcement, provided a minimum amount of such reinforcement is present. Thus, only the concrete strength or the member size can be modified if the shear strength of the beam-column joint is inadequate. The strength reduction factor ϕ for shear in joints is 0.85 (9.3.4).

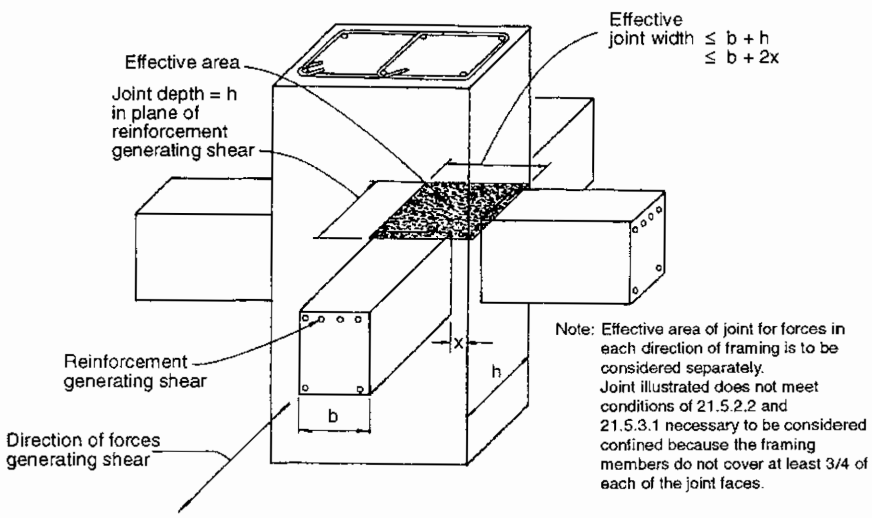


Figure 29-11 Effective Area of Joint (A_j)

The larger the tension force in the steel, the greater the shear in the joint (Fig. 29-12). Thus, the tensile force in the reinforcement is conservatively taken as $1.25f_yA_s$. The multiplier of 1.25 takes into account the likelihood that due to strain-hardening and actual strengths higher than the specified yield strengths, a larger tensile force may develop in the bars, resulting in a larger shear force.

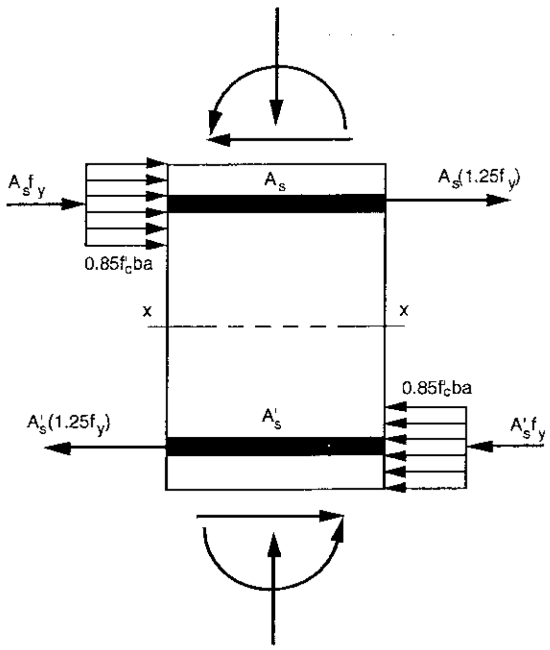


Figure 29-12 Horizontal Shear in Beam-Column Joint

21.5.4 Development Length of Bars in Tension

A standard 90-degree hook located within the confined core of a column or boundary element is depicted in Fig. 29-13. Equation (21-6), based on the requirements of 12.5, includes the factors for hooks enclosed in ties (0.8), satisfaction of minimum cover requirements (0.7), a cyclic load factor (1.1), and a factor of 1.25 for overstrength in the reinforcing steel. The equation for the development length in 12.5.2 $\left[\left(0.02\beta\lambda f_y / \sqrt{f_c'}\right) d_b\right]$ is multiplied by these factors to obtain the equation that is given in 21.5.4.1 for uncoated reinforcing bars with $f_y = 60{,}000$ psi embedded in normal weight concrete:

$$\ell_{dh} = \frac{0.8 \times 0.7 \times 1.25 \times 1.1 \times 0.02 \times 1.0 \times 1.0 \times 60{,}000 \times d_b}{\sqrt{f_c'}}$$

$$= \frac{924 d_b}{\sqrt{f_c'}} = \frac{f_y d_b}{65\sqrt{f_c'}}$$

For bar sizes No. 3 through No. 11, the development length ℓ_{dh} for a bar with a standard 90-degree hook in normal- weight aggregate concrete shall not be less than the largest of $8d_b$, 6 in., and the length obtained from Eq. (21-6) shown above. For lightweight aggregate concrete, the development length shall be increased by 25%.

The development length for No. 11 and smaller straight bars is determined by multiplying the development length for hooked bars required by 21.5.4.1 by (a) two-and-a-half (2.5) if the depth of the concrete cast in one lift beneath the bar does not exceed 12 in., and (b) three-and-a-half (3.5) if the depth of the concrete cast in one lift beneath the bar exceeds 12 in. (21.5.4.2). If a portion of a straight bar is not located within the confined core of a column or boundary element, the length of that bar shall be increased by an additional 60%. Provisions for epoxy-coated bars are given in 21.5.4.4.

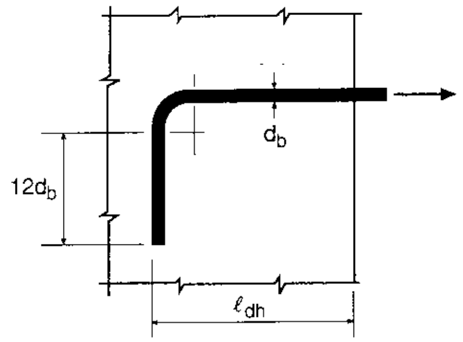


Figure 29-13 Standard 90-Degree Hook

21.6 SPECIAL MOMENT FRAMES CONSTRUCTED USING PRECAST CONCRETE

In addition to the requirements of 21.2 through 21.5, special moment frames constructed using precast concrete must satisfy the requirements of 21.6. The detailing provisions in 21.6.1 for frames with ductile connections and 21.6.2 for frames with strong connections are intended to produce frames that respond to design displacements essentially like cast-in-place special moment frames. Section 21.6.3 provides a design procedure for special moment frames that do not satisfy the appropriate prescriptive requirements of Chapter 21.

21.6.1 Special Moment Frames with Ductile Connections

Special moment frames with ductile connections are designed and detailed so that flexural yielding occurs within the connection regions. Type II mechanical splices or any other technique that provides development in tension and compression of at least the specified tensile strength of the bars and $1.25f_y$, respectively, can be used to make the reinforcement continuous in the connections.

According to 21.6.1(a), the nominal shear strength V_n at the connection must be computed in accordance with the shear-friction design method of 11.7.4. In order to help prevent sliding at the faces of the connection, V_n must be greater than or equal to $2V_e$, where V_e is the design shear force in the beams that is computed according to 21.3.4.1 or the design shear force in the columns that is computed according to 21.4.5.1. Since the ductile connections may be at locations that are not adjacent to the joints, using V_e may be conservative.

Mechanical splices of beam reinforcement must satisfy the requirements of 21.2.6 and must be located at least h/2 from the face of the joint, where h is the overall depth of the beam. This additional requirement is intended to avoid strain concentrations over a short length of reinforcement adjacent to a splice device.

21.6.2 Special Moment Frames with Strong Connections

Special moment frames with strong connections are designed and detailed so that flexural yielding occurs away from the connection regions. Examples of beam-to-beam, beam-to-column, and column-to-footing connections are shown in Fig. R21.6.2.

According to 21.6.2(a), the geometric constraint in 21.3.1.2 related to the clear span to effective depth ratio must be satisfied for any segment between locations where flexural yielding is intended to occur due to the design displacements.

To ensure that strong connections remain elastic and do not slip following the formation of plastic hinges, the design strength of the connection, fS_n, in both flexure and shear must be greater than or equal to the bending moment and shear force, S_e, respectively, corresponding to the development of probable flexural or shear strengths at intended locations of flexural or shear yielding (21.6.2(b)). These provisions are illustrated in Figs. 29-14 and 29-15 for a beam-to-beam and a beam-to-column strong connection, respectively, with sidesway to the right. Sidesway to the left must also be considered.

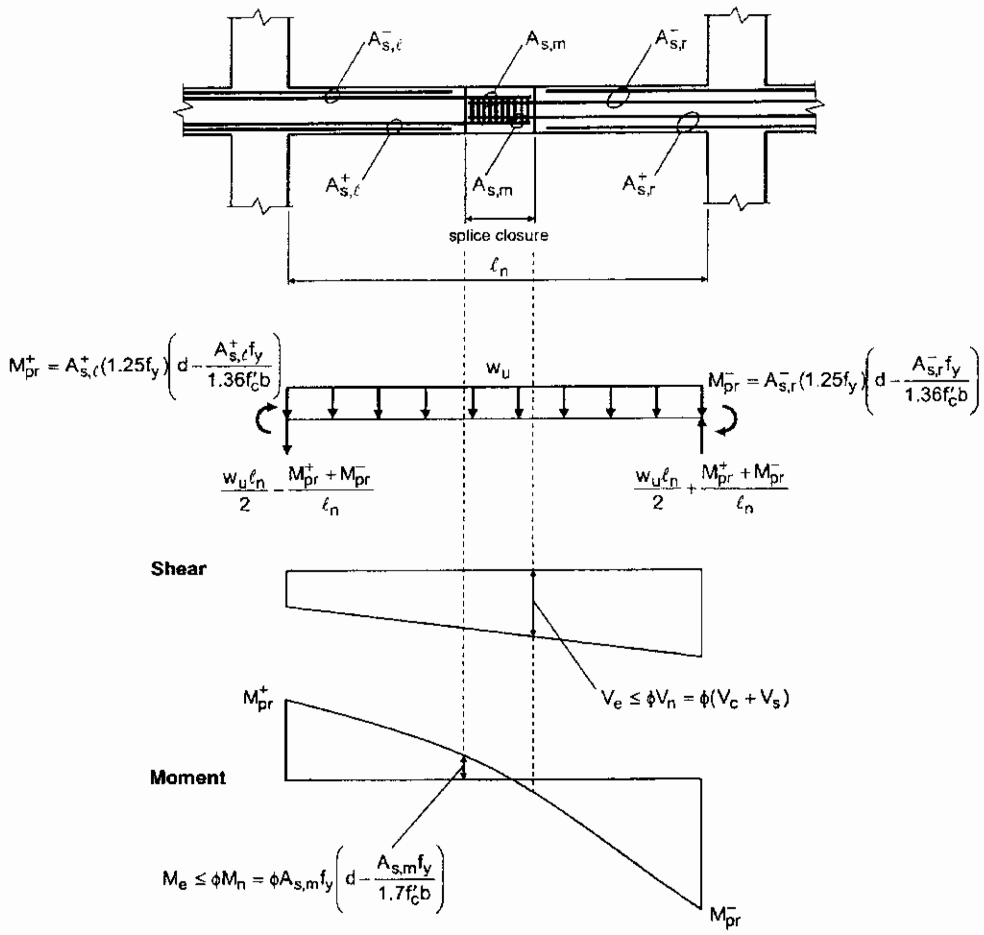
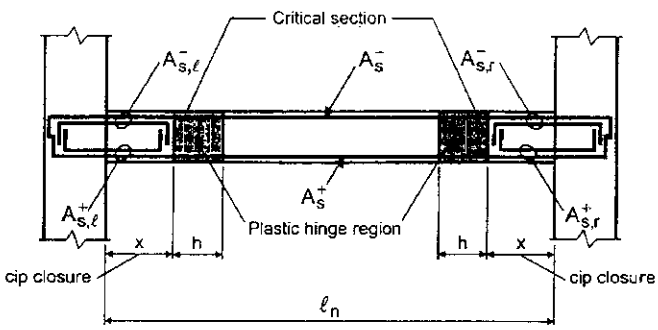


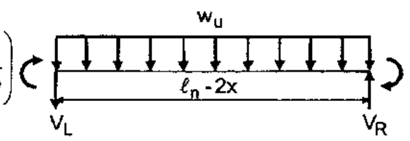
Fig. 29-14 Design Requirements for Beam-to-Beam Strong Connections near Midspan

Section 21.6.2(c) requires that primary longitudinal reinforcement be continuous across connections and be developed outside both the strong connection and the plastic hinge region. Laboratory tests of precast beam-column connections showed that strain concentrations caused brittle fracture of reinforcing bars at the faces of mechanical splices. To avoid this premature fracture, designers should carefully select the locations of strong connections or take other measures, such as using debonded reinforcement in highly stressed regions.

The column-to-column connection requirements of 21.6.2(d) are provided to avoid hinging and strength deterioration of these connections. For columns above the ground floor level, the moments at a joint may be limited by the flexural strengths of the beams framing into that joint (21.4.2.2). Dynamic inelastic analysis and studies of strong ground motion have shown that for a strong column-weak beam deformation mechanism, the beam end moments are not equally divided between the top and bottom columns, even where columns have equal stiffness.[29.4] From an elastic analysis, the moments would be distributed as shown in Fig. 29-16, while the actual distribution is likely to be as shown in Fig. 29-17.



$$M_{pr}^+ = A_s^+(1.25f_y)\left(d - \frac{A_s^+ f_y}{1.36f_c' b}\right)$$



$$M_{pr}^- = A_s^-(1.25f_y)\left(d - \frac{A_s^- f_y}{1.36f_c' b}\right)$$

$$V_L = \frac{w_u(\ell_n - 2x)}{2} - \frac{M_{pr}^+ + M_{pr}^-}{\ell_n - 2x}$$

$$V_R = \frac{w_u(\ell_n - 2x)}{2} + \frac{M_{pr}^+ + M_{pr}^-}{\ell_n - 2x}$$

$$M_{e,\ell}^+ = M_{pr}^+ + V_L x$$

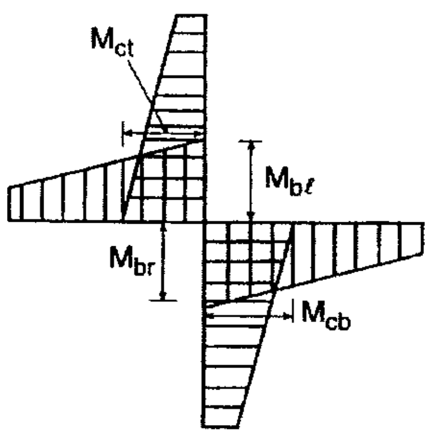
$$M_{e,r}^- = M_{pr}^- + V_R x$$

$$\phi M_{n,\ell}^+ = \phi A_{s,\ell}^+ f_y \left(d - \frac{A_{s,\ell}^+ f_y}{1.7f_c' b}\right) \geq M_{e,\ell}^+$$

$$\phi M_{n,r}^- = \phi A_{s,r}^- f_y \left(d - \frac{A_{s,r}^- f_y}{1.7f_c' b}\right) \geq M_{e,r}^-$$

Fig. 29-15 Design Requirements for Beam-to-Column Strong Connections

Fig. 29-16 Bending Moments at Beam-to-Column Connection – Elastic Analysis



$$M_{b\ell} + M_{br} = M_{cb} + M_{ct}$$

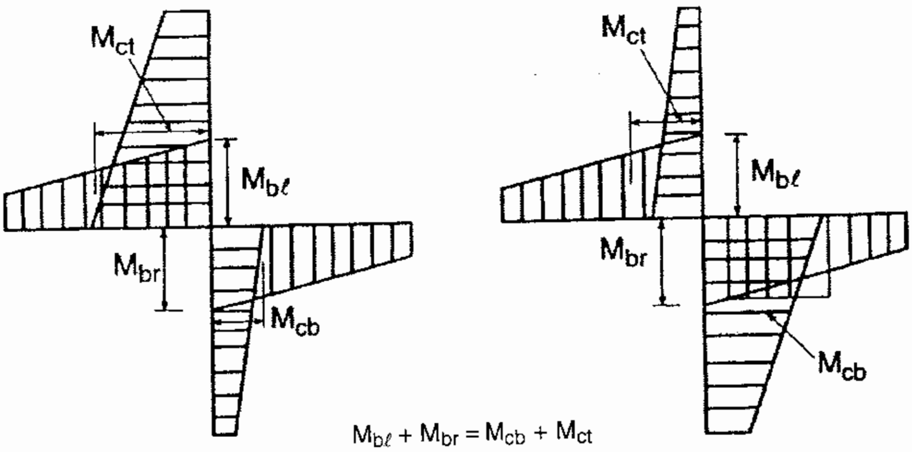


Fig. 29-17 Bending Moments at Beam-to-Column Connection – Inelastic Analysis

Figure 29-18 shows the distribution of the elastic moments M_E (dashed lines) due to the seismic forces and the corresponding envelopes of dynamic moments ωM_E (solid lines) over the full column height, where ω is a dynamic amplification factor. In regions outside of the middle third of the column height, ω is to be taken as 1.4 (21.6.2(d)); thus, connections within these regions must be designed such that $\phi M_n \geq 1.4M_E$. For connections located within the middle third of the column height, 21.6.2(d) requires that $\phi M_n \geq 0.4M_{pr}$, where M_{pr} is the maximum probable flexural strength of the column within the story height. Also, the design shear strength ϕV_n of the connection must be greater than or equal to the design shear force V_e computed according to 21.4.5.1.

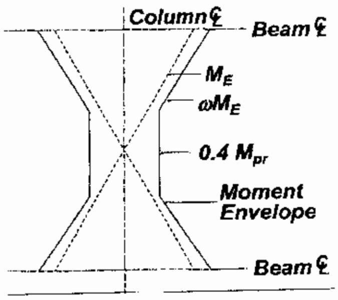


Fig. 29-18 Moment Envelope at Column-to-Column Connection

21.6.3 Non-emulative Design

It has been demonstrated in experimental studies that special moment frames constructed using precast concrete that do not satisfy the provisions of 21.6.1 for frames with ductile connections or 21.6.2 for frames with strong connections can provide satisfactory seismic performance characteristics. For these frames, the requirements of ACI T1.1 Acceptance Criteria for Moment Frames Based on Structural Testing [29.5], as well as the provisions of 21.6.3(a) and 21.6.3(b), must be satisfied.

ACI T1.1 defines minimum acceptance criteria for weak beam/strong column moment frames designed for regions of high seismic risk that do not satisfy the prescriptive requirements of Chapter 21 of ACI 318-99. According to ACI T1.1, acceptance of such frames as special moment frames must be validated by analysis and laboratory tests.

Prior to testing, a design procedure must be developed for prototype moment frames that have the same generic form as those for which acceptance is sought (see 4.0 of ACI T1.1). The design procedure should account for the effects of material nonlinearity (including cracking), deformations of members and connections, and reversed cyclic loading, and must be used to proportion the test modules (see 5.0 for requirements for the test modules). It is also important to note that the overstrength factor (column-to-beam strength ratio) used for the columns of the prototype frame should not be less than 1.2, which is specified in 21.4.2.2 of ACI 318-99.

The test method is described in 7.0. In short, the test modules are to be subjected to a sequence of displacement-controlled cycles that are representative of the drifts expected during the design earthquake for the portion of the frame that is represented by the test module. Figure R5.1 of ACI T1.1 illustrates connection configurations for interior and exterior one-way joints and, if applicable, corner joints that must be tested as a minimum.

The first loading cycle shall be within the linear elastic response range of the module. Subsequent drift ratios are to be between 1.25 and 1.5 times the previous drift ratio, with 3 fully reversed cycles applied at each drift ratio. Testing continues until the drift ratio equals or exceeds 0.035. Cyclic deformation history that satisfies 7.0 is illustrated in Fig. R7.0. Drift ratio is defined in Fig. R2.1.

Section 9.0 provides the detailed acceptance criteria that apply to each module of the test program. The performance of the test module is deemed satisfactory when these criteria are met for both directions of response.

The first criterion is that the test module must attain a lateral resistance greater than or equal to the calculated nominal lateral resistance E_n (see 1.0 for definition of E_n) before the drift ratio exceeds the allowable story drift limitation of the governing building code (see Fig. R9.1). This criterion helps provide adequate initial stiffness.

In order to provide weak beam/strong column behavior, the second criterion requires that the maximum lateral resistance E_{max} recorded in the test must be less than or equal to λE_n where λ is the specified overstrength factor for the test column, which must be greater than or equal to 1.2. Commentary section R9.1.2 provides a detailed discussion on this requirement. Also see Fig. R9.1.

The third criterion requires that the characteristics of the third complete cycle for each test module, at a drift ratio greater than or equal to 0.035, must satisfy 3 criteria regarding peak force value, relative energy dissipation ratio, and drift at zero stiffness. The first of these criteria limits the level of strength degradation, which is inevitable at high drift ratios under revised cyclic loading. A maximum strength degradation of $0.25E_{max}$ is specified (see Fig. R9.1). The second of these criteria sets a minimum level of damping for the frame as a whole by requiring that the relative energy dissipation ratio β be greater than or equal to 1/8. If β is less than 1/8, oscillations may continue for a long time after an earthquake, resulting in low-cycle fatigue effects and possible excessive displacements. The definition of β is illustrated in Fig. R2.4. The third of these criteria helps ensure adequate stiffness around zero drift ratio. The structure would be prone to large displacements following a major earthquake if this stiffness becomes too small. A hysteresis loop for the third cycle between peak drift ratios of 0.035, which has the form shown in Fig. R9.1, is acceptable. An unacceptable hysteresis loop form is shown in Fig. R9.1.3 where the stiffness around the zero drift ratio is unacceptably small for positive, but not for negative, loading.

As noted above, 21.6.3 has additional requirements to those in ACI T1.1. According to 21.6.3(a), the details and materials used in the test specimen shall be representative of those used in the actual structure. Section 21.6.3(b) stipulates additional requirements for the design procedure. Specifically, the design procedure must identify the load path or mechanism by which the frame resists the effects due to gravity and earthquake forces and shall establish acceptable values for sustaining that mechanism. Any portions of the mechanism that deviate from code requirements shall be contained in the test specimens and shall be tested to determine upper bounds for acceptance values. In other words, deviations are acceptable if it can be demonstrated that they do not adversely affect the performance of the framing system.

21.7 SPECIAL REINFORCED CONCRETE STRUCTURAL WALLS AND COUPLING BEAMS

When properly proportioned so that they possess adequate lateral stiffness to reduce interstory distortions due to earthquake-induced motions, structural walls (also called shearwalls) reduce the likelihood of damage to the nonstructural elements of a building. When used with rigid frames, a system is formed that combines the gravity-load-carrying efficiency of the rigid frame with the lateral-load-resisting efficiency of the structural wall.

Observations of the comparative performance of rigid-frame buildings and buildings stiffened by structural walls during earthquakes have pointed to the consistently better performance of the latter. The performance of buildings stiffened by properly designed structural walls has been better with respect to both safety and damage control. The need to ensure that critical facilities remain operational after a major tremor and the need to reduce economic losses from structural and nonstructural damage, in addition to the primary requirement of life safety (i.e., no collapse), has focused attention on the desirability of introducing greater lateral stiffness into earthquake-resistant multistory structures. Structural walls, which have long been used in designing for wind resistance, offer a logical and efficient solution to the problem of lateral stiffening of multistory buildings.

Structural walls are normally much stiffer than regular frame elements and are therefore subjected to correspondingly greater lateral forces due to earthquake motions. Because of their relatively greater depth, the lateral deformation capacities of walls are limited, so that, for a given amount of lateral displacement, structural walls tend to exhibit greater apparent distress than frame members. However, over a broad period range, a structure with structural walls, which is substantially stiffer and hence has a shorter period than a structure with frames, will suffer less lateral displacement than the frame, when subjected to the same ground motion intensity. Structural walls with a height-to-horizontal length ratio, h_w/ℓ_w, in excess of 2 behave essentially as vertical cantilever beams and should therefore be designed as flexural members, with their strength governed by flexure rather than by shear.

Isolated structural walls or individual walls connected to frames will tend to yield first at the base where the bending moment is the greatest. Coupled walls, i.e., two or more walls linked by short, rigidly-connected beams at the floor levels, on the other hand, have the desirable feature that significant energy dissipation through inelastic action in the coupling beams can be made to precede hinging at the bases of the walls.

The left-hand column of Table 29-4 contains the requirements for special reinforced concrete structural walls (recall that special reinforced concrete structural wall are required in regions of high seismic hazard or for structures assigned to high seismic performance or design categories). For comparison purposes, the requirements of ordinary reinforced concrete structural walls are also contained in Table 29-4.

21.7.2 Reinforcement

Special reinforced concrete structural walls are to be provided with reinforcement in two orthogonal directions in the plane of the wall (see Fig. 29-19). The minimum reinforcement ratio for both the longitudinal and the transverse reinforcement is 0.0025, unless the design shear force is less than or equal to $A_{cv}\sqrt{f_c'}$, where A_{cv} is the area of concrete bounded by the web thickness and the length of the wall in the direction of analysis, in which case, the minimum reinforcement must not be less than that given in 14.3. The reinforcement provided for shear strength must be continuous and distributed uniformly across the shear plane with a maximum spacing of 18 in. At least two curtains of reinforcement are required if the in-plane factored shear force assigned to the wall exceeds $2A_{cv}\sqrt{f_c'}$. This serves to reduce fragmentation and premature deterioration of the concrete under load reversals into the inelastic range. Uniform distribution of reinforcement across the height and horizontal length of the wall helps control the width of the inclined (diagonal) cracks.

Section 21.5.4 modifies the development length requirements of Chapter 12 for longitudinal beam bars terminating at exterior beam-column joints of structures assigned to high seismic design categories. But then 21.7.2.3

Table 29-4 Structural Walls

	Special Reinforced Concrete Structural Wall	Ordinary Reinforced Concrete Structural Wall
Reinforcement	The distributed web reinforcement ratios ρ_ℓ and ρ_t shall not be less than 0.0025. If the design shear force $V_u \leq A_{cv}\sqrt{f'_c}$, provide minimum reinforcement per 14.3. **21.7.2.1**	Minimum vertical reinforcement ratio = 0.0012 for No. 5 bars or smaller = 0.0015 for No. 6 bars or larger Minimum horizontal reinforcement ratio = 0.0020 for No. 5 bars or smaller = 0.0025 for No. 6 bars or larger **14.3**
	At least two curtains of reinforcement shall be used in a wall if the in-plane factored shear force (V_u) assigned to the wall exceeds $2\,A_{cv}\sqrt{f'_c}$. **21.7.2.2**	Walls more than 10 in. thick require two curtains of reinforcement (except basement walls). **14.3.4**
	Reinforcement spacing each way shall not exceed 18 in. **21.7.2.1**	Reinforcement spacing shall not exceed: 3 × wall thickness 18 in. **14.3.5**
	Reinforcement in structural walls shall be developed or spliced for f_y in tension in accordance with Chapter 12, except: (a) The effective depth shall be permitted to be $0.8\,\ell_w$ for walls. (b) The requirements of 12.11, 12.12 and 12.13 need not apply. (c) At locations where yielding of longitudinal reinforcement may occur as a result of lateral displacements, development lengths of such reinforcement must be 1.25 times the values calculated for f_y in tension. (d) Mechanical and welded splices of reinforcement must conform to 21.2.6 and 21.2.7, respectively. **21.7.2.3**	The development lengths, spacing, and anchorage of reinforcement shall be as per Chapters 12, 14, and 15.

— continued on next page —

A_{cv} = gross area of concrete section bounded by web thickness and length of section in the direction of shear force considered

A_{cw} = area of concrete section of an individual pier

A_g = gross area of section

b_w = width of web

d = effective depth of section

f'_c = specified compressive strength of concrete

f_{yh} = specified yield strength of transverse reinforcement

h = overall thickness of member

h_w = height of entire wall or segment of wall considered

ℓ_w = length of entire wall or segment of wall in direction of shear force

s = spacing of transverse reinforcement

s_2 = spacing of horizontal reinforcement in walls

ρ_t = ratio of area of distributed transverse reinforcement to gross concrete area perpendicular to that reinforcement

ρ_ℓ = ratio of area of distributed longitudinal reinforcement to gross concrete area perpendicular to that reinforcement

Table 29-4 Structural Walls (cont'd)

	Special Reinforced Concrete Structural Wall	Ordinary Reinforced Concrete Structural Wall
Shear Strength	The nominal shear strength (V_n) for structural walls shall not exceed: $$V_n = A_{cv}\left(\alpha_c\sqrt{f'_c} + \rho_t f_y\right)$$ where α_c is 3.0 for $h_w / \ell_w \le 1.5$, is 2.0 for $h_w / \ell_w \ge 2.0$, and varies linearly between 3.0 and 2.0 for h_w / ℓ_w between 1.5 and 2.0. **21.7.4.1**	The nominal shear strength (V_n) for walls can be calculated using the following methods: $$V_c = 3.3\sqrt{f'_c}\,hd + \frac{N_u d}{4\ell_w}$$ or $$V_c = \left[0.6\sqrt{f'_c} + \frac{\ell_w\left(1.25\sqrt{f'_c} + 0.2\dfrac{N_u}{\ell_w h}\right)}{\dfrac{M_u}{V_u} - \dfrac{\ell_w}{2}}\right]hd$$ where $\dfrac{M_u}{V_u} - \dfrac{\ell_w}{2} \ge 0$ $$V_s = \frac{A_v f_y d}{s}$$ $$V_n = V_c + V_s$$ **11.10**
	Walls shall have distributed shear reinforcement providing resistance in two orthogonal directions in the plane of the wall. If the ratio (h_w / ℓ_w) does not exceed 2.0, reinforcement ratio (ρ_ℓ) shall not be less than reinforcement ratio (ρ_t). **21.7.4.3**	Ratio (ρ_t) of horizontal shear reinforcement area to gross concrete area of vertical section shall not be less than 0.0025. **11.10.9.2** Spacing of horizontal shear reinforcement shall not exceed the smallest of $\ell_w/5$, $3h$, and 18 in. **11.10.9.3** The minimum vertical reinforcement ratio (ρ_ℓ) is a function of (h_w / ℓ_w) and of the horizontal reinforcement as shown below: $$\rho_n = 0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)(\rho_t - 0.0025) \ge 0.0025$$ Also, the vertical shear reinforcement ratio need not exceed the required horizontal shear reinforcement ratio. **11.10.9.4** Spacing of vertical shear reinforcement shall not exceed the smallest of $\ell_w/3$, $3h$, and 18 in. **11.10.9.5**
	Nominal shear strength of all wall piers sharing a common lateral force shall not be assumed to exceed $8A_{cv}\sqrt{f'_c}$, and the nominal shear strength of any one of the individual wall piers shall not be assumed to exceed $10A_{cw}\sqrt{f'_c}$. **21.7.4.4**	No similar requirement.
	Nominal shear strength of horizontal wall segments and coupling beams shall not be assumed to exceed $10A_{cw}\sqrt{f'_c}$. **21.7.4.5**	This limitation also exists for ordinary walls, except A_{cw} is replaced by hd where d may be taken equal to $0.8\ell_w$. **11.10.3**

of ACI 318-02 required that all continuous reinforcement in structural walls must be anchored or spliced in accordance with the provisions for reinforcement in tension in 21.5.4. This was very confusing to the user, because 21.5.4 is really not applicable to situations covered by 21.7.2.3. This problem existed with ACI 318 editions prior to 2002 as well.

In a very significant and beneficial change, the requirements of 21.7.2.3 have been modified to remove the reference to beam-column joints in 21.5.4. Because actual forces in longitudinal reinforcement of structural walls may exceed calculated forces, it is now required that reinforcement in structural walls be developed or spliced for f_y in tension in accordance with Chapter 12. The effective depth of member referenced in 12.10.3 is permitted to be taken as $0.8l_w$ for walls. Requirements of 12.11, 12.12, and 12.13 need not be satisfied, because they address issues related to beams and do not apply to walls. At locations where yielding of longitudinal reinforcement is expected, 1.25 f_y is required to be developed in tension, to account for the likelihood that the actual yield strength exceeds the specified yield strength, as well as the influence of strain-hardening and cyclic load reversals. Where transverse reinforcement is used, development lengths for straight and hooked bars may be reduced as permitted in 12.2 and 12.5, respectively, because closely spaced transverse reinforcement improves the performance of splices and hooks subjected to repeated cycles of inelastic deformation. The requirement that mechanical splices of reinforcement conform to 21.2.6, and welded splices to 21.2.7, has now been placed in 21.7.2.3. Consequently, 21.7.6.4(f) of 21.7.6.6 of ACI 318-02 have been deleted.

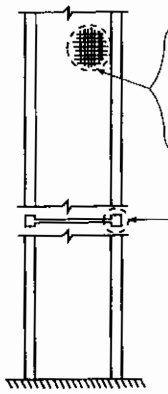


Figure 29-19 Structural Wall Design and Detailing Requirements

21.7.3 Design Forces

A condition similar to that used for the shear design of beams and columns is not as readily established for structural walls, primarily because the shear force at any section is significantly influenced by the forces and deformations at the other sections. Unlike the flexural behavior of beams and columns in a frame, with the forces and deformations determined primarily by the displacements in the end joints, the flexural deformation at any section of a structural wall is substantially influenced by the displacements at locations away from the section under consideration. Thus, for structural walls, the design shear force is determined from the lateral load analysis in accordance with the factored load combinations (21.7.3). The possibility of local yielding, as in the portion of a wall between two window openings, must also be considered; the actual shear forces may be much greater than that indicated by the lateral load analysis based on the factored design forces.

21.7.4 Shear Strength

The nominal shear strength V_n of structural walls is given in 21.7.4.1. The equation for V_n recognizes the higher shear strength of walls with high ratios of shear to moment. Additional requirements for wall segments and wall piers are contained in 21.7.4.2, 21.7.4.4, and 21.7.4.5.

The strength reduction factor f is determined in accordance with 9.3.4. Note that ϕ for shear must be 0.60 for any structural member that is designed to resist earthquake effects if its nominal shear strength is less than the shear corresponding to the development of the nominal flexural strength of the member. This is applicable to brittle members, such as low-rise walls or portions of walls between openings, which are impractical to reinforce to raise their nominal shear strength above the nominal flexural strength for the pertinent loading conditions.

Walls are to be provided with distributed shear reinforcement in two orthogonal directions in the plane of the wall (21.7.4.3). If the ratio of the height of the wall to the length of the wall is less than or equal to 2.0, the reinforcement ratio ρ_ℓ shall be greater than or equal to the reinforcement ratio ρ_t.

21.7.5 Design for Flexural and Axial Loads

Structural walls subjected to combined flexural and axial loads shall be designed in accordance with 10.2 and 10.3, excluding 10.3.6 and the nonlinear strain requirements of 10.2.2 (21.7.5.1). This procedure is essentially the same as that commonly used for columns. Reinforcement in boundary elements and distributed in flanges and webs must be included in the strain compatibility analysis. Openings in walls must also be considered.

Provisions for the influence of flanges for wall sections forming L-, T-, C-, or other cross-sectional shapes are in 21.7.5.2. Effective flange widths shall be assumed to extend from the face of the web a distance equal to the smaller of one-half the distance to an adjacent wall web and 25 percent of the total wall height.

21.7.6 Boundary Elements of Special Reinforced Concrete Structural Walls

Two approaches for evaluating the need for special boundary elements at the edges of structural walls are provided in 21.7.6. Section 21.7.6.2 allows the use of a displacement-based approach. In this method, the wall is displaced an amount equal to the expected design displacement, and special boundary elements are required to confine the concrete when the calculated neutral axis depth exceeds a certain critical value. Confinement is required over a horizontal length equal to a portion of the neutral axis depth (21.7.6.4). This approach is applicable to walls or wall piers that are essentially continuous in cross-section over the entire height of the wall and designed to have one critical section for flexure and axial loads, i.e., where the inelastic response of the wall is dominated by flexure at a critical, yielding section (21.7.6.2).

According to 21.7.6.2, compression zones must include special boundary elements when

$$c \geq \frac{\ell_w}{600(\delta_u/h_w)}, \quad \delta_u/h_w \geq 0.007 \qquad \textit{Eq. (21-8)}$$

where c = distance from the extreme compression fiber to the neutral axis per 10.2.7 calculated for the factored axial force and nominal moment strength, consistent with the design displacement δ_u, resulting in the largest neutral axis depth

ℓ_w = length of the entire wall or segment of wall considered in the direction of the shear force

δ_u = design displacement

h_w = height of entire wall or of the segment of wall considered

The design displacement δ_u is the total lateral displacement expected for the design-basis earthquake, as specified by the governing code for earthquake-resistant design. In the *International Building Code*, ASCE 7 starting with its 1998 edition , and the NEHRP Provisions (1997 edition onwards), the design-basis earthquake is two-thirds of the maximum considered earthquake (MCE), which, in most of the country, has a two percent chance of being exceeded in 50 years. In these documents, the design displacement is computed using a static or dynamic linear-elastic analysis under code-specified actions. Considered in the analysis are the effects of cracking, torsion, P-Δ effects, and modification factors to account for expected inelastic response. In particular, δ_u is determined by multiplying the deflections from an elastic analysis under the prescribed seismic forces by a deflection amplification factor, which is given in the governing code. The deflection amplification factor, which depends on the type of seismic force-resisting system, is used to increase the elastic deflections to levels that would be expected for the design-basis earthquake. The lower limit of 0.007 on the quantity δ_u / h_w is specified to require a moderate wall deformation capacity for stiff buildings.

Typically, the reinforcement for a structural wall section is determined first for the combined effects of bending and axial load, and shear forces in accordance with the provisions outlined above for all applicable load combinations. The distance c can then be obtained from a strain compatibility analysis for each load combination that includes seismic effects, considering sidesway to the left and to the right. The largest c is used in Eq. (21-8) to determine if special boundary elements are required.

When special boundary elements are required, they must extend horizontally from the extreme compression fiber a distance not less than the larger of $c - 0.1\ell_w$ and c/2 (21.7.6.4(a); see Fig. 29-20). In the vertical direction, the special boundary elements must extend from the critical section a distance greater than or equal to the larger of ℓ_w or $M_u/4V_u$ (21.7.6.2). This distance is based on upper bound estimates of plastic hinge lengths, and is beyond the zone over which concrete spalling is likely to occur.

The second approach for evaluating the need for special boundary elements is contained in 21.7.6.3. These provisions have been retained from earlier editions of the code since they are conservative for assessing transverse reinforcement requirements at wall boundaries for many walls. Compression zones shall include special boundary elements where the maximum extreme fiber stress corresponding to the factored forces, including earthquake effects, exceeds $0.2 f'_c$ (see Fig. 29-21). Special boundary elements can be discontinued where the compressive stress is less than $0.15 f'_c$. Note that the stresses are calculated assuming a linear response of the gross concrete section. The extent of the special boundary element is the same as when the approach of 21.7.6.2 is followed.

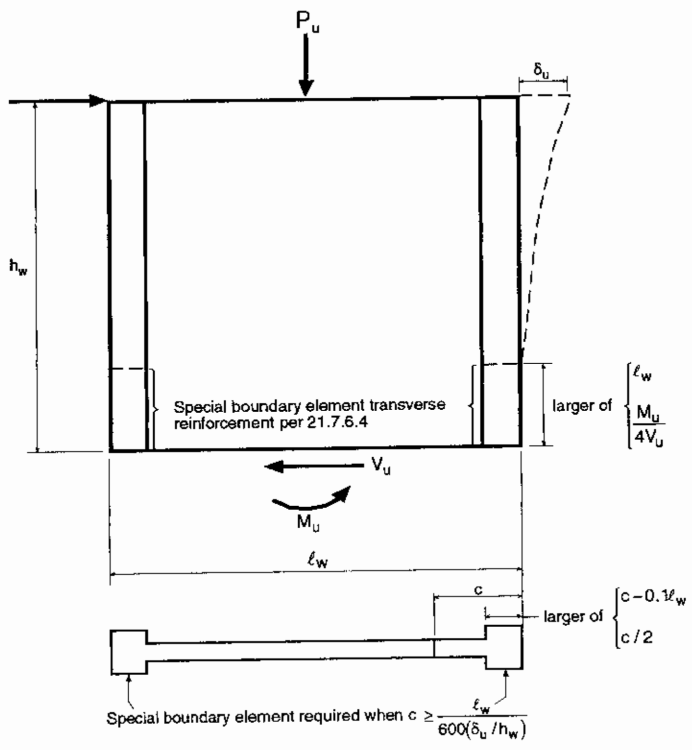


Figure 29-20 Special Boundary Element Requirements per 21.7.6.2

Section 21.7.6.4 contains the details of the reinforcement when special boundary elements are required by 21.7.6.2 or 21.7.6.3. The transverse reinforcement must satisfy the same requirements as for special moment frame members subjected to bending and axial load (21.4.4.1 through 21.4.4.3), excluding Eq. (21-3) (21.7.6.4(c); see Fig. 29-22). Also, the transverse reinforcement shall extend into the support a distance not less than the development length of the largest longitudinal bar in the special boundary element; for footings or mats, the transverse reinforcement shall extend at least 12 in. into the footing or mat (21.7.6.4(d)). Horizontal reinforcement in the wall web shall be anchored within the confined core of the boundary element to develop its specified yield strength (21.7.6.4(e)). To achieve this anchorage, 90-degree hooks or mechanical anchorages are recommended. Mechanical splices and welded splices of the longitudinal reinforcement in the boundary elements shall conform to 21.2.6 and 21.2.7, respectively (21.7.6.4(f)).

When special boundary elements are not required, the provisions of 21.7.6.5 must be satisfied. For the cases when the longitudinal reinforcement ratio at the wall boundary is greater than $400/f_y$, transverse reinforcement, spaced not more than 8 in. on center, shall be provided that satisfies 21.4.4.1(c), 21.4.4.3, and 21.7.6.4(c) (21.7.6.5(a)). This requirement helps in preventing buckling of the longitudinal reinforcement that can be caused by cyclic load reversals. The longitudinal reinforcement ratio to be used includes only the reinforcement at the end of the wall as indicated in Fig. R21.7.6.5. Horizontal reinforcement terminating at the edges of structural walls must be properly anchored per 21.7.6.5(b) in order for the reinforcement to be effective in resisting shear and to help in preventing buckling of the vertical edge reinforcement. The provisions of 21.7.6.5(b) are not required to be satisfied when the factored shear force V_u is less than $A_{cv}\sqrt{f'_c}$.

21.7.7 Coupling Beams

When adequately proportioned and detailed, coupling beams between structural walls can provide an efficient means of energy dissipation under seismic forces, and can provide a higher degree of overall stiffness to the structure. Due to their relatively large depth to clear span ratio, ends of coupling beams are usually subjected to large inelastic rotations. Adequate detailing and shear reinforcement are necessary to prevent shear failure and to ensure ductility and energy dissipation.

Coupling beams with $\ell_n/h \geq 4$ must satisfy the requirement of 21.3 for flexural members of special moment frames, excluding 21.3.1.3 and 21.3.1.4(a) if it can be shown that the beam has adequate lateral stability (21.7.7.1). When $\ell_n/h < 4$, coupling beams with two intersecting groups of diagonally-placed bars symmetrical about the midspan is permitted (21.7.7.2). The diagonal bars are required for deep coupling beams ($\ell_n/h < 2$) with a factored shear force V_u greater than $4\sqrt{f'_c}A_{cw}$, unless it can be shown otherwise that safety and stability are not compromised (21.7.7.3). Experiments have shown that diagonally oriented reinforcement is effectve only if the bars can be placed at a large inclination.

Note that in the 2002 code, h replaces d in the definition of the aspect ratio (clear span/depth) and A_{cw} (formerly A_{cp}) replaced b_wd in the shear equations. The first change simplified the code requirements, since d is not always readily known for beams with multiple layers of reinforcement. The second change removed an inconsistency between 21.6.4.5 and 21.6.7.4 of the 1999 code; A_{cw} is now consistently used in 21.7.4.5 and 21.7.7.4.

Section 21.7.7.4 contains the reinforcement details for the two intersecting groups of diagonally placed bars. Figure 29-23 provides a summary of these requirements. The requirement on side dimensions of the cage and its core is to provide adequate toughness and stability when the bars are stressed beyond yielding. The nominal shear strength of a coupling beam is computed from the following (21.7.7.4(b)):

$$V_n = 2A_{vd}f_y\sin\alpha \leq 10\sqrt{f'_c}A_{cw} \qquad \textit{Eq. (21-9)}$$

The additional reinforcement specified in 21.7.7.4(f) is used to confine the concrete outside of the diagonal cores.

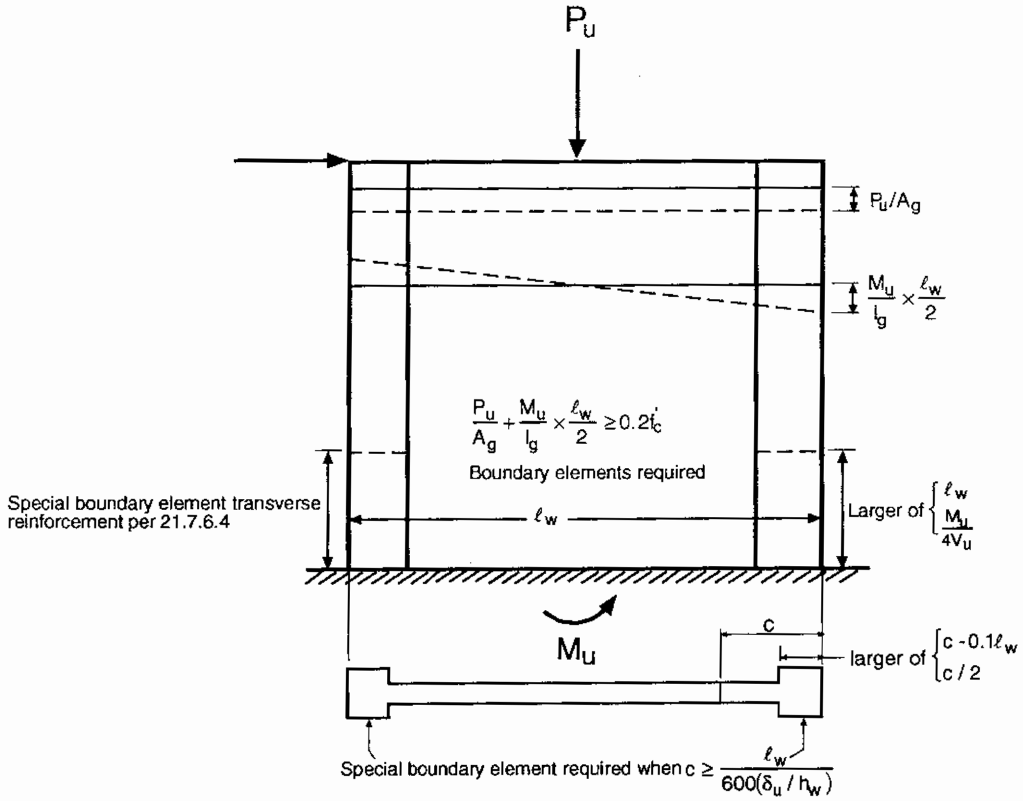


Figure 29-21 Special Boundary Element Requirements per 21.7.6.3

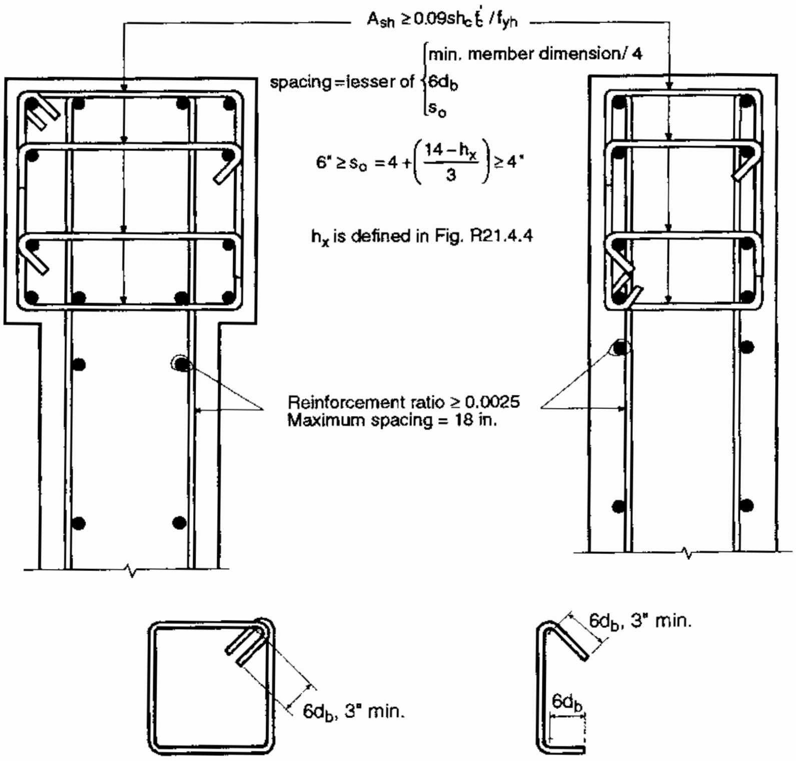


Fig. 29-22 Reinforcement Details for Special Boundary Elements

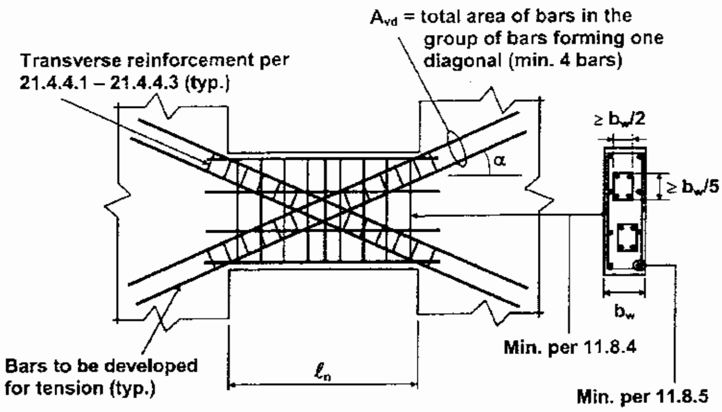


Figure 29-23 Coupling Beam with Diagonally Oriented Reinforcement

21.8 SPECIAL STRUCTURAL WALLS CONSTRUCTED USING PRECAST CONCRETE

According to 21.8.1, special structural walls constructed using precast concrete shall satisfy all requirements of 21.7 for cast-in-place special structural walls and the requirements in 21.13.2 and 21.13.3 for intermediate precast structural walls. Thus, the left-hand column of Table 29-4 may be utilized. The provisions for intermediate precast structural walls are discussed later. Note that the provisions of 21.8 do not apply to tilt-up walls.

21.9 STRUCTURAL DIAPHRAGMS AND TRUSSES

In building construction, diaphragms are structural elements, such as floor or roof slabs, that perform some or all of the following functions:

- Provide support for building elements such as walls, partitions, and cladding, and resist horizontal forces but not act as part of the vertical lateral-force-resisting system
- Transfer lateral forces to the vertical lateral-force-resisting system
- Interconnect various components of the lateral-force-resisting system with appropriate strength, stiffness, and toughness to permit deformation and rotation of the building as a unit

Section 21.9.4 prescribes a minimum thickness of 2 in. for concrete slabs and composite topping slabs serving as diaphragms used to transmit earthquake forces. The minimum thickness is based on what is currently used in joist and waffle slab systems and composite topping slabs on precast floor and roof systems. A minimum of 2.5 in. is required for topping slabs placed over precast floor or roof systems that do not act compositely with the precast system to resist the seismic forces.

Sections 21.9.2 and 21.9.3 provide design criteria for cast-in-place diaphragms. For the case of a cast-in-place composite topping slab on a precast floor or roof system, bonding is required so that the floor or roof system can provide restraint against slab buckling; also, reinforcement is required to ensure shear transfer across the precast joints. Composite action is not required for a cast-in-place topping slab on a precast floor or roof system, provided the topping slab acting alone is designed to resist the seismic forces.

21.9.5 Reinforcement

The minimum reinforcement ratio for structural diaphragms is the same as that required by 7.12 for temperature and shrinkage reinforcement. The maximum reinforcement spacing of 18 in. is intended to control the width of inclined cracks. Sections 21.9.5.1 and 21.9.5.2 contain provisions for welded wire reinforcement used in topping slabs placed over precast floor and roof elements and bonded prestressing tendons used as primary reinforcement in diaphragm chords or collectors, respectively.

According to 21.9.5.3, structural truss elements, struts, ties, diaphragm chords, and collector elements must have transverse reinforcement as specified in 21.4.4.1 through 21.4.4.3 when the compressive stress at any section exceeds $0.2\,f'_c$. Note that compressive stress is calculated for the factored forces using a linearly elastic model and gross section properties. The special transverse reinforcement is no longer required where the compressive stress is less than $0.15\,f'_c$.

In recent seismic codes and standards, collector elements of diaphragms are required to be designed for forces amplified by a factor Ω_0, to account for the overstrength in the vertical elements of the seismic-force-resisting system. The amplification factor Ω_0, ranges between 2 and 3 for concrete structures, depending upon the document selected and on the type of seismic system. To account for this, 21.9.5.3 now additionally states that where design forces have been amplified to account for the overstrength of the vertical elements of the seismic-force-resisting system. The limits of $0.2\,f'_c$ and $0.15\,f'_c$ shall be increased to $0.5\,f'_c$ and $0.4\,f'_c$, respectively.

Section 21.5.4 modifies the development length requirements of Chapter 12 for longitudinal beam bars terminating at exterior beam-column joints of structures assigned to high seismic design categories. But then 21.9.5.4 of ACI 318-02 required that all continuous reinforcement in diaphragms, tresses, ties, chords, and collector elements be anchored or spliced in accordance with the provisions for reinforcement in tension as specified in 21.5.4. This was very confusing to the user, because 21.5.4 is really not applicable to situations covered by 21.9.5.4. This problem existed with ACI 318 editions prior to 2002 as well.

In a very significant and beneficial change, the requirements of 21.9.5.4 have now been modified to remove the reference to beam-column joints in 21.5.4. All reference now is directly to the provisions of Chapter 12. Section 21.9.5.4 now requires that all continuous reinforcement in diaphragms, trusses, struts, ties, chords, and collector elements be developed or spliced for f_y in tension. This is a change similar to that made in 21.7.2.3.

21.9.7 Shear Strength

The shear strength requirements for monolithic structural diaphragms are similar to those for structural walls. In particular, the nominal shear strength V_n is computed from:

$$V_n = A_{cv}\left(2\sqrt{f'_c} + \rho_t f_y\right) \le 8A_{cv}\sqrt{f'_c} \qquad \textit{Eq. (21-10)}$$

where A_{cv} is the thickness times the width of the diaphragm. Shear reinforcement should be placed perpendicular to the span of the diaphragm.

The nominal shear strength V_n for topping slab diaphragms does not include the contribution from the concrete:

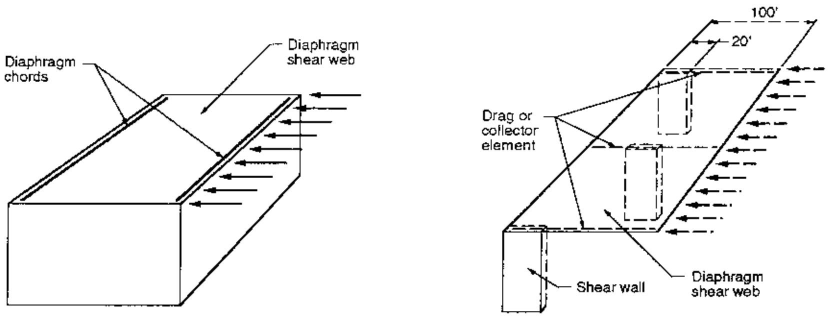
$$V_n = A_{cv}\rho_t f_y \le 8A_{cv}\sqrt{f'_c} \qquad \textit{Eq. (21-11)}$$

Typically, topping slabs are scored immediately above the boundary between the flanges of adjacent precast floor members to control the location of shrinkage cracks. Thus, these weakened sections of the diaphragm are cracked under service load conditions, and the contribution of the concrete to the overall shear strength of the diaphragm may have already been reduced before the design earthquake occurs. For this reason, the contribution of the concrete is taken as zero. The required web reinforcement must be uniformly distributed in both directions.

21.9.8 Boundary Elements of Structural Diaphragms

According to 21.9.8.1, boundary elements of structural diaphragms are to be proportioned to resist the sum of the factored axial forces acting in the plane of the diaphragm and the force obtained from dividing the factored moment at the section by the distance between the boundary elements of the diaphragm at that section. It is assumed that the factored flexural moments are resisted entirely by chord forces acting at opposite edges of the diaphragm (see Fig. 29-24). It is essential that splices of tensile reinforcement located in the chord and collector elements be fully developed and adequately confined (21.9.8.2). Mechanical and welded splices must conform to 21.2.6 and 21.2.7, respectively. If chord reinforcement is located within a wall, the joint between the diaphragm and the wall should be provided with adequate shear strength to transfer the shear forces.

Reinforcement details for chords and collectors at splices and anchorage zones are given in 21.9.8.3.



Diaphragm chords resist tension at edges of diaphragm

Drag or collector elements transfer forces from diaphragm to vertical elements

Figure 29-24 Diaphragm Chord and Collector Elements

21.10 FOUNDATIONS

Requirements for foundations supporting buildings in regions of high seismic risk or assigned to high seismic performance or design categories are contained in 21.10. It is important to note that the foundations must also comply with all other applicable provisions of the code. For piles, drilled piers, caissons, and slabs on grade, the provisions of 21.10 supplement other applicable design and construction criteria (see also 1.1.5 and 1.1.6).

21.10.2 Footings, Foundation Mats, and Pile Caps

Detailing requirements are contained in 21.10.2.1 through 21.10.2.4 for footings, mats, and pile caps supporting columns or walls, and are illustrated in Fig. 29-25.

21.10.3 Grade Beams and Slabs on Grade

Grade beams that are designed as ties between pile caps or footings must have continuous reinforcement that is developed within or beyond the supported column, or must be anchored within the pile cap or footing at discontinuities (21.10.3.1).

Section 21.10.3.2 contains geometrical and reinforcement requirements. The smallest cross-sectional dimension of the grade beam shall be greater than or equal to the clear spacing between the connected columns divided by 20; however, this dimension need not be greater than 18 in. Closed ties shall be provided over the length of the beam spaced at a maximum of one-half the smallest orthogonal cross-sectional dimension of the beam or 12 in., whichever is smaller. Both of these provisions are intended to provide reasonable beam proportions.

According to 21.10.3.3, grade beams and beams that are part of a mat foundation that is subjected to flexure from columns that are part of the lateral-force-resisting system shall have reinforcing details conforming to 21.3 for flexural members of special moment frames.

Slabs on grade shall be designed as diaphragms according to the provisions of 21.9 when they are subjected to seismic forces from walls or columns that are part of the lateral-force-resisting system (21.10.3.4). Such slabs shall be designated as structural members on the design drawings for obvious reasons.

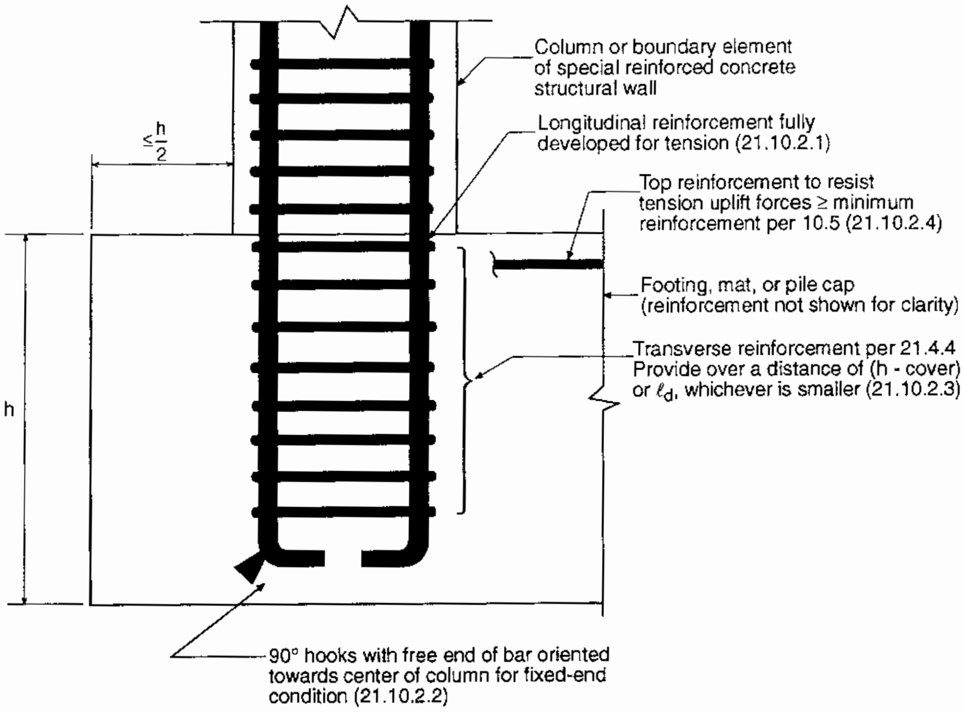


Figure 29-25 Reinforcement Details for Footings, Mats, and Pile Caps per 21.10.2

21.10.4 Piles, Piers, and Caissons

When piles, piers, or caissons are subjected to tension forces from earthquake-induced effects, a proper load path is required to transfer these forces from the longitudinal reinforcement of the column or boundary element through the pile cap to the reinforcement of the pile or caisson. Thus, continuous longitudinal reinforcement is required over the length resisting the tensile forces, and it must be properly detailed to transfer the forces through the elements (21.10.4.2). When grouted or post-installed reinforcing bars are used to transfer tensile forces between the pile cap or mat foundation and a precast pile, a test must be performed to ensure that the grouting system can develop at least 125 percent of the specified yield strength of the reinforcing bar, (21.10.4.3). In lieu of a test, reinforcing bars can be cast in the upper portion of a pile, exposed later by chipping away the concrete, and then mechanically connected or welded to achieve the proper extension.

Transverse reinforcement in accordance with 21.4.4 is required at the top of piles, piers, and caissons over a length equal to at least 5 times the cross-sectional dimension of the member, but not less than 6 ft below the bottom of the pile cap (21.10.4.4(a)). This requirement is based on numerous failures that were observed in earthquakes just below the pile cap, and provides ductility in this region of the pile. Also, for portions of piles in soil that is not capable of providing lateral support, or for piles in air or water, the entire unsupported length plus the length specified in 21.10.4.4(a) must be confined by transverse reinforcement per 21.4.4 (21.10.4.4(b)). Additional requirements for precast concrete driven piles, foundations supporting one- and two-story stud bearing wall construction, and pile caps with batter piles are contained in 21.10.4.5 through 21.10.4.7.

21.11 FRAME MEMBERS NOT PROPORTIONED TO RESIST FORCES INDUCED BY EARTHQUAKE MOTIONS

In regions of high seismic risk or for structures assigned to high seismic performance or design categories, frame members that are assumed not to contribute to lateral resistance shall comply with the requirements of 21.11. Specifically, these members are detailed depending on the magnitude of the moments and shears that are induced when they are subjected to the design displacements. This requirement is intended to enable the gravity load system to maintain its vertical load carrying capacity when subjected to the maximum lateral displacement of the lateral-force-resisting system expected for the design-basis earthquake.

The following summarizes the requirements of 21.11:

(1) Compute moments and shears (E) in all elements that are not part of the lateral-force-resisting system due to the design displacement δ_u. The displacement δ_u is determined based on the provisions of the governing building code. In the IBC, in ASCE 7 starting with its 1998 edition, and in the NEHRP Provisions (1997 and subsequent editions), δ_u is determined from the design-basis earthquake, (two-thirds of the Maximum Considered Earthquake, which for most of the country is an earthquake having a 98% probability of non-exceedance in 50 years) using a static or dynamic linear-elastic analysis, and considering the effects of cracked sections, torsion, and P-Δ effects. δ_u is determined by multiplying the deflections from an elastic analysis under the prescribed seismic forces by a deflection amplification factor, which accounts for expected inelastic response and which is given in the governing code for various seismic-force-resisting systems.

(2) Determine the factored moment M_u and the factored shear V_u in each of the elements that are not part of the lateral-force-resisting system from the more critical of the following load combinations:

$$U = 1.2D + 1.0L + 0.2S + E$$

$$U = 0.9D + E$$

The local factor on L can be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where $L > 100$ psf.

Note that the E-values (moments and shears) in the above expressions are determined in step 1 above.

(3) If $M_u \le \phi M_n$ and $V_u \le \phi V_n$ for an element that is not part of the lateral-force-resisting system, and if such an element is subjected to factored gravity axial forces $P_u \le A_g f_c'/10$, it must satisfy the longitudinal reinforcement requirements in 21.3.2.1; in addition, stirrups spaced at no more than d/2 must be provided throughout the length of the member. If such an element is subjected to $P_u > A_g f_c'/10$ where $P_u \le 0.35P_o$ (P_o is the nominal axial load strength at zero eccentricity), it must conform to 21.4.3, 21.4.4.1(c), 21.4.4.3, and 21.4.5. In addition, ties at a maximum spacing of s_o must be provided throughout the height of the column, where s_o must not exceed the smaller of six times the smallest longitudinal bar diameter and 6 in. If the factored gravity axial force $P_u > 0.35P_o$, the requirements of 21.11.2.3 must be satisfied and the amount of transverse reinforcement provided shall be one-half of that required by 21.4.4.1, with the spacing not exceeding s_o for the full column height.

(4) If M_u or V_u determined in step 2 for an element that is not part of the lateral-force-resisting system exceeds ϕM_n or ϕV_n, or if induced moments and shears due to the design displacements are not calculated, then the structural materials must satisfy 21.2.4 and 21.2.5, and the splices of reinforcement must satisfy 21.2.6 and 21.2.7. If such an element is subjected to $P_u \le A_g f_c'/10$, it must conform to 21.3.2.1 and 21.3.4; in addition, stirrups spaced at no more than d/2 must be provided throughout the length of the member. If such an element is subjected to $P_u > A_g f_c'/10$, it must be provided with full ductile detailing in conformance with 21.4.3.1, 21.4.4, 21.4.5, and 21.5.2.1. Note that the requirements of 21.4.3.1 used to be requirements of 21.4.3 before 2002. This change was meant to be made in 21.11.2, was inadvertently made in 21.11.3, and needs to be corrected in the future.

Precast concrete frame members assumed not to contribute to lateral resistance must also conform to 21.11.1 through 21.11.3. In addition, the following requirements of 21.11.4 must be satisfied: (a) ties specified in 21.11.2.2 must be provided over the entire column height, including the depth of the beams; (b) structural integrity reinforcement of 16.5 must be provided in all members; and (c) bearing length at the support of a beam must be at least 2 in. longer than the computed bearing length according to 10.17. The 2 in. increase in bearing length is based on an assumed 4% story drift ratio and a 50 in. beam depth, and is considered to be conservative for ground motions expected in high seismic zones.

In a very significant change, provisions for shear reinforcement at slab-column joints have been added in a new section 21.11.5, to reduce the likelihood of punching shear failure in two-way slabs without beams. A prescribed amount and detailing of shear reinforcement is required unless either 21.11.5(a) or (b) is satisfied.

Section 21.11.5(a) requires calculation of shear stress due to the factored shear force and induced moment according to 11.12.6.2. The induced moment is the moment that is calculated to occur at the slab-column joint where subjected to the design displacement defined in 21.1. Section 13.5.1.2 and the accompanying commentary provide guidance on selection of the slab stiffness for the purpose of this calculation.

Section 21.11.5(b) does not require the calculation of induced moments, and is based on research[29.5, 29.6] that identifies the likelihood of punching shear failure considering interstory drift and shear due to gravity loads. The requirement is illustrated in the newly added Fig. R21.11.5. The requirement can be satisfied in several ways: adding slab shear reinforcement, increasing slab thickness, designing a structure with more lateral stiffness to decrease interstory drift, or a combination of two or more of these.

If column capitals, drop panels, or other changes in slab thickness are used, the requirements of 21.1.5 must be evaluated at all potential critical sections.

21.12 REQUIREMENTS FOR INTERMEDIATE MOMENT FRAMES

For comparison purposes with the requirements for special moment frames, the provisions for beams (21.12.4) and columns (21.12.5) in intermediate moment frames have been presented in Table 29-1 and Table 29-2, respectively. The shear provisions of 21.12.3 are also included in those tables.

As was noted above, hoops instead of stirrups are now required at both ends of beams for a distance not less than 2h from the faces of the supports. The likelihood of spalling and loss of shell concrete in some regions of the frame are high. Both observed behavior under actual earthquakes and experimental research have shown that the transverse reinforcement will open at the ends and lose the ability to confine the concrete unless it is bent around the longitudinal reinforcement and its ends project into the core of the element. Similar provisions are now given in 21.12.5 for columns.

Two-way slabs without beams are acceptable lateral-force-resisting systems in regions of low or moderate seismic risk, or for structures assigned to low or moderate seismic performance or design categories. They are not permitted in regions of high seismic risk or for structures assigned to high seismic performance or design categories. Table 29-5, Fig. 29-26, and Fig. 29-27 summarize the detailing requirements for two-way slabs of intermediate moment frames. Provisions for two-way slabs of ordinary moment frames are also presented in Table 29-5.

The provisions of 21.12.6.2 for the band width within which flexural moment transfer reinforcement must be placed at edge and corner slab-column connections are new in the 2002 code. For these connections, flexural moment-transfer reinforcement perpendicular to the edge is not considered fully effective unless it is placed within the specified narrow band width (see Fig. R21.12.6.1).

The shear strength requirements of 21.12.6.8 are also new in the 2002 code. Slab-column frames are susceptible to punching shear failures during earthquakes if the shear stresses due to gravity loads are high. Thus, a limit was inroduced on the allowable shear stress caused by gravity loads, which in turn permits the slab-column connection to have adequate toughness to withstand the anticipated inelastic moment transfer.

21.13 INTERMEDIATE PRECAST STRUCTURAL WALLS

This new section applies to intermediate precast structural walls used to resist forces induced by the design earthquake.

Connections between precast wall panels or between wall panels and the foundation are required to resist forces due to earthquake motions and must provide for yielding that is restricted to steel elements or reinforcement (21.13.2). When Type 2 mechanical splices are used for connecting the primary reinforcement, the strength of the splice should be greater than or equal to 1.5 times f_y of the reinforcement (21.13.3).

Note that the provisions of 21.13, like those of 21.8, do not apply to tilt-up walls.

*Table 29-5 Two-Way Slabs Without Beams**

Intermediate — All reinforcement provided to resist M_{slab}, the portion of slab moment balanced by the support moment, must be placed within the column strip defined in 13.2.1. **21.12.6.1** Ordinary — The middle strip is allowed to carry a portion of the unbalanced moment.
Intermediate — The fraction, defined by Eq. (13-1), of the moment M_{slab} shall be resisted by reinforcement placed within the band width specified in 13.5.3.2. Band width for edge and corner connections shall not extend beyond the column face a distance greater than c_t measured perpendicular to the slab span. **21.12.6.2** Ordinary — Similar requirement, except band width restriction for edge and corner connections does not apply.
Intermediate — Not less than one-half of the reinforcement in the column strip at the support shall be placed within the effective slab width specified in 13.5.3.2. **21.12.6.3** Ordinary — No similar requirement.
Intermediate — Not less than one-fourth of the top reinforcement at the support in the column strip shall be continuous throughout the span. **21.12.6.4** Ordinary — No similar requirement.
Intermediate — Continuous bottom reinforcement in the column strip shall not be less than one-third of the top reinforcement at the support in the column strip. **21.12.6.5** Intermediate — Not less than one-half of all middle strip bottom reinforcement and all column strip bottom reinforcement at midspan shall be continuous and shall develop its yield strength at the face of the support as defined in 13.6.2.5. **21.12.6.6** Ordinary — All bottom bars within the column strip shall be continuous or spliced with Class A tension splices or with mechanical or welded splices satisfying 12.14.3. **13.3.8.5**
Intermediate — At discontinuous edges of the slab, all top and bottom reinforcement at the support shall be developed at the face of support as defined in 13.6.2.5. **21.12.6.7** Ordinary — Positive moment reinforcement perpendicular to a discontinuous edge shall extend to the edge of the slab and have embedment of at least 6 in. in spandrel beams, columns, or walls. Negative moment reinforcement perpendicular to a discontinuous edge must be anchored and developed at the face of the support according to provisions in Chapter 12. **13.3.3, 13.3.4**
Intermediate — At the critical sections for columns defined in 11.12.1.2, two-way shear caused by factored gravity loads shall not exceed $0.4\phi V_c$ where V_c is calculated by 11.12.2.1 for nonprestressed slabs and 11.12.2.2 for prestressed slabs. This requirement may be waived if the contribution of the earthquake-induced factored two-way shear stress transferred by eccentricity of shear in accordance with 11.12.6.1 and 11.12.6.2 at the point of maximum stress does not exceed $(\phi v_n$ permitted by 11.12.6.2)/2. **21.12.6.8** Ordinary — No similar requirement.

** Not permitted as part of the lateral-force-resisting system in regions of high seismic risk or for structures assigned to high seismic performance or design categories.*

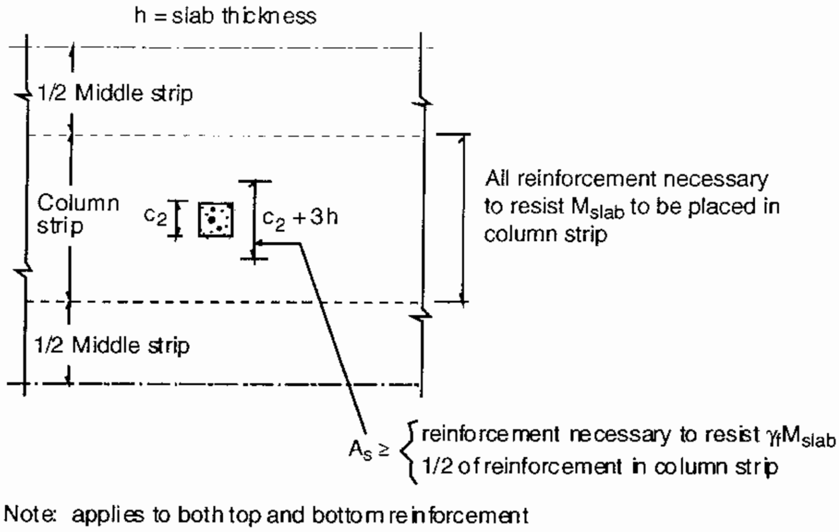
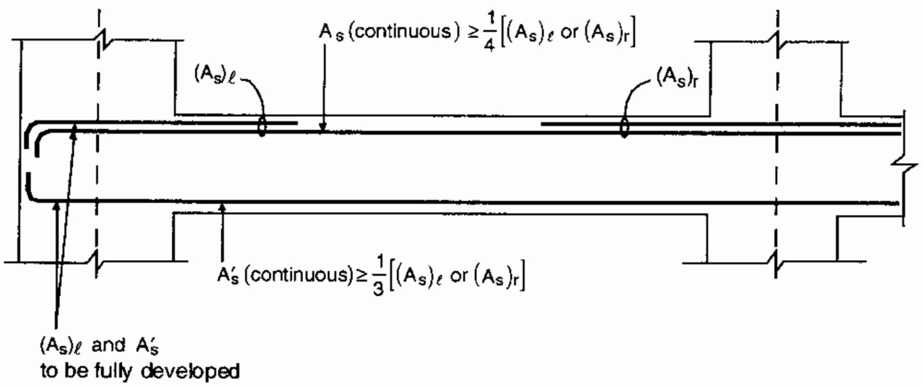
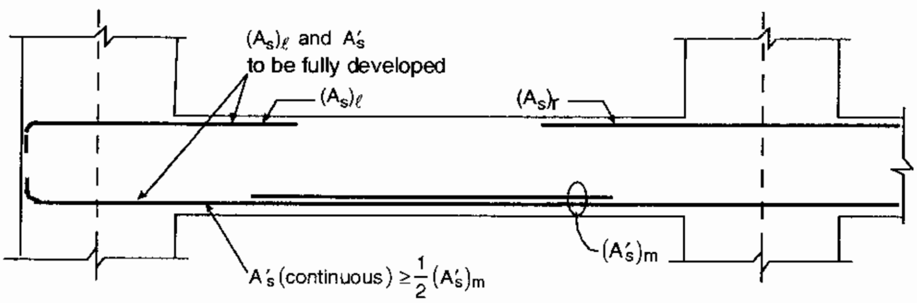


Figure 29-26 Location of Reinforcement in Two-way Slabs without Beams



(a) Column Strip



(b) Middle Strip

Figure 29-27 Details of Reinforcement in Two-way Slabs without Beams

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Example 29.1—Design of a 12-Story Cast-in-Place Frame-Shearwall Building and its Components

This example, and the 5 examples that follow, illustrate the design and detailing requirements for typical members of a 12-story cast-in-place concrete building.

A typical plan and elevation of the structure are shown in Figs. 29-28(a) and (b) respectively. The columns and structural walls have constant cross-sections throughout the height of the building*, and the bases of the lowest story segments are assumed fixed. The beams and the slabs also have the same dimensions at all floor levels. Although the member dimensions in this example are within the practical range, the structure itself is a hypothetical one, and has been chosen mainly for illustrative purposes. Other pertinent design data are as follows:

Material properties:

Concrete:	$f'_c = 4000$ psi, $w_c = 145$ pcf
Reinforcement:	$f_y = 60{,}000$ psi

Service loads:

Live load:	Floors = 50 psf Additional average value to allow for heavier load on corridors = 25 psf Total average live load (floors) = 75 psf Roof = 20 psf
Superimposed dead load:	Average for partitions = 20 psf Ceiling and mechanical = 10 psf Total average superimposed dead load (floors) = 30 psf Roof = 10 psf

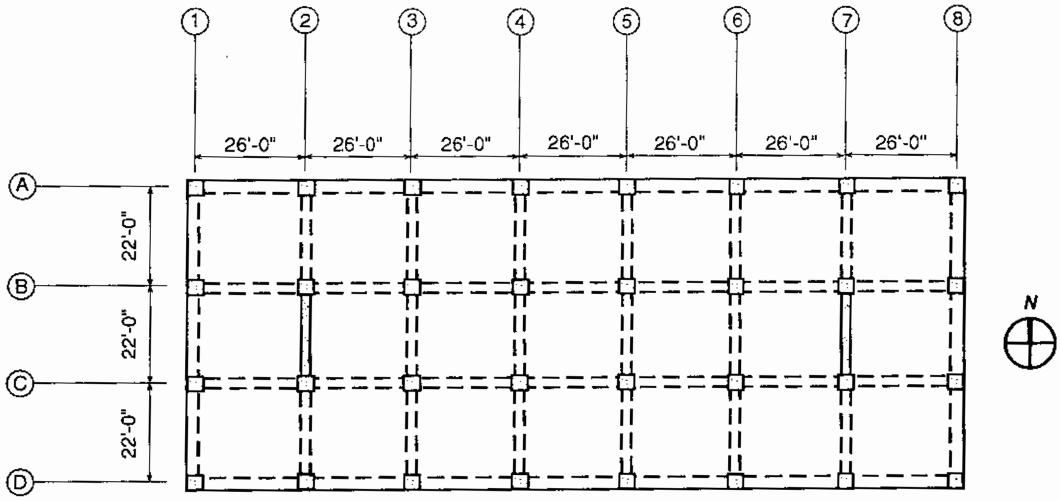
Seismic design data:

The building is located in a region of high seismic risk, and is assigned to a high seismic design or performance category.

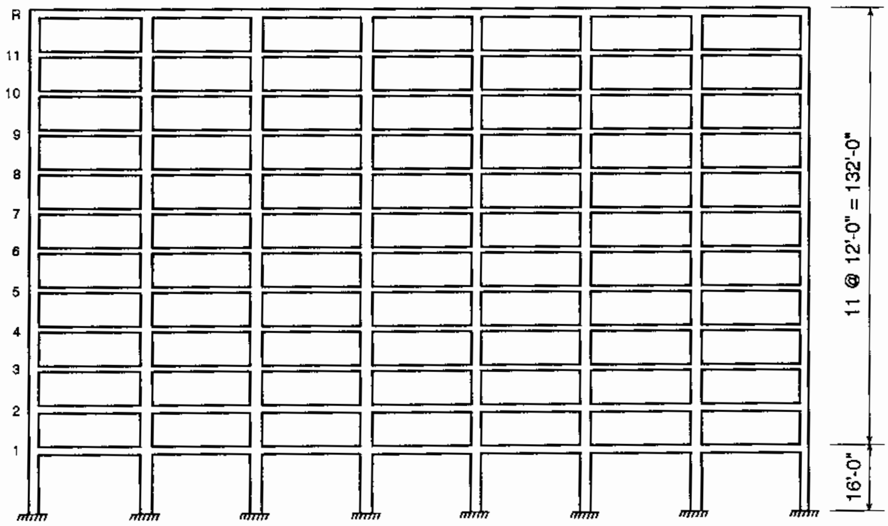
Dual system (special reinforced concrete structural walls with special moment frames) in the N-S direction

Special moment frames in the E-W direction

* The uniformity in member dimensions used in this example has been adopted mainly for simplicity.



(a) Typical floor plan



Exterior columns: 22 x 22 in.
Interior columns: 26 x 26 in.
Beams: 20 x 24 in.
Slab: 8 in.
Walls: 18 in. web + 32 x 32 in. boundary elements

(b) Longitudinal section

Figure 29-28 Example Building

1. Lateral analysis

   The computation of the seismic and wind design forces is beyond the scope of this example.

   A three-dimensional analysis of the building was performed in both the N-S and E-W directions for both seismic and wind load cases. The effects of the seismic forces governed; thus, load combinations containing the effects of wind loads are not considered in the following examples.

2. Gravity analysis

   The Equivalent Frame Method of 13.7 was used to determine the gravity load moments in the members.

   Cumulative service axial loads for the columns and walls were computed considering live load reduction according to ASCE 7.

Example 29.2—Proportioning and Detailing of Flexural Members of Building in Example 29.1

Design a beam on the first floor of a typical interior E-W frame of the example building (Fig. 29-28). The beam has dimensions of b = 20 in. and h = 24 in. (d = 21.5 in.). The slab is 8 in. thick. Use $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Calculations and Discussion	Code Reference

1. Check satisfaction of limitations on section dimensions.

   Factored axial compressive force on beams is negligible. O.K. — *21.3.1.1*

   $$\frac{\ell_n}{d} = \frac{(26 \times 12) - 30}{21.5} = 13.1 > 4 \quad \text{O.K.}$$ *21.3.1.2*

   $$\frac{\text{width}}{\text{depth}} = \frac{20}{24} = 0.83 > 0.3 \quad \text{O.K.}$$ *21.3.1.3*

   width = 20 in. > 10 in. O.K. — *21.3.1.4*

   < width of supporting column + (1.5 × depth of beam)

   < 24 + (1.5 × 24) = 60 in. O.K.

2. Determine required flexural reinforcement.

   The required reinforcement for the beams on the first floor level is shown in Table 29-6. The provided areas of steel are within the limits specified in 21.3.2.1. Also given in Table 29-6 are the design moment strengths ϕM_n at each section. The positive moment strength at a joint face must be at least equal to 50% of the negative moment strength provided at that joint. At the exterior negative location, this provision is satisfied since the positive design moment strength of 220.8 ft-kips is greater than 351.2/2 = 175.6 ft-kips. The provision is also satisfied at the interior negative location since 220.8 ft-kips is greater than 414.0/2 = 207.0 ft-kips. *21.3.2.2*

   Neither the negative nor the positive moment strength at any section along the length of the member shall be less than 25% of the maximum moment strength provided at the face of either joint. In this case, 25% of the maximum design moment strength is equal to 414.0/4 = 103.5 ft-kips. Providing at least 2-No. 8 bars (ϕM_n = 147.9 ft-kips) or 2-No. 7 bars (ϕM_n = 113.2 ft-kips) at any section will satisfy this requirement. However, to satisfy the minimum reinforcement requirement of 21.3.2.1 (i.e., minimum A_s = 1.43 in.^{2}), a minimum of 2-No. 8 bars (A_s = 1.58 in.^{2}) or 3-No. 7 bars (A_s = 1.80 in.^{2}) must be provided at any section. This also automatically satisfies the requirement that 2 bars be continuous at both the top and the bottom of any section. *21.3.2.2* *21.3.2.1*

Table 29-6 Required Reinforcement for Beam of Typical E-W Frame on Floor Level 6

Location		M_u (ft-kips)	Required A_s* (in.^{2})	Reinforcement*	ϕM_n** (ft-kips)
End Span	Ext. Neg.	-291.9	3.23	5-No. 8	351.2
		138.7	1.48	4-No. 7	220.8
	Positive	145.3	1.55	4-No. 7	220.8
	Int. Neg.	-366.2	4.14	6-No. 8	414.0
		120.1	1.43	4-No. 7	220.8
Interior Span	Positive	125.1	1.43	4-No. 7	220.8
	Negative	-354.3	3.99	5-No. 8	351.2
		135.7	1.45	4-No. 7	220.8

Max A_s = 0.025 x 20 x 21.5 = 10.75 in.^{2} (21.3.2.1)
Min. A_s = $\sqrt{4,000}$ x 20 x 21.5/60,000 = 1.36 in.^{2}
= 200 x 20 x 21.5/60,000 = 1.43 in.^{2} (governs)
***Does not include slab reinforcement.*

3. Calculate required length of anchorage of flexural reinforcement in exterior column.

Beam longitudinal reinforcement terminated in a column shall be extended to the far face of the confined column core and shall be anchored in tension according to 21.5.4 and in compression according to Chapter 12. *21.5.1.3*

Minimum development length ℓ_{dh} for a bar with a standard 90-degree hook in normal-weight concrete is

$$\ell_{dh} = \frac{f_y d_b}{65\sqrt{f_c'}}$$ *21.5.4.1*

$$\geq 8d_b$$

$$\geq 6 \text{ in.}$$

A standard hook is defined as a 90-degree bend plus a $12d_b$ extension at the free end of the bar. *7.1.2*

For the No. 8 top bars (bend diameter ≥ $6d_b$): *7.2.1*

$$\ell_{dh} = \begin{cases} (60{,}000 \times 1.00)/\left(65\sqrt{4000}\right) = 14.6 \text{ in.} & \text{(governs)} \\ 8 \times 1.00 = 8 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

For the No. 7 bottom bars (bend diamter ≥ $6d_b$):

$$\ell_{dh} = \begin{cases} (60{,}000 \times 0.875)/\left(65\sqrt{4000}\right) = 12.8 \text{ in.} & \text{(governs)} \\ 8 \times 0.875 = 7 \text{ in.} \\ 6 \text{ in.} \end{cases}$$

Note that the development length ℓ_{dh} is measured from the near face of the column to the far edge of the vertical 12-bar-diameter extension (see Fig. 29-29).

When reinforcing bars extend through a joint, the column dimension must be at least 20 times the diameter of the largest longitudinal bar for normal weight concrete. In this case, the minimum required column dimension is 20 × 1.0 = 20 in., which is less than each of the two column widths that is provided. *21.5.1.4*

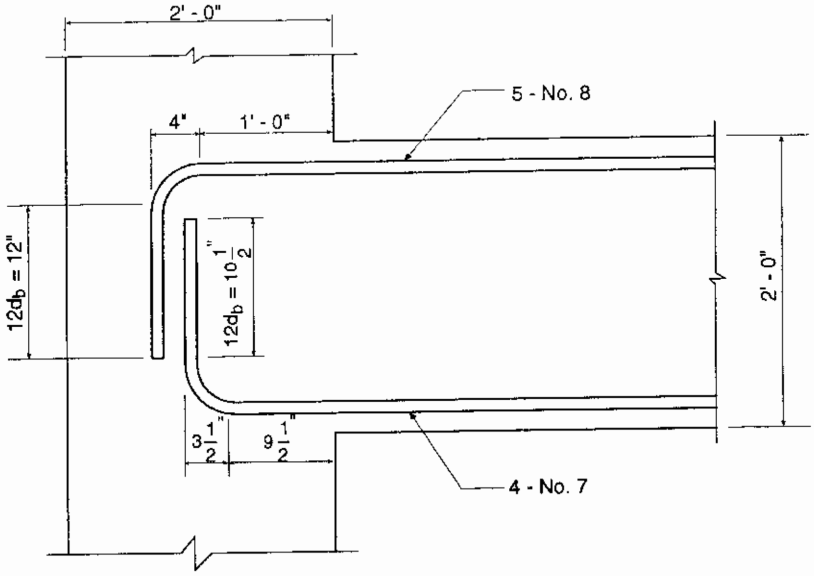


Figure 29-29 Detail of Flexural Reinforcement Anchorage at Exterior Column

4. Determine shear reinforcement requirements.

Design for shear forces corresponding to end moments that are calculated by assuming the stress in the tensile flexural reinforcement equal to $1.25f_y$ and a strength reduction factor, $\phi = 1.0$ (probable flexural strength), plus shear forces due to factored tributary gravity loads. *21.3.4.1*

The following equation can be used to compute M_{pr}*:

$$M_{pr} = A_s\,(1.25f_y)\left(d - \frac{a}{2}\right)$$

$$\text{where } a = \frac{A_s(1.25f_y)}{0.85f'_c b}$$

* The slab reinforcement within the effective slab width defined in 8.10 is not included in the calculation of M_{pr} (note that this reinforcement must be included when computing the flexural strength of the beam when checking the requirements of 21.4.2). It is unlikely that all or even most of the reinforcement within the slab effective width away from the beam will yield when subjected to the forces generated from the design-basis earthquake. Furthermore, including the slab reinforcement in the calculation of M_{pr} would result in a major deviation from how members have been designed in the past. In particular, the magnitude of the negative probable moment strength of the beam would significantly increase if the slab reinforcement were included. This in turn would have a significant impact on the shear strength requirements of the beam (21.3.4) and most likely the columns framing into the joint as well (21.4.5). Such significant increases seem unwarranted when compared to the appropriate provisions in previous editions of the ACI Code and other codes.

For example, for sidesway to the right, the interior joint must be subjected to the negative moment M_{pr} which is determined as follows:

For 6-No. 8 top bars, $A_s = 6 \times 0.79 = 4.74$ in.^{2}

$$a = \frac{A_s(1.25f_y)}{0.85f'_c b} = \frac{4.74 \times 1.25 \times 60}{0.85 \times 4 \times 20} = 5.23 \text{ in.}$$

$$M_{pr} = A_s(1.25f_y)\left(d - \frac{a}{2}\right) = 4.74 \times 1.25 \times 60 \times \left(21.5 - \frac{5.23}{2}\right) = 6713.6 \text{ in.-kips} = 559.5 \text{ ft-kips}$$

Similarly, for the exterior joint, the positive moment M_{pr} based on the 4-No. 7 bottom bars is equal to 302.6 ft-kips. The probable flexural strengths for sidesway to the left can be obtained in a similar fashion.

The factored gravity load at midspan is:

$$w_D = \left[\frac{8}{12}(145) + 30\right] \times 22 + \frac{16 \times 20}{144}(145) = 3109 \text{ lbs/ft}$$

$$w_L = 75 \times 22 = 1650 \text{ lbs/ft}$$

$$w_u = 1.2^* w_D + 0.5w_L = 4.56 \text{ kips/ft}$$ *Eq. (9-5)*

Figure 29-30 shows the exterior beam span and the shear forces due to the gravity loads. Also shown are the probable flexural strengths M_{pr} at the joint faces for sidesway to the right and to the left and the corresponding shear forces due to these moments. Note that the maximum combined design shear forces are larger than those obtained from the structural analysis. *21.3.4.2*

The shear strength of concrete V_c is to be taken as zero when the earthquake-induced shear force calculated in accordance with 21.3.4.1 is greater than or equal to 50% of the total shear force and the factored axial compressive force is less than $A_g f'_c/20$ where A_g is the gross cross-sectioned area of the beam. The beam carries negligible axial forces, and the maximum earthquake-induced shear force, which is equal to 36.3 kips (see Fig. 29-30), is greater than one-half the total design shear force which is equal to $0.5 \times 68.1 = 34.1$ kips. Thus, V_c must be taken equal to zero. The maximum shear force V_s is:

$$\phi V_s = V_u - \phi V_c$$

$$V_s = \frac{V_u}{\phi} - V_c$$

$$= \frac{68.1}{0.75} - 0 = 90.8 \text{ kips}$$

* Note that in seismic design complying with the IBC, the factor would be $(1.2 + 0.2S_{DS})$, where S_{DS} is the design spectral response acceleration at short periods at the site of the structure.

where the strength reduction factor ϕ is 0.75. *9.3.4*

Shear strength contributed by shear reinforcement must not exceed $(V_s)_{max}$: *11.5.6.9*

$$(V_s)_{max} = 8\sqrt{f'_c}b_wd = 8\sqrt{4000} \times 20 \times 21.5/1000 = 217.6 \text{ kips} > 90.8 \text{ kips} \qquad \text{O.K.}$$

Also, V_s is less than $4\sqrt{f'_c}b_wd = 108.8$ kips. *11.5.4.3*

Required spacing of No. 3 closed stirrups (hoops) for a factored shear force of 90.8 kips is: *11.5.6.2*

$$s = \frac{A_vf_yd}{V_s} = \frac{(4 \times 0.11) \times 60 \times 21.5}{90.8} = 6.3 \text{ in.}$$

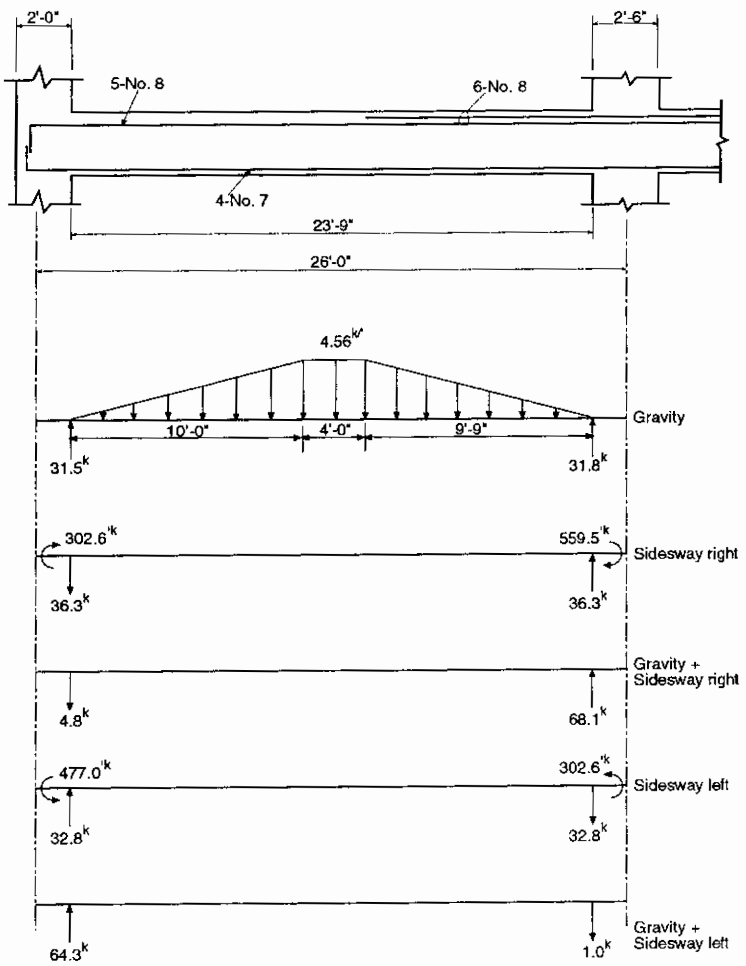


Figure 29-30 Design Shear Forces for Exterior Beam Span of Typical E-N Frame on Floor Level 1

Note that 4 legs are required for lateral support of the longitudinal bars. *21.3.3.3*

Maximum allowable hoop spacing (s_{max}) within a distance of $2h = 2 \times 24 = 48$ in. from the face of the support is the smallest of the following: *21.3.2*

$$s_{max} = \frac{d}{4} = \frac{21.5}{4} = 5.4 \text{ in. (governs)}$$

$= 8 \times$ (diameter of smallest longitudinal bar) $= 8 \times 0.875 = 7.0$ in.

$= 24 \times$ (diameter of hoop bar) $= 24 \times 0.375 = 9.0$ in.

$= 12$ in.

Therefore, hoops must be spaced at 5 in. on center with the first one located at 2 in. from the face of the support. Eleven hoops are to be placed at this spacing.

Where hoops are no longer required, stirrups with seismic hooks at both ends may be used. At a distance of 52 in. from the face of the interior support, $V_u = 63.7$ kips. *21.3.3.4*

With $V_c = 2\sqrt{4000} \times 20 \times 21.5/1000 = 54.4$ kips, the spacing required for No. 3 stirrups with two legs is 9.3 in. < d/2 = 10.8 in.

A 9 in. spacing, starting at 52 in. from the face of the support will be sufficient for the remaining portion of the beam.

5. Negative reinforcement cutoff points.

For the purpose of determining cutoff points for the negative reinforcement at the interior support, a moment diagram corresponding to the probable flexural strengths at the beam ends and 0.9* times the dead load on the span will be used. The cutoff point for four of the six No. 8 bars at the top will be determined. *Eq. (9-7)*

With the design flexural strength of a section with 2-No. 8 top bars = 147.9 ft-kips (calculated using $f_s = f_y = 60$ ksi and $\phi = 0.9$, since a section with such light reinforcement will be tension-controlled), the distance from the face of the support to where the moment under the loading considered equals 147.9 ft-kips is readily obtained by summing moments about section a-a in Fig. 29-31, and equating these to 147.9 ft-kips:

$$\frac{x}{2}\left(\frac{2.8x}{9.75}\right)\left(\frac{x}{3}\right) - 55.8x + 559.5 = 147.9$$

* Note that in seismic design complying with the IBC, the factor would be $(1.2 + 0.2S_{DS})$, where S_{DS} is the design spectral response acceleration at short periods at the site of the structure.

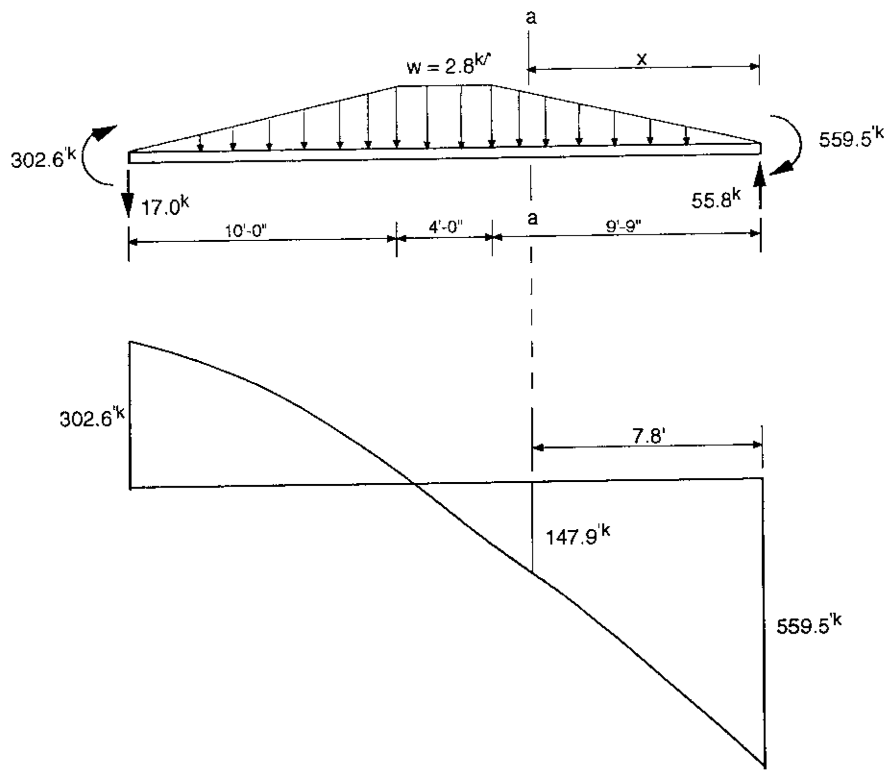


Figure 29-31 Moment Diagram for Cutoff Location of Negative Bars at Interior Support

Solving for x gives a distance of 7.8 ft. The 4-No. 8 bars must extend a distance d = 21.5 in. or $12d_b = 12 \times 1.0 = 12$ in. beyond the distance x. Thus, from the face of the support, the total bar length must be at least equal to 7.8 + (21.5/12) = 9.6 ft. Also, the bars must extend a full development length ℓ_d beyond the face of the support: *12.10.3* *12.10.4*

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s\lambda}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b$$ *Eq. (12-1)*

where ψ_t = reinforcement location factor = 1.3 (top bar)

ψ_e = coating factor = 1.0 (uncoated reinforcement)

ψ_s= reinforcement size factor = 1.0 (No. 8 bar)

λ = lightweight aggregate concrete factor = 1.0 (normal weight concrete)

$$c = \text{spacing or cover dimension} = \begin{cases} 1.5 + 0.375 + \dfrac{1.0}{2} = 2.375 \text{ in.} \\ \dfrac{20-2(1.5 + 0.375)-1.0}{2 \times 5} = 1.525 \text{ in. (governs)} \end{cases}$$

K_{tr} = transverse reinforcement index = 0 (conservative)

$$\frac{c + K_{tr}}{d_b} = \frac{1.525 + 0}{1.0} = 1.525$$

$$\ell_d = \frac{3}{40} \times \frac{60{,}000}{\sqrt{4000}} \times \frac{1.3 \times 1.0 \times 1.0 \times 1.0}{1.525} \times 1.0 = 60.7 \text{ in.} = 5.1 \text{ ft} < 9.6 \text{ ft}$$

The total required length of the 4-No. 8 bars must be at least 9.6 ft beyond the face of the support.

Flexural reinforcement shall not be terminated in a tension zone unless one or more of the conditions of 12.10.5 are satisfied. In this case, the point of inflection is approximately 11.25 ft from the face of the right support which is greater than 9.6 ft. The 4-No. 8 bars can not be terminated here unless one of the conditions of 12.10.5 is satisfied.

Check if the factored shear force V_u at the cutoff point does not exceed two-thirds of ϕV_n. For No. 3 stirrups spaced at 9 in. on center that are provided in this region: *12.10.5.1*

$$\phi V_n = \phi(V_s + V_c) = 0.75 \times \left(\frac{0.22 \times 60 \times 21.5}{9} + 54.4\right) = 64.5 \text{ kips}$$

$$\frac{2}{3}\phi V_n = 43.0 \text{ kips} > V_u = 42.7 \text{ kips at 9.6 ft from face of support}$$

Since $2\phi V_n/3 > V_u$, the cutoff point for the 4-No. 8 bars can be 9.6 ft beyond the face of the interior support.

The cutoff point for three of the 5-No. 8 bars at the exterior support can be determined in a similar fashion. These bars can be cut off at 8.4 ft from the face of the exterior support.

6. Flexural reinforcement splices.

Lap splices of flexural reinforcement must not be placed within a joint, within a distance 2h from faces of supports or within regions of potential plastic hinging. Note that all lap splices have to be confined by hoops or spirals with a maximum spacing or pitch of d/4 or 4 in. over the length of the lap. Lap splices will be determined for the No. 7 bottom bars. *21.3.2.3*

Since all of the bars will be spliced within the required length, use a Class B splice. *12.15.2*

Required length of splice = $1.3\ell_d \geq 12$ in. *12.15.1*

where

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s\lambda}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b$$ *Eq. (12-1)*

reinforcement location factor ψ_t = 1.0 (other than top bars) 12.2.4

coating factor ψ_e = 1.0 (uncoated bars)

reinforcement size factor ψ_s = 1.0 (No. 7 bar)

lightweight aggregate concrete factor λ = 1.0 (normal weight concrete)

$$c = 1.5 + 0.375 + \frac{0.875}{2} = 2.31 \text{ in.} \quad \text{(governs)}$$

$$= \frac{1}{2}\left[\frac{20 - 2(1.5 + 0.375) - 0.875}{3}\right] = 2.56 \text{ in.}$$

$$K_{tr} = \frac{A_{tr}f_{yt}}{1500sn} = \frac{(2\times 0.11)(60{,}000)}{1500\times 4.0\times 4} = 0.55$$

$$\frac{c + K_{tr}}{d_b} = \frac{2.31 + 0.55}{0.875} = 3.3 > 2.5, \text{ use } 2.5$$

Therefore,

$$\ell_d = \frac{3}{40}\times\frac{60{,}000}{\sqrt{4000}}\times\frac{1.0\times 1.0\times 1.0\times 1.0}{2.5}\times 0.875 = 24.9 \text{ in.}$$

Class B splice length = 1.3 × 24.9 = 32.4 in.

7. Reinforcement details for the beam are shown in Fig. 29-32.

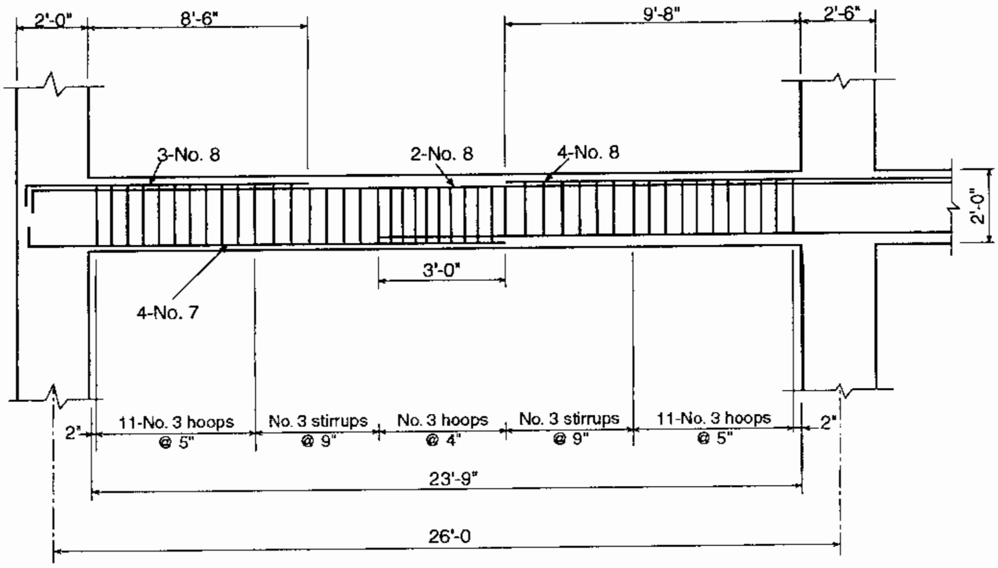


Figure 29-32 Reinforcement Details for Exterior Beam on Floor Level 1

Example 29.3—Proportioning and Detailing of Columns of Building in Example 29.1

Determine the required reinforcement for an edge column supporting the first floor of a typical E-W interior frame. The column dimensions have been established at 24-in. square. Use $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

Calculations and Discussion	Code Reference

Table 29-7 contains a summary of the factored axial loads and bending moments for an edge column in the first floor level for seismic forces in the E-W direction.

From Table 29-7, maximum $P_u = 1012$ kips *21.4.1*

$P_u = 1012 \text{ kips} > A_g f'_c/10 = (24 \times 24) \times 4/10 = 230 \text{ kips}$

Thus, the provisions of 21.4 governing special moment frame members subjected to bending and axial load apply.

Table 29-7 Summary of Factored Axial Loads and Bending Moments for an Edge Column in the First Story for Seismic Forces in the E-W Direction

Load Combination	Axial Load, P_u (kips)	Bending Moment, M_u (ft-kips)
1.2D + 1.6L	1002.9	-78.2
1.2D + 0.5L + E	722.8	166.4
1.2D + 0.5L − E	1012.0	-275.6
0.9D + E	459.8	188.1
0.9D − E	749.0	-253.9

1. Check satisfaction of limitations on section dimensions.

   • Shortest cross-sectional dimension = 24 in. > 12 in. O.K. *21.4.1.1*

   • Ratio of shortest cross-sectional dimension to perpendicular dimension = 1.0 > 0.4 O.K. *21.4.1.2*

2. Determine required longitudinal reinforcement.

   Based on the load combinations in Table 29-7, a 24 × 24 in. column with 8-No. 8 bars ($\rho_g = 1.10\%$) is adequate for the column supporting the first floor level.

   Note that $0.01 < \rho_g \le 0.06$ O.K. *21.4.3.1*

3. Nominal flexural strength of columns relative to that of beams in E-W direction.

$$\Sigma M_c \text{ (columns)} \geq \frac{6}{5} \Sigma M_g \text{ (beams)}$$ 21.4.2.2

The nominal negative flexural strength M_n^- of the beam framing into the column must include the slab reinforcement within an effective slab width equal to: 21.4.2.2

$(16 \times 8) + 20 = 148$ in.
$22 \times 12 = 264$ in. 8.10.2
$(26 \times 12)/4 = 78$ in. (governs)

The minimum required A_s in the 78-in. effective width is equal to $0.0018 \times 78 \times 8 = 1.12$ in.^{2}, which corresponds to 6-No. 4 bars @ 78/6 = 13 in. spacing. This spacing is less than the maximum bar spacing (= 2h = 16 in.). Provide No. 4 @ 13 in. at both the top and bottom of the slab (according to Fig. 13.3.8, 100 percent of both the top and the bottom reinforcement in the column strip must be continuous or anchored at the support).

A strain compatibility analysis of the section yields M_n^- of the beam equal to 632 ft-kips.

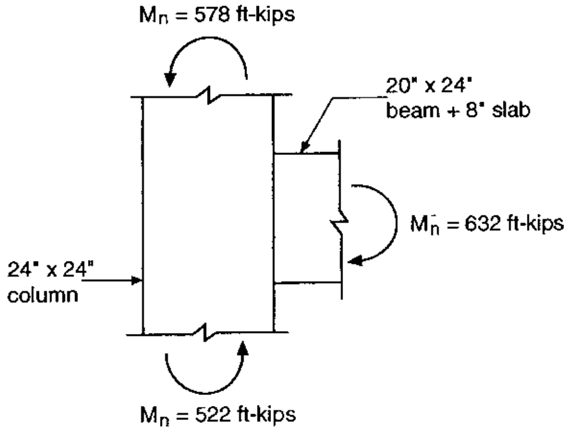
For the lower end of the upper column framing into the joint, the minimum nominal flexural strength is 578 ft-kips, which corresponds to $P_u = 922$ kips. Similarly, the minimum M_n is 522 ft-kips for the upper end of the lower column framing into the joint; this corresponds to $P_u = 1012$ kips.

Therefore,

$\Sigma M_c = 578 + 522 = 1100$ ft-kips

$\Sigma M_g = 632$ ft-kips

$1100 \text{ ft-kips} > \frac{6}{5} \times 632 = 758$ ft-kips O.K.



4. Nominal flexural strength of columns relative to that of beams in the N-S direction.

The beams in the N-S direction framing into columns at the first floor level require 4-No. 7 bars at both the top and the bottom of the section.

The nominal negative flexural strength M_n^- of the beams framing into the column must include the slab reinforcement within an effective slab width equal to:

$(22 \times 12)/12 + 20 = 42$ in. (governs)

$(6 \times 8) + 20 = 68$ in. *8.10.3*

$(23.75 \times 12)/2 + 20 = 162.5$ in.

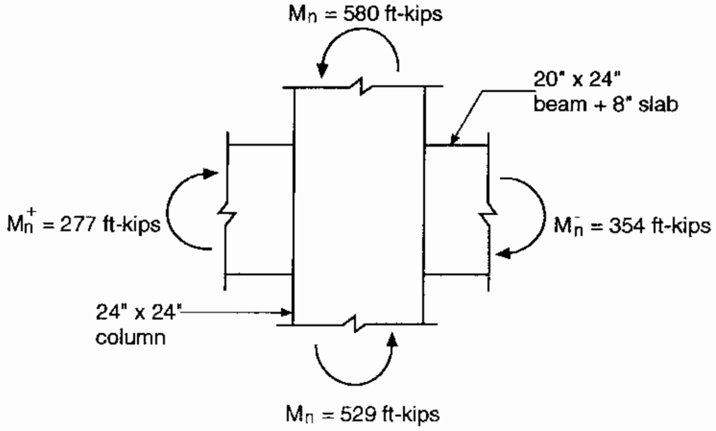
The minimum A_s in the 42-in. effective width is equal to $0.0018 \times 42 \times 8 = 0.6$ in.^{2}, which corresponds to 3-No. 4 bars @ 42/3 = 14 in. spacing. This spacing is less than the maximum bar spacing (= 2h = 16 in.). Provide No. 4 @ 14 in. at both the top and the bottom of the slab (according to Fig. 13.3.8, 100 percent of both the top and the bottom reinforcement in the column strip must be continuous or anchored at the support).

A strain compatibility analysis of the section yields $M_n^- = 354$ ft-kips and $M_n^+ = 277$ ft-kips.

For the lower end of the upper column framing into the joint, the minimum nominal flexural strength is 580.4 ft-kips, which corresponds to $P_u = 918$ kips. Similarly, the minimum M_n is 528.6 ft-kips for the upper end of the lower column framing into the joint; this corresponds to $P_u = 1{,}003$ kips.

$\Sigma M_g = 354 + 277 = 631$ ft-kips

$\Sigma M_c = 580 + 529 = 1{,}109 \text{ ft-kips} > \frac{6}{5}\Sigma M_g = \frac{6}{5} \times 631 = 757$ ft-kips O.K. *21.4.4.4 Eq. (21-1)*



5. Determine transverse reinforcement requirements.

   a. Confinement reinforcement (see Fig. 29-7(b)).

   Transverse reinforcement for confinement is required over a distance ℓ_o from the column ends where

$$\ell_o \geq \begin{cases} \text{depth of member} = 24 \text{ in.} \\ 1/6\,(\text{clear height}) = (14 \times 12)/6 = 28 \text{ in. (governs)} \\ 18 \text{ in.} \end{cases}$$ 21.4.4.4

Maximum allowable spacing of rectangular hoops assuming No. 4 hoops with a crosstie in each direction: 21.4.4.2

$$s_{max} = 0.25\,(\text{smallest dimension of column}) = 0.25 \times 24 = 6 \text{ in.}$$

$$= 6\,(\text{diameter of longitudinal bar}) = 6 \times 1.0 = 6 \text{ in.}$$

$$= s_o = 4 + \left(\frac{14 - h_x}{3}\right) = 4 + \left(\frac{14 - 11}{3}\right) = 5 \text{ in.} \quad < 6 \text{ in.} \quad \text{(governs)}$$

$$> 4 \text{ in.}$$

$$\text{where } h_x = \frac{24 - 2\left(1.5 + 0.5 + \frac{1.0}{2}\right)}{2} + 2\left(\frac{1.0}{2} + \frac{0.5}{2}\right) = 11 \text{ in.}$$

Required cross-sectional area of confinement reinforcement in the form of hoops:

$$A_{sh} \geq \begin{cases} 0.3sb_c\left[\frac{A_g}{A_{ch}} - 1\right]\frac{f'_c}{f_{yt}} & \text{Eq. (21-3)} \\ 0.09sb_c\,\frac{f'_c}{f_{yt}} & \text{Eq. (21-4)} \end{cases}$$

where

s = spacing of transverse reinforcement (in.)

b_c = cross-sectional dimension of column core, measured center-to-center of confining reinforcement (in.) = 24 - 2(1.5 + 0.25) = 20.5 in.

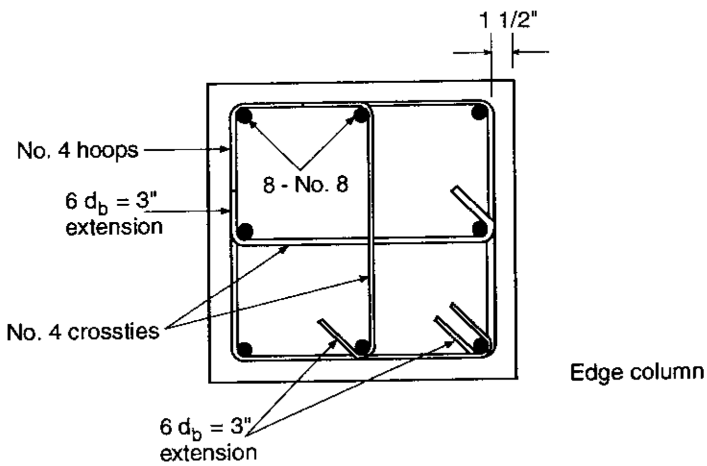
A_{ch} = core area of column section, measured outside to-outside of transverse reinforcement (in.^{2}) = [24 - (2 × 1.5)]^2 = 441 in.^{2}

f_{yt} = specified yield strength of transverse reinforcement (psi)

For a hoop spacing of 5 in. and f_{yt} = 60,000 psi, the required cross-sectional area is:

$$A_{sh} \geq \begin{cases} (0.3 \times 5 \times 20.5)\left(\dfrac{576}{441} - 1\right)\dfrac{4000}{60{,}000} = 0.63\,\text{in.}^2 & \text{(governs)} \\ \\ (0.09 \times 5 \times 20.5)\dfrac{4000}{60{,}000} = 0.62\,\text{in.}^2 \end{cases}$$

No. 4 hoops with one crosstie, as shown in the sketch below, provides $A_{sh} = 3 \times 0.20 = 0.60$ in.^{2} < 0.63 in.^{2} Either accept or reduce hoop spacing to 4 in. so that governing A_{sh} = 0.50 in.^{2} < provided A_{sh} = 0.60 in.^{2} *21.4.4.3*



b. Transverse reinforcement for shear.

As in the design of shear reinforcement for beams, the design shear for columns is based not on the factored shear forces obtained from a lateral load analysis but rather on the nominal flexural strengths provided in the columns. The column design shear forces shall be determined from the consideration of the maximum forces that can be developed at the faces of the joints, with the probable flexural strengths calculated for the factored axial compressive forces resulting in the largest moments acting at the joint faces. *21.4.5*

The largest probable flexural strength that may develop in the column can conservatively be assumed to correspond to the balanced point of the column interaction diagram.

With the strength reduction factors equal to 1.0 and $f_y = 1.25 \times 60 = 75$ ksi, the moment corresponding to balanced failure is 742 ft-kips. Thus, $V_u = (2 \times 742)/14 = 106$ kips.

The shear force need not exceed that determined from joint strengths based on the probable flexural strengths M_{pr} of the members framing into the joint. For seismic forces in the E-W direction, the negative probable flexural strength of the beam framing into the joint at the face of the edge column is 477.0 ft-kips (see Fig. 29-30). *21.4.5.1*

Distribution of this moment to the columns is proportional to EI/ℓ of the columns above and below the joint. Since the columns above and below the joint have the same cross-section, reinforcement, and concrete strength, EI is a constant, and the moment is distributed according to $1/\ell$. Therefore, the moment at the top of the first story column is

$$477.0\left(\frac{12}{12 + 16}\right) = 204.4 \text{ ft-kips}$$

It is possible for the base of the first story column to develop the probable flexural strength of 742.0 ft-kips. Thus, the shear force is

$$V_u = \frac{204.4 + 742.0}{14} = 67.6 \text{ kips}$$

For seismic forces in the N-S direction, the negative probable flexural strength of the beam framing into one side of the column is 302.6 ft-kips (4-No. 7 top bars). The positive probable flexural strength of the beam framing into the other side of the column is also 302.6 ft-kips (4-No. 7 bottom bars). Therefore, at the top of the first story column, the moment is

$$(2 \times 302.6)\left(\frac{12}{12 + 16}\right) = 259.4 \text{ ft-kips}$$

The shear force is

$$V_u = \frac{259.4 + 742.0}{14} = 71.5 \text{ kips}$$

Both of these shear forces are greater than those obtained from analysis.

Since the factored axial forces are greater than $A_g f'_c / 20 = 115$ kips, the shear strength of the concrete may be used: *21.4.5.2*

$$V_c = 2\sqrt{f'_c}\,bd\left(1 + \frac{N_u}{2000A_g}\right)$$ *Eq. (11-4)*

Conservatively using the minimum axial load from Table 29-7,

$$V_c = \frac{2\sqrt{4000}\,(24 \times 17.7)}{1000}\left[1 + \frac{459{,}800}{2000 \times (24)^2}\right] = 75.2 \text{ kips}$$

$$V_s = \frac{A_v f_y d}{s} = \frac{(3 \times 0.20) \times 60 \times 17.7}{4.5} = 141.6 \text{ kips}$$

$$\phi(V_c + V_s) = 0.75(75.2 + 141.6) = 162.6 \text{ kips} > V_u = 71.5 \text{ kips O.K.}$$

Thus, the transverse reinforcement spacing over the distance ℓ_o = 28 in. near the column ends required for confinement is also adequate for shear.

The remainder of the column length must contain hoop reinforcement with center-to-center spacing not to exceed either six times the diameter of the column longitudinal bars (= 6 × 1.0 = 6.0 in.) or 6 in. *21.4.4.6*

Use No. 4 hoops and crossties spaced at 4 in. within a distance of 28 in. from the column ends and No. 4 hoops spaced at 6 in. or less over the remainder of the column.

6. Minimum length of lap splices of column vertical bars.

The location of lap splices of column bars must be within the center half of the member length. Also, the splices are to be designed as tension splices. If all the bars are spliced at the same location, the splices need to be Class B. Transverse reinforcement at 4.5 in. is to be provided over the full lap splice length. *21.4.3.2*

Required length of Class B splice = $1.3\ell_d$ *12.15.1*

where

$$\ell_d = \left(\frac{3}{40}\,\frac{f_y}{\sqrt{f'_c}}\,\frac{\psi_t\psi_e\psi_s\lambda}{\left(\dfrac{c_b + K_{tr}}{d_b}\right)}\right)d_b \qquad \textit{Eq. (12-1)}$$

reinforcement location factor ψ_t = 1.0 (other than top bars) *12.2.4*

coating factor ψ_e = 1.0 (uncoated bars)

reinforcement size factor ψ_s = 1.0 (No. 7 and larger bars)

lightweight aggregate concrete factor λ = 1.0 (normal weight concrete)

$$c = 1.5 + 0.5 + \frac{1.0}{2} = 2.5 \text{ in.} \quad \text{(governs)}$$

$$= \frac{1}{2}\left[\frac{24-2(1.5+0.5)-1.0}{2}\right] = 4.75 \text{ in.}$$

$$K_{tr} = \frac{A_{tr}f_{yt}}{1500sn} = \frac{(3\times0.20)(60{,}000)}{1500\times4.5\times3} = 1.8$$

where A_{tr} is for 3-No. 4 bars, s (the maximum spacing of transverse reinforcement within ℓ_d) = 4.5 in., and n (number of bars being developed) = 3

$$\frac{c+K_{tr}}{d_b} = \frac{2.5+1.8}{1.0} = 4.3 > 2.5\text{, use } 2.5$$

Therefore,

$$\ell_d = \frac{3}{40}\times\frac{60{,}000}{\sqrt{4000}}\times\frac{1.0\times1.0\times1.0\times1.0}{2.5}\times 1.0 = 28.5 \text{ in.}$$

Class B splice length = 1.3 × 28.5 = 37.1 in.

Use a 3 ft-2 in. splice length.

7. Reinforcement details for the column are shown in Fig. 29-33. Note that for practical purposes, a 4-in. hoop spacing is used over the entire length of the column.

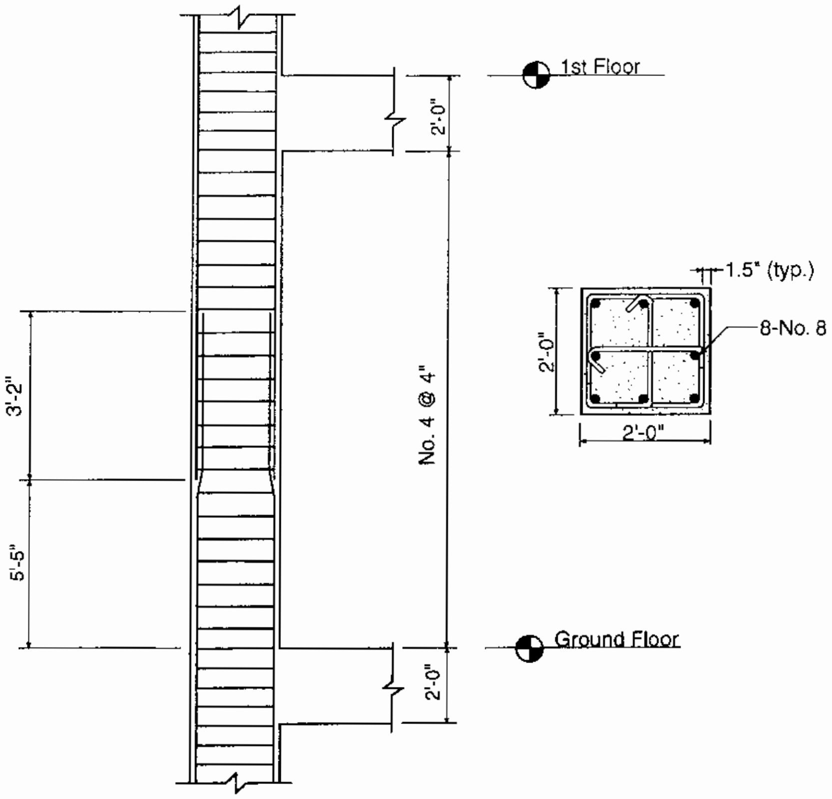


Figure 29-33 Reinforcement Details for Edge Column Supporting Level 1

Example 29.4—Proportioning and Detailing of Exterior Beam-Column Connection of Building in Example 29.1

Determine the transverse reinforcement and shear strength requirements for an exterior beam-column connection between the beam considered in Example 29.2 and the column of Example 29.3. Assume the joint to be located at the first floor level.

Calculations and Discussion	Code Reference

1. Transverse reinforcement for confinement.

   Section 21.5.2.1 requires the same amount of confinement reinforcement within the joint as for the length ℓ_o at column ends, unless the joint is confined by beams framing into all vertical faces of the column. A member that frames into a face is considered to provide confinement if at least three-quarters of the face of the joint is covered by the framing member.

   In the case of the beam-column joint considered here, beams frame into only three sides of the column. In Example 29.3, confinement requirements at column ends were satisfied by No. 4 hoops with crossties spaced at 4 in.

2. Check shear strength of joint in E-W direction.

   The shear force across section x-x (see Fig. 29-34) of the joint is obtained as the difference between the tensile force from the top flexural reinforcement of the framing beam (stressed to $1.25f_y$) and the horizontal shear from the column above. *21.5.1.1*

   $T = A_s (1.25f_y) = (5 \times 0.79)(1.25 \times 60) = 296 \text{ kips}$

   An estimate of the horizontal shear from the column, V_h, can be obtained by assuming that the beams in the adjoining floors are also deformed so that plastic hinges form at their junctions with the column, with M_{pr} (beam) = 477.0 ft-kips (see Fig. 29-30). By further assuming that the end moments in the beams are resisted by the columns above and below the joint inversely proportional to the column lengths, the average horizontal shear in the column is approximately:

   $$V_h = \frac{2 \times 477.0}{12 + 16} = 34.1 \text{ kips}$$

   Thus, the net shear at section x-x of the joint is $V_u = 296 - 34.1 = 261.9$ kips. Section 21.5.3.1 gives the nominal shear strength of a joint as a function of the area of the joint cross-section, A_j, and the degree of confinement by framing beams. For the joint confined on three faces considered here (note: beam width = 20 in. > 0.75 (column width) = $0.75 \times 24 = 18$ in.) :

   $$\phi V_c = \phi 15\sqrt{f'_c}\, A_j$$ *21.5.3.1* *9.3.4*

   $$= 0.85 \times 15\sqrt{4000} \times 24^2 / 1000 = 464.5 \text{ kips} > 261.9 \text{ kips} \quad \text{O.K.}$$

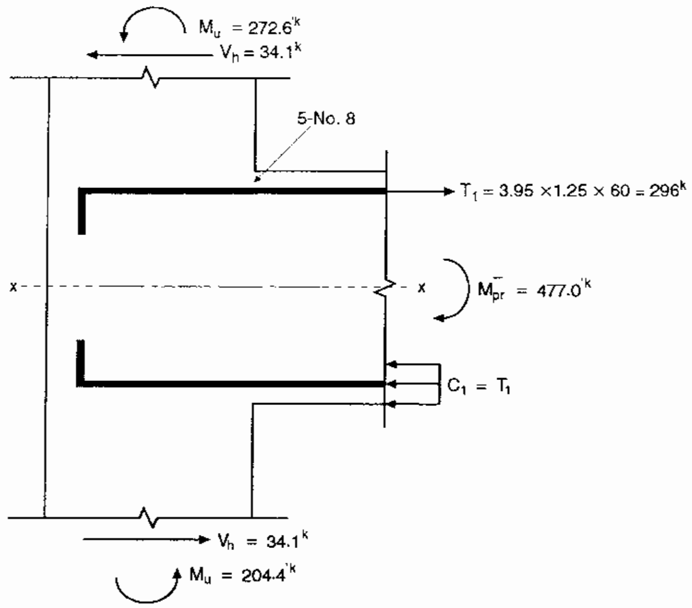


Figure 29-34 Shear Analysis of Exterior Beam-Column Joint in E-W Direction

3. Check shear strength of joint in N-S direction.

The shear force across section x-x (see Fig. 29-35) of the joint is determined as follows:

$T_1 = A_s(1.25f_y) = (4 \times 0.60)(1.25 \times 60) = 180$ kips

The negative and positive probable flexural strengths at the joint are 302.6 ft-kips (4-No. 7 bars top and bottom).

The average horizontal shear in the column is approximately:

$$V_u = \frac{2(302.6 + 302.6)}{12 + 16} = 43.2 \text{ kips}$$

Thus, the net shear at section x-x of the joint is

$$V_u = T_1 + C_2 - V_u = 180 + 180 - 43.2 = 316.8 \text{ kips} < \phi V_c = 464.5 \text{ kips O.K.}$$

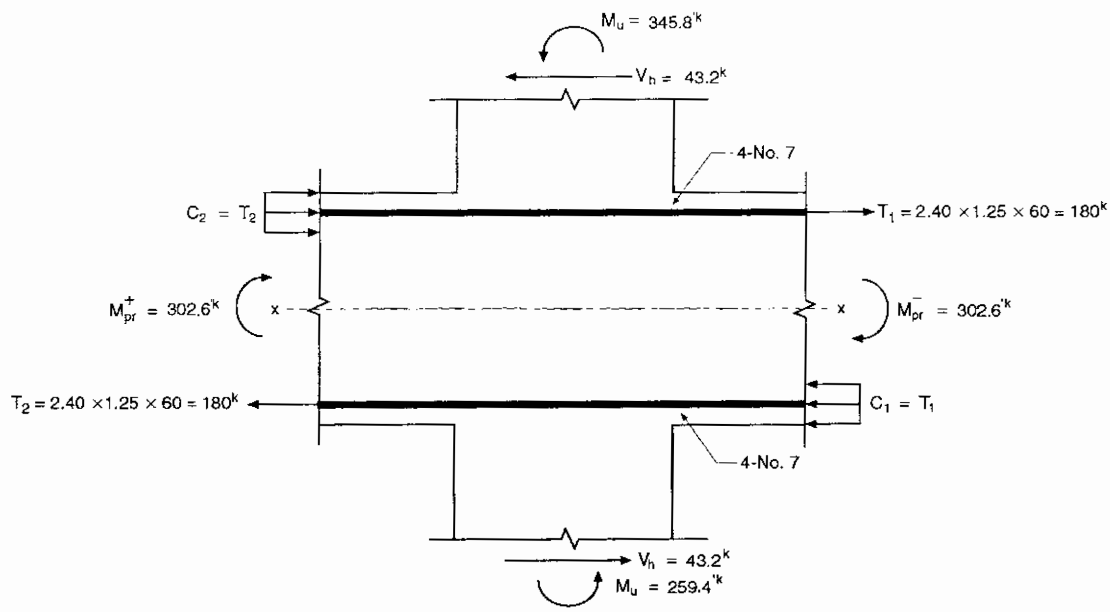


Figure 29-35 Shear Analysis of Exterior Beam-Column Joint in N-S Direction

Note that if the shear strength of the concrete in the joint as calculated above were inadequate, any adjustment would have to take the form of either an increase in the column cross-section (and hence A_j) or an increase in the beam depth (to reduce the amount of flexural reinforcement required and hence the tensile force T) since transverse reinforcement is considered not to have a significant effect on shear strength.

4. Reinforcement details for the exterior joint are shown in Fig. 29-36.

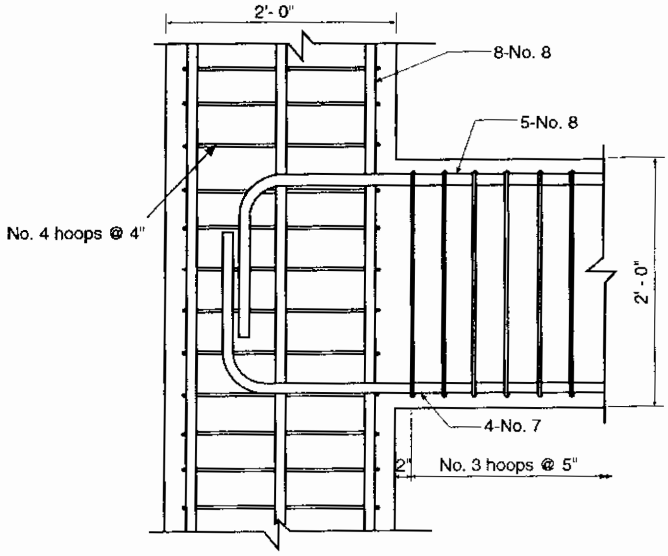


Figure 29-36 Reinforcement Details of Exterior Joint

Example 29.5—Proportioning and Detailing of Interior Beam-Column Connection of Building in Example 29.1

Determine the transverse reinforcement and shear strength requirements for the interior beam-column connection at the first floor of the interior E-W frame considered in the previous examples. The column is 30-in. square and is reinforced with 12-No. 8 bars. The beams have dimensions of b = 20 in. and d = 21.5 in. and are reinforced as noted in Example 29.2 (see Fig. 29-32).

Calculations and Discussion	Code Reference

1. Determine transverse reinforcement requirements.

   a. Confinement reinforcement

   Maximum allowable spacing of rectangular hoops assuming No. 4 hoops with two crossties in both directions:

   s_{max} = 0.25 (least dimension of column) = 0.25 × 30 = 7.5 in. *21.4.4.2*

   = 6 (diameter of longitudinal bar) = 6 × 1.00 = 6 in.

   $$= s_o = 4 + \left(\frac{14 - h_x}{3}\right) = 4 + \left(\frac{14 - 9.83}{3}\right) = 5.4 \text{ in.} < 6 \text{ in.} \quad \text{(governs)}$$

   $$> 4 \text{ in.}$$

   $$\text{where } h_x = \frac{30 - 2\left(1.5 + 0.5 + \dfrac{1.00}{2}\right)}{3} + 2\left(\frac{1.00}{2} + \frac{1}{4}\right) = 9.83 \text{ in.}$$

   With a hoop spacing of 5 in., the required cross-sectional area of confinement reinforcement in the form of hoops is:

   $$A_{sh} \geq \begin{cases} 0.3sb_c\left(\dfrac{A_g}{A_{ch}} - 1\right)\dfrac{f'_c}{f_{yt}} = (0.3 \times 5 \times 26.5)\left(\dfrac{900}{729} - 1\right)\dfrac{4000}{60{,}000} = 0.62 \text{ in.}^2 & \textit{Eq.(21-3)} \\ 0.09sb_c\dfrac{f'_c}{f_{yt}} = (0.09 \times 5 \times 26.5)\dfrac{4000}{60{,}000} = 0.80 \text{ in.}^2 \quad \text{(governs)} & \textit{Eq.(21-4)} \end{cases}$$

   Since the joint is framed by beams having widths = 20 in. < 3/4 width of column = 22.5 in. on all four sides, it is not considered confined and a 50% reduction in the amount of confinement reinforcement indicated above is not allowed. *21.5.2.2*

   No. 4 hoops spaced at 5 in. on center provide A_{sh} = 0.20 × 4 = 0.80 in.^{2}

   b. Transverse reinforcement for shear

   Following the same procedure in Example 29.4, the shear forces in the column are obtained for seismic forces in the E-W and N-S directions.

The largest probable flexural strength that may develop in the column can conservatively be assumed to correspond to the balanced point of the column interaction diagram.

With the strength reduction factor equal to 1.0 and $f_y = 1.25 \times 60 = 75$ psi, the moment corresponding to balanced failure is 1438 ft-kips. Thus, $V_u = (2 \times 1{,}438)/14 = 205.4$ kips.

The shear force need not exceed that determined from joint strengths based on the M_{pr} of the beams framing into the joint. *21.4.5.1*

For seismic forces in the E-W direction, M^-_{pr} of the beam framing into the joint at the face of the interior column is 477.0 ft-kips (5-No. 8 top bars). The M^+_{pr} is 302.6 ft-kips (4-No. 7 bottom bars) based on the beam framing into the other face of the joint. Distributing the moment to the columns in proportion to $1/\ell$, the moment at the top of the first story column is:

$$(477.0 + 302.6)\left(\frac{12}{12 + 16}\right) = 334.1 \text{ ft-kips}$$

It is possible for the base of the first story column to develop M_{pr} of the column. Thus, the shear force is:

$$V_u = \frac{334.1 + 1438}{14} = 334.1 \text{ft-kips}$$

For seismic forces in the N-S direction, M^-_{pr} of the beam is 302.6 ft-kips (4-No. 7 top bars) and M^+_{pr} is 302.6 ft-kips (4-No. 7 bottom bars). Therefore, at the top of the first story column, the moment is approximately:

$$(2 \times 302.6)\left(\frac{12}{12 + 16}\right) = 259.4 \text{ ft-kips}$$

The shear force is:

$$V_u = \frac{259.4 + 1438}{14} = 121.2 \text{ kips}$$

Both of these shear forces are greater than those obtained from analysis.

Since the factored axial forces are greater than $A_g f'_c/20 = 180$ kips, the shear strength of the concrete may be used: *21.4.5.2*

$$V_c = 2\sqrt{f'_c}\,bd\left(1 + \frac{N_u}{2000\,A_g}\right)$$ *Eq. (11-4)*

Conservatively using the minimum axial force on the column:

$$V_c = \frac{2\sqrt{4000}\,(30 \times 27.5)}{1000}\left[1 + \frac{1{,}192{,}700}{2000 \times (30)^2}\right] = 173.5 \text{ kips}$$

$\phi V_c = 0.75 \times 173.5 = 130.1 \text{ kips} > V_u = 126.6 \text{ kips}$ O.K.

Thus, the transverse reinforcement spacing over the distance $\ell_o = 28$ in. near the column ends required for confinement is also adequate for shear.

Use No. 4 hoops and crossties spaced at 5 in. at the ends of the column.

2. Check shear strength of joint in E-W direction

Following the same procedure as used in Example 29.4, the forces affecting the horizontal shear across a section near mid-depth of the joint shown in Fig. 29-37 are obtained.

Net shear force across section x-x $= T_1 + C_2 - V_h = 296.3 + 180.0 - 55.7 = 420.6 \text{ kips} = V_u$

Shear strength of joint, noting that the joint is not confined on all faces is (i.e., beam width = 20 in. < 0.75 (column width) = 0.75 × 30 = 22.5 in.):

$\phi V_c = \phi 12\sqrt{f'_c}A_j$

$= 0.85 \times 12\sqrt{4000} \times 30^2 / 1000 = 580.6 \text{ kips} > 420.6 \text{ kips}$ O.K. 21.5.3.1

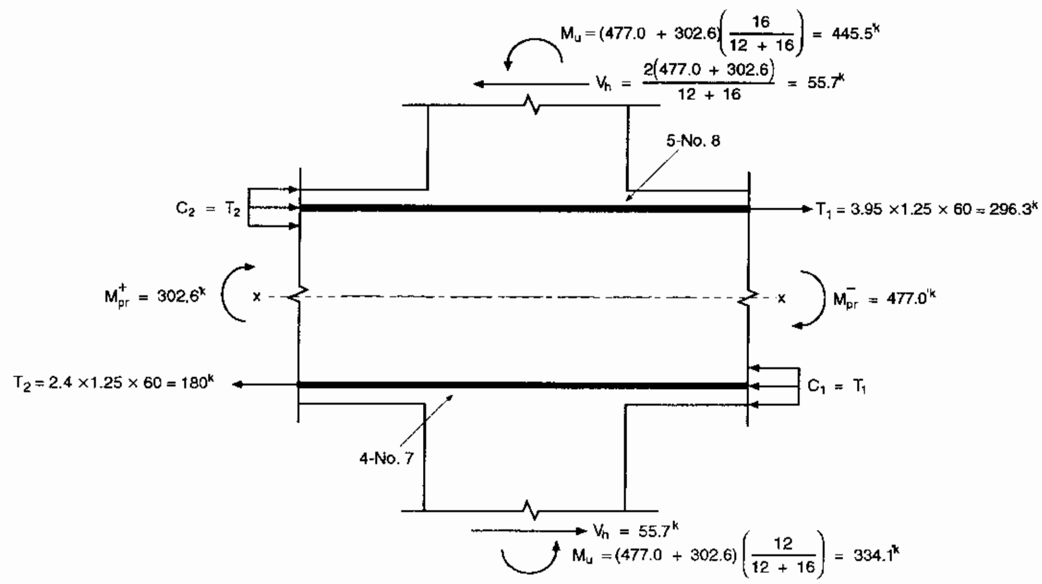


Figure 29-37 Shear Analysis of Interior Beam-Column Joint in E-W Direction

3. Check shear strength of joint in N-S direction

At both the top and the bottom of the beam, 4-No. 7 bars are required (M_{pr} = 302.6 ft-kips).

Net shear force across section x-x = $T_1 + C_2 - V_u$ = 180 + 180 - 43.2 = 316.8 kips = V_u

where $T_1 = C_2 = 2.4 \times 1.25 \times 60 = 180$ kips

$V_u = 2(302.6 + 302.6)/(12 + 16) = 43.2$ kips

$\phi V_c = \phi 12\sqrt{f'_c}A_j = 580.6 \text{ kips} > V_u = 316.8 \text{ kips}$ O.K.

Example 29.6—Proportioning and Detailing of Structural Wall of Building in Example 29.1

Design the wall section at the first floor level of the building in Example 29.1. At the base of the wall, $M_u = 49{,}142$ ft-kips and $V_u = 812$ kips.

Calculations and Discussion	Code Reference

1. Determine minimum longitudinal and transverse reinforcement requirements in the wall.

   a. Check if two curtains of reinforcement are required.

   Two curtains of reinforcement shall be provided in a wall if the in-plane factored shear force assigned to the wall exceeds $2A_{cv}\sqrt{f'_c}$, where A_{cv} is the cross-sectional area bounded by the web thickness and the length of section in the direction of the shear force considered. *21.7.2.2*

   $2A_{cv}\sqrt{f'_c} = 2 \times 18 \times 24.5 \times 12 \times \sqrt{4000}/1000 = 669 \text{ kips} < V_u = 812 \text{ kips}$

   Therefore, two curtains of reinforcement are required.

   Note that $V_u = 812$ kips < upper limit on shear strength = $\phi 8A_{cv}\sqrt{f'_c} = 2{,}008$ kips O.K. *21.7.4.4*

   b. Required longitudinal and transverse reinforcement in wall.

   Minimum distributed web reinforcement ratios = 0.0025 with max. spacing = 18 in. *21.7.2.1*

   With A_{cv} (per foot of wall) = $18 \times 12 = 216$ in.^{2}, minimum required area of reinforcement in each direction per foot of wall = $0.0025 \times 216 = 0.54$ in.^{2}/ft

   Assuming No. 5 bars in two curtains ($A_s = 2 \times 0.31 = 0.62$ in.^{2}), required spacing is

   $$s = \frac{0.62}{0.54} \times 12 = 13.8 \text{ in.} < 18 \text{ in.}$$

3. Determine reinforcement requirements for shear. *21.7.4*

   Assume two curtains of No. 5 bars spaced at 12 in. on center. Shear strength of wall:

   $$\phi V_n = \phi A_{cv}\left(\alpha_c\sqrt{f'_c} + \rho_t f_y\right) \qquad \textit{Eq. (21-7)}$$

   where $\phi = 0.75$ and $\alpha_c = 2.0$ for $h_w/\ell_w = 148/24.5 = 6 > 2$

   $A_{cv} = 18 \times 24.5 \times 12 = 5292$ in.^{2}

   $$\rho_t = \frac{0.62}{18 \times 12} = 0.0029$$

   $\phi V_n = (0.75 \times 5292)\,[2\sqrt{4000} + (0.0029 \times 60{,}000)]\,/\,1000 = 1193 \text{ kips} > 812 \text{ kips}$ O.K.

Therefore, use two curtains of No. 5 bars spaced at 12 in. on center in the horizontal direction.

The reinforcement ratio ρ_ℓ shall not be less than the ratio ρ_t when the ratio h_w/ℓ_w is less than 2.0. Since h_w/ℓ_w is equal to 6.0, the minimum reinforcement ratio will be used. *21.7.4.3*

Use 2 curtains of No. 5 bars spaced at 12 in. on center in the vertical direction.

3. Determine reinforcement requirements for combined flexural and axial loads.

Structural walls subjected to combined flexural and axial loads shall be designed in accordance with 10.2 and 10.3 except that 10.3.6 and the nonlinear strain requirements of 10.2.2 do not apply. *21.7.5.1*

Assume that each 30 × 30 in. column at the end of the wall is reinforced with 24-No. 11 bars. It was determined above that 2-No. 5 bars at a spacing of 12 in. are required as vertical reinforcement in the web. With this reinforcement, the wall is adequate to carry the factored load combinations per 9.2.

4. Determine if special boundary elements are required.

The need for special boundary elements at the edges of structural walls shall be evaluated in accordance with 21.7.6.2 or 21.7.6.3. The provisions of 21.7.6.2 are used in this example. *21.7.6.1*

Compression zones shall be reinforced with special boundary elements where

$$c \geq \frac{\ell_w}{600(\delta_u / h_w)}, \quad \delta_u / h_w \geq 0.007 \qquad \textit{Eq. (21-8)}$$

In this case, $\ell_w = 24.5$ ft = 294 in., $h_w = 148$ ft = 1776 in., $\delta_u = 13.5$ in. and $\delta_u / h_w = 0.0076 > 0.007$. Therefore, special boundary elements are required if c is greater than or equal to $294/(600 \times 0.0076) = 64.5$ in.

The distance c to be used in Eq. (21-8) is the largest neutral axis depth calculated for the factored axial force and nominal moment strength consistent with the design displacement δ_u. From a strain compatibility analysis, the largest c is equal to 68.1 in. corresponding to an axial load of 3649 kips and nominal moment strength of 97,302 ft-kips, which is greater than 64.5 in. Thus, special boundary elements are required.

The special boundary element shall extend horizontally from the extreme compression fiber a distance not less than $c - 0.1\ell_w = 68.1 - (0.1 \times 294) = 38.7$ in. (governs) or $c/2 = 68.1/2 = 34.1$ in. Considering the placement of the vertical bars in the web, confine 45 in. at both ends of the wall. *21.7.6.4(a)*

5. Determine special boundary element transverse reinforcement.

Transverse reinforcement shall satisfy the requirements of 21.4.4.1 through 21.4.4.3 except Eq. (21-3) need not be satisfied. *21.7.6.4(c)*

- Confinement of 30 × 30 in. boundary elements

  Maximum allowable spacing of rectangular hoops assuming No. 4 hoops and crossties around every longitudinal bar in both directions of the 30 × 30 in. boundary elements:

$$s_{max} = 0.25 \text{ (minimum member dimension)} = 0.25 \times 30 = 7.5 \text{ in.}$$
$$= 6 \text{ (diameter of longitudinal bar)} = 6 \times 1.41 = 8.5 \text{ in.}$$
$$= s_o = 4 + \left(\frac{14 - h_x}{3}\right) = 4 + \left(\frac{14 - 6.0}{3}\right) = 6.7 \text{ in.} \geqslant 6.0 \text{ in.; use 6 in. (governs)}$$

  where h_x = maximum horizontal spacing of hoop or crosstie legs on all faces of the 30 × 30 in. boundary element.

  Required cross-sectional area of transverse reinforcement in the 30 × 30 in. boundary elements, assuming s = 6.0 in.:

$$A_{sh} = \frac{0.09sh_c f'_c}{f_y} = \frac{0.09 \times 6.0 \times [30 - (2 \times 1.5) - 0.5] \times 4}{60} = 0.95 \text{ in.}^2$$

  No. 4 hoops with crossties around every longitudinal bar in the 30 × 30 in. boundary elements provide $A_{sh} = 7 \times 0.2 = 1.40$ in.^{2} > 0.95 in.^{2}

- Confinement of web

  Maximum allowable spacing of No. 5 transverse reinforcement:

$$s_{max} = 0.25 \text{ (minimum member dimension)} = 0.25 \times (45 - 30) = 3.75 \text{ in.} \quad \text{(governs)}$$
$$= 6 \text{ (diameter of longitudinal bar)} = 6 \times 0.625 = 3.75 \text{ in.}$$
$$= s_o = 4 + \left(\frac{14 - 13.25}{3}\right) = 4.25 \text{ in.}$$

  For confinement in the direction parallel to the wall, assuming s = 3.0 in.:

  $b_c = 18 - (2 \times 1.5) - 0.625 = 14.375$ in.

$$A_{sh} = \frac{0.09 \times 3.0 \times 14.375 \times 4}{60} = 0.26 \text{ in.}^2$$

  Using 2-No. 5 horizontal bars, $A_{sh} = 2 \times 0.31 = 0.62$ in.^{2} > 0.26 in.^{2}

  For confinement in the direction perpendicular to the wall:

  $b_c = 45 - 30 = 15$ in.

$$A_{sh} = \frac{0.09 \times 3.0 \times 15 \times 4}{60} = 0.27 \text{ in.}^2$$

  With a No. 5 hoop and crosstie, $A_{sh} = 2 \times 0.31 = 0.62$ in.^{2} > 0.27 in.^{2}

The transverse reinforcement of the boundary element shall extend vertically a distance of ℓ_w = 24.5 ft (governs) or $M_u/4V_u$ = 49,142/(4 × 812) = 15.1 ft from the critical section. *21.7.6.2(b)*

6. Determine required development and splice lengths.

Reinforcement in structural walls shall be developed or spliced for f_y in tension in accordance with Chapter 12, except that at locations where yielding of longitudinal reinforcement is likely to occur as a result of lateral displacements, development of longitudinal reinforcement shall be 1.25 times the values calculated for f_y in tension. *21.7.2.3*

a. Lap splice for No. 11 vertical bars in boundary elements.*

Class B splices are designed for the No. 11 vertical bars.

Required length of Class B splice = $1.3\ell_d$ *12.15.1*

where

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s\lambda}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b$$ *Eq. (12-1)*

ψ_t = 1.3 for top bars; a = 1.0 for other bars *12.2.4*

ψ_e = 1.0 for uncoated bars

ψ_s = 1.0 for No. 7 and larger bars

λ = 1.0 for normal weight concrete (1.3 for lightweight concrete)

Assume no more than 50% of the bars spliced at any one location.

$$c = 1.5 + 0.5 + \frac{1.41}{2} = 2.7 \text{ in.} \quad \text{(governs)}$$

$$= \frac{1}{2}\left[\frac{30 - 2(1.5 + 0.5) - 1.41}{3}\right] = 4.1 \text{ in.}$$

$$K_{tr} = \frac{A_{tr}f_{yt}}{1500sn} = \frac{(4 \times 0.20)(60{,}000)}{1500 \times 6.0 \times 4} = 1.3$$

where A_{tr} is for 4-No. 5 bars, s = 6.0 in., and n (number of bars being developed) = 4 in one layer at one location.

$$\frac{(c_b + K_{tr})}{d_b} = \frac{(2.7 + 1.3)}{1.41} = 2.8 > 2.5, \text{ use } 2.5$$

* The use of mechanical connectors may be considered as an alternative to lap splices for these large bars.

Therefore,

$$\ell_d = \frac{3}{40} \times \frac{1.25 \times 60{,}000}{\sqrt{4000}} \times \frac{1.0}{2.5} \times 1.41 = 50.2 \text{ in.}$$

Class B splice length $= 1.3 \times 50.2 = 65.3$ in.

Use a 5 ft-6 in. splice length.

Note that splices beyond the first story can be 25% shorter, or 4 ft-6 in. long, as long as the same reinforcement continues.

b. Lap splice for No. 5 vertical bars in wall web.

Again assuming no more than 50% of bars spliced at any one location, the length of the Class B splice is determined as follows:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \lambda}{\left(\dfrac{c_b + K_{tr}}{d_b} \right)} \right) d_b$$ *Eq. (12-1)*

reinforcement location factor $\psi_t = 1.0$ (other than top bars) *12.2.4*

coating factor $\psi_e = 1.0$ (uncoated bars)

reinforcement size factor $\psi_s = 0.8$ (No. 6 and smaller bars)

lightweight aggregate concrete factor $\lambda = 1.0$ (normal weight concrete)

$$c = 0.75 + 0.625 + \frac{0.625}{2} = 1.7 \text{ in.} \quad \text{(governs)}$$

$$= \frac{1}{2} \times 12 = 6 \text{ in.}$$

$K_{tr} = 0$

$$\frac{(c_b + K_{tr})}{d_b} = \frac{1.7}{0.625} = 2.7 > 2.5, \text{ use } 2.5$$

Therefore,

$$\ell_d = \frac{3}{40} \times \frac{1.25 \times 60{,}000}{\sqrt{4000}} \times \frac{0.8}{2.5} \times 0.625 = 17.8 \text{ in.}$$

Class B splice length $= 1.3 \times 17.8 = 23.1$ in.

Use a 2 ft-0 in. splice length.

Although all the No. 5 bars will not yield at the base, it is simpler to base the splice lengths of all No. 5 bars on possible yielding. Beyond the first story, the splice lengths may be reduced to 1 ft-8 in.

c. Development length for No. 5 horizontal bars in wall assuming no hooks are used within boundary element. *21.5.4*

$$\ell_d = \left[\frac{3}{40}\frac{1.25f_y}{\sqrt{f_c'}}\frac{\psi_t\psi_e\psi_s\lambda}{\left(\frac{c+K_{tr}}{d_b}\right)}\right]d_b \qquad \textit{Eq. (12-1)}$$

Since it is reasonable to assume that the depth of concrete cast in one lift beneath a horizontal bar will be greater than 12 in.,

reinforcement factor $\psi_t = 1.0$

coating factor $\psi_e = 1.0$ (uncoated bars)

reinforcement size factor $\psi_s = 0.8$ (No. 6 and smaller bars)

lightweight aggregate concrete factor $\lambda = 1.0$ (normal weight concrete)

$$c = 0.75 + \frac{0.625}{2} = 1.06 \text{ in.(governs)}$$

$$= \frac{1}{2} \times 12 = 6 \text{ in.}$$

$$K_{tr} = 0$$

$$\frac{(c_b + K_{tr})}{d_b} = \frac{1.06}{0.625} = 1.7 < 2.5$$

Therefore,

$$\ell_d = \frac{3}{40} \times \frac{1.25 \times 60{,}000}{\sqrt{4000}} \times \frac{1.3 \times 0.8}{1.7} \times 0.625 = 34.0 \text{ in.}$$

This length cannot be accommodated within the confined core of the boundary element, thus hooks are needed.

Anchor horizontal bars to longitudinal reinforcement in boundary element. *21.7.6.4(e)*

No lap splices would be required for the No. 5 horizontal bars (full length bars weigh approximately 25 lbs. and are easily installed).

7. Reinforcement details for structural wall are shown in Fig. 29-38.

   Note that the No. 5 bars at 3 in. that are required for confinement in the direction parallel to the web are developed into the boundary element and into the web beyond the face of the 2 ft-6 in. boundary element [see Fig. 29-38(b)].

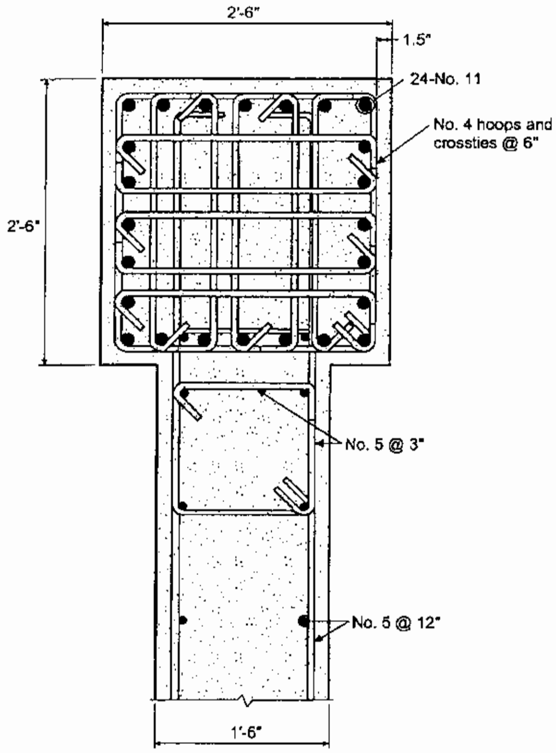


Figure 29-38(a) Reinforcement Details for the Structural Wall

Example 29.7—Design of 12-Story Precast Frame Building using Strong Connections*

This example illustrates the design and detailing requirements for typical beam-to-beam, column-to-column, and beam-to-column connections for the precast building shown in Fig. 29-39. In particular, details are developed for: (1) a strong connection near midspan of an interior beam that is part of an interior frame on the third floor level, (2) a column-to-column connection at mid-height between levels 2 and 3 of an interior column stack that is part of an interior frame, and (3) a strong connection at the interface between an exterior beam at the second floor level of an exterior frame and the continuous corner column to which it is connected. Pertinent design data are as follows:

Material Properties:

Concrete (w_c = 150 pcf): f'_c = 6000 psi for columns in the bottom six stories
= 4000 psi elsewhere

Reinforcement: f_y = 60,000 psi

Service Loads:

Live load = 50 psf
Superimposed dead load = 42.5 psf

Member Dimensions:

Beams in N-S direction:	24 × 26 in.
Beams in E-W direction:	24 × 20 in.
Columns:	24 × 24 in.
Slab:	7 in.

Calculations and Discussion	Code Reference

1. Seismic design forces

The computation of the seismic design forces is beyond the scope of this example. Traditional analysis methods can be used for precast frames, although care should be taken to approximate the component stiffness in a way that is appropriate for the precast components being used. For emulation design (as illustrated in this example), it is reasonable to model the beams and columns as if they were monolithic concrete.

2. Strong connection near beam midspan

a. Required flexural reinforcement

Special moment frames with strong connections constructed using precast concrete shall satisfy all requirements for special moment frames constructed with cast-in-place concrete, in addition to the provisions of 21.6.2. *21.6.2*

The required reinforcement for the beams on the third floor level is shown in Table 29-8. The design moments account for all possible load combinations per 9.2.1, and the provided areas of steel are within the limits specified in 21.3.2.1. Also given in Table 29-8 are the flexural moment strengths ϕM_n at each section. Note that at each location, the section is tension-controlled, so that $\phi = 0.9$. *9.3.2.1*

*This example has been adapted from: Ghosh, S.K., Nakaki, S.D., and Krishnan, K., "Precast Structures in Regions of High Seismicity: 1997 UBC Design Provisions", PCI Journal, Vol. 42, No. 6, November-December 1997, pp. 76-93.

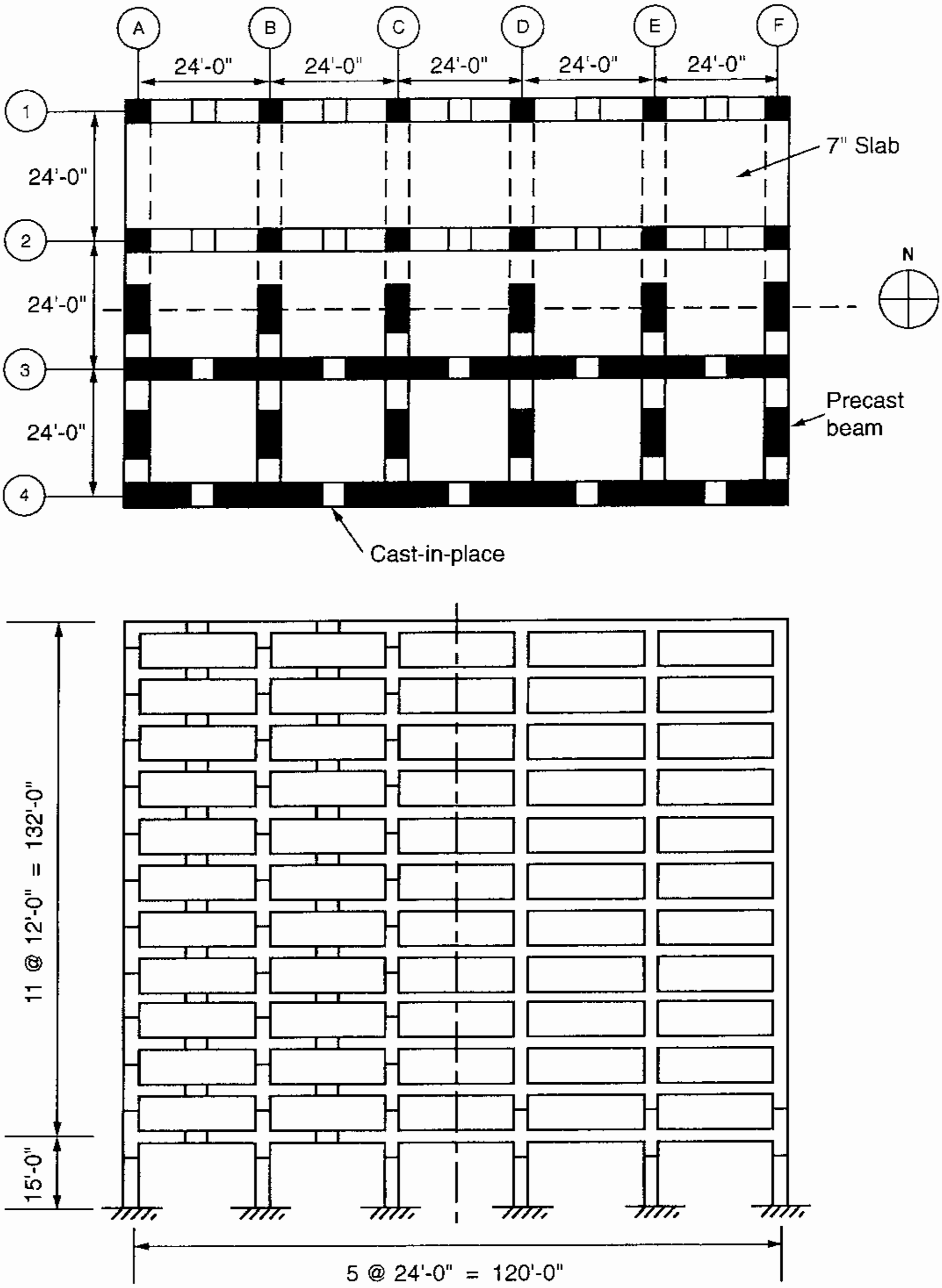


Fig 29-39 Example Building

Table 29-8 Required Reinforcement for E-W Third Floor Beam

Location	M_u (ft-kips)	A_s* (in.^{2})	Reinforcement	ϕM_n (ft-kips)
Supports	-510.8	7.79	8-No. 9	-522.0
	+311.7	4.38	5-No. 9	+351.0
Interior	+63.0	1.40	2-No. 9	+150.3

*Max. A_s = $0.025 \times 24 \times 17.44 = 10.46$ in.^{2} (21.3.2.1)
Min. A_s = $3\sqrt{4000} \times 24 \times 17.44/60{,}000 = 1.32$ in.^{2} (10.5.1)
= $200 \times 24 \times 17.44/60{,}000 = 1.40$ in.^{2} (governs)

Three No. 9 bars are made continuous at the top and bottom throughout the spans, providing negative and positive design moment strengths of 220.6 ft-kips.

Check provisions of 21.3.2.2 for moment strength along span of beam:

At the supports, ϕM_n^+ (5-No. 9) = 351.0 ft-kips > $\phi M_n^-/2$ = 261.0 ft-kips O.K. *21.3.2.2*

At other sections, ϕM_n (3-No. 9) = 220.6 ft-kips > $\phi M_n^-/4$ = 130.5 ft-kips O.K.

b. Lap splice length *12.2.1*

Lap splices of flexural reinforcement must not be placed within a joint, within a distance 2h from faces of supports or within regions of potential plastic hinging. Note that all lap splices must be confined by hoops or spirals with a maximum spacing or pitch of d/4 = 4.4 in. or 4 in. (governs) over the length of the lap. Lap splice lengths will be determined for the No. 9 top and bottom bars. *21.3.2.3*

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \lambda}{\dfrac{c_b + K_{tr}}{d_b}} \right) d_b$$ *Eq. (12-1)*

where $\dfrac{c + K_{tr}}{d_b} \leq 2.5$

$\psi_t = 1.3$ for top bars; $\alpha = 1.0$ for other bars *12.2.4*

$\psi_e = 1.0$ for uncoated bars

$\psi_s = 1.0$ for No. 7 and larger bars

$\lambda = 1.0$ for normal weight concrete (1.3 for lightweight concrete)

$$c = 1.5 + 0.5 + \frac{1.128}{2} = 2.56 \text{ in. (governs)}$$

$$= \frac{24 - 2(1.5 + 0.5) - 1.128}{2 \times 2} = 4.72 \text{ in.}$$

$$\frac{c}{d_b} = \frac{2.56}{1.128} = 2.27, \text{ which makes it reasonable to take } \frac{c_b + K_{tr}}{d_b} = 2.5$$

Thus, for top bars:

$$\ell_d = \frac{3}{40} \frac{60{,}000}{\sqrt{4000}} \frac{1.3}{2.5} d_b = 37d_b = 37 \times 1.128 = 41.7 \text{ in.}$$

For bottom bars:

$$\ell_d = \frac{3}{40} \frac{60{,}000}{\sqrt{4000}} \frac{1.0}{2.5} d_b = 28.5d_b = 28.5 \times 1.128 = 32.2 \text{ in.}$$

Note that 2-No. 9 top bars are adequate in the interior of the span, i.e., ϕM_n(2-No. 9) = 150.3 ft-kips > $\phi M_n^- /4$ = 130.5 ft-kips. Thus, the top bar development length can be reduced by an excess reinforcement factor of (A_s required/A_s provided) = 2/3: *12.2.5*

ℓ_d = 2/3 × 41.7 = 27.8 in.

Since all of the reinforcement is spliced at the same location, Type B splices are to be used for both the top and bottom bars. *12.15.2*

Type B splice length = 1.3 × 32.2 = 41.9 in. > 12 in. for the bottom bar *12.15.1*

Provide 3 ft-6 in. splice length for both the top and the bottom bars.

c. Reinforcing bar cutoff points

For the purpose of determining the cutoff points for the reinforcement, a moment diagram corresponding to the probable moment strengths at the beam ends and 0.9 times the dead load on the span will be used, since this will result in the longest bar lengths. The cutoff point for 5 of the 8-No. 9 bars at the top will be determined.

Determine probable moment strengths M_{pr}^+ and M_{pr}^- with $f_s = 1.25f_y$ = 75 ksi and ϕ = 1.0, ignoring compression steel. *21.0*

For 5-No.9 bottom bars:

$$a = \frac{A_s f_s}{0.85 f'_c b} = \frac{5 \times 75}{0.85 \times 4 \times 24} = 4.6 \text{ in.}$$

$$M_{pr}^+ = A_s f_s\left(d - \frac{a}{2}\right) = (5 \times 75)\left(17.44 - \frac{4.6}{2}\right)/12 = 473.1 \text{ ft-kips}$$

where d = 20 – 1.5 (clear cover) – 0.5 (diameter of No. 4 stirrup) – 0.564 (diameter of No. 9 bar/2) = 17.44 in.

Similarly, for 8-No. 9 top bars: M_{pr}^- = 688.2 ft-kips

Dead load on beam:

$$w_D = \left(\frac{7}{12} \times 0.150 \times 24\right) + (0.0425 \times 24) + \left(\frac{24 \times 13 \times 0.150}{144}\right) = 3.45 \text{ kips/ft at midspan}$$

$0.9w_D = 0.9 \times 3.45 = 3.11$ kips/ft

The distance from the face of the interior support to where the moment under the loading considered equals ϕM_n(3-No. 9) = 220.6 ft-kips is readily obtained by summing moments about section A-A (see Fig. 29-40):

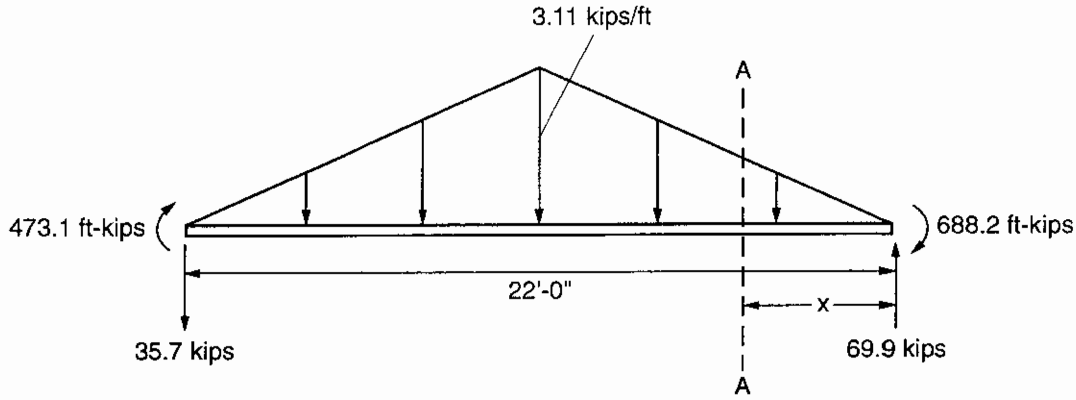


Fig. 29-40 Cutoff Location of Negative Bars

$$\frac{x}{2}\left(\frac{3.11x}{11}\right)\left(\frac{x}{3}\right) + 688.2 - 220.6 - 69.9x = 0$$

Solving for x gives a distance of 6.91 ft from the face of the support.

The 5-No. 9 bars must extend a distance d = 17.44 in. or $12d_b$ = 13.54 in. beyond the distance x. Thus, from the face of the support, the total bar length must be at least equal to 6.91 + (17.44/12) = 8.4 ft. Also, the bars must extend a full development length ℓ_d beyond the face of the support: *12.10.3* *12.10.4*

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s\lambda}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b \qquad \textit{Eq. (12-1)}$$

where $\frac{c + K_{tr}}{d_b} \le 2.5$

$\psi_t = 1.3$ for top bars *12.2.4*

$\psi_e = 1.0$ for uncoated bars

$\psi_s = 1.0$ for No. 7 and larger bars

$\lambda = 1.0$ for normal weight concrete

$$c = 2.56 \text{ in. or } \frac{24-2(1.5+0.5)-1.128}{2 \times 7} = 1.35 \text{ in. (governs)}$$

$K_{tr} = 0$ (conservative)

$$\frac{c+K_{tr}}{d_b} = \frac{1.35+0}{1.128} = 1.2$$

$$\ell_d = \frac{3}{40}\frac{60{,}000}{\sqrt{4000}}\frac{1.3}{1.2}d_b = 77d_b = 77 \times 1.128 = 86.9 \text{ in.} = 7.2 \text{ ft} < 8.4 \text{ ft}$$

The total required length of the 5-No. 9 bars must be at least 8.4 ft beyond the face of the support.

Flexural reinforcement shall not be terminated in a tension zone unless one or more of the conditions of 12.10.5 are satisfied. In this case, the point of inflection is approximately 10.7 ft from the face of the right support, which is greater than 8.4 ft. The 5-No. 9 bars can not be terminated here unless one of the conditions of 12.10.5 is satisfied.

Check if the factored shear force V_u at the cutoff point does not exceed two-thirds of ϕV_n. In this region of the beam, it can be shown that No. 4 stirrups @ 8 in. are required. However, No. 4 stirrups @ 6 in. will be provided to satisfy 12.10.5.1.

$$\phi V_n = \phi(V_c + V_s) = 0.75 \times \left(2\sqrt{4000} \times 24 \times 17.44 + \frac{0.4 \times 60{,}000 \times 17.44}{6}\right)/1000 = 92.0 \text{ kips}$$

$$\frac{2}{3}\phi V_n = 61.3 \text{ kips} > V_u = 60.0 \text{ kips at 8.4 ft from face of support}$$

Since $2\phi V_n/3 > V_u$, the cutoff point for the 5-No. 9 bars can be 8.4 ft beyond the face of the interior support.

The cutoff point for 2 of the 5-No. 9 bottom bars can be determined in a similar fashion. These bars can be cut off at 8.4 ft from the face of the exterior support as well, which is short of the splice closure.

d. Check connection strength

For strong connections: $\phi S_n \geq S_e$ *21.6.2(b)*

where S_n = nominal flexural or shear strength of the connection *21.0*

S_e = moment or shear at connection corresponding to development of probable strength at intended yield locations, based on the governing mechanism of inelastic lateral deformation, considering both gravity and earthquake load effects

At the connection,

$$\phi V_n = \phi(V_c + V_s) = 0.75 \times \left(2\sqrt{4000} \times 24 \times 17.44 + \frac{0.4 \times 60{,}000 \times 17.44}{4}\right)/1000 = 118.2 \text{ kips}$$

Gravity load on beam:

$1.2w_D + 0.5w_L = (1.2 \times 3.45) + (0.5 \times 0.05 \times 24) = 4.74$ kips/ft *Eq. (9-5)*

Maximum shear force at connection due to gravity and earthquake load effects occurs at 9.125 ft from face of right support (see Fig. 29-41):

$$V_e = 78.9 - \left(\frac{1}{2} \times 9.125 \times \frac{4.74 \times 9.125}{11}\right) = 61.0 \text{ kips} < \phi V_n = 118.2 \text{ kips} \quad \text{O.K.}$$

At the connection, ϕM_n (3-No. 9) = 220.6 ft-kips

Maximum moment at connection due to gravity and earthquake load effects occurs at 9.125 ft from face of left support (see Fig. 29-41):

$$M_e = 473.1 - (26.8 \times 9.125) - \left(\frac{1}{2} \times 9.125\right)\left(\frac{4.74 \times 9.125}{11}\right)\left(\frac{1}{3} \times 9.125\right) = 174.0 \text{ ft-kips}$$

$$< \phi M_n = 220.6 \text{ ft-kips} \quad \text{O.K.}$$

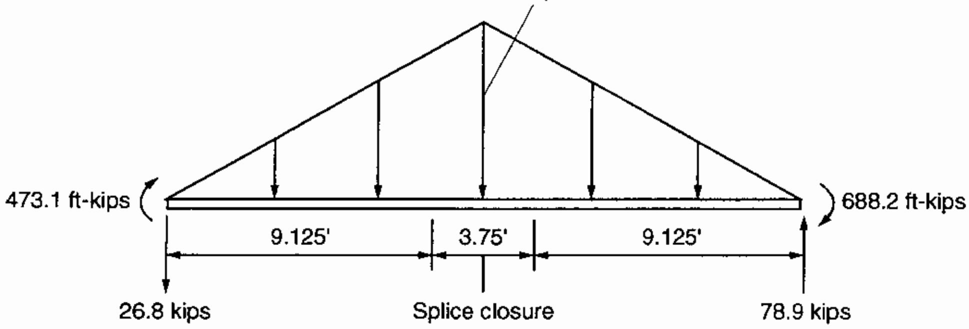
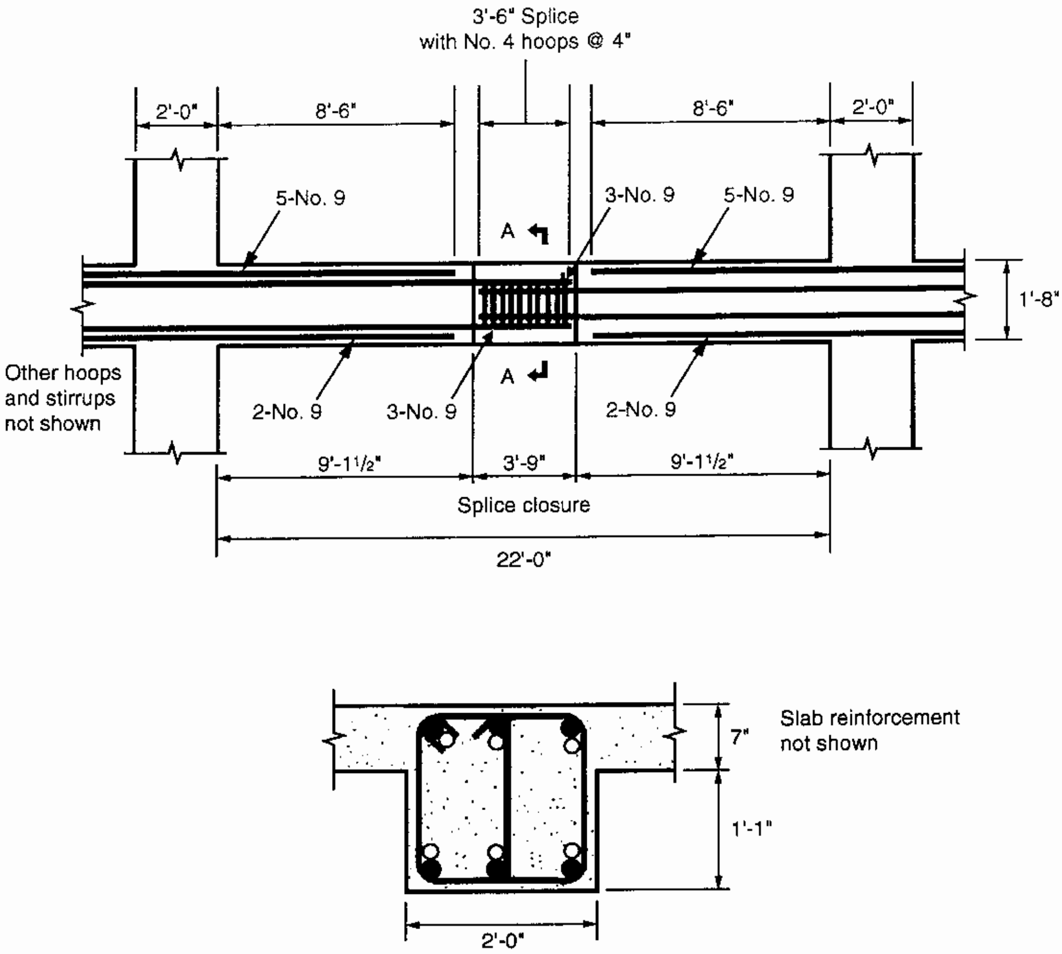


Fig. 29-41 Connection Strength

e. Reinforcement details

The reinforcement details for the beam are shown in Fig. 29-42.



Section A-A

Fig. 29-42 Reinforcement Details for Beam-to-Beam Connection

3. Column-to-column connection at mid-height

   a. Determine required longitudinal reinforcement

   A summary of the design forces for the interior column between levels 2 and 3, which is part of an interior longitudinal frame, is contained in Table 29-9. The design forces account for all possible load combinations per 9.2.1.

Table 29-9 Design Forces for Interior Column between the Second and Third Floors

Load Combination	Axial load, P_u (kips)	Moment, M_u (ft-kips)		Shear, V_u (kips)
		Top	Bottom	
1.2D + 1.6L	1402.6	-8.0	5.5	1.2
1.2D + 0.5L + E	1609.8	-408.3	467.5	70.7
1.2D + 0.5L - E	1195.4	392.3	-456.5	73.1
0.9D + E	1125.8	-405.5	466.3	71.1
0.9D - E	711.4	395.1	-457.8	72.7

It can be shown that 12-No. 10 bars are adequate for all load combinations.

Check longitudinal reinforcement ratio:

$$\rho_g = \frac{A_{st}}{bh} = \frac{12 \times 1.27}{24 \times 24} = 0.0265$$

$\rho_{min} = 0.01 < \rho_g = 0.0265 < \rho_{max} = 0.06$ O.K. *21.4.3.1*

b. Nominal flexural strength of columns relative to that of beams

ΣM_c (columns) $\geq \frac{6}{5}\Sigma M_g$ (beams) *21.4.2.2*

For the top end of the lower column framing into the joint between the second and the third floor levels, M_n = 1182.3 ft-kips, which corresponds to P_u = 711.4 kips. Similarly, for the bottom end of the upper column framing into the same joint, M_n = 1168.4 ft-kips, which corresponds to P_u = 655.5 kips.

Thus,

$\Sigma M_c = 1182.3 + 1168.4 = 2350.7$ ft-kips

The nominal negative flexural strength M_n^- of the beam framing into the column must include the slab reinforcement within an effective slab width equal to:

16(slab thickness) + beam width = $(16 \times 7) + 24 = 136$ in. *8.10.2*
Center-to-center beam spacing = $24 \times 12 = 288$ in.
Span/4 = $(24 \times 12)/4 = 72$ in. (governs)

The minimum required A_s in the 72-in. effective width = $0.0018 \times 72 \times 7 = 0.91$ in.^{2} which corresponds to 5-No. 4 bars @ 72/5 = 14.4 in. Since maximum bar spacing = 2h = 14 in., provide No. 4 @ 14 in. at both the top and the bottom of the slab (according to ACI 318 Fig. 13.3.8, 100 percent of both the top and the bottom reinforcement in the column strip must be continuous or anchored at the support).

From a strain compatibility analysis, $M_n^- = 736.0$ ft-kips and $M_n^+ = 459.0$ ft-kips

Thus,

$\Sigma M_g = 736.0 + 459.0 = 1195.0$ ft-kips

$2350.7 \text{ ft-kips} > \frac{6}{5} \times 1195.0 = 1434.0 \text{ ft-kips}$ O.K. *Eq. (21-1)*

The intent of 21.4.2.2 is to prevent a story mechanism, rather than prevent local yielding in a column. The 6/5 factor is clearly insufficient to prevent column yielding if the adjacent beams both hinge. Therefore, confinement reinforcement is required in the potential hinge regions of a frame column.

c. Minimum connection strength

At column-to-column connections, $\phi M_n \geq 0.4 M_{pr}$ when bars are spliced within the middle third of the clear column height. *21.6.2(d)*

For the column between the second and the third floor levels with $P_u = 711.4$ kips, it can be shown from a strain compatibility analysis that $M_{pr} = 1244.1$ ft-kips.

Also, as indicated above, $M_n = 1182.3$ ft-kips for $P_u = 711.4$ kips. From a strain compatibility analysis, $\varepsilon_t = 0.00223$, so that $\phi = 0.48 + (83 \times 0.00223) = 0.67$. *9.3.2.2*

Therefore,

$\phi M_n = 0.67 \times 1182.3 = 792.1 \text{ ft-kips} > 0.4 M_{pr} = 0.4 \times 1244.1 = 497.6 \text{ ft-kips}$ O.K.

Splice all twelve bars at mid-height, as shown in Fig. 29-43.

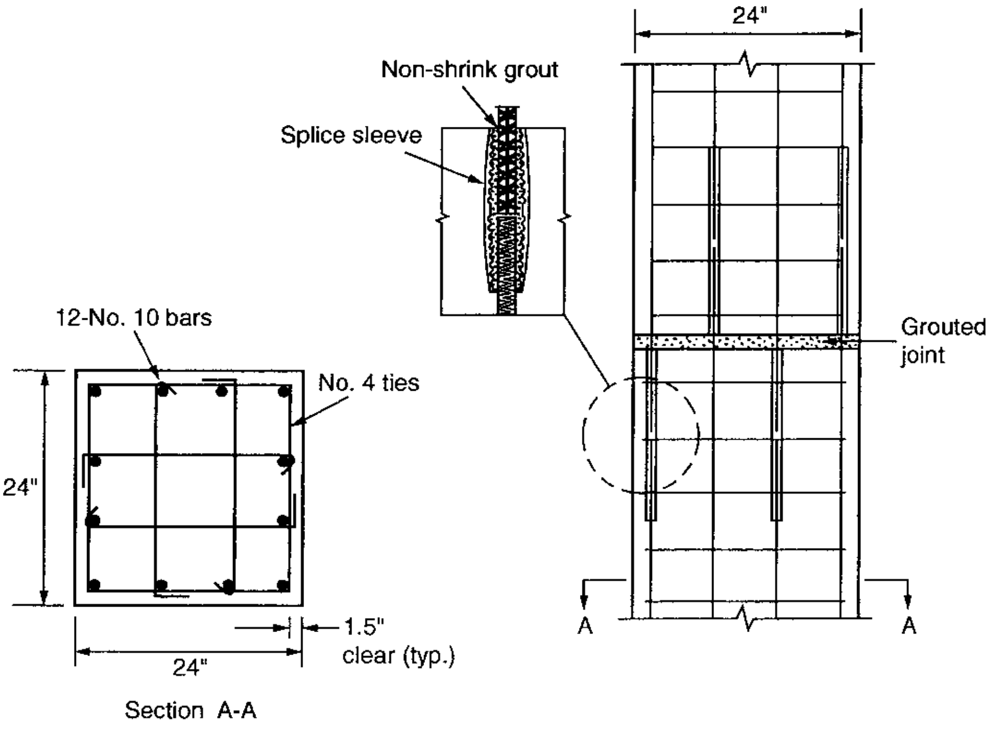


Fig. 29-43 Reinforcement Details for Column-to-Column Connection

4. Column-face strong connection in beam

A strong connection is to be designed at the interface between a precast beam at the second floor level of the building that forms the exterior span of an exterior transverse frame and the continuous corner column to which it is connected.

a. Required flexural reinforcement

From the combined effects of gravity and earthquake forces, the required flexural reinforcement at the top of the beam is 5-No. 9 bars and is 4-No. 9 bars at the bottom. All possible load combinations of 9.2.1 were considered.

b. Strength design of connection

The beam-to-column connection similar to the one depicted in ACI 318 Fig. R21.6.2(c) will be provided.

The strong connection must be designed for the probable moment strength of the beam plus the moment at the face of the column due the shear force at the critical section. *21.6.2*

Determine probable moment strengths M_{pr}^{+} and M_{pr}^{-} with $f_s = 1.25f_y = 75$ ksi and $\phi = 1.0$, ignoring compression steel. *21.0*

For 4-No. 9 bottom bars:

$$a = \frac{A_s f_s}{0.85 f'_c b} = \frac{4 \times 75}{0.85 \times 4 \times 24} = 3.7 \text{ in.}$$

$$M_{pr}^{+} = A_s f_s\left(d - \frac{a}{2}\right) = (4 \times 75)\left(23.44 - \frac{3.7}{2}\right)/12 = 539.8 \text{ ft-kips}$$

where d = 26 – 1.5 (clear cover) – 0.5 (diameter of No. 4 stirrup) – 0.564 (diameter of No. 9 bar/2) = 23.44 in.

Similarly, for 5-No. 9 top bars: $M_{pr}^{-} = 660.6$ ft-kips

Assuming a 2 ft-6 in. cast-in-place closure, the shear forces at the critical sections, and the moments at the connections can be determined for the two governing load combinations as follows (see Fig. 29-44).

Load combination 1: U = 1.2D + 0.5L + E *Eq. (9-5)*

$$w_D = \left(\frac{7}{12} \times 0.150 \times 13\right) + (0.0425 \times 13) + \left(\frac{24 \times 19 \times 0.150}{144}\right) = 2.17 \text{ kips/ft at midspan}$$

$$w_L = 0.05 \times 13 = 0.65 \text{ kips/ft at midspan}$$

$$w_{u,\,mid} = (1.2 \times 2.17) + (0.5 \times 0.65) = 2.93 \text{ kips/ft}$$

and

$$w_{u,\,end} = 2.93 \times \frac{2.5}{11} = 0.67 \text{ kips/ft}$$

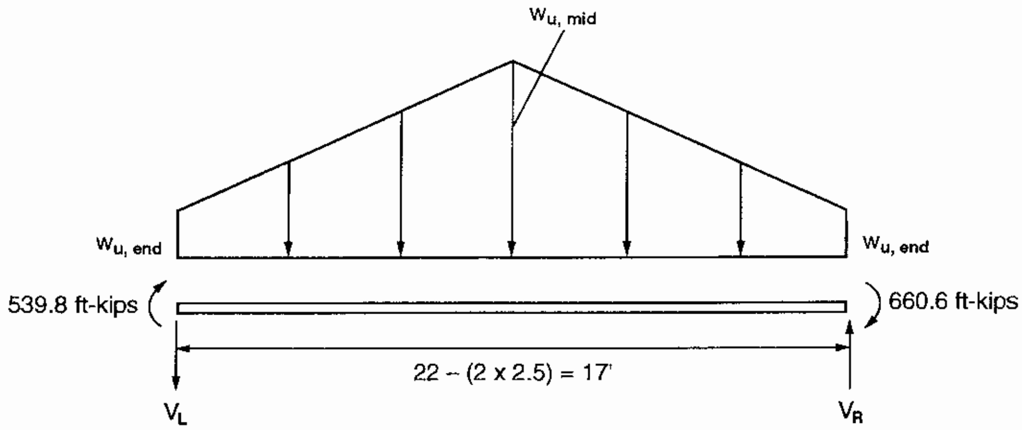


Fig. 29-44 Shear Forces at Critical Sections

From Fig. 29-44:

$$V_R(17) = \left(0.67 \times 17 \times \frac{17}{2}\right) + \left[\frac{1}{2} \times 17 \times (2.93 - 0.67) \times \frac{17}{2}\right] + 539.8 + 660.6$$

or, $V_R = 85.9$ kips

$$V_L = 85.9 - (0.67 \times 17) - \frac{1}{2}[(2.93 - 0.67) \times 17] = 55.3 \text{ kips}$$

$$M^+_{e,\ell} = 539.8 + (55.3 \times 2.5) = 678.1 \text{ ft-kips}$$

$$M^-_{e,r} = 660.6 + (85.9 \times 2.5) = 875.4 \text{ ft-kips}$$

Load combination 2: U = 0.9D + E *Eq. (9-7)*

$$w_{u,mid} = 0.9w_D = 0.9 \times 2.17 = 1.95 \text{ kips/ft}$$

and

$$w_{u,end} = 1.95 \times \frac{2.5}{11} = 0.44 \text{ kips/ft}$$

From Fig. 29-44:

$$V_R(17) = \left(0.44 \times 17 \times \frac{17}{2}\right) + \left[\frac{1}{2} \times 17 \times (1.95 - 0.44) \times \frac{17}{2}\right] + 539.8 + 660.6$$

or, $V_R = 80.8$ kips

$$V_L = 80.8 - (0.44 \times 17) - \frac{1}{2}[(1.95 - 0.44) \times 17] = 60.5 \text{ kips}$$

$$M^{+}_{e,\ell} = 539.8 + (60.5 \times 2.5) = 691.1 \text{ ft-kips}$$

$$M^{-}_{e,r} = 660.6 + (80.8 \times 2.5) = 862.6 \text{ ft-kips}$$

Thus, the governing moments at the connections are

$$M^{+}_{e,\ell} = 691.1 \text{ ft-kips and } M^{-}_{e,r} = 875.4 \text{ ft-kips}$$

At the bottom of the connection, provide an additional 4-No. 9 bars to the 4-No. 9 bars (2 layers) and at the top of the section, provide an additional 5-No. 9 bars to the 5-No. 9 bars (2 layers). From a strain compatibility analysis considering all of the reinforcement in the section:

$$\phi M^{+}_{n} = 729.3 \text{ ft-kips} > M^{+}_{e,\ell} = 691.1 \text{ ft-kips} \quad \text{O.K.}$$

$$\phi M^{-}_{n} = 888.2 \text{ ft-kips} > M^{-}_{e,r} = 875.4 \text{ ft-kips} \quad \text{O.K.}$$

For both the positive and negative moment capacities, the strain in the extreme tension steel was determined from the strain compatibility analysis to be greater than 0.005 so that the section is tension-controlled.

$$\text{Maximum reinforcement ratio} = \frac{10 \times 1.0}{24 \times 22.33} = 0.019 < 0.025 \quad \text{O.K.}$$ *21.3.2.1*

where the effective depth d was determined from the strain compatibility analysis.

c. Anchorage and splices

Per 21.5.4.1, the minimum development length for a bar with a standard 90-degree hook in normal weight aggregate concrete is:

$$\ell_{dh} = \frac{f_y d_b}{65\sqrt{f'_c}} = \frac{(60{,}000)(1.128)}{65\sqrt{4000}} = 16.5 \text{ in.}$$ *Eq. (21-6)*

Figure 29-45 shows the reinforcement details for the connection.

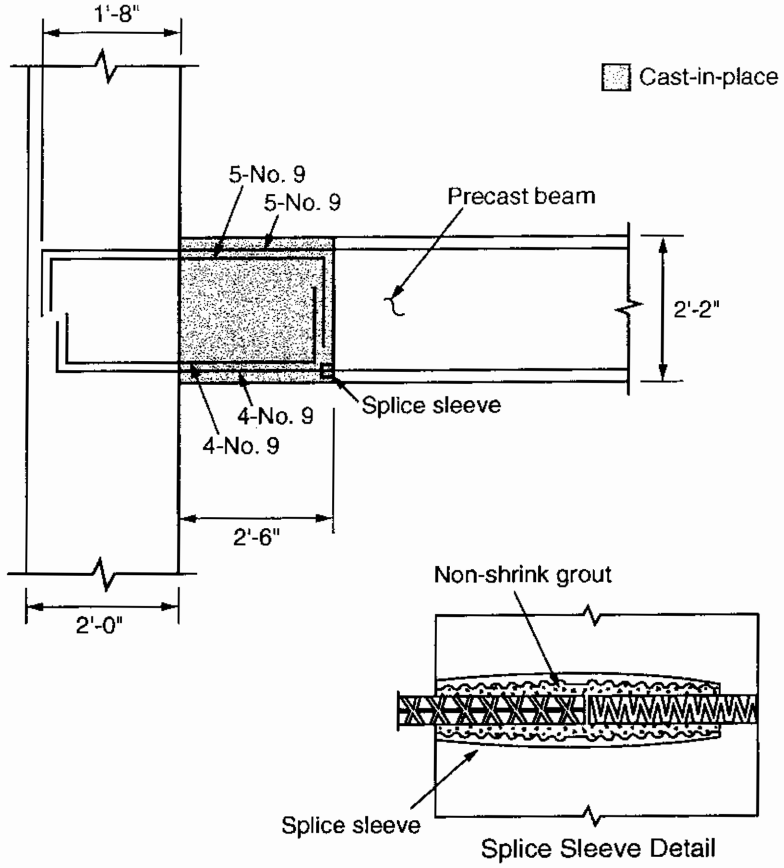


Fig. 29-45 Reinforcement Details for Connection

Example 29.8—Design of Slab Column Connections According to 21.11.5

Figure 29.46 shows the partial plan of a 5-story building assigned to Seismic Design Category D (i.e., high seismic design category). The seismic-force-resisting system consists of a building frame, where shear walls (not shown in figure) resist the seismic forces. Check the slab-column connections at columns B1 and B2 for the provisions of 21.11.5 assuming that the induced moments transferred between the slab and column under the design displacement are not computed.

Material Properties:

Concrete (w_c = 150 pcf): f'_c = 4000 psi

Reinforcement: f_y = 60 ksi

Service Loads:

Live load = 50 psf

Superimposed dead load = 30 psf

Member Dimensions:

Slab thickness = 9 in.

Columns = 24 × 24 in.

Additional Data:

Story height = 10 ft

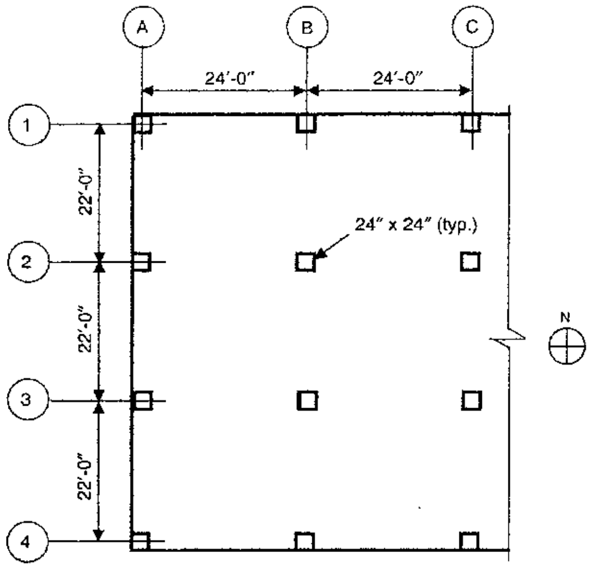


Figure 29-46 Partial Floor Plan

Design displacements and story drifts in the N-S direction:

Story	Design Displacement (in.)	Story Drift (in.)*
5	1.5	0.3
4	1.2	0.4
3	0.8	0.3
2	0.5	0.3
1	0.2	0.2

* Story drift = design displacement at top of story — design displacement at bottom of story

Calculations and Discussion	Code Reference

1. Column B1

   a. Determine factored shear force V_u due to gravity loads on slab critical section for two-way action

   $w_D = (9/12) \times 0.15 + 0.03 = 0.143$ ksf

$w_L = 0.05$ ksf

$w_u = 1.2w_D + 0.5w_L = 1.2 \times 0.143 + 0.5 \times 0.05 = 0.2$ ksf — *21.11.5, 9.2.1(a)*

Critical section dimensions: — *11.12.1.2*

Use average $d = 9 - 1.25 = 7.75$ in.

$b_1 = 24 + (7.75/2) = 27.875$ in.

$b_2 = 24 + 7.75 = 31.75$ in.

$V_u = 0.2[(24 \times 12) - (27.875 \times 31.75/144)] = 56$ kips

b. Determine two-way shear design strength ϕV_c

For square columns, Eq. (11-35) governs: — *11.12.21*

$$V_c = 4\sqrt{f_c'}\,b_o d$$

where $b_o = b_2 + 2b_1 = 31.75 + 2 \times 27.875 = 87.5$ in.

Thus,

$$V_c = 4\sqrt{4000} \times 87.5 \times 7.75/1000 = 172 \text{ kips}$$

$\phi V_c = 0.75 \times 172 = 129$ kips — *9.3.2.3*

c. Check criterion in 21.11.5(b)

Since induced moments are not computed, the requirements of 21.11.5(b) must be satisfied.

Maximum story drift at 4th floor level = 0.4 in.

Design story drift ratio = story drift/story height = $0.4/(10 \times 12) = 0.003$

Limiting design story drift ratio:

$0.035 - 0.05(V_u/\phi V_c) = 0.035 - 0.05(56/129) = 0.013 > 0.005$

Since the design story drift ratio = $0.003 < 0.013$, slab shear reinforcement satisfying the requirements of 21.11.5 need not be provided.

2. Column B2

a. Determine factored shear force V_u due to gravity loads on slab critical section for two-way action

Example 29.8 (cont'd)	Calculations and Discussion	Code Reference

$w_u = 1.2w_D + 0.5w_L = 1.2 \times 0.143 + 0.5 \times 0.05 = 0.2$ ksf — *21.11.5, 9.2.1(a)*

Critical section dimensions: — *11.12.1.2*

$b_1 = b_2 = 24 + 7.75 = 31.75$ in.

$V_u = 0.2[(24 \times 22) - (31.75^2/144)] = 104$ kips

b. Determine two-way shear design strength ϕV_c

For square columns, Eq. (11-35) governs: — *11.12.21*

$$V_c = 4\sqrt{f_c'}b_o d$$

where $b_o = 4 \times 31.75 = 127.0$ in.

Thus,

$$V_c = 4\sqrt{4000} \times 127.0 \times 7.75/1000 = 249 \text{ kips}$$

$\phi V_c = 0.75 \times 249 = 187$ kips — *9.3.2.3*

c. Check criterion in Section 21.11.5(b)

Maximum story drift at 4th floor level = 0.4 in.

Design story drift ratio = story drift/story height = $0.4/(10 \times 12) = 0.003$

Limiting design story drift ratio:

$0.035 - 0.05(V_u/\phi V_c) = 0.035 - 0.05(104/187) = 0.007 > 0.005$

Since the design story drift ratio = 0.003 < 0.007, slab shear reinforcement satisfying the requirements of 21.11.5 need not be provided.

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Structural Plain Concrete

BACKGROUND

With publication of the 1983 edition of ACI 318, provisions for structural plain concrete were incorporated into the code by reference. The document referenced was ACI 318.1, *Building Code Requirements for Structural Plain Concrete*. This method of regulating plain concrete continued with the 1989 edition of ACI 318. For the 1995 edition, the provisions formally contained in the ACI 318.1 standard were incorporated into Chapter 22 of the code and publication of ACI 318.1 was discontinued. While the presentation of some provisions is different, few technical changes have been made since the 1989 edition of ACI 318.1. Technical changes that were made are discussed at the appropriate location in this part.

22.1, 22.2 SCOPE AND LIMITATIONS

By definition, structural plain concrete is concrete in members that either contains no reinforcement or contains less reinforcement than the minimum amount specified for reinforced concrete in other chapters of ACI 318 and Appendices A through C (22.2.1). The designer should take special note of 22.2.2. Since the structural integrity of structural plain concrete members depends solely on the properties of the concrete, it limits the use of plain concrete to: members that are continuously supported by soil or by other structural members capable of providing vertical support continuous throughout the length of the plain concrete member; members in which arch action assures compression under all conditions of loading; and walls and pedestals. Chapter 22 of ACI 318 contains specific design provisions for structural plain concrete walls, footings and pedestals.

Section 22.1.1.2 indicates that sidewalks and other slabs-on-grade are not regulated by the code unless they transmit vertical loads or lateral forces from other parts of the structure to the soil. Section 1.1.6 also stipulates that if a slab transmits vertical loads or lateral forces from the structure to the soil, it also must comply with the code. In addition, 22.2.3 points out that the design and construction of portions of structural plain concrete foundation piers and cast-in-place piles embedded in ground capable of providing adequate lateral support are not governed by Chapter 22. Provisions for these elements are typically found in the general building code.

22.3 JOINTS

Structural plain concrete members must be small enough or provided with contraction (control) joints so as to create elements that are flexurally discontinuous (22.3.1). This requires that the build-up of tensile stresses due to external loads and internal loads, such as from drying shrinkage, temperature and moisture changes, and creep, must be limited to permissible values. Section 22.3.2 emphasizes several items that will influence the size of elements and, consequently, the spacing of contraction joints. These include: climatic conditions; selection and proportioning of materials; mixing, placing and curing of concrete; degree of restraint to movement; stresses due to external and internal loads to which the element is subjected; and construction techniques. Where contraction joints are provided, the member thickness must be reduced a minimum of 25% if the joint is to be effective. For additional information on drying shrinkage of concrete, other causes of volume changes of concrete, and the use of contraction joints to relieve stress build-up, see Refs. 30.1 through 30.3.

While not a part of the provisions, R22.3 gives an exception to the above requirement for contraction joints. It indicates that where random cracking due to creep, shrinkage and temperature effects will not affect the structural integrity, and is otherwise acceptable, such as transverse cracks in a continuous wall footing, contraction joints are not necessary.

22.4 DESIGN METHOD

As for reinforced concrete designed in accordance with Chapters 1 through 21, the provisions of Chapter 22 are based on the strength design methodology. Load combinations and load factors are found in 9.2, and are the same as those used for the design of reinforced concrete. The load combinations and load factors in the 1999 and earlier ACI codes were replaced with load combinations and load factors from ASCE 7-98 into the 2002 code. The strength reduction factor, ϕ, is found in 9.3.5. It was reduced from 0.65 (found in the 1999 and earlier editions of the ACI code) to 0.55 in the 2002 code,and applies for all stress conditions (i.e., flexure, compression, shear and bearing). Everything else remaining the same, the reduction in ϕ results in a 15.4% decrease in the design strength. Although some load factors in 9.2 are less than those in C.2, they have not been reduced enough to completely compensate for the lower design strength. While each case needs to be investigated, generally speaking a more economical design will be obtained by using the load and strength reduction factors of Appendix C. If snow loads or roof live loads are included in the controlling gravity load, depending on the magnitude of these loads with respect to floor live load, use of the load and strength reduction factors of Chapter 9 may be more economical.

To quickly determine which one of the two sets of load and strength reduction factors should be used, compute the governing load/load effect (e.g., P_u or M_u) using the load factors in 9.2 and C.2. These values can then be divided by the corresponding strength reduction factors from 9.3.5 and C.3.5, respectively, to determine the nominal loads/load effects. Satisfying the lower nominal load/load effect may be more economical.

Numerous figures and tables are provided in the main body of this part to assist the user in designing structural members of plain concrete. They are based on the load factors and strength reduction factor (0.55) of Chapter 9. An appendix to this part contains similar figures and tables based on the load factors and strength reduction factor (0.65) found in Appendix C. To facilitate comparing companion figures and tables, their assigned numbers are the same except those corresponding to the appendix, are prefaced with the letter "C."

A linear stress-strain relationship in both tension and compression is assumed for members subject to flexure and axial loads. The allowable stress design procedures contained in Appendix A - Alternate Design Method of the 1999 and earlier editions of the code do not apply to structural members of plain concrete. That Appendix was removed from the 2002 code.

Where the provisions for contraction joints and/or size of members have been observed in accordance with 22.3, tensile strength of plain concrete is permitted to be considered (22.4.5). Tension is not to be considered beyond the outside edges of the panel, contraction joints or construction joints, nor is flexural tension allowed to be assumed between adjacent structural plain concrete elements (22.4.7).

Section 22.4.8 permits the entire cross-section to be considered effective in resisting flexure, combined flexure and axial load, and shear; **except that for concrete cast on the ground, such as a footing, the overall thickness, h, shall be assumed to be 2 in. less than actual**. The commentary indicates that this provision is necessary to allow for unevenness of the excavation and for some contamination of the concrete adjacent to the soil. No strength shall be assigned to any steel reinforcement that may be present (22.4.6).

As in the past, 22.2.4, through its reference to 1.1.1, requires that the minimum specified compressive strength of concrete, f'_c, used in design of structural plain concrete elements shall not be less than 2500 psi. This provision is considered necessary due to the fact that safety and load-carrying capability is based solely on the strength and quality of the concrete.

22.5 STRENGTH DESIGN

Permissible stresses of ACI 318.1-89 were replaced with formulas for calculating nominal strengths for flexure, compression, shear and bearing. The nominal moment strength, M_n, is given by:

$$M_n = 5\sqrt{f'_c}\,S_m \qquad \text{Eq. (22-2)}$$

for flexural tension controlled sections, and

$$M_n = 0.85 f'_c S_m \qquad \text{Eq. (22-3)}$$

for flexural compression controlled sections.

The nominal axial compression strength, P_n, is given by:

$$P_n = 0.60 f'_c \left[1 - \left(\frac{\ell_c}{32h}\right)^2\right] A_1 \qquad \text{Eq. (22-5)}$$

Note that the effective length factor, k, is missing from the numerator of the ratio $\ell_c/32h$. This change from ACI 318.1 was made because it was felt that it is always conservative to assume $k = 1$, which is based on both ends being fixed against translation. Also, it was recognized that it is difficult to obtain fixed connections in typical types of construction utilizing structural plain concrete walls. If a connection fixed against rotation is provided at one or both ends, the engineer can always assume $k = 0.8$ as in the past. However, before doing so the engineer should verify that the member providing rotational restraint has a flexural stiffness EI/ℓ at least equal to that of the wall.

For members subject to combined flexural and axial compression, two interaction equations are given and both must be satisfied. For the compression face:

$$\frac{P_u}{\phi P_n} + \frac{M_u}{\phi M_n} \leq 1 \qquad \text{Eq. (22-6)}$$

where $M_n = 0.85 f'_c S_m$

and for the tension face:

$$\frac{M_u}{Sm} - \frac{P_u}{A_g} \leq 5\phi\sqrt{f'_c} \qquad \text{Eq. (22-7)}$$

The nominal moment strength, M_n, for use in Eq. (22-6) (i.e., $0.85 f'_c S_m$) is more conservative than in the 1989 edition of ACI 318.1 in which it was $f'_c S_m$. Other nominal strengths of Chapter 22 are consistent with those calculated using permissible stresses of ACI 318.1-89.

The nominal shear strength, V_n, is given by:

$$V_n = \frac{4}{3}\sqrt{f'_c}\,b_w h \qquad \text{Eq. (22-9)}$$

for beam action, and by

$$V_n = \left[\frac{4}{3} + \frac{8}{3\beta}\right]\sqrt{f'_c}\,b_o h \leq 2.66\sqrt{f'_c}\,b_o h \qquad \text{Eq. (22-10)}$$

for two-way action, or punching shear.

In Eq. (22-10), the expression $[4/3 + 8/(3\beta)]$ reduces the nominal shear strength for concentrated loads with long-to-short-side ratios β greater than 2. Where the ratio is equal to or less than 2, the expression takes on the maximum permitted value of 2.66.

The equations for computing nominal flexural and shear strengths apply to normal weight aggregate concrete. If lightweight aggregate concrete is used, the strengths may need to be reduced in accordance with 22.5.6. Where the average splitting tensile strength of lightweight concrete, f_{ct}, is specified and concrete is proportioned in accordance with 5.2 of ACI 318, $\sqrt{f'_c}$ shall be replaced with $f_{ct}/6.7$, but the value of $f_{ct}/6.7$ shall not exceed $\sqrt{f'_c}$. Where f_{ct} is not specified, the value of $\sqrt{f'_c}$ shall be multiplied by 0.75 for all lightweight aggregate concrete and by 0.85 for sand-lightweight aggregate concrete, with linear interpolation permitted for mix designs using partial sand replacement.

Nominal bearing strength, B_n, is given by:

$$B_n = 0.85 f'_c A_1 \qquad \textit{Eq. (22-12)}$$

where A_1 is the loaded area. If the supporting surface is wider on all sides than A_1, the bearing strength may be increased by $\sqrt{A_2/A_1}$, but by not more than 2. A_2 is the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, A_1, and having side slopes of 1 vertical to 2 horizontal. See Part 6 for determination of A_2.

22.6 WALLS

22.6.5 Empirical Design Method

The provisions offer two alternatives for designing plain concrete walls. The simpler of the two is referred to as the *empirical design method.* It is only permitted for walls of solid rectangular cross-section where the resultant of all factored loads falls within the middle one-third of the overall thickness of the wall. In determining the effective eccentricity, the moment induced by lateral loads must be considered in addition to any moment induced by the eccentricity of the axial load. Limiting the eccentricity to one-sixth the wall thickness assures that all portions of the wall remain under compression. Under the empirical design method, the nominal axial load strength, P_n, is determined from:

$$P_n = 0.45 f'_c A_g \left[1 - \left(\frac{\ell_c}{32h}\right)^2\right] \qquad \textit{Eq. (22-14)}$$

This is a single-strength equation considering only the axial load. Moments due to eccentricity of the applied axial load and/or lateral loads can be ignored since an eccentricity not exceeding h/6 is assumed.

To assist the code user in the design of plain concrete walls using the empirical design method, Fig. 30-1 has been provided. By entering the figure with the required axial load strength, one can select the wall thickness that will yield a design axial load strength, ϕP_n, that is equal to or greater than required. For intermediate values of f'_c, the required wall thickness can be determined by interpolation.

22.6.3 Combined Flexure and Axial Load

The second method, which may be used for all loading conditions, must be used where the resultant of all factored loads falls outside of the middle one-third of the wall thickness (i.e., $e > h/6$). In this procedure the wall must be proportioned to satisfy the provisions for combined flexure and axial loads of interaction Eqs. (22-6) and (22-7). Where the effective eccentricity is less than 10% of the wall thickness, h, an assumed eccentricity of not less than 0.10h is required.

To utilize this method, one must generally proceed on a trial and error basis by assuming a wall thickness and specified compressive strength of concrete, f'_c, and determine if the two interaction equations are satisfied. This

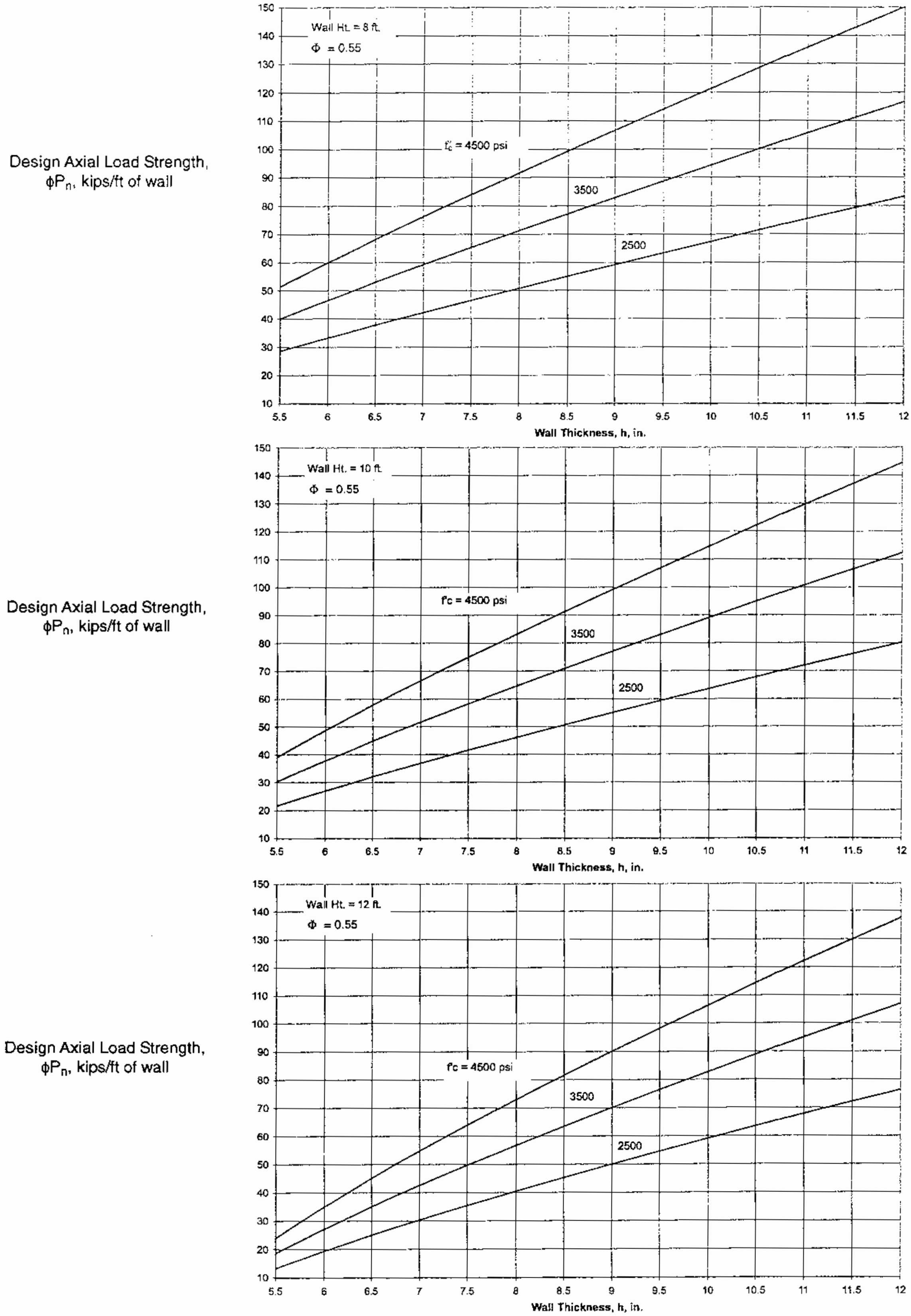


Figure 30-1 Design Axial Load Strength, P_n, of Plain Concrete Walls using the Empirical Design Method

process can proceed on a more structured basis if it is first determined whether Eq. (22-6) or (22-7) will control. Each equation can be rearranged to solve for M_u. Then, by setting the two equations equal to one another, rearranging to get both terms with P_u on the left side, and introducing constants to make units consistent, the resulting equation (1), which is shown below, can be solved for the axial load, P_u. The computed value of P_u is the axial load at which the moment strength is greatest and is the same regardless of whether Eq. (22-6) or (22-7) is used. If the required axial load strength is less than the computed value of P_u, the design is governed by Eq. (22-7). Conversely, if the required axial load strength is greater than the computed valued of P_u, the design is governed by Eq. (22-6). The equation for solving for P_u is:

$$\frac{S_m P_u}{12A_g} + \frac{M_n P_u}{P_n} = \phi M_n - \frac{5\phi\sqrt{f_c'}S_m}{12{,}000} \qquad (1)$$

where M_n is determined from Eq. (22-3), axial loads are in kips, moments are in ft-kips, section modulus is in in.^{3}, area is in in.^{2}, and $\sqrt{f_c'}$ is in psi.

If it is determined that Eq. (22-6) governs, equation (1) can be rearranged, with A_g and S_m expressed in terms of h. Quadratic equation (2) can be solved for the required wall thickness h:

If Eq. (22-6) governs, the required wall thickness is best determined by trial and error. Several iterations may be necessary before the most economical design solution is achieved.

$$0.06\phi\sqrt{f_c'}\,h^2 + P_u h - 72M_u = 0 \qquad (2)$$

where the axial load is in kips, the moment is in ft-kips, $\sqrt{f_c'}$ is in psi, and the thickness is in inches. If the required wall thickness is more than that assumed, another iteration is necessary. If it is significantly less than assumed, it may be advisable to repeat the process to determine if a more economical thickness and/or concrete strength can be justified.

The design process can be greatly simplified with the use of axial load-moment strength curves such as those shown in Figs. 30-2 and 30-3. To use the curves, enter with the known factored axial load, P_u, and determine if the design moment strength, ϕM_n, equals or exceeds the required factored moment strength, M_u. Of course, the curves can also be used by entering with the required factored moment strength, M_u, and determining if the design axial load strength, ϕP_n, equals or exceeds the required factored axial load strength, P_u.

If the effective eccentricity due to all factored loads is less than 0.10h, the design axial load strength, ϕP_n, is determined by projecting horizontally to the left from the intersection of the line labeled "e = h/10" and the curve representing the specified compressive strength of concrete, f_c'. For example, Fig. 30-2 shows that for an 8-in. wall, 8 ft in height constructed of concrete with a specified compressive strength, f_c', of 2500 psi, the design axial load strength, ϕP_n, is approximately 68 kips/ft of wall. This assumes that the wall is loaded concentrically and there are no lateral loads to induce moments (i.e., $\phi M_n = 0$). However, when the axial load is applied at the required minimum eccentricity of 0.10h, the design axial load strength is reduced to approximately 50 kips/ft of wall. The moment corresponding to the 50-kip load being applied at the minimum eccentricity of 0.10h is approximately 3.3 ft-kips/ft of wall.

A line labeled "e = h/6" has also been included on Figs. 30-2 and 30-3 to assist the user in identifying when the effective eccentricity exceeds this value. If the intersection of the axial load, P_u, and moment, M_u, lies to the right of the line, a portion of the wall is under tension due to the induced moment.

Walls of plain concrete are typically used as basement walls and above grade walls in residential and small commercial buildings. In most cases the axial loads are small compared to the design axial load compressive strength, ϕP_n, of the wall. Therefore, Figs. 30-4 through 30-6 have been developed which include only the lower range of values of axial loads from Figs. 30-2 and 30-3. Where small axial loads are acting in conjunction with moments, the design is governed by flexural tension [Eq. (22-7)] rather than by combined axial and flexural compression [Eq. (22-6)]. An

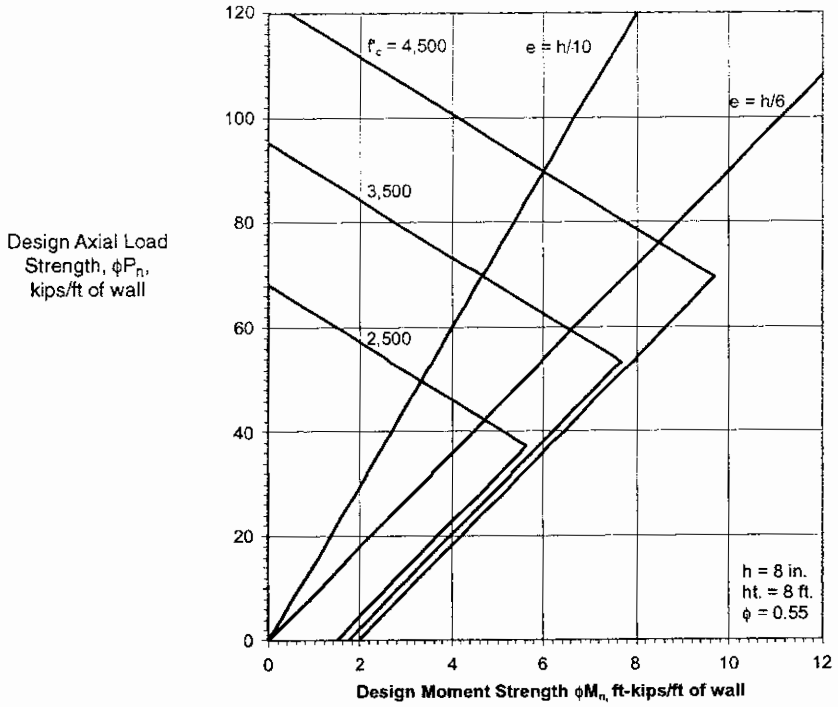


Figure 30-2 Design Strength Interaction Diagrams for 8.0-in. Wall, 8 ft in Height

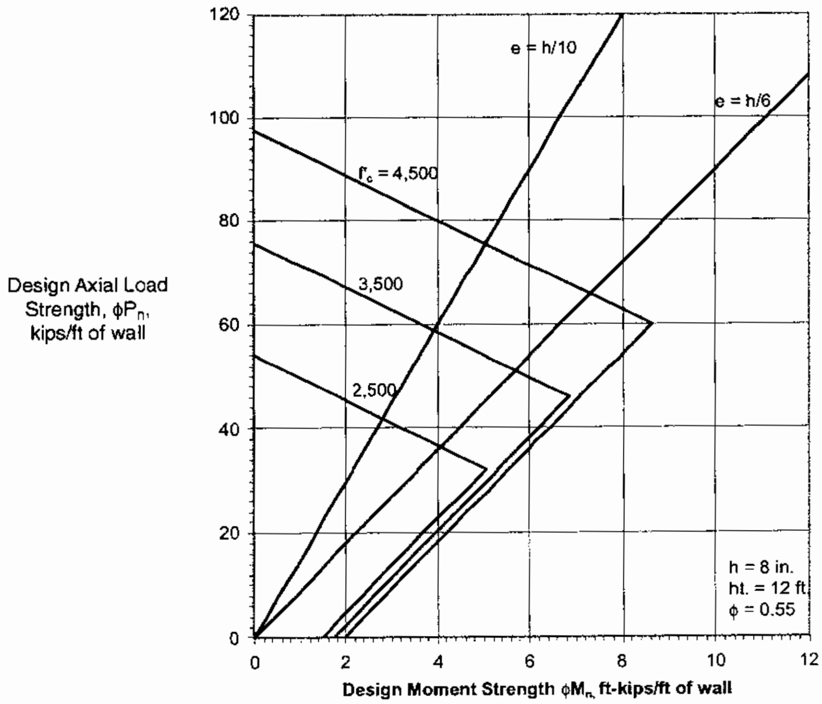


Figure 30-3 Design Strength Interaction Diagrams for 8.0-in. Wall, 12 ft in Height

examination of Eq. (22-7) will reveal that the design moment strength for lightly-loaded walls is not a function of the wall's height; therefore, the format of Figs. 30-4 through 30-6 is somewhat different than that of Figs. 30-2 and 30-3. To assist the user in verifying that the wall being designed is controlled by Eq. (22-7) instead of Eq. (22-6), Figs. 30-7 through 30-9 have been provided. These figures show the value of the design axial load strength, ϕP_n, that corresponds to the maximum value of the design moment strength, ϕM_n. For example, Fig. 30-7 shows an 8-in. wall 8 ft high has a design axial load strength, ϕP_n, of approximately 37.4 kips/ft of wall when a moment equal to the maximum design moment strength, ϕM_n, is applied. From Fig. 30-2 the maximum design moment strength, ϕM_n, is approximately 5.6 ft-kips/ft of wall when a factored load of approximately 37 kips/ft of wall is applied. When using Figs. 30-4 through 30-6, the user should always verify that the required axial load strength, P_u, is less than the value determined from Figs. 30-7 through 30-9. Also, the provisions of 22.6.6.2 should not be overlooked. They require that the thickness of the wall be not less than the larger of 1/24 the unsupported height or length of the wall and 5-1/2 in. A close examination of Figs. 30-7 through 30-9 will show that in almost every case covered, the design axial load strength exceeds 15 kips/ft of wall, which is significantly greater than the factored load on typical walls in low-rise residential buildings.

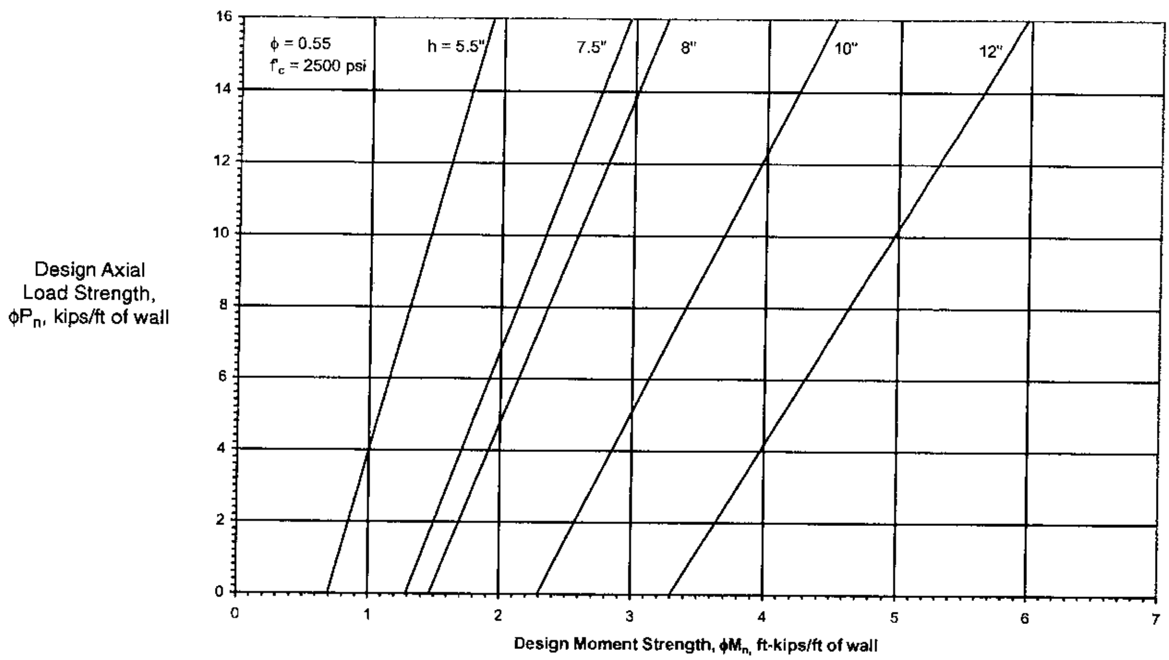


Figure 30-4 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f_c' = 2500 psi)

In typical construction of basement or foundation walls retaining unbalanced backfill, some walls of the building are generally nonload bearing walls. In this case, especially where the wall may be backfilled before all the dead load that will eventually be on the wall is in place, it is prudent to design the wall assuming no axial load is acting in conjunction with the lateral soil load. For this condition, equation (2) simplifies to:

$$0.06\phi\sqrt{f_c'}\,h^2 - 72M_u = 0 \qquad (3)$$

In this form, the equation can be rearranged to solve for required wall thickness:

$$h = (72M_u/0.06\phi\sqrt{f_c'})^{1/2} \qquad (4)$$

or to solve for required specified compressive strength of concrete:

$$f_c' = (72M_u/0.06\phi h^2)^2 \qquad (5)$$

In equations (3), (4), and (5), f_c' is in psi, h is in inches, and M_u is in ft-kips/ft of wall

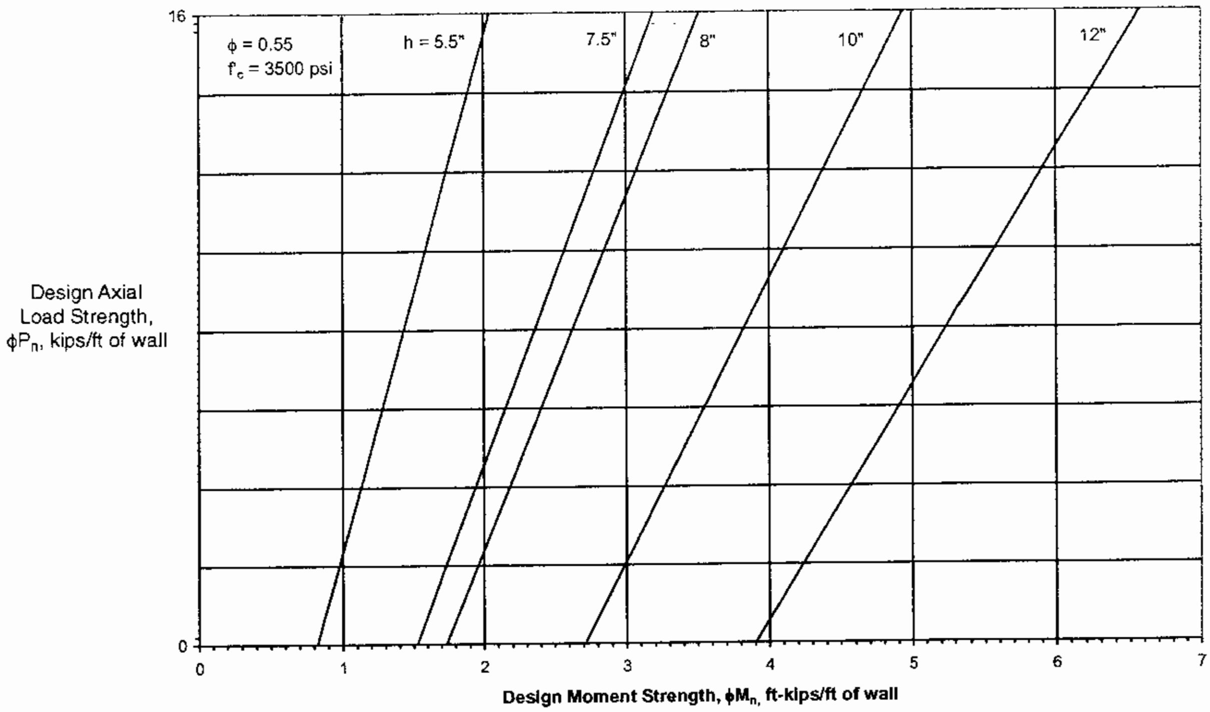


Figure 30-5 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f_c' = 3500 psi)

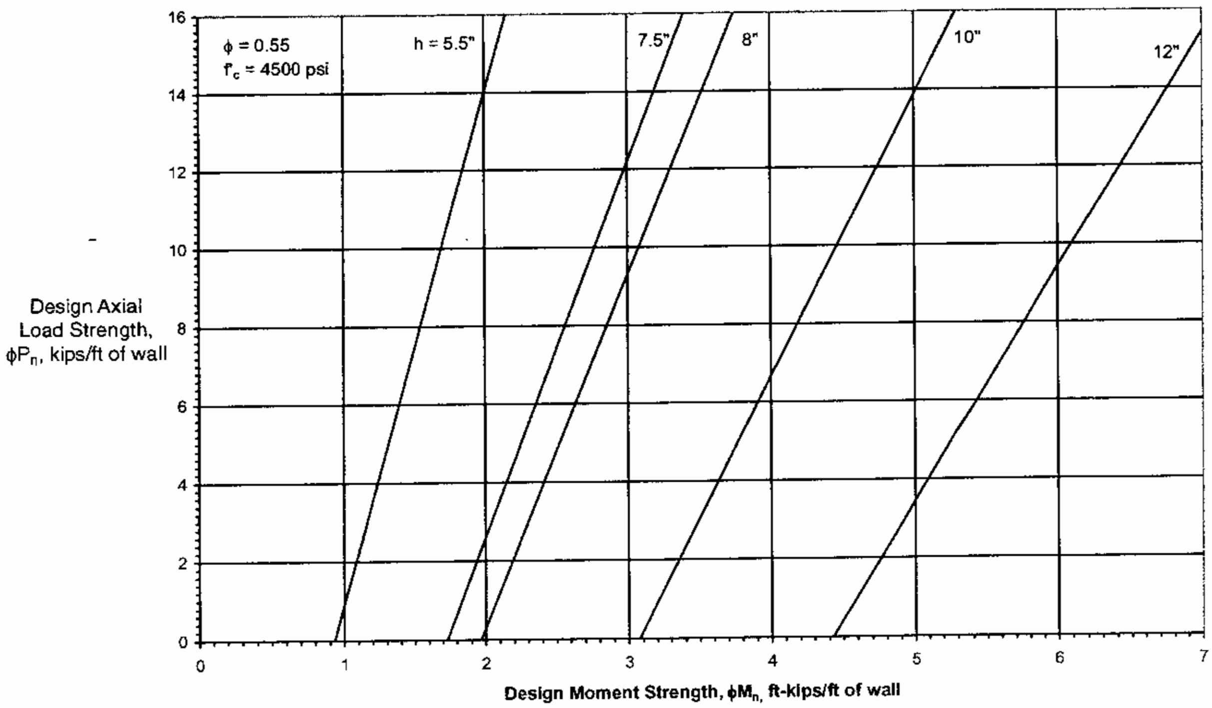


Figure 30-6 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f_c' = 4500 psi)

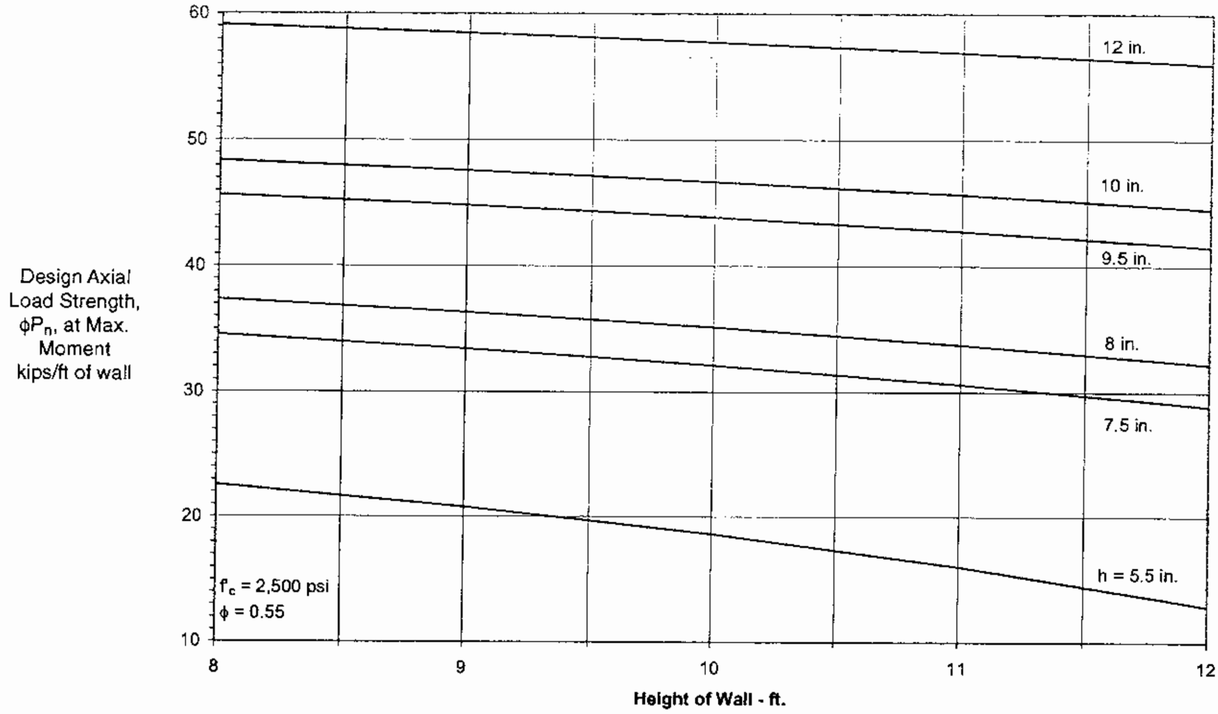


Figure 30-7 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 2500 psi)

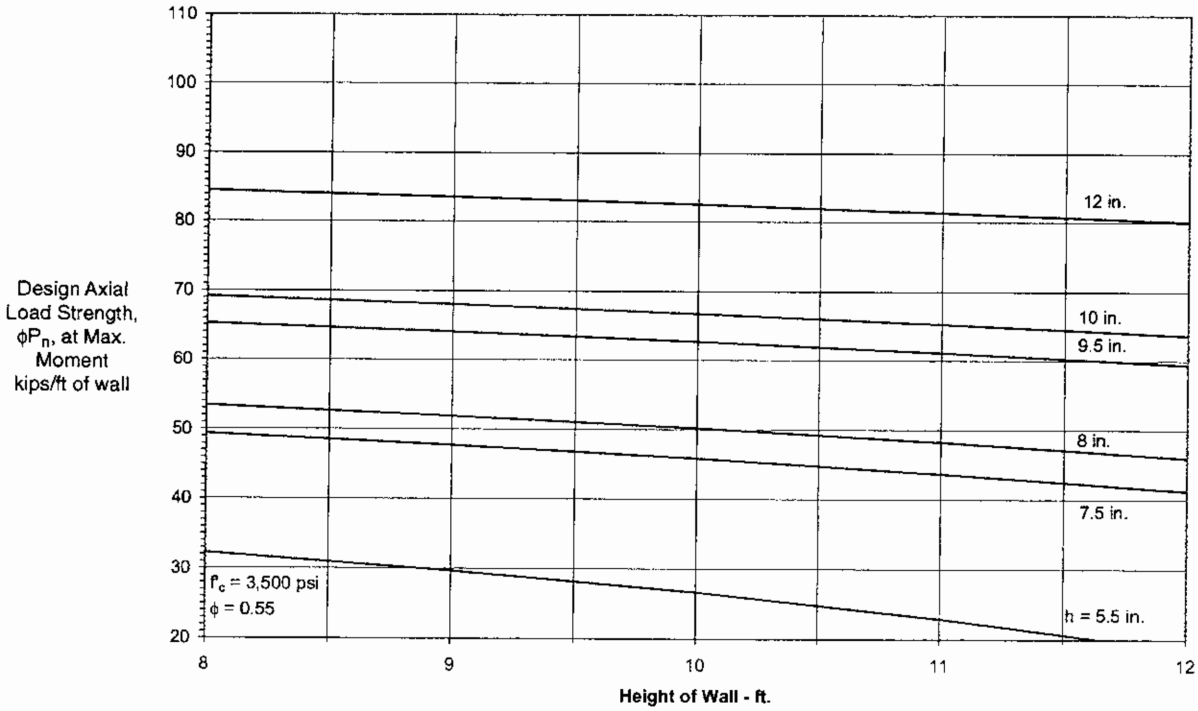


Figure 30-8 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 3500 psi)

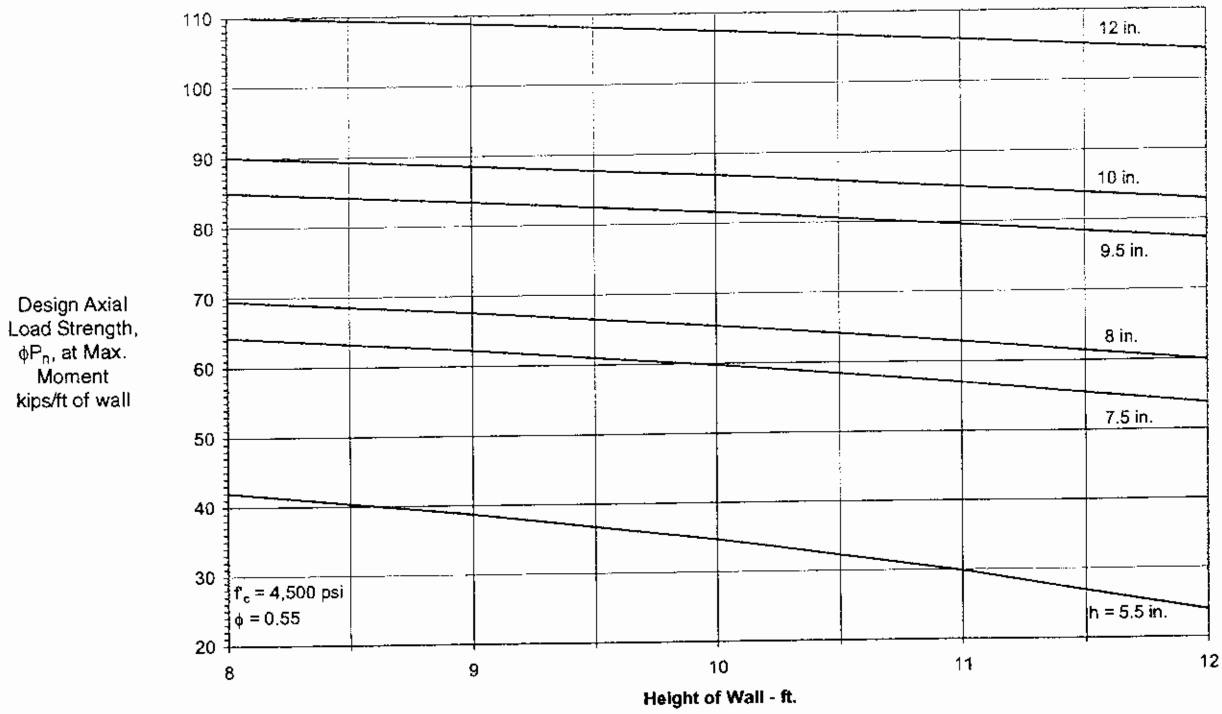


Figure 30-9 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 4500 psi)

Comparison of the Two Methods

Since 22.6.5.1 limits use of the empirical method to cases where the resultant of all factored loads falls in the middle one-third of the wall, one would assume that this method is more conservative for effective eccentricities less that one-sixth the wall thickness. However, this is not the case. The empirical method has an implied eccentricity factor [ratio of strength given by Eq. (22-14) to that given by Eq. (22-5)] of 0.75 (i.e., 0.45/0.60). However, the curves of Figs. 30-2 and 30-3 have an eccentricity factor (ratio of strength where load is applied at eccentricity of 0.10h to that of load applied concentrically) of 0.725 for 2500 psi concrete. Therefore, for effective eccentricities of less than one-sixth the wall thickness, the empirical method will yield a greater nominal axial load strength.

Section 22.6.6.3 requires that exterior basement and foundation walls must be not less than 7-1/2 in. thick. Note though, some building codes permit these walls to be 5-1/2 in. in thickness. Section 22.6.6.2 requires the thickness of other walls to be not less than 5-1/2 in., but not less than 1/24 the unsupported height or length of the wall, whichever is shorter.

Other limitations of 22.6.6 must also be observed. They include: the wall must be braced against lateral translation (22.6.6.4); and not less than 2-No. 5 bars, extending at least 24 in. beyond the corners, shall be provided around all door and window openings (22.6.6.5).

To facilitate the design of simply-supported walls subject to lateral loads from wind and/or soil, Tables 30-1 and 30-2 have been provided. The tables give factored moments due to various combinations of wind and soil lateral loads, and varying backfill heights. The tables also accommodate exterior walls completely above grade (no lateral load due to backfill), as well as walls that are not subject to lateral loading from wind. Tables 30-1 and 30-2 are to be used with the load factors of 9.2, and Tables C30-1 and C30-2 are to be used with the load factors of C.2. Note that the only difference between the two tables in each set is that the first table was developed using a load factor on wind of 1.6; whereas, the second table utilizes a load factor of 1.3. In each set of tables the load factor on the lateral load due to the soil is the same; either 1.6 or 1.7. Table 30-1 or C30-1 is to be used where load combinations in 9.2. or C.2, respectively, are being investigated where the wind load has been reduced by a directionality factor as in the IBC and ASCE 7-98 [9.2.1(b)]. Table 30-2 or C30-2 is to be used where load combinations in 9.2 or C.2, respectively, are being investigated where the wind load has not been reduced by a directionality factor, such as in the NBC, SBC, and UBC, and in editions of ASCE 7 prior to 1998.

For exterior walls partially above and below grade, the moments in the table assume that the wind load is acting in the same direction as the lateral load due to the soil (i.e., inward). Most contemporary wind design standards require that exterior walls be designed for both inward and outward acting pressures due to wind. Generally, the higher absolute value of wind loading occurs where the wall is under negative pressure (i.e., the force is acting outward on the wall). Section 9.2.1(d) stipulates that where earth pressure counteracts wind load, which is the case with wind acting outward, the load factor on H must be set equal to zero in load combination Eq. (9-6). Except for situations where the backfill height is small compared to the overall wall height, and depending upon the relative magnitude of the design wind pressure and lateral soil pressure, the moment due to lateral soil loads and inward-acting wind of Tables 30-1 and 30-2 will generally apply. For situations where outward-acting wind controls, it is simpler to design the wall as though the full height of the wall is exposed to the outward-acting wind pressure. Tables 30-1, 30-2, C30-1, and C30-2 can be used in this manner by assuming the backfill height is zero.

Before designing a structural plain concrete wall that will be resisting wind uplift and/or overturning forces, the appropriate load combinations of 9.2 or C.2 need to be investigated. If the entire wall cross-section will be in tension due to the factored axial and lateral forces, the wall must be designed as a reinforced concrete wall, or other means must be employed to transfer the uplift forces to the foundation. This condition can occur frequently in the design of walls supporting lightweight roof systems subject to net uplift forces from wind loads.

22.7 FOOTINGS

It is common practice throughout the United States, including high seismic risk areas, to use structural plain concrete footings for the support of walls for all types of structures. In addition, plain concrete is frequently used for footings supporting columns and pedestals, particularly in residential construction. These uses of structural plain concrete are permitted by Chapter 22; however, 22.7.3 prohibits the use of structural plain concrete footings which are supported on piles. In addition, contemporary building codes also have limitations on the use of structural plain concrete footings for structures in regions of moderate seismic risk, or for structures assigned to intermediate seismic performance or design categories. See section below on 22.10 for limitations on the use of structural plain concrete footings for structures in regions of high seismic risk, or for structures assigned to high seismic performance or design categories.

Many architects and engineers specify that two No. 4 or No. 5 longitudinal bars be included in footings supporting walls. However, typically these footings have no reinforcement in the transverse direction, or the amount provided is less than that required by the code to consider the footing reinforced. Such footings must be designed as structural plain concrete, since in the transverse direction the footing is subjected to flexural and possibly shear stresses due to the projection of the footing beyond the face of the supported member.

The base area of footings must be determined from unfactored loads and moments, if any, using permissible soil bearing pressures. Once the base area of the footing is selected, factored loads and moments are used to proportion the thickness of the footing to satisfy moment and, where applicable, shear strength requirements. Sections 22.7.5 and 22.7.6 define the critical sections for computing factored moments and shears. The locations are summarized in Table 30-3. Figure 22-2 illustrates the location of the critical sections for beam action and two-way action shear for a footing supporting a column or pedestal.

Footings must be proportioned to satisfy the requirements for moment in accordance with Eq. (22-2). For footings supporting columns, pedestals or concrete walls, if the projection of the footing beyond the face of the supported member does not exceed the footing thickness, h, it is not necessary to check for beam action shear since the location of the critical section for calculating shear falls outside the footing. Where beam action shear must be considered, the requirements of Eq. (22-9) must be satisfied. In addition, for footings supporting columns, pedestals or other concentrated loads, if the projection of the footing beyond the critical section exceeds h/2, it is necessary to determine if the requirements of Eq. (22-10) are satisfied for two-way action (punching) shear. Generally, flexural strength will govern the thickness design of plain concrete footings; however, the engineer should not overlook the possibility that beam action shear or two-way action shear may control. It must be remembered that the provisions of 22.4.8 require that for plain concrete members cast on soil, the thickness, h, used to compute flexural and shear strengths is the overall thickness minus 2 in. Thus, for a footing with an overall thickness of 8 in. (which is the minimum overall thickness permitted by 22.7.4), the thickness, h, used to compute strengths is 6 in. Some building codes permit 6-in. thick footings for residential and other small buildings. In this case the thickness, h, for strength computation purposes is 4 in.

Table 30-1 Factored Moments Induced in Walls by Lateral Soil and/or Wind Load (ft-kips/linear ft)
(For Use with Chapter 9 Load Factors: Soil 1.6, Wind 1.6)

Wall Ht. (ft.)	Back fill Ht. (ft.)	Unfactored Design Lateral Soil Load (psf per foot of depth)																			
		30	30	30	30	30	45	45	45	45	45	60	60	60	60	60	100	100	100	100	100
		Unfactored Design Wind Pressure (psf)																			
		0	10	20	40	80	0	10	20	40	80	0	10	20	40	80	0	10	20	40	80
8	0	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02
8	1	0.01	0.13	0.25	0.50	1.00	0.01	0.13	0.25	0.50	1.00	0.01	0.13	0.26	0.50	1.00	0.02	0.14	0.26	0.51	1.01
8	2	0.05	0.14	0.26	0.48	0.93	0.08	0.16	0.27	0.50	0.95	0.10	0.18	0.29	0.51	0.96	0.17	0.23	0.34	0.56	1.00
8	3	0.15	0.21	0.29	0.48	0.85	0.23	0.28	0.35	0.53	0.90	0.31	0.36	0.42	0.59	0.95	0.51	0.56	0.62	0.75	1.10
8	4	0.33	0.37	0.41	0.51	0.78	0.49	0.53	0.57	0.66	0.90	0.65	0.69	0.73	0.82	1.02	1.09	1.12	1.16	1.25	1.42
8	5	0.57	0.59	0.62	0.67	0.79	0.85	0.87	0.90	0.95	1.06	1.13	1.16	1.18	1.23	1.34	1.88	1.91	1.93	1.98	2.09
8	6	0.86	0.88	0.89	0.91	0.96	1.30	1.31	1.32	1.34	1.39	1.73	1.74	1.75	1.78	1.83	2.88	2.89	2.90	2.93	2.98
8	7	1.21	1.21	1.21	1.22	1.23	1.81	1.81	1.82	1.82	1.84	2.41	2.42	2.42	2.43	2.44	4.02	4.03	4.03	4.04	4.05
8	8	1.58	1.58	1.58	1.58	1.58	2.36	2.36	2.36	2.36	2.36	3.15	3.15	3.15	3.15	3.15	5.26	5.26	5.26	5.26	5.26
9	0	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30
9	1	0.01	0.16	0.32	0.64	1.27	0.01	0.16	0.32	0.64	1.27	0.01	0.17	0.32	0.64	1.27	0.02	0.17	0.33	0.65	1.28
9	2	0.05	0.18	0.32	0.62	1.20	0.08	0.20	0.34	0.63	1.22	0.10	0.21	0.36	0.65	1.23	0.17	0.27	0.40	0.69	1.27
9	3	0.16	0.24	0.36	0.61	1.12	0.24	0.31	0.42	0.67	1.17	0.32	0.39	0.48	0.72	1.23	0.53	0.60	0.68	0.88	1.37
9	4	0.34	0.40	0.47	0.65	1.05	0.51	0.57	0.63	0.78	1.17	0.69	0.74	0.80	0.94	1.30	1.14	1.20	1.26	1.38	1.66
9	5	0.60	0.65	0.69	0.78	1.01	0.91	0.95	0.99	1.08	1.28	1.21	1.25	1.29	1.38	1.57	2.01	2.05	2.09	2.18	2.36
9	6	0.94	0.96	0.99	1.04	1.16	1.41	1.43	1.46	1.51	1.62	1.88	1.90	1.93	1.98	2.09	3.13	3.15	3.18	3.23	3.33
9	7	1.33	1.35	1.36	1.38	1.43	2.00	2.01	2.03	2.05	2.10	2.67	2.68	2.69	2.72	2.77	4.45	4.46	4.47	4.50	4.55
9	8	1.78	1.78	1.78	1.79	1.80	2.66	2.67	2.67	2.68	2.69	3.55	3.56	3.56	3.57	3.58	5.92	5.92	5.93	5.93	5.95
9	9	2.24	2.24	2.24	2.24	2.24	3.37	3.37	3.37	3.37	3.37	4.49	4.49	4.49	4.49	4.49	7.48	7.48	7.48	7.48	7.48
10	0	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60
10	1	0.01	0.20	0.40	0.79	1.57	0.01	0.20	0.40	0.79	1.57	0.01	0.20	0.40	0.79	1.58	0.02	0.21	0.41	0.80	1.58
10	2	0.05	0.22	0.40	0.77	1.51	0.08	0.23	0.42	0.78	1.52	0.11	0.25	0.43	0.80	1.54	0.18	0.30	0.48	0.84	1.58
10	3	0.16	0.28	0.44	0.76	1.43	0.25	0.35	0.50	0.82	1.48	0.33	0.42	0.56	0.87	1.53	0.55	0.64	0.74	1.03	1.67
10	4	0.36	0.44	0.54	0.80	1.35	0.54	0.61	0.70	0.93	1.47	0.71	0.79	0.87	1.08	1.60	1.19	1.27	1.34	1.52	1.96
10	5	0.64	0.70	0.76	0.91	1.31	0.95	1.01	1.08	1.22	1.55	1.27	1.33	1.39	1.53	1.82	2.12	2.18	2.24	2.37	2.64
10	6	1.00	1.04	1.09	1.18	1.39	1.50	1.54	1.59	1.68	1.87	2.00	2.04	2.09	2.18	2.36	3.33	3.38	3.42	3.51	3.69
10	7	1.44	1.47	1.49	1.55	1.66	2.16	2.19	2.22	2.27	2.38	2.88	2.91	2.94	2.99	3.10	4.81	4.83	4.86	4.91	5.02
10	8	1.95	1.96	1.97	2.00	2.05	2.92	2.93	2.95	2.97	3.02	3.89	3.91	3.92	3.94	3.99	6.49	6.50	6.52	6.54	6.59
10	9	2.50	2.50	2.51	2.51	2.53	3.75	3.75	3.76	3.76	3.78	5.00	5.00	5.01	5.01	5.03	8.33	8.34	8.34	8.35	8.36
10	10	3.08	3.08	3.08	3.08	3.08	4.62	4.62	4.62	4.62	4.62	6.16	6.16	6.16	6.16	6.16	10.26	10.26	10.26	10.26	10.26
12	0	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30
12	1	0.01	0.29	0.57	1.14	2.28	0.01	0.29	0.57	1.14	2.28	0.01	0.29	0.58	1.14	2.28	0.02	0.30	0.58	1.15	2.29
12	2	0.06	0.30	0.58	1.12	2.21	0.08	0.32	0.59	1.14	2.22	0.11	0.34	0.61	1.15	2.24	0.18	0.39	0.65	1.20	2.28
12	3	0.17	0.36	0.61	1.12	2.13	0.26	0.43	0.67	1.17	2.18	0.34	0.50	0.73	1.23	2.23	0.57	0.70	0.90	1.38	2.38
12	4	0.38	0.51	0.71	1.15	2.06	0.57	0.69	0.86	1.28	2.18	0.76	0.88	1.02	1.42	2.30	1.26	1.38	1.51	1.83	2.66
12	5	0.69	0.80	0.92	1.25	2.01	1.03	1.14	1.25	1.53	2.25	1.37	1.48	1.59	1.84	2.51	2.29	2.39	2.50	2.73	3.26
12	6	1.10	1.19	1.28	1.49	2.03	1.65	1.74	1.83	2.02	2.46	2.20	2.28	2.37	2.56	2.97	3.66	3.75	3.84	4.02	4.40
12	7	1.61	1.68	1.75	1.89	2.20	2.42	2.49	2.55	2.69	2.99	3.23	3.29	3.36	3.50	3.78	5.38	5.45	5.51	5.65	5.92
12	8	2.22	2.27	2.31	2.41	2.61	3.34	3.38	3.43	3.52	3.71	4.45	4.49	4.54	4.63	4.82	7.41	7.46	7.50	7.59	7.78
12	9	2.92	2.94	2.97	3.03	3.14	4.37	4.40	4.43	4.48	4.59	5.83	5.86	5.89	5.94	6.05	9.72	9.75	9.77	9.83	9.94
12	10	3.68	3.69	3.70	3.73	3.78	5.51	5.53	5.54	5.56	5.62	7.35	7.36	7.38	7.40	7.45	12.25	12.27	12.28	12.30	12.35
12	11	4.48	4.49	4.49	4.50	4.51	6.73	6.73	6.73	6.74	6.75	8.97	8.97	8.98	8.98	8.99	14.95	14.95	14.95	14.96	14.97
12	12	5.32	5.32	5.32	5.32	5.32	7.98	7.98	7.98	7.98	7.98	10.64	10.64	10.64	10.64	10.64	17.74	17.74	17.74	17.74	17.74

Table 30-2 Factored Moments Induced in Walls by Lateral Soil and/or Wind Load (ft-kips/linear ft)
(For Use with Chapter 9 Load Factors: Soil 1.6, Wind 1.3)

Wall Ht. (ft.)	Backfill Ht. (ft.)	Unfactored Design Lateral Soil Load (psf per foot of depth)																			
		30	30	30	30	30	45	45	45	45	45	60	60	60	60	60	100	100	100	100	100
		Unfactored Design Wind Pressure (psf)																			
		0	10	20	40	80	0	10	20	40	80	0	10	20	40	80	0	10	20	40	80
8	0	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83
8	1	0.01	0.10	0.21	0.41	0.81	0.01	0.11	0.21	0.41	0.81	0.01	0.11	0.21	0.41	0.81	0.02	0.11	0.21	0.42	0.82
8	2	0.05	0.12	0.21	0.40	0.76	0.08	0.14	0.23	0.41	0.78	0.10	0.16	0.25	0.43	0.79	0.17	0.22	0.30	0.47	0.83
8	3	0.15	0.20	0.26	0.41	0.71	0.23	0.27	0.32	0.46	0.76	0.31	0.35	0.40	0.52	0.81	0.51	0.55	0.60	0.70	0.96
8	4	0.33	0.36	0.39	0.47	0.68	0.49	0.52	0.55	0.62	0.80	0.65	0.68	0.72	0.78	0.94	1.09	1.12	1.15	1.21	1.35
8	5	0.57	0.59	0.61	0.65	0.74	0.85	0.87	0.89	0.93	1.02	1.13	1.15	1.17	1.21	1.30	1.88	1.90	1.92	1.96	2.05
8	6	0.86	0.87	0.88	0.90	0.94	1.30	1.31	1.32	1.34	1.38	1.73	1.74	1.75	1.77	1.81	2.88	2.89	2.90	2.92	2.96
8	7	1.21	1.21	1.21	1.22	1.23	1.81	1.81	1.82	1.82	1.83	2.41	2.42	2.42	2.43	2.44	4.02	4.03	4.03	4.04	4.05
8	8	1.58	1.58	1.58	1.58	1.58	2.36	2.36	2.36	2.36	2.36	3.15	3.15	3.15	3.15	3.15	5.26	5.26	5.26	5.26	5.26
9	0	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05
9	1	0.01	0.13	0.26	0.52	1.03	0.01	0.13	0.26	0.52	1.03	0.01	0.14	0.26	0.52	1.04	0.02	0.14	0.27	0.53	1.04
9	2	0.05	0.15	0.27	0.51	0.98	0.08	0.17	0.29	0.52	1.00	0.10	0.19	0.30	0.54	1.01	0.17	0.24	0.35	0.58	1.06
9	3	0.16	0.22	0.32	0.52	0.93	0.24	0.30	0.38	0.57	0.98	0.32	0.38	0.44	0.63	1.04	0.53	0.59	0.65	0.80	1.18
9	4	0.34	0.39	0.44	0.58	0.90	0.51	0.56	0.61	0.72	1.02	0.69	0.73	0.78	0.88	1.15	1.14	1.19	1.23	1.33	1.55
9	5	0.60	0.64	0.67	0.75	0.92	0.91	0.94	0.97	1.04	1.20	1.21	1.24	1.27	1.34	1.49	2.01	2.05	2.08	2.15	2.29
9	6	0.94	0.96	0.98	1.02	1.11	1.41	1.43	1.45	1.49	1.58	1.88	1.90	1.92	1.96	2.05	3.13	3.15	3.17	3.21	3.29
9	7	1.33	1.34	1.35	1.37	1.42	2.00	2.01	2.02	2.04	2.08	2.67	2.68	2.69	2.71	2.75	4.45	4.46	4.47	4.49	4.53
9	8	1.78	1.78	1.78	1.79	1.80	2.66	2.67	2.67	2.68	2.69	3.55	3.56	3.56	3.56	3.57	5.92	5.92	5.93	5.93	5.94
9	9	2.24	2.24	2.24	2.24	2.24	3.37	3.37	3.37	3.37	3.37	4.49	4.49	4.49	4.49	4.49	7.48	7.48	7.48	7.48	7.48
10	0	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30
10	1	0.01	0.16	0.32	0.64	1.28	0.01	0.17	0.32	0.64	1.28	0.01	0.17	0.33	0.65	1.28	0.02	0.17	0.33	0.65	1.29
10	2	0.05	0.18	0.33	0.63	1.23	0.08	0.20	0.35	0.65	1.24	0.11	0.22	0.36	0.66	1.26	0.18	0.27	0.41	0.71	1.30
10	3	0.16	0.25	0.38	0.64	1.18	0.25	0.32	0.44	0.70	1.23	0.33	0.40	0.50	0.75	1.28	0.55	0.62	0.70	0.92	1.43
10	4	0.36	0.42	0.50	0.70	1.14	0.54	0.60	0.67	0.84	1.27	0.71	0.78	0.84	0.99	1.40	1.19	1.25	1.31	1.45	1.77
10	5	0.64	0.69	0.74	0.85	1.15	0.95	1.00	1.05	1.16	1.41	1.27	1.32	1.37	1.48	1.71	2.12	2.17	2.22	2.32	2.53
10	6	1.00	1.04	1.07	1.15	1.31	1.50	1.54	1.57	1.64	1.80	2.00	2.04	2.07	2.14	2.29	3.33	3.37	3.40	3.47	3.62
10	7	1.44	1.46	1.48	1.53	1.62	2.16	2.18	2.21	2.25	2.34	2.88	2.90	2.93	2.97	3.06	4.81	4.83	4.85	4.89	4.98
10	8	1.95	1.96	1.97	1.99	2.03	2.92	2.93	2.94	2.96	3.00	3.89	3.90	3.91	3.93	3.98	6.49	6.50	6.51	6.53	6.57
10	9	2.50	2.50	2.51	2.51	2.52	3.75	3.75	3.75	3.76	3.77	5.00	5.00	5.00	5.01	5.02	8.33	8.34	8.34	8.34	8.35
10	10	3.08	3.08	3.08	3.08	3.08	4.62	4.62	4.62	4.62	4.62	6.16	6.16	6.16	6.16	6.16	10.26	10.26	10.26	10.26	10.26
12	0	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87
12	1	0.01	0.23	0.47	0.93	1.85	0.01	0.24	0.47	0.93	1.85	0.01	0.24	0.47	0.93	1.85	0.02	0.24	0.47	0.94	1.86
12	2	0.06	0.25	0.47	0.92	1.80	0.08	0.27	0.49	0.93	1.82	0.11	0.29	0.51	0.95	1.83	0.18	0.34	0.55	0.99	1.87
12	3	0.17	0.32	0.52	0.93	1.75	0.26	0.39	0.58	0.98	1.80	0.34	0.46	0.64	1.04	1.85	0.57	0.68	0.82	1.19	2.00
12	4	0.38	0.48	0.63	0.98	1.72	0.57	0.67	0.79	1.12	1.84	0.76	0.86	0.97	1.26	1.97	1.26	1.36	1.46	1.69	2.33
12	5	0.69	0.77	0.87	1.12	1.73	1.03	1.12	1.21	1.41	1.97	1.37	1.46	1.55	1.74	2.24	2.29	2.37	2.46	2.64	3.05
12	6	1.10	1.17	1.24	1.40	1.80	1.65	1.72	1.79	1.94	2.28	2.20	2.27	2.34	2.49	2.81	3.66	3.73	3.80	3.95	4.25
12	7	1.61	1.67	1.72	1.84	2.08	2.42	2.47	2.53	2.64	2.87	3.23	3.28	3.34	3.44	3.67	5.38	5.43	5.49	5.59	5.82
12	8	2.22	2.26	2.30	2.37	2.53	3.34	3.37	3.41	3.48	3.64	4.45	4.48	4.52	4.60	4.75	7.41	7.45	7.49	7.56	7.71
12	9	2.92	2.94	2.96	3.00	3.10	4.37	4.40	4.42	4.46	4.55	5.83	5.85	5.88	5.92	6.01	9.72	9.74	9.76	9.81	9.90
12	10	3.68	3.69	3.70	3.72	3.76	5.51	5.52	5.53	5.55	5.60	7.35	7.36	7.37	7.39	7.43	12.25	12.26	12.27	12.29	12.33
12	11	4.48	4.49	4.49	4.49	4.51	6.73	6.73	6.73	6.74	6.75	8.97	8.97	8.97	8.98	8.99	14.95	14.95	14.95	14.96	14.97
12	12	5.32	5.32	5.32	5.32	5.32	7.98	7.98	7.98	7.98	7.98	10.64	10.64	10.64	10.64	10.64	17.74	17.74	17.74	17.74	17.74

Figure 30-10 has been provided to aid in the selection of footing thickness to satisfy flexural strength requirements. The figure is entered with the *factored* soil bearing pressure. Project vertically upward to the curve that represents the length that the footing projects beyond the critical section at which the moment must be calculated (see Table 30-3). Read horizontally to the left to determine the minimum required footing thickness. Two (2) in. must be added to this value to satisfy 22.4.8. The thicknesses in the figure are based on a specified compressive strength of concrete, f'_c of 2500 psi. For higher strength concrete, the thickness can be reduced by multiplying by the factor:

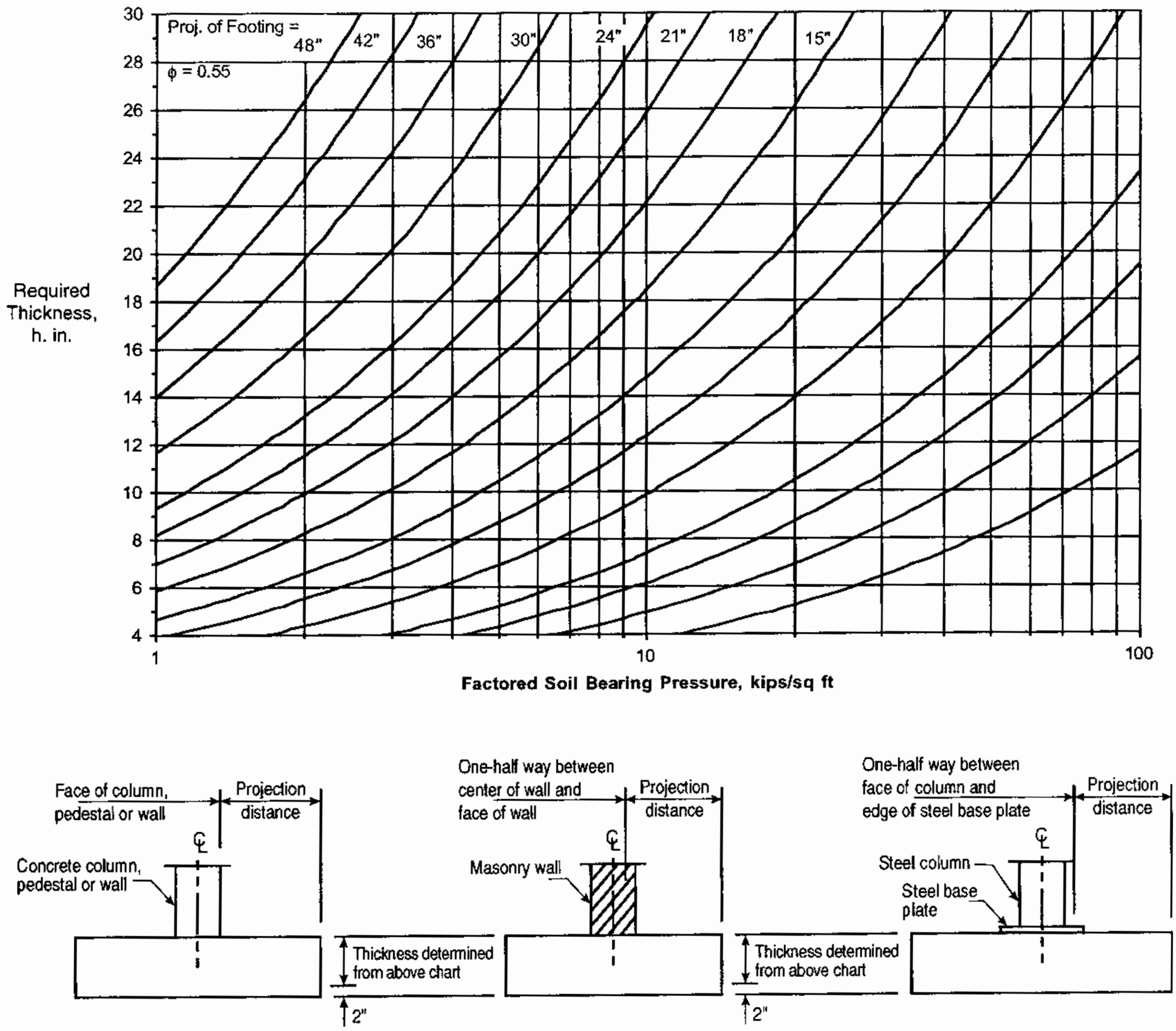
$$(2500/\text{specified compressive strength of concrete})^{0.25}$$

As the exponent in the equation suggests, a large increase in concrete strength results in only a small decrease in footing thickness. For example, doubling the concrete strength only reduces the thickness 16 percent.

*Table 30-3 Locations for Computing Moments and Shears in Footings**

Supported Member	Moment	Shear – Beam Action	Shear – Two-Way (punching)
Concrete Wall	at face of wall	h from face of wall	Not Applicable
Masonry Wall	1/2 way between center of wall and face of wall	h from face of wall	Not Applicable
Column or Pedestal	at face of column or pedestal	h from face of column or pedestal	h/2 from face of column or pedestal
Column with Steel Base Plate	1/2 way between face of column and edge of steel base plate	h from 1/2 way between face of column and edge of steel base plate	h/2 from 1/2 way between face of column edge of steel base plate

* h = thickness of footing for moment and shear computation purposes.



* For f'_c greater than 2500 psi, multiple thickness determined from above chart by $(2500/f'_c)^{0.25}$

Figure 30-10 Thickness of Footing Required to Satisfy Flexural Strength for Various Projection Distances, in. (f'_c = 2500 psi)*

22.8 PEDESTALS

Pedestals of plain concrete are permitted by 22.8.2 provided the unsupported height does not exceed three times the average least plan dimension. The design must consider all vertical and lateral loads to which the pedestal will be subjected. The nominal bearing strength, B_n, must be determined from Eq. (22-12). Where moments are induced due to eccentricity of the axial load and/or lateral loads, the pedestal shall be designed for both flexural and axial loads and satisfy interaction Eqs. (22-6) and (22-7). In Eq. (22-6), P_n is replaced with B_n, nominal bearing strength.

Pedestal-like members with heights exceeding three times the least lateral dimension are defined as *columns* by the code and must be designed as reinforced concrete members. Columns of structural plain concrete are prohibited by Chapter 22.

Some contemporary building codes prohibit the use of structural plain concrete pedestals to resist seismic lateral forces in structures at moderate seismic risk or assigned to intermediate seismic performance or design categories. See Table 1-3 and section below on 22.10.

22.10 PLAIN CONCRETE IN EARTHQUAKE-RESISTING STRUCTURES

The '99 edition of the code included a new section 22.10 to address a seismic design issue not covered previously. This concerns the use of plain concrete elements in structures subject to earthquake ground motions intense enough to cause significant structural damage to the elements or partial or total collapse of the structure. By default, the model building codes in use in the U.S. had assumed responsibility for this subject. The requirements, based on similar provisions in *The BOCA National Building Code*[30.4] and *Standard Building Code*[30.5], prohibit the use of structural plain concrete foundation elements for structures in regions at high seismic risk, or for structures assigned to high seismic performance or design categories, except for three specific cases cited in the provisions. See Table 1-3 for an explanation of how seismic risk assigned by the model building codes can be correlated to the requirements of ACI 318.

The provisions prohibit the use of structural plain concrete foundation elements for structures assigned a "high" seismic risk in accordance with Table 1-3, except for the following specific cases. They are:

1. In detached one-and two-family dwellings not exceeding three stories in height and constructed with wood or steel stud bearing walls, the following are permitted:
    a. plain concrete footings supporting walls, columns or pedestals; and
    b. plain concrete foundation or basement walls provided
        i. the wall is not less than 7-1/2 in. thick, and
        ii. it retains no more than 4 ft of unbalanced fill.

2. In structures other than covered by 1 above, plain concrete footings supporting cast-in-place reinforced concrete walls or reinforced masonry walls are permitted provided the footing has at least two continuous No. 4 longitudinal reinforcing bars that provide an area of steel of not less than 0.002 times the gross transverse cross-sectional area of the footing. Continuity of reinforcement must be provided at corners and intersections. In the 2002 ACI code, the requirement was added to limit the use of this provision to situations where the supported wall is either of cast-in-place reinforced concrete or reinforced masonry.

Although Chapter 22 of the code has no limitations on the use of structural plain concrete elements for structures in areas of moderate seismic risk, or for structures assigned to intermediate seismic performance or design categories in accordance with Table 1-3, model building codes in use in the U.S. either prohibit their use or generally require that some reinforcement be included to provide some ductility and tie the elements together. Where construction is contemplated in these areas, the legally adopted building code should be consulted to determine the specific limitations.

REFERENCES

30.1 *Joints in Walls Below Grade,* CR059 Portland Cement Association, Skokie, IL, 1982.

30.2 *Building Movements and Joints,* Portland Cement Association, Skokie, IL, 1982.

30.3 Kosmatka, Steven H., Kerkhoff, Beatrix, and Panarese, William C.; *Design and Control of Concrete Mixtures,* EB001, Fourteenth Edition, Portland Cement Association, Skokie, IL, 2002.

30.4 *The BOCA National Building Code,* Building Officials and Code Administrators International, Country Club Hills, IL, 1999.

30.5 *Standard Building Code,* Southern Building Code Congress International, Birmingham, AL 1999.

APPENDIX 30A

This Appendix includes figures and tables that parallel those of Part 30. The figures of this Appendix are compatible with the load factors and strength reduction factor ($\phi = 0.65$) of ACI 318-02 and ACI 318-05, Appendix C. (In the body of Part 30, figures and tables are compatible with load factors and strength reduction factor ($\phi = 0.55$) of ACI 318-02 and ACI 318-05, Chapter 9.)

Included are the following:

Table C30-1 *Factored Moments Induced in Walls by Lateral Soil and/or Wind Loads (ft/kips/linear ft)*
(For Use with Appendix C Load Factors: Soil 1.7, Wind 1.6)

Wall Ht. (ft.)	Back fill Ht. (ft.)	Unfactored Design Lateral Soil Load (psf per foot of depth)																			
		30	30	30	30	30	45	45	45	45	45	60	60	60	60	60	100	100	100	100	100
		Unfactored Design Wind Pressure (psf)																			
		0	10	20	40	80	0	10	20	40	80	0	10	20	40	80	0	10	20	40	80
8	0	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02	0.00	0.13	0.26	0.51	1.02
8	1	0.01	0.13	0.25	0.50	1.00	0.01	0.13	0.25	0.50	1.00	0.02	0.13	0.26	0.50	1.00	0.03	0.14	0.26	0.51	1.01
8	2	0.05	0.15	0.26	0.48	0.93	0.08	0.17	0.28	0.50	0.95	0.11	0.19	0.29	0.52	0.96	0.18	0.24	0.34	0.56	1.01
8	3	0.16	0.22	0.30	0.48	0.86	0.25	0.30	0.37	0.54	0.91	0.33	0.38	0.44	0.60	0.97	0.55	0.60	0.65	0.78	1.12
8	4	0.35	0.39	0.43	0.53	0.80	0.52	0.56	0.60	0.69	0.92	0.69	0.73	0.77	0.86	1.06	1.15	1.19	1.23	1.31	1.49
8	5	0.60	0.63	0.65	0.70	0.82	0.90	0.93	0.95	1.00	1.11	1.20	1.23	1.25	1.30	1.41	2.00	2.03	2.05	2.10	2.20
8	6	0.92	0.93	0.94	0.97	1.02	1.38	1.39	1.40	1.43	1.48	1.84	1.85	1.86	1.88	1.93	3.06	3.07	3.08	3.11	3.16
8	7	1.28	1.29	1.29	1.30	1.31	1.92	1.93	1.93	1.94	1.95	2.57	2.57	2.57	2.58	2.59	4.28	4.28	4.28	4.29	4.30
8	8	1.68	1.68	1.68	1.68	1.68	2.51	2.51	2.51	2.51	2.51	3.35	3.35	3.35	3.35	3.35	5.58	5.58	5.58	5.58	5.58
9	0	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30	0.00	0.16	0.32	0.65	1.30
9	1	0.01	0.16	0.32	0.64	1.27	0.01	0.16	0.32	0.64	1.27	0.02	0.17	0.32	0.64	1.27	0.03	0.17	0.33	0.65	1.28
9	2	0.06	0.18	0.33	0.62	1.20	0.08	0.20	0.34	0.64	1.22	0.11	0.22	0.36	0.65	1.24	0.19	0.27	0.41	0.70	1.28
9	3	0.17	0.25	0.37	0.62	1.13	0.26	0.33	0.43	0.68	1.18	0.34	0.41	0.50	0.74	1.24	0.57	0.63	0.71	0.91	1.39
9	4	0.36	0.42	0.49	0.66	1.07	0.55	0.60	0.66	0.81	1.19	0.73	0.78	0.84	0.98	1.33	1.21	1.27	1.33	1.45	1.73
9	5	0.64	0.68	0.73	0.82	1.04	0.96	1.00	1.05	1.13	1.33	1.28	1.32	1.37	1.45	1.64	2.14	2.18	2.22	2.31	2.48
9	6	1.00	1.02	1.05	1.10	1.21	1.49	1.52	1.55	1.60	1.71	1.99	2.02	2.04	2.10	2.20	3.32	3.35	3.37	3.42	3.53
9	7	1.42	1.43	1.44	1.47	1.52	2.13	2.14	2.15	2.18	2.23	2.84	2.85	2.86	2.88	2.93	4.73	4.74	4.75	4.77	4.82
9	8	1.89	1.89	1.89	1.90	1.91	2.83	2.83	2.84	2.84	2.86	3.77	3.78	3.78	3.79	3.80	6.29	6.29	6.30	6.30	6.32
9	9	2.39	2.39	2.39	2.39	2.39	3.58	3.58	3.58	3.58	3.58	4.77	4.77	4.77	4.77	4.77	7.95	7.95	7.95	7.95	7.95
10	0	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60	0.00	0.20	0.40	0.80	1.60
10	1	0.01	0.20	0.40	0.79	1.57	0.01	0.20	0.40	0.79	1.57	0.02	0.20	0.40	0.79	1.58	0.03	0.21	0.41	0.80	1.58
10	2	0.06	0.22	0.40	0.77	1.51	0.09	0.24	0.42	0.79	1.52	0.11	0.26	0.44	0.80	1.54	0.19	0.31	0.49	0.85	1.59
10	3	0.18	0.29	0.44	0.77	1.43	0.26	0.36	0.51	0.83	1.49	0.35	0.44	0.57	0.89	1.54	0.58	0.67	0.77	1.06	1.70
10	4	0.38	0.46	0.56	0.82	1.37	0.57	0.65	0.73	0.96	1.50	0.76	0.83	0.92	1.11	1.63	1.26	1.34	1.42	1.59	2.02
10	5	0.68	0.74	0.80	0.95	1.34	1.01	1.07	1.14	1.27	1.60	1.35	1.41	1.47	1.60	1.90	2.25	2.31	2.37	2.50	2.77
10	6	1.06	1.11	1.15	1.24	1.45	1.59	1.64	1.68	1.77	1.96	2.13	2.17	2.21	2.30	2.49	3.54	3.59	3.63	3.72	3.89
10	7	1.53	1.56	1.58	1.64	1.75	2.30	2.32	2.35	2.40	2.51	3.06	3.09	3.12	3.17	3.28	5.11	5.13	5.16	5.21	5.32
10	8	2.07	2.08	2.09	2.12	2.17	3.10	3.12	3.13	3.15	3.20	4.14	4.15	4.16	4.19	4.24	6.90	6.91	6.92	6.95	7.00
10	9	2.66	2.66	2.66	2.67	2.68	3.98	3.99	3.99	4.00	4.01	5.31	5.32	5.32	5.33	5.34	8.85	8.86	8.86	8.87	8.88
10	10	3.27	3.27	3.27	3.27	3.27	4.91	4.91	4.91	4.91	4.91	6.54	6.54	6.54	6.54	6.54	10.91	10.91	10.91	10.91	10.91
12	0	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30	0.00	0.29	0.58	1.15	2.30
12	1	0.01	0.29	0.57	1.14	2.28	0.01	0.29	0.57	1.14	2.28	0.02	0.29	0.58	1.14	2.28	0.03	0.30	0.58	1.15	2.29
12	2	0.06	0.31	0.58	1.12	2.21	0.09	0.32	0.60	1.14	2.23	0.12	0.34	0.61	1.16	2.24	0.19	0.39	0.66	1.20	2.29
12	3	0.18	0.37	0.62	1.12	2.13	0.27	0.44	0.68	1.18	2.19	0.37	0.51	0.74	1.24	2.25	0.61	0.74	0.93	1.40	2.40
12	4	0.40	0.53	0.73	1.17	2.07	0.60	0.73	0.89	1.31	2.20	0.81	0.93	1.07	1.46	2.34	1.34	1.46	1.59	1.89	2.72
12	5	0.73	0.84	0.96	1.29	2.04	1.09	1.20	1.32	1.58	2.30	1.46	1.56	1.68	1.92	2.57	2.43	2.54	2.64	2.87	3.39
12	6	1.17	1.26	1.35	1.55	2.08	1.75	1.84	1.93	2.12	2.55	2.34	2.42	2.51	2.70	3.10	3.89	3.98	4.07	4.25	4.62
12	7	1.71	1.78	1.85	1.99	2.30	2.57	2.64	2.70	2.84	3.13	3.43	3.50	3.56	3.70	3.98	5.72	5.78	5.85	5.98	6.26
12	8	2.36	2.41	2.45	2.55	2.74	3.54	3.59	3.63	3.73	3.92	4.72	4.77	4.82	4.91	5.10	7.87	7.92	7.97	8.06	8.24
12	9	3.10	3.13	3.15	3.21	3.32	4.65	4.67	4.70	4.76	4.87	6.20	6.22	6.25	6.31	6.42	10.33	10.35	10.38	10.44	10.55
12	10	3.91	3.92	3.93	3.96	4.01	5.86	5.87	5.88	5.91	5.96	7.81	7.82	7.84	7.86	7.91	13.02	13.03	13.04	13.07	13.12
12	11	4.76	4.77	4.77	4.78	4.79	7.15	7.15	7.15	7.16	7.17	9.53	9.53	9.54	9.54	9.56	15.88	15.89	15.89	15.89	15.91
12	12	5.65	5.65	5.65	5.65	5.65	8.48	8.48	8.48	8.48	8.48	11.31	11.31	11.31	11.31	11.31	18.84	18.84	18.84	18.84	18.84

Table C30-2 Factored Moments Induced in Walls by Lateral Soil and/or Wind Loads (ft-kips/linear ft)
(For Use with Appendix C Load Factors: Soil 1.7, Wind 1.3)

Wall Ht. (ft.)	Backfill Ht. (ft.)	Unfactored Design Lateral Soil Load (psf per foot of depth)																			
		30	30	30	30	30	45	45	45	45	45	60	60	60	60	60	100	100	100	100	100
		Unfactored Design Wind Pressure (psf)																			
		0	10	20	40	80	0	10	20	40	80	0	10	20	40	80	0	10	20	40	80
8	0	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83	0.00	0.10	0.21	0.42	0.83
8	1	0.01	0.11	0.21	0.41	0.81	0.01	0.11	0.21	0.41	0.81	0.02	0.11	0.21	0.41	0.81	0.03	0.12	0.22	0.42	0.82
8	2	0.05	0.13	0.22	0.40	0.76	0.08	0.15	0.23	0.42	0.78	0.11	0.17	0.25	0.43	0.80	0.18	0.23	0.30	0.48	0.84
8	3	0.16	0.21	0.27	0.41	0.72	0.25	0.29	0.34	0.47	0.77	0.33	0.37	0.42	0.54	0.83	0.55	0.59	0.63	0.73	0.99
8	4	0.35	0.38	0.41	0.49	0.69	0.52	0.55	0.58	0.65	0.82	0.69	0.72	0.76	0.82	0.98	1.15	1.18	1.22	1.28	1.42
8	5	0.60	0.62	0.64	0.68	0.78	0.90	0.92	0.94	0.98	1.07	1.20	1.22	1.24	1.28	1.37	2.00	2.02	2.04	2.08	2.17
8	6	0.92	0.93	0.94	0.96	1.00	1.38	1.39	1.40	1.42	1.46	1.84	1.85	1.86	1.88	1.92	3.06	3.07	3.08	3.10	3.14
8	7	1.28	1.29	1.29	1.29	1.30	1.92	1.93	1.93	1.93	1.95	2.57	2.57	2.57	2.58	2.59	4.28	4.28	4.28	4.29	4.30
8	8	1.68	1.68	1.68	1.68	1.68	2.51	2.51	2.51	2.51	2.51	3.35	3.35	3.35	3.35	3.35	5.58	5.58	5.58	5.58	5.58
9	0	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05	0.00	0.13	0.26	0.53	1.05
9	1	0.01	0.13	0.26	0.52	1.03	0.01	0.13	0.26	0.52	1.03	0.02	0.14	0.27	0.52	1.04	0.03	0.14	0.27	0.53	1.04
9	2	0.06	0.15	0.27	0.51	0.98	0.08	0.17	0.29	0.53	1.00	0.11	0.19	0.31	0.54	1.02	0.19	0.25	0.36	0.59	1.06
9	3	0.17	0.23	0.32	0.52	0.94	0.26	0.31	0.39	0.58	0.99	0.34	0.40	0.46	0.65	1.05	0.57	0.62	0.68	0.83	1.21
9	4	0.36	0.41	0.46	0.59	0.91	0.55	0.59	0.64	0.75	1.05	0.73	0.77	0.82	0.93	1.18	1.21	1.26	1.31	1.40	1.62
9	5	0.64	0.68	0.71	0.78	0.95	0.96	1.00	1.03	1.10	1.25	1.28	1.32	1.35	1.42	1.57	2.14	2.17	2.21	2.27	2.41
9	6	1.00	1.02	1.04	1.08	1.17	1.49	1.52	1.54	1.58	1.67	1.99	2.01	2.03	2.08	2.16	3.32	3.34	3.36	3.41	3.49
9	7	1.42	1.43	1.44	1.46	1.50	2.13	2.14	2.15	2.17	2.21	2.84	2.85	2.86	2.88	2.92	4.73	4.74	4.75	4.77	4.81
9	8	1.89	1.89	1.89	1.90	1.91	2.83	2.83	2.84	2.84	2.85	3.77	3.78	3.78	3.79	3.80	6.29	6.29	6.30	6.30	6.31
9	9	2.39	2.39	2.39	2.39	2.39	3.58	3.58	3.58	3.58	3.58	4.77	4.77	4.77	4.77	4.77	7.95	7.95	7.95	7.95	7.95
10	0	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30	0.00	0.16	0.33	0.65	1.30
10	1	0.01	0.16	0.32	0.64	1.28	0.01	0.17	0.32	0.64	1.28	0.02	0.17	0.33	0.65	1.28	0.03	0.17	0.33	0.65	1.29
10	2	0.06	0.18	0.33	0.63	1.23	0.09	0.20	0.35	0.65	1.25	0.11	0.22	0.37	0.67	1.26	0.19	0.28	0.42	0.71	1.31
10	3	0.18	0.26	0.38	0.65	1.18	0.26	0.34	0.45	0.71	1.24	0.35	0.42	0.52	0.77	1.30	0.58	0.65	0.73	0.94	1.45
10	4	0.38	0.44	0.52	0.72	1.16	0.57	0.63	0.70	0.87	1.29	0.76	0.82	0.89	1.03	1.43	1.26	1.33	1.39	1.52	1.84
10	5	0.68	0.73	0.78	0.89	1.18	1.01	1.06	1.11	1.22	1.46	1.35	1.40	1.45	1.55	1.78	2.25	2.30	2.35	2.45	2.66
10	6	1.06	1.10	1.13	1.21	1.37	1.59	1.63	1.66	1.74	1.89	2.13	2.16	2.20	2.27	2.41	3.54	3.58	3.61	3.68	3.83
10	7	1.53	1.55	1.57	1.62	1.71	2.30	2.32	2.34	2.38	2.47	3.06	3.08	3.11	3.15	3.24	5.11	5.13	5.15	5.19	5.28
10	8	2.07	2.08	2.09	2.11	2.15	3.10	3.11	3.12	3.14	3.18	4.14	4.15	4.16	4.18	4.22	6.90	6.91	6.92	6.94	6.98
10	9	2.66	2.66	2.66	2.67	2.68	3.98	3.99	3.99	3.99	4.01	5.31	5.31	5.32	5.32	5.33	8.85	8.86	8.86	8.86	8.87
10	10	3.27	3.27	3.27	3.27	3.27	4.91	4.91	4.91	4.91	4.91	6.54	6.54	6.54	6.54	6.54	10.91	10.91	10.91	10.91	10.91
12	0	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87	0.00	0.23	0.47	0.94	1.87
12	1	0.01	0.24	0.47	0.93	1.85	0.01	0.24	0.47	0.93	1.85	0.02	0.24	0.47	0.93	1.85	0.03	0.25	0.48	0.94	1.86
12	2	0.06	0.26	0.48	0.92	1.80	0.09	0.27	0.49	0.94	1.82	0.12	0.29	0.51	0.95	1.84	0.19	0.35	0.56	1.00	1.88
12	3	0.18	0.33	0.53	0.93	1.75	0.27	0.40	0.59	0.99	1.81	0.37	0.48	0.65	1.05	1.87	0.61	0.71	0.85	1.22	2.02
12	4	0.40	0.51	0.65	1.00	1.73	0.60	0.70	0.82	1.15	1.86	0.81	0.90	1.01	1.30	2.00	1.34	1.44	1.54	1.77	2.39
12	5	0.73	0.82	0.91	1.15	1.76	1.09	1.18	1.27	1.48	2.02	1.46	1.54	1.63	1.83	2.31	2.43	2.52	2.60	2.78	3.18
12	6	1.17	1.24	1.31	1.47	1.85	1.75	1.82	1.89	2.05	2.38	2.34	2.41	2.48	2.63	2.94	3.89	3.96	4.03	4.18	4.48
12	7	1.71	1.77	1.82	1.94	2.18	2.57	2.63	2.68	2.79	3.02	3.43	3.48	3.54	3.65	3.87	5.72	5.77	5.82	5.93	6.15
12	8	2.36	2.40	2.44	2.51	2.67	3.54	3.58	3.62	3.69	3.85	4.72	4.76	4.80	4.87	5.02	7.87	7.91	7.95	8.02	8.17
12	9	3.10	3.12	3.14	3.19	3.28	4.65	4.67	4.69	4.74	4.83	6.20	6.22	6.24	6.28	6.37	10.33	10.35	10.37	10.42	10.50
12	10	3.91	3.92	3.93	3.95	3.99	5.86	5.87	5.88	5.90	5.94	7.81	7.82	7.83	7.85	7.89	13.02	13.03	13.04	13.06	13.10
12	11	4.76	4.77	4.77	4.78	4.79	7.15	7.15	7.15	7.16	7.17	9.53	9.53	9.53	9.54	9.55	15.88	15.88	15.89	15.89	15.90
12	12	5.65	5.65	5.65	5.65	5.65	8.48	8.48	8.48	8.48	8.48	11.31	11.31	11.31	11.31	11.31	18.84	18.84	18.84	18.84	18.84

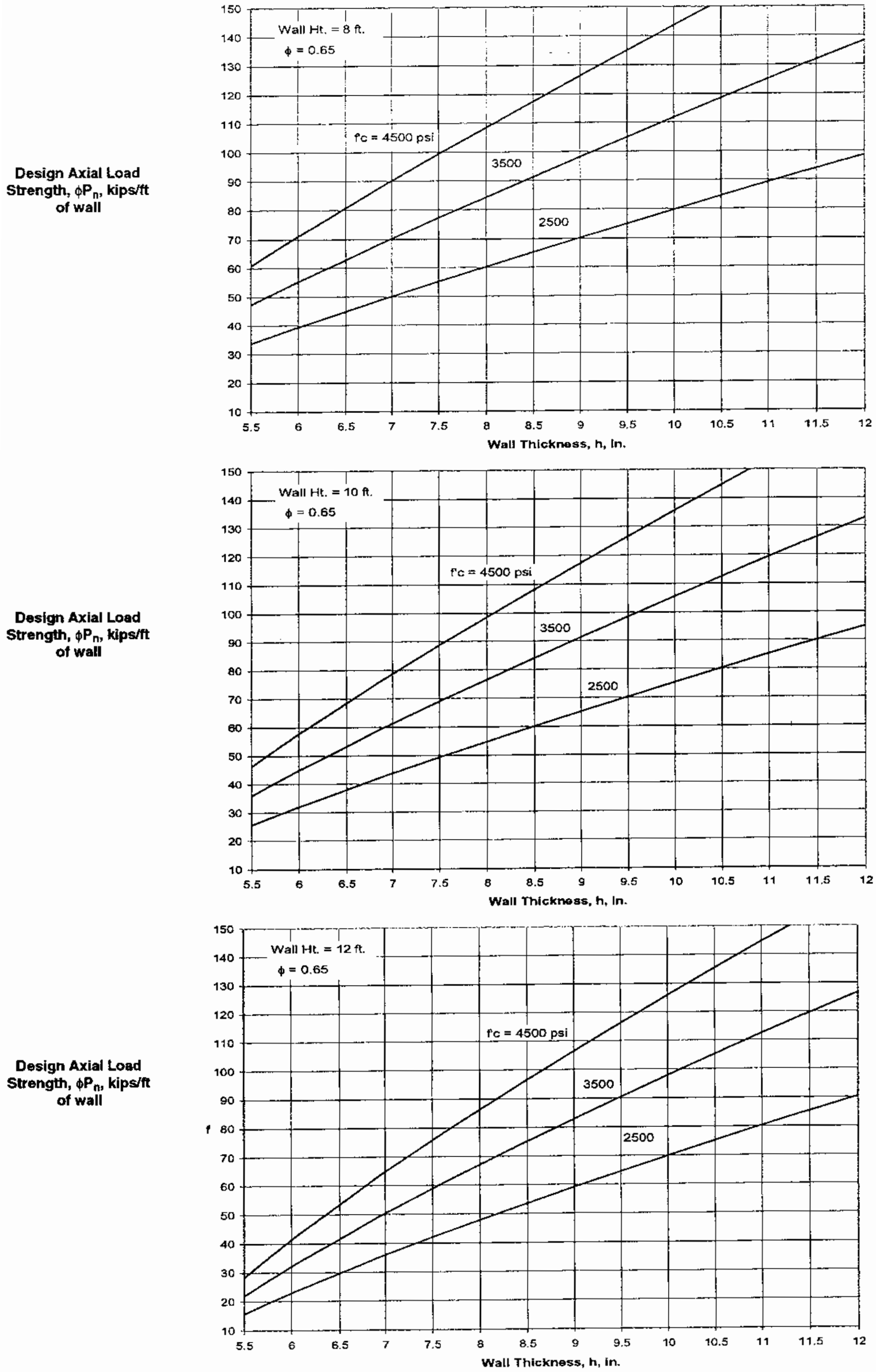


Figure C30-1 Design Axial Load Strength, P_n, of Plain Concrete Walls using the Empirical Design Method

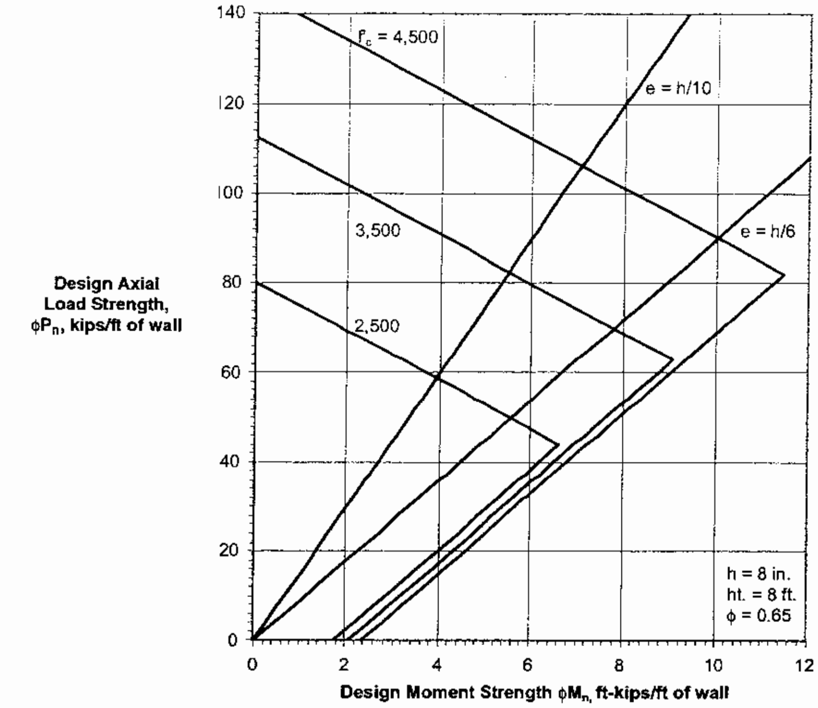


Figure C30-2 Design Strength Interaction Diagrams for 8.0-in. Wall, 8 ft in Height

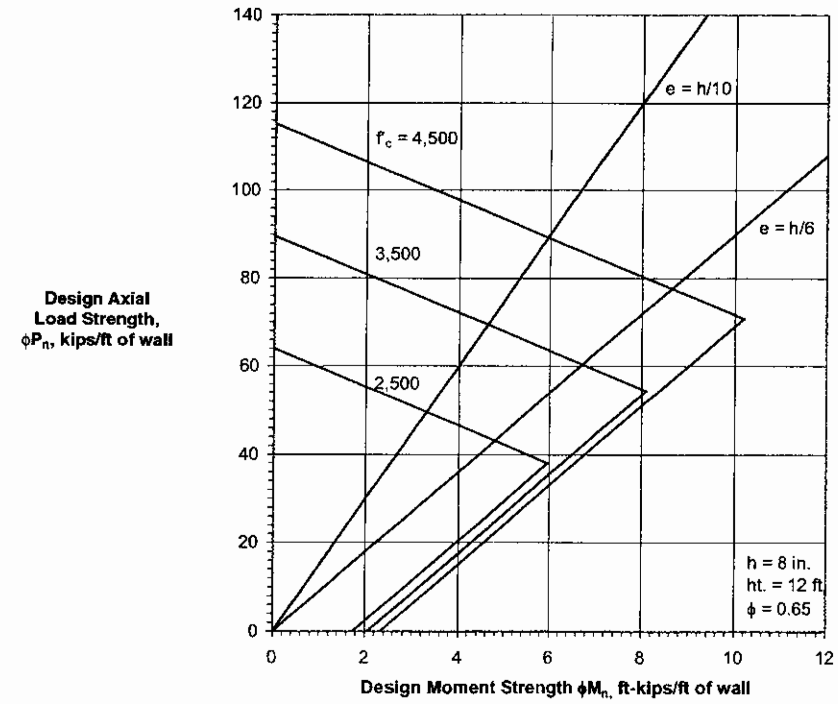


Figure C30-3 Design Strength Interaction Diagrams for 8.0-in. Wall, 12 ft in Height

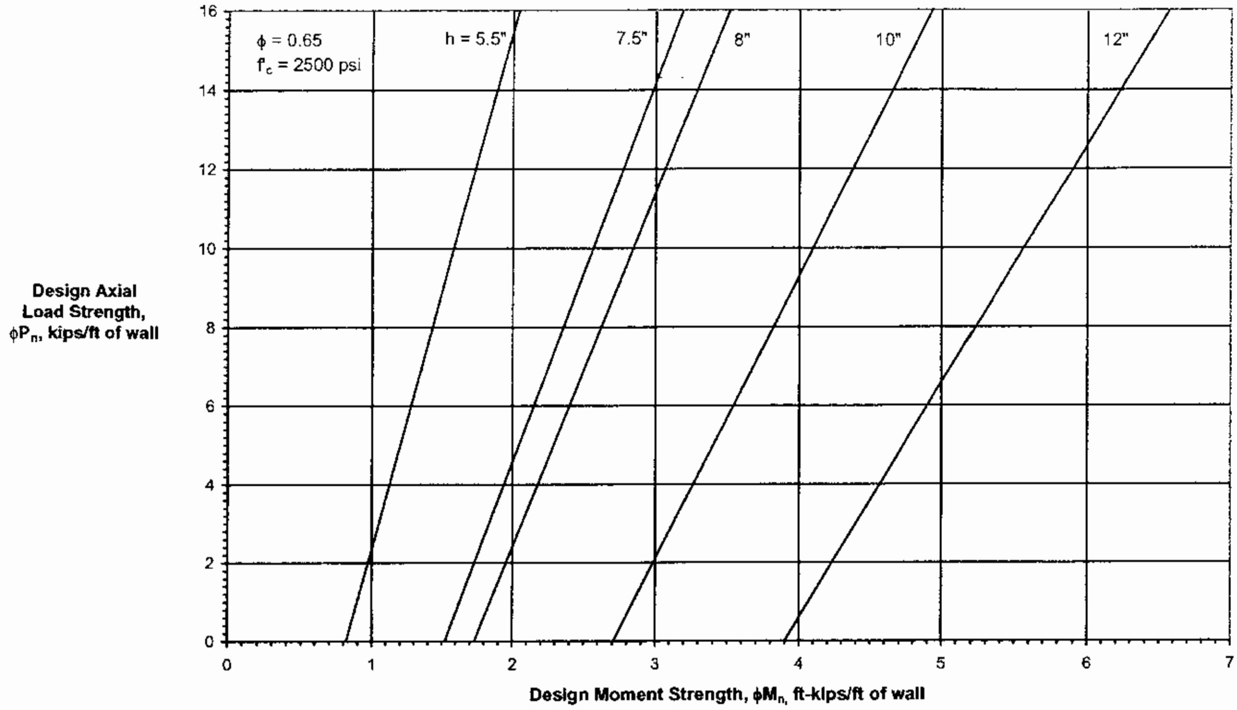


Figure C30-4 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 2500 psi)

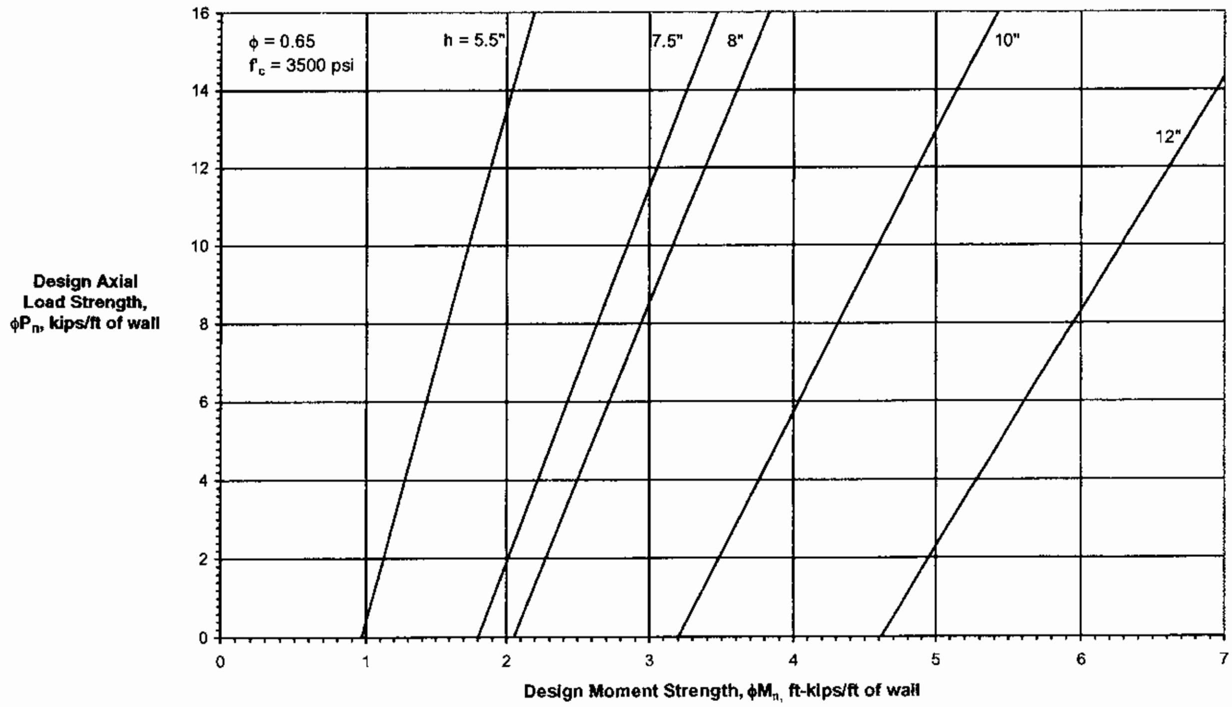


Figure C30-5 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls (f'_c = 3500 psi)

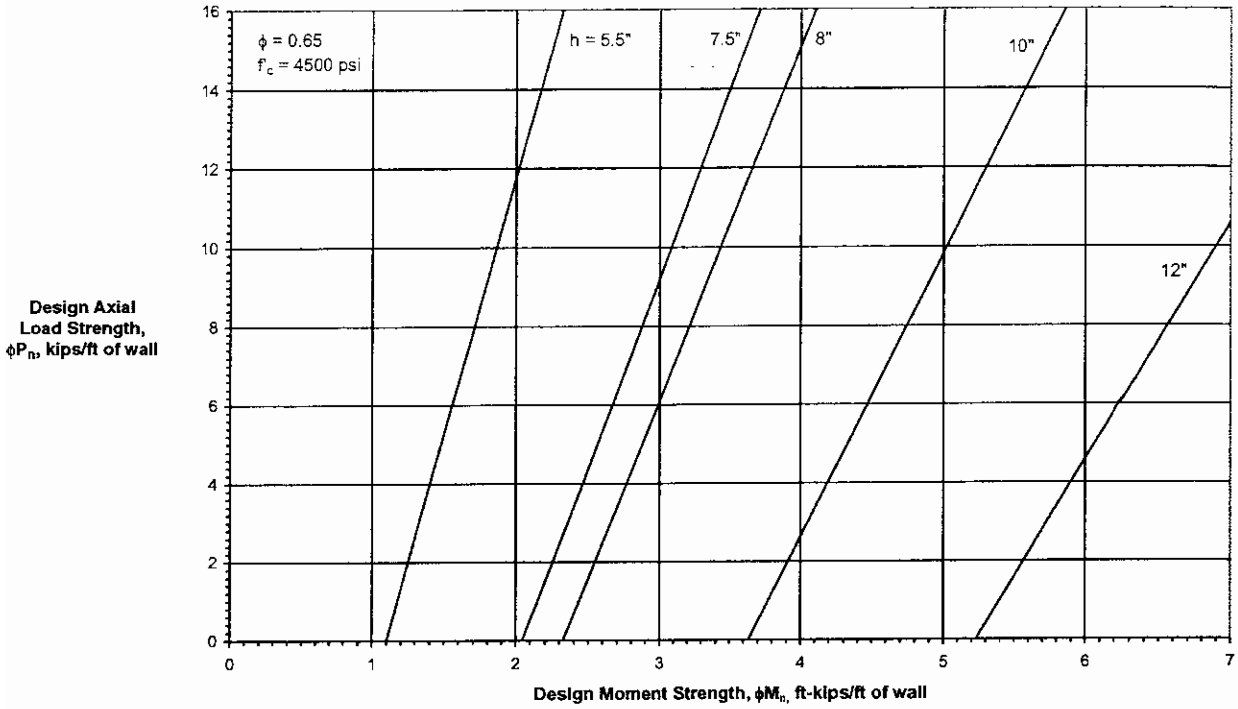


Figure C30-6 Design Strength Interaction Diagrams for Lightly-Loaded Plain Concrete Walls ($f'_c = 4500$ psi)

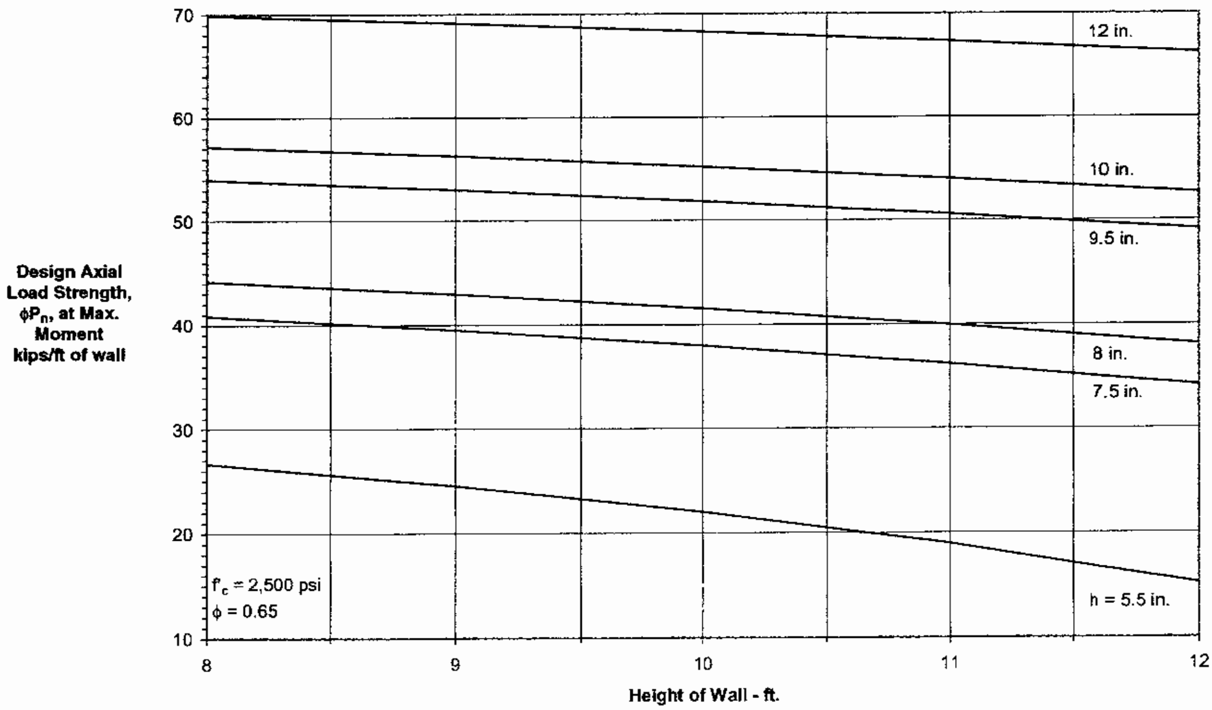


Figure C30-7 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength ($f'_c = 2500$ psi)

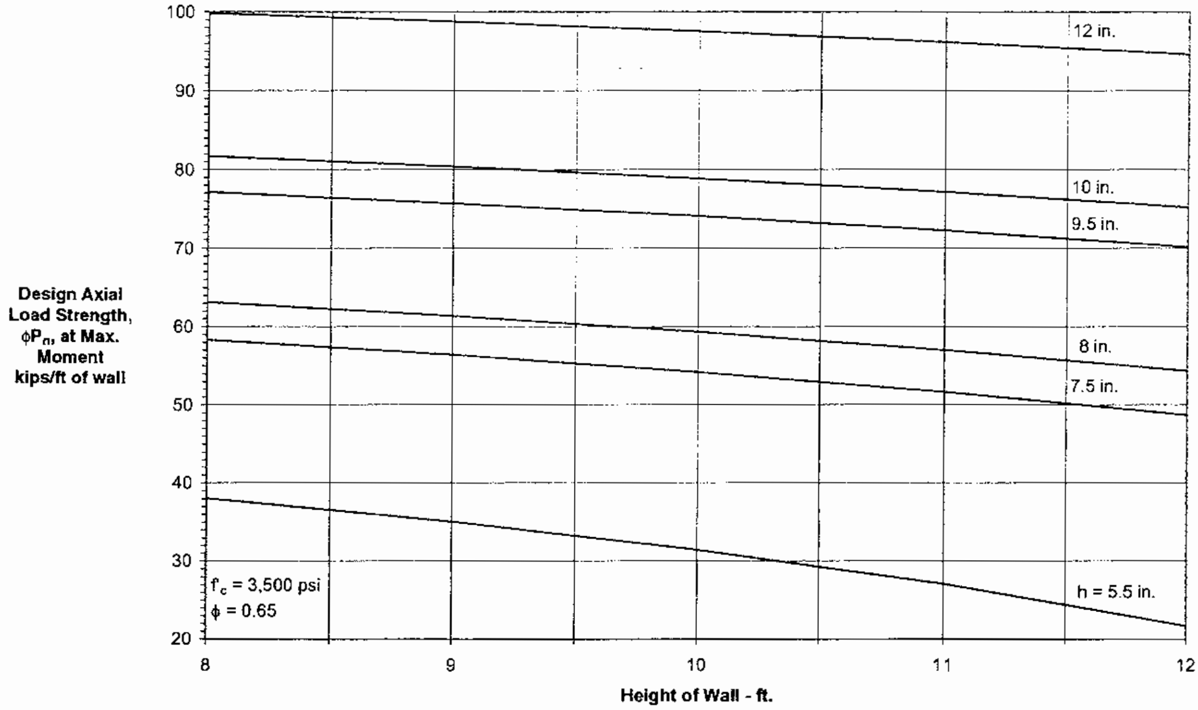


Figure C30-8 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 3500 psi)

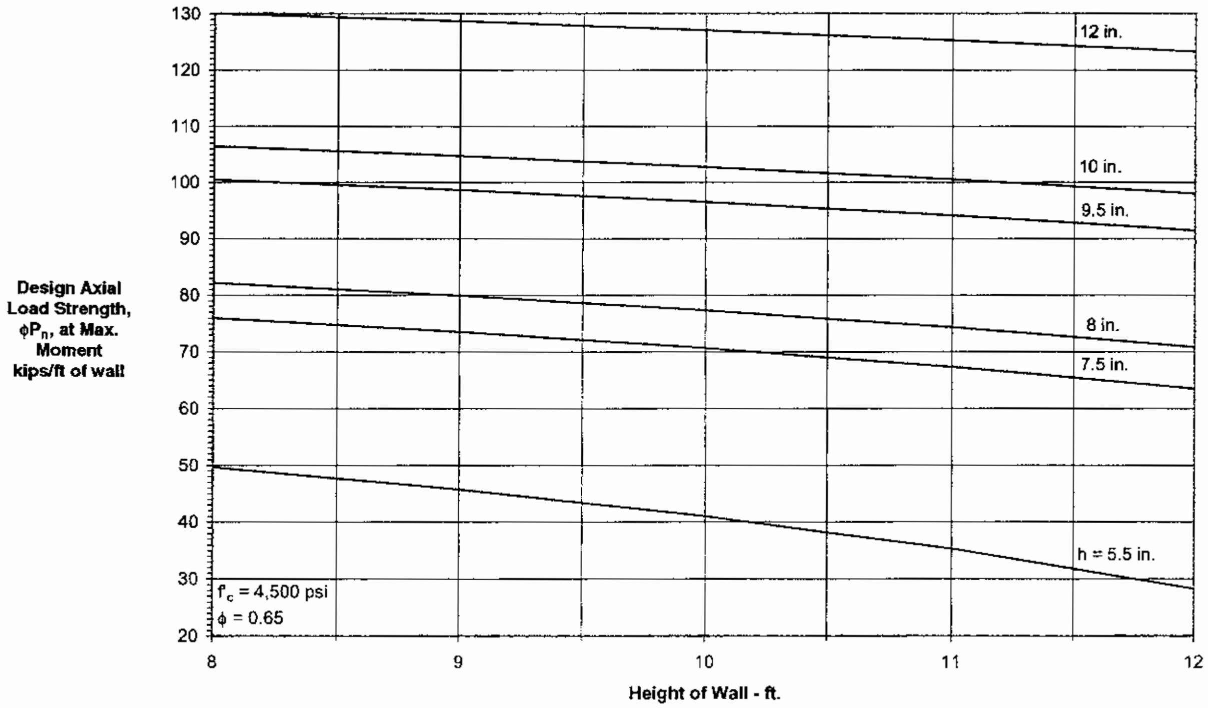
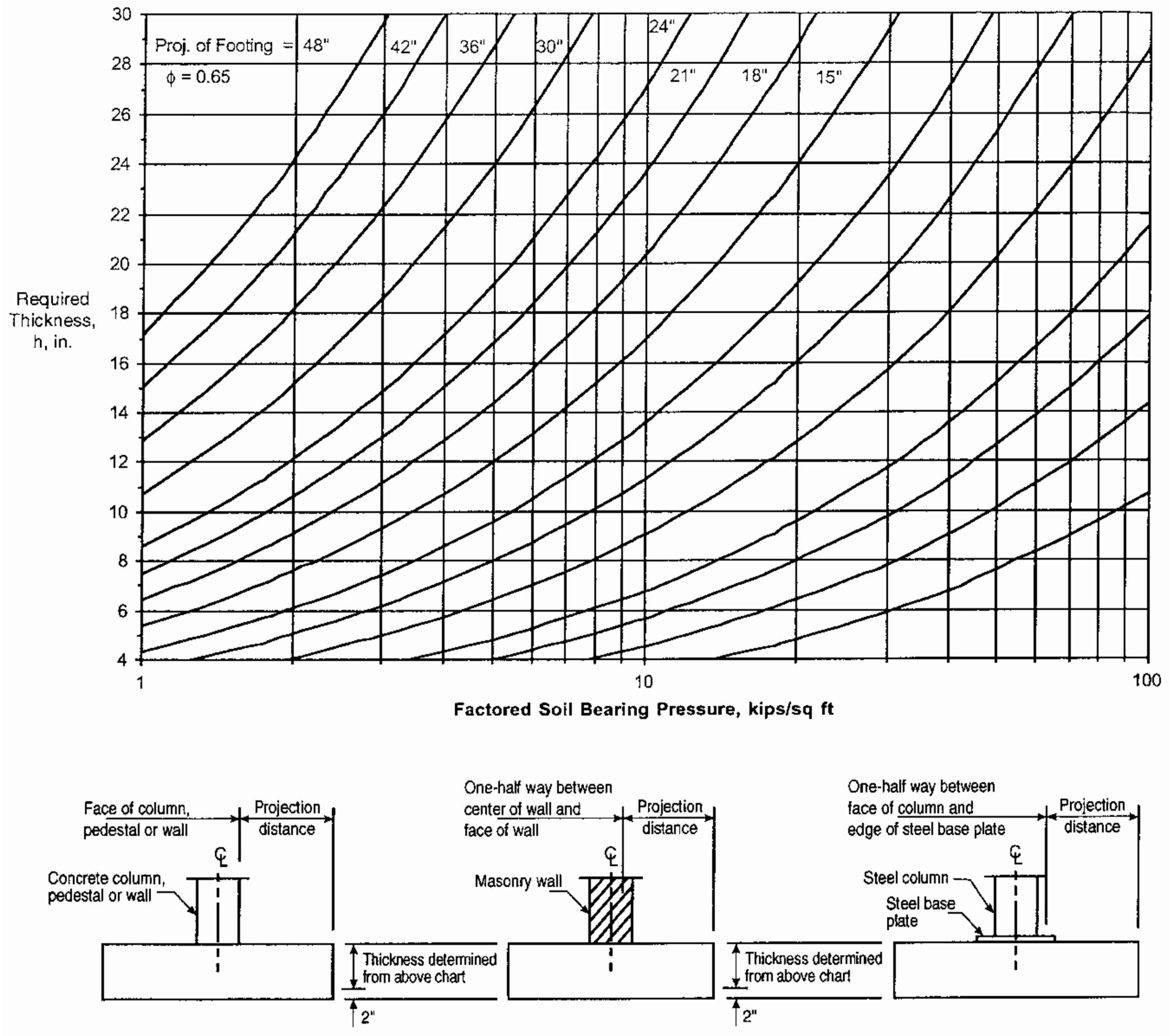


Figure C30-9 Design Axial Load Strength of Plain Concrete Walls at Maximum Design Moment Strength (f'_c = 4500 psi)



*For f'_c greater than 2500 psi, multiple thickness determined from above chart by $(2500/f'_c)^{0.25}$

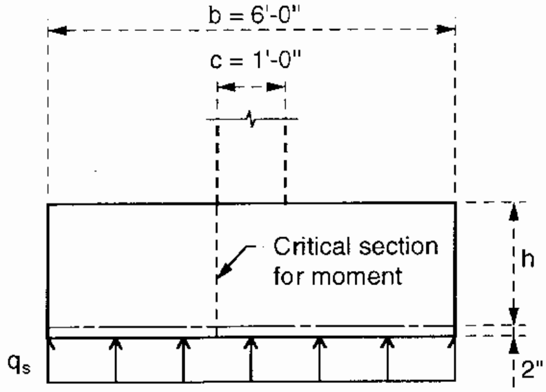
Figure C30-10 Thickness of Footing Required to Satisfy Flexural Strength for Various Projection Distances, in. (f'_c = 2500 psi)*

Example 30.1—Design of Plain Concrete Footing and Pedestal

Proportion a plain concrete square footing with pedestal for a residential occupancy building. Design in conformance with Chapter 22 using the load and strength reduction factors of Chapter 9. Perform a second design using the alternate load and strength reduction factors of Appendix C to determine which results in a more economical design.

Design data:

Service dead load = 40 kips
Service floor live load = 40 kips
Service roof live load = 7.5 kips
Service roof snow load = 10 kips
Service surcharge = 0
Pedestal dimensions = 12 × 12 in.
Permissible soil bearing pressure = 2.5 ksf
f'_c = 2500 psi



Calculations and Discussion	Code Reference

1. Determine base area of footing:

The base area is determined by using unfactored service gravity loads and the permissible soil bearing pressure. *22.7.2*

$$A_f = \frac{40 + 40 + 10}{2.5} = 36 \text{ ft}^2$$

Use 6 × 6 ft square footing ($A_f = 36 \text{ ft}^2$)

2. Determine the applicable load combinations that must be considered.

To proportion the footing for strength, factored loads must be used. Two sets of load factors are provided; one in 9.2 and the other in C.2. Because of the significant difference between strength reduction factor, ϕ, to be used with each set of load factors (0.55 in 9.3.5 versus 0.65 in C.3.5), a design in accordance with each set of factors should be evaluated to determine which alternate will provide the more economical solution. *22.7.1* *9.2* *9.3.5* *C.2* *C.3.5*

The required strength must at least equal the largest factored load determined from applicable load combinations. One of the following load combinations will govern:

1. $U = 1.2D + 1.6L + 0.5S$ *Eq. (9-2)*
2. $U = 1.2D + 0.5L + 1.6S$ *Eq. (9-3)*
3. $U = 1.2D + 0.5L + 0.5S$ *Eq. (9-4)*

Note that in Combination 1, T is being neglected. In Combinations 2 and 3 the factor on L is 0.5 in accordance with 9.2.1(a). *9.2.1(a)*

3. Calculate the factored axial load, P_u, for each load combination.

   By observation it can be seen that either Combination 1 or 2 will yield the largest factored axial load.

   1. $P_u = 1.2D + 1.6L + 0.5S = 1.2(40) + 1.6(40) + 0.5(10) = 117$ kips — *Eq. (9-2)*

   2. $P_u = 1.2D + 0.5L + 1.6S = 1.2(40) + 0.5(40) + 1.6(10) = 64.8$ kips — *Eq. (9-3)*

   Use $P_u = 117$ kips

   Upon reviewing the applicable load combination of C.2, it is obvious that Eq. (C-1) will control:

   $P_u = 1.4D + 1.7L = 1.4\ (40) + 1.7\ (50) = 141$ kips — *Eq. (C-1)*

   To quickly determine which one of the two sets of load and strength reduction factors to use, compare the nominal axial load strength, P_n, required by the factored loads of 9.2 to that required by C.2. — *9.3.5, C.3.5*

   Chapter 9 $P_n = P_u/\phi = 117/0.55 = 212.7$ kips

   Appendix C $P_n = P_u/\phi = 141/0.65 = 216.9$ kips

   Since the nominal axial load strength required by the load and strength reduction factors of Chapter 9 is less than the corresponding load required by Appendix C, design according to Chapter 9 will be more economical. Regardless of the load and strength reduction factors used, the design procedures will be the same.

4. Calculate the factored soil bearing pressure.

   Since the footing must be proportioned for strength by using factored loads and induced reactions, the factored soil bearing pressure must be used. — *22.7.1*

   $$q_s = \frac{P_u}{A_f} = \frac{117}{36} = 3.25 \text{ ksf}$$

5. Determine the footing thickness required to satisfy moment strength.

   For plain concrete, flexural strength will usually control thickness. The critical section for calculating moment is at the face of the concrete pedestal (see figure above). — *22.7.5(a)*

   $$M_u = q_s\,(b)\left(\frac{b - c}{2}\right)\left(\frac{b - c}{4}\right)$$

   $$= 3.25\ (3)\ (2.5)^2 = 60.9 \text{ ft-kips}$$

   $\phi M_n \geq M_u$ — *Eq. (22-1)*

$\phi = 0.55$ for all stress conditions *9.3.5*

$$\phi M_n = 5\phi\sqrt{f'_c}\,S_m \quad \textit{Eq. (22-2)}$$

$$= \frac{5\,(0.55)\left(\sqrt{2500}\right)(6)\,(12)h^2}{(1000)\,(6)} \geq 60.9 \text{ ft-kips}$$

Solving for h:

$$h \geq \left[\frac{60.9\,(12)\,(1000)\,(6)}{5\,(0.55)\left(\sqrt{2500}\right)(6)\,(12)}\right]^{0.5} = 21.0 \text{ in.}$$

Alternate solution using Fig. 30-10:

Enter figure with factored soil bearing pressure (3.25 kips/sq. ft). Project upward to the distance that the footing projects beyond the face of the pedestal, which is 30 in. (36 - 6). Then project horizontally and read approximately 21 in. as the required footing thickness.

For concrete cast on the soil, the bottom 2 in. of concrete cannot be considered for strength computations (the reduced overall thickness is to allow for unevenness of the excavation and for some contamination of the concrete adjacent to the soil). *22.4.8*

Use overall footing thickness of 24 in.

6. Check for beam action shear. Use effective thickness of h = 22 in. = 1.83 ft.

The critical section for beam action shear is located a distance equal to the thickness, h, away from the face of the pedestal, or 0.67 ft (3 - 0.5 - 1.83) from the edge of the footing. *22.7.6.1* *22.7.6.2(a)*

$$V_u = q_s b\left[\left(\frac{b}{2}\right) - \left(\frac{c}{2}\right) - h\right] = 3.25\,(6)\,(0.67) = 13.07 \text{ kips}$$

$$\phi V_n \geq V_u \quad \textit{Eq. (22-8)}$$

$$V_n = \left(\frac{4}{3}\right)\sqrt{f'_c}\,b_w h \quad \textit{Eq. (22-9)}$$

$$\phi V_n = \frac{4\,(0.55)\left(\sqrt{2500}\right)(72)\,(22)}{(3)\,(1000)} = 58.08 \text{ kips} > 13.07 \text{ kips} \quad \text{O.K.}$$

7. Check for two-way action (punching) shear.

The critical section for two-way action shear is located a distance equal to one-half the footing thickness, h, away from the face of the pedestal. *22.7.6.1* *22.7.6.2(b)*

$$V_u = q_s\,[b^2 - (c + h)^2] = 3.25\left[6^2 - \left(1 + \frac{22}{12}\right)^2\right] = 90.91 \text{ kips}$$

$$\phi V_n \geq V_u$$ *Eq. (22-8)*

$$V_n = \left[\frac{4}{3} + \frac{8}{3\beta}\right]\sqrt{f_c'}\,b_o h \leq 2.66\sqrt{f_c'}\,b_o h$$ *Eq. (22-10)*

where β_c is the ratio of the long-to-short side of the supported load. In this case, $\beta_c = 1$.

Since $\left[\left(\frac{4}{3}\right) + \left(\frac{8}{3}\right)\right] = 4.0 > 2.66$,

$$V_n = 2.66\sqrt{f_c'}\,b_o h$$

$$\phi V_n = \frac{2.66\ (0.55)\left(\sqrt{2500}\right)(34)\ (4)\ (22)}{1000} = 218.87 \text{ kips} > 90.91 \text{ kips} \quad \text{O.K.}$$

8. Check bearing strength of pedestal. *22.8.3*

P_u = 141 kips (from Step 3)

$$\phi B_n \geq P_u$$ *Eq. (22-11)*

$$B_n = 0.85 f_c' A_1$$ *Eq. (22-12)*

$$\phi B_n = \frac{0.85\ (0.55)\ (2500)\ (12 \times 12)}{1000} = 168.3 \text{ kips} > 117 \text{ kips} \quad \text{O.K.}$$

Example 30.2—Design of Plain Concrete Basement Wall

A plain concrete basement wall is to be used to support a 2-story residential occupancy building of wood frame construction with masonry veneer. The height of the wall is 10 ft (distance between the top of the concrete slab and the wood-framed floor, both of which provide lateral support of the wall). The backfill height is 7 ft and the wall is laterally restrained at the top. Design of the wall is required in accordance with Chapter 22 using the load and strength reduction factors of Chapter 9. Perform a second design using the alternate load and strength reduction factors of Appendix C to determine which results in a more economical design.

Design data:

Service dead load = 1.6 kips per linear foot
Service floor live load = 0.8 kips per linear foot
Service roof live load = 0.4 kips per linear foot
Service roof snow load = 0.3 kips per linear foot
Service lateral earth pressure = 60 psf/ft of depth
Service wind pressure = 20 psf inward, 25 psf outward
Assume factored roof dead load plus factored wind uplift load on roof (Eq. 9-6) = 0
Eccentricity of axial loads = 0

Calculations and Discussion	Code Reference

Design using load and strength reduction factors of Chapter 9.

1. The wall must be designed for vertical, lateral, and other loads to which it will be subjected. Therefore, determine the applicable load combinations that must be considered. — *22.6.2*, *9.2*

   1. $U = 1.2D + 1.6L + 0.5L_r + 1.6H$ — *Eq. (9-2)*
   2. $U = 1.2D + 0.5L + 1.6L_r + 1.6H$ — *Eq. (9-3)*
   3. $U = 1.2D + 0.5L + 0.5L_r\ 1.6W + 1.6H$ — *Eq. (9-4)*
   4. $U = 0.9D + 1.6W + 1.6H$ — *Eq. (9-6)*
   5. $U = 1.6H$ — *Eq. (9-6)*

   Note that in Combination 1, T is being neglected. In Combinations 2 and 3 the factor on L is 0.5 in accordance with 9.2.1(a). In Combinations 2 and 4, 1.6H has been included since the lateral soil load will always be acting. In Combination 5, D has been omitted since this condition may occur during construction.

2. Calculate the axial load, P_u, for each load combination.

   1. $P_u = 1.2D + 1.6L + 0.5L_r = 1.2(1.6) + 1.6(0.8) + 0.5(0.4) = 3.40$ kips/ft — *Eq. (9-2)*
   2. $P_u = 1.2D + 0.5L + 1.6L_r = 1.2(1.6) + 0.5(0.8) + 1.6(0.4) = 2.96$ kips/ft — *Eq. (9-3)*
   3. $P_u = 1.2D + 0.5L + 0.5L_r = 1.2(1.6) + 0.5(0.8) + 0.5(0.4) = 2.52$ kips/ft — *Eq. (9-4)*

4. $P_u = 0.9D = 0.9(1.6) = 1.44$ kips/ft *Eq. (9-6)*

5. $P_u = 0$ *Eq. (9-6)*

3. Calculate the moment, M_u, for load combination.

1. M_u due to 1.6H *Eq. (9-2)*

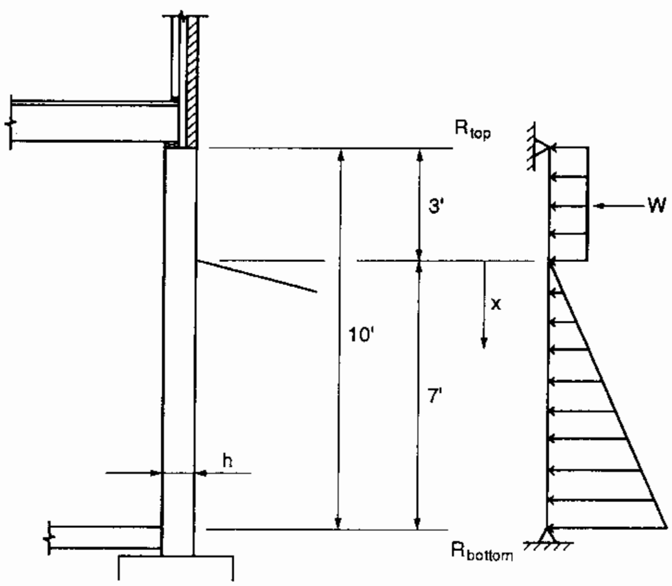
2. M_u due to 1.6H *Eq. (9-3)*

3. M_u due to 1.6W + 1.6H *Eq. (9-4)*

4. M_u due to 1.6W + 1.6H *Eq. (9-6)*

5. M_u due to 1.6H *Eq. (9-6)*

The maximum moment occurs at the location of zero shear. To determine this location with respect to the top of the wall, first calculate the reaction at the top of the wall. If wind is acting and in the same direction as the lateral soil load, and the resultant of the wind load, W, is greater than the reaction at the top of the wall, the location of zero shear is some distance "X" below the top of the wall (above the top of the backfill). Otherwise, the zero shear location will be some distance "X" below the top of the backfill. Next, sum the horizontal forces above "X" (the location of zero shear). Finally, solve for "X." See figure below.



For load Combinations 3 and 4, the reaction at the top of the wall is:

$$R_{top} = \frac{1.6\ (20)\ (3)\ (8.5)\ +\ [1.6\ (60)7^3]\ /\ 6}{10} = 630.4 \text{ plf}$$

$W = 1.6(20)(3) = 96$ plf

$W < R_{top}$

Therefore, location of zero shear is below top of backfill.

Sum horizontal forces at "X":

$630.4 - 1.6\,(20)\,(3) - [1.6\,(60)\,(X^2)]/2 = 0$

$630.4 - 96 - 48X^2 = 0$

Solving for distance "X":

$$X = \left(\frac{630.4 - 96}{48}\right)^{0.5} = 3.37 \text{ ft}$$

The point of zero shear is $3.0 + 3.37 = 6.37$ ft from the top of the wall.

Note: It is generally simpler to compute the location of zero shear with respect to the top of the wall since doing so with respect to the bottom will involve solving a quadratic equation.

Compute the maximum moment, M_u, due to the wind and lateral soil loads:

$$M_u = \frac{630.4\,(6.37) - 1.6\,(20)\,(3)\,(6.37 - 1.5) - [1.6\,(60)\,(3.37)^3]/6}{1000} = 2.94 \text{ ft-kips/ft}$$

Alternately, the maximum moment can be determined from Table 30-1 since the load factor on wind is 1.6.

From Table 30-1, for a 10-ft high wall with 7 ft of backfill, for unfactored wind and soil loads of 20 psf and 60 psf/ft, respectively, the moment, M_u, is 2.94 ft-kips/ft, which is the same as calculated above.

Next determine from the moment Table 30-1 for load Combinatios 1, 2 and 5 (with no wind acting). From Table 30-1 for a 10-ft high wall with 7 ft of backfill, for unfactored wind and soil loads of 0 psf and 60 psf/ft, respectively, the moment, M_u, is 2.88 ft-kips/ft. Note that either Table 30-1 or 30-2 can be used since this load combination does not include wind. The moment is read from the zero column for "unfactored design wind pressure."

4. Calculate the effective eccentricities for load Combinations 1, 3, and 4 to determine if the wall can be designed by the empirical method (i.e., $e \leq h/6$). Assume a conservative wall thickness of 12 in. *22.6.3* *22.6.5*

Allowable $e = 12/6 = 2$ in $= 0.167$ ft

For load Combination 1:

$$e = \frac{2.88}{3.40} = 0.85 \text{ ft} > 0.167 \text{ ft}$$

For load Combination 3:

$$e = \frac{2.94}{2.52} = 1.17 \text{ ft} > 0.167 \text{ ft}$$

For load Combination 4:

$$e = \frac{2.94}{1.44} = 2.04 \text{ ft} > 0.167 \text{ ft}$$

Since the effective eccentricity exceeds h/6, the wall cannot be designed by the empirical method. The wall must be designed taking into consideration both flexure and axial compression. *22.5.3* *22.6.3*

5. Determine the required wall thickness to satisfy the axial load and induced moments by using the appropriate interaction equation. The factored axial loads and moments for the various load combinations are summarized in the following table. *Eq. (22-6)* *Eq. (22-7)*

Load Combination	Axial Load, P_u, kips/ft	Moment, M_u, ft-kips/ft
1	3.40	2.88
2	2.96	2.88
3	2.52	2.94
4	1.44	2.94
5	0	2.88

By observation, the combination of very low axial load and relatively high moment will be governed by interaction Eq. (22-7). *22.5.3*

$$\frac{M_u}{S} - \frac{P_u}{A_g} \leq 5\phi\sqrt{f_c'}$$ *Eq. (22-6)*

By rearranging the equation and substituting for S and A_g in terms of "h," the required thickness can be determined by solving the following quadratic equation (3):

$$0.06\,\phi\sqrt{f_c'}\,h^2 + P_u h - 72M_u = 0 \qquad (3)$$

Determine the required wall thickness, h, to satisfy load Combination 5 ($\phi = 0.55$, $P_u = 0$ and $M_u = 2.88$ ft-kips/ft), since this combination has the highest moment and lowest axial load. Since $P_u = 0$, equation (3) simplifies to equation (4). Use equation (4) to solve for required, "h," assuming $f_c' = 4000$ psi.

$$h = (72M_u/0.06\phi\sqrt{f_c'})^{1/2} = [((72)(2.88)) / ((0.6)(0.55)(\sqrt{4000}))]^{1/2} = 9.97 \text{ in.} \qquad (4)$$

Assume a 10-inch wall with $f_c' = 4000$ psi.

Check preliminary wall selection for all load combinations by interpolating between values from Figs. 30-5 and 30-6 for $f_c' = 3500$ and 4500 psi, respectively. The following table summarizes the required axial load, P_u, and moment, M_u, strengths for the various load combinations, and indicates the approximate design moment strength, ϕM_n, determined from the figures based on a wall thickness of 10 in.

Load Combination	Axial Load, P_u kips/ft	Moment, M_u ft-kips/ft	Approximate Design Moment Strength,[1] ϕM_n ft-kips/ft	$\phi M_n / M_u$
1	3.40	2.88	Note 2	—
2	2.96	2.88	(3.1 + 3.5)/2 = 3.3	1.15
3	2.52	2.94	Note 2	—
4	1.44	2.94	(2.9 + 3.3)/2 = 3.1	1.05
5	0	2.88	(2.7 + 3.1)/2 = 2.9	1.01

[1] Values interpolated from Figs. 30-5 and 30-6.
[2] There is no need to evaluate this combination since another combination has the same moment and a lower axial load. Given equal moments, the combination with the lower axial load will govern.

Since the ratio of design moment strength, ϕM_n, to required moment strength, M_u, exceeds 1 in all cases, the 10-in. wall is adequate for axial loading and induced moments.

Tentatively, use 10-in. wall with concrete $f'_c = 4000$ psi and check for shear.

6. Check for shear strength.

Shear strength will rarely govern the design of a wall; nevertheless, it should not be overlooked. The shear will be greatest at the bottom of the wall. The critical section for calculating shear is located at wall thickness, h, above the top of the floor slab. 22.5.4 22.6.4

For shear, load Combinations 3 and 4, which are the same, will control. Calculate the reaction at the bottom of the wall.

$$R_{bottom} = \frac{[(1.6)(20)(3)^2/2] + [((1.6)(60)(7)^2/2)(3 + (2/3)(7))]}{10} = 1818 \text{ plf}$$

$$V_u = 1818 - 1.6(60)\{[(7 - 10/12)(10/12)] + [(10/12)^2/2]\}$$

$$= 1291 \text{ plf} = 1.29 \text{ klf}$$

$$\phi V_n \geq V_u \qquad \text{Eq. (22-8)}$$

$$\phi V_n = \frac{4\phi\sqrt{f'_c}\,b_w h}{3} \qquad \text{Eq. (22-9)}$$

$$= \frac{4(0.55)\left(\sqrt{4000}\right)(12)(10)}{3(1000)} = 5.57 \text{ klf} > 1.29 \text{ klf} \quad \text{O.K.}$$

7. Use 10-in. wall with specified compressive strength of concrete, $f'_c = 4000$ psi.

Design using load and strength reduction factors of Appendix C.

C1. The wall must be designed for vertical, lateral, and other loads to which it will be subjected. Therefore, determine the applicable load combinations that must be considered. *22.6.2* *C.2*

1. $U = 0.75(1.4D + 1.7L) + 1.6W + 1.7H$ *Eq. (C-2)*
2. $U = 0.9D + 1.6W + 1.7H$ *Eq. (C-3)*
3. $U = 1.4D + 1.7L + 1.7H$ *Eq. (C-4)*
4. $U = 0.9D + 1.7H$ *Eq. (C-3)*
5. $U = 1.7H$ *Eq. (C-3)*

In Combinations 1 and 2, 1.7H has been added since lateral earth pressure will always be acting and for consistency with Eq. (9-2) and (9-3). For consistency with Eq. (9-6) and since dead load, D, may not be acting during early phases of construction, it has been omitted.

C2. Calculate the factored axial load, P_u, for each load combinations.

1. $P_u = 0.75\ (1.4D + 1.7L) = 0.75((1.4)(1.6) + (1.7)(1.2)) = 3.21$ kips/ft *Eq. (C-2)*
2. $P_u = 0.9D = 0.9(1.6) = 1.44$ kips/ft *Eq. (C-3)*
3. $P_u = 1.4D + 1.7L = 1.4(1.6) + 1.7(1.2) = 4.28$ kips/ft *Eq. (C-4)*
4. $P_u = 0.9D = 0.9(1.6) = 1.44$ kips/ft *Eq. (C-3)*
5. $P_u = 0$ *Eq. (C-3)*

C3. Calculate the factored moment, M_u, for each load combinations using Table 30-3.

1. $M_u = 1.6W + 1.7H = 3.12$ ft-kips/ft *Eq. (C-2)*
2. $M_u = 1.6W + 1.7H = 3.12$ ft-kips/ft *Eq. (C-3)*
3. $M_u = 1.7H = 3.06$ ft-kips/ft *Eq. (C-4)*
4. $M_u = 1.7H = 3.06$ ft-kips/ft *Eq. (C-3)*
5. $M_u = 1.7H = 3.06$ ft-kips/ft *Eq. (C-3)*

C4. By observation, the effective eccentricities exceed one-sixth the wall thickness; therefore, the empirical design procedure cannot be used. *22.6.5.1*

C5. Determine the required wall thickness by using the appropriate interaction equation. Since lightly-loaded walls are governed by Eq. (22-7), Figures 30-4 through C30-6 will be used to design the wall for axial loads and flexure. The following table shows for each load combination the factored axial load and moment, and corresponding approximate design moment strength, ϕM_n, and ratio of over- or under-strength based on the assumed wall thickness and concrete strength. Assume a wall thickness of 10 in. and $f'_c = 3000$ psi. *22.5.3* *Eq. (22-6)* *Eq. (22-7)*

Load Combination	Axial Load, P_u kips/ft	Moment, M_u ft-kips/ft	Approximate Design Moment Strength,[1] ϕM_n ft-kips/ft	$\phi M_n/M_u$
1	3.21	3.12	Note 2	—
2	1.44	3.12	(2.9 + 3.4)/2 = 3.15	1.031
3	4.28	3.06	(3.3 + 3.8)/2 = 3.55[3]	1.16
4	1.44	3.06	Note 2	—
5	0	3.06	(2.7 + 3.2)/2 = 2.95	0.96

[1] Values interpolated from Figs. C30-4 and C30-5 based on 10-inch wall with 3000 psi concrete.
[2] There is no need to evaluate this combination since another combination has the same moment and a lower axial load. Given equal moments, the combination with the lower axial load will govern.
[3] While it was not necessary to check this load combination, it was done to illustrate that given equal induced moments, the combination with the higher axial, P_u, load yields a greater design moment strength, ϕM_n.

Since the 10-in. wall of 3000 psi concrete exceeds the requirements for all combinations except where no dead load is present (Combination 5), and in that case it is less than 4% under-strength, tentatively use these parameters.

C6. Check for shear strength.

As indicated for the first part of this design example in which the load and strength reduction factors of Chapter 9 were used, shear strength will rarely govern the design of a wall. Since in that example the wall was greatly over-designed for shear, there is no need to check for shear in this example. *22.5.4* *22.6.4*

C7. Use 10-in. wall with concrete specified compressive strength, f'_c = 3000 psi.

This example showed that use of the load and strength reduction factors of Appendix C resulted in a more economical design. To quickly determine which one of the two sets of load and strength reduction factors should be used, compare the nominal moment strength required by the factored loads of 9.2 to that required by C.2. The set requiring the lower nominal strength will generally be more economical.

Chapter 9 $M_n = M_u/\phi = 2.88/0.55 = 5.24$ ft-kips/ft

Appendix C $M_n = M_u/\phi = 3.06/0.65 = 4.71$ kips

Since the nominal moment strength required by the load and strength reduction factors of Appendix C is less than the corresponding strength required by Chapter 9, a design according to Appendix C will be more economical as the example shows. Regardless of the load and strength reduction factors used, the design procedures are the same.

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31

Alternate (Working Stress) Design Method

INTRODUCTION

Although the Working Stress Design (WSD) was deleted from the code in the 2002 edition, the current Commentary Section R1.1 states, "The Alternate Design Method of the 1999 code may be used in place of applicable sections of this code." Note that the Commentary is not mandatory language and, thus, does not bear legal status. Therefore, in jurisdictions that adopt the current code, designers that intend to design by the 1999 WSD are cautioned to first seek approval of the local building official of the jurisdiction where the structure will be built.

GENERAL CONSIDERATIONS

Prior to the 1956 edition of the code, the working stress design method, which was very similar to the alternate design method of Appendix A, was the only method available for design of reinforced concrete members. The (ultimate) strength design method was introduced as an appendix to the 1956 code. In the next edition of the code (1963), strength design was moved to the body of the code as an alternative to working stress design. Because of the widespread acceptance of the strength design method, the 1971 code covered the working stress method in less than one page. The working stress method was moved out of the body of the code and into an appendix with the 1983 edition of the code. The method then became referred to as the "alternate design method." It remained an appendix through the 1999 code.

The alternate design method presented in Appendix A of the 1999 code is a method that seeks to provide adequate structural safety and serviceability by limiting stresses at service loads to certain prescribed limits. These "allowable stresses" are well within the range of elastic material behavior for concrete in compression and steel in tension (and compression). Concrete is assumed to be cracked and provide no resistance in tension. The stress in the concrete is represented by a linear elastic stress distribution. The steel is generally transformed into an equivalent area of concrete for design.

The alternate design method is identical to the "working stress design method" used prior to 1963 for members subject to flexure without axial loads. The procedures for the design of compression members with flexure, shear design, and bond stress and development of reinforcement follow the procedures of the strength design method of the body of the code with factors applied to reflect design at service loads. The design procedures of the alternate design method have not been updated as thoroughly as the remainder of the code.

The replacement of the working stress design method and alternate design method by the strength design method can be attributed to several factors including:

- the uniform treatment of all types of loads, i.e., all load factors are equal to unity. The different variability of different types of loads (dead and live load) is not acknowledged.

- the unknown factor of safety against failure (as discussed below)
- and the typically more conservative designs, which generally require more reinforcement or larger member sizes for the same design moments when compared to the strength design method.

It should be noted that in general, reinforced concrete members designed using working stresses, or the alternate design method, are less likely to have cracking and deflection problems than members designed using strength methods with Grade 60 reinforcement. This is due to the fact that with strength design using Grade 60 reinforcement, the stresses at service loads have increased significantly from what they were with working stress design.

Therefore, crack widths and deflection control are more critical in members designed using strength design methods because these factors are directly related to the stress in the reinforcement.

Today, the alternate design method is rarely used, except for a few special types of structures or by designers who are not familiar with strength design. Footings seem to be the members most often designed using the alternate design method. Note that ACI 350, Environmental Engineering Concrete Structures, governs the design of water retaining structures.

COMPARISON OF WORKING STRESS DESIGN WITH STRENGTH DESIGN

To illustrate the variability of the factor of safety against failure by the working stress design versus the strength design method, a rectangular and a T-section with dimensions shown in Figs. 31-1 and 31-2, respectively, were analyzed. In both cases, $f_c' = 4000$ psi, $f_y = 60$ ksi, and amount of reinforcement was varied between minimum flexural reinforcement per 10.5.1 and a maximum of $0.75\rho_b$ per Appendix B. Flexural strengths were computed using three procedures:

1. Nominal flexural strength, M_n, using the rectangular stress block of 10.2.7. Results are depicted by the solid lines.

2. Nominal flexural strength based on equilibrium and compatibility. This detailed analysis was performed using program Response 2000[31.1] assuming representative stress-strain relationships for concrete and reinforcing steel are shown in Fig. 31.1. Results are depicted by symbol "+."

3. Working stress analysis using linear elastic stress-strain relationships for concrete and reinforcement, and permissible service load stresses of Appendix A of the 1999 code, as noted below. The results are depicted by the dashed lines for M_s.

Obervations:

(a) Flexural strength based on the rectangular stress block, M_n, is very similar to the prediction based on detailed analysis using strain compatibility and equilibrium.

(b) The factor of safety, represented by the ratio $\phi M_n/M_s$ is highly variable. For the rectangular sections, that ratio ranges between 2.3 and 2.6 while for the T-section it ranges between 2.3 and 2.4. In comparison, for flexural design using Chapter 9 load and strength reduction factors, the factor of safety (L.F./ϕ) ranges between 1.2/0.9 = 1.33 where dead load dominates, and 1.6/0.9 = 1.78 where live load dominates. For Appendix C load and strength reduction factors, those ratios are 1.4/0.9 = 1.56 and 1.7/0.9 = 1.89, respectively.

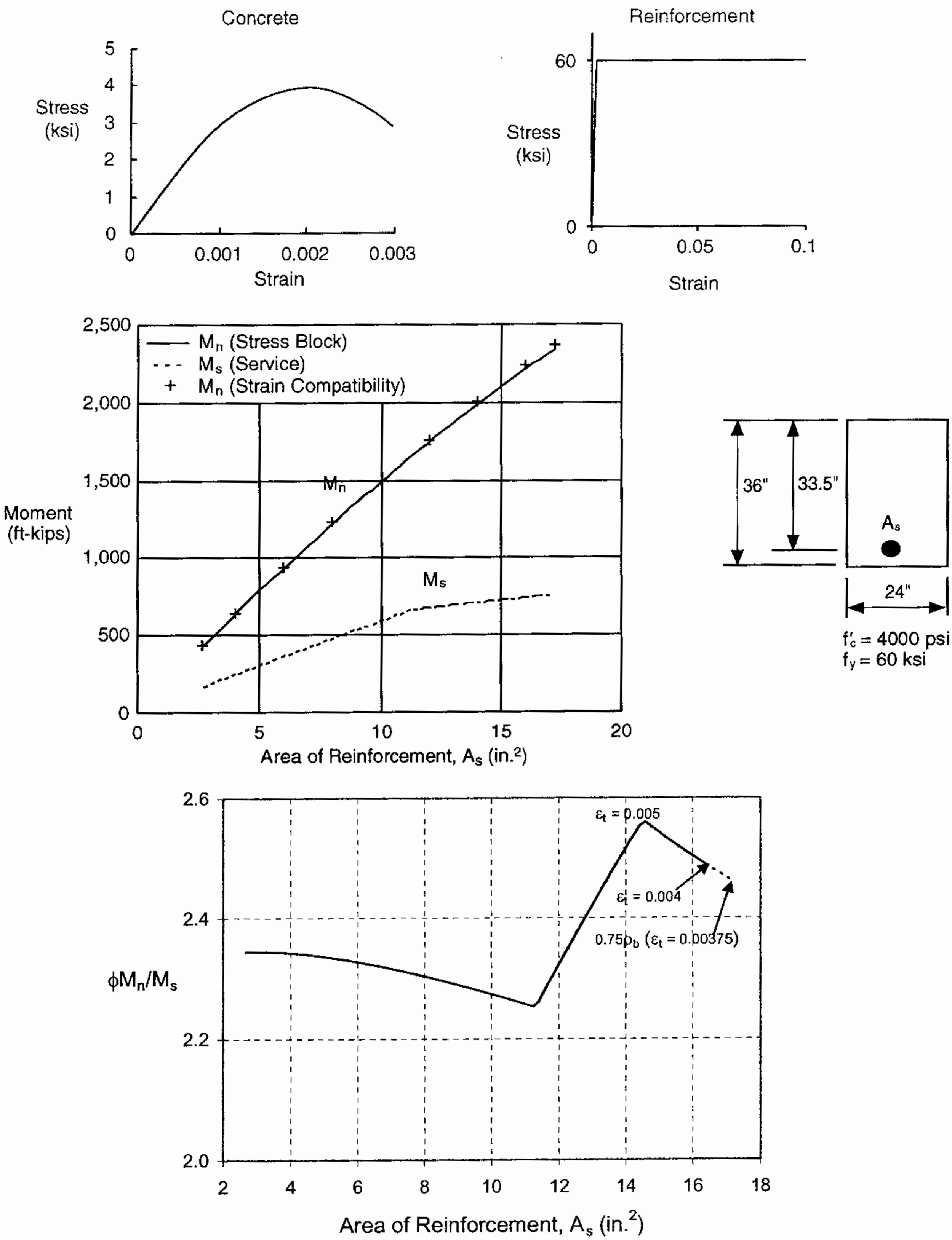
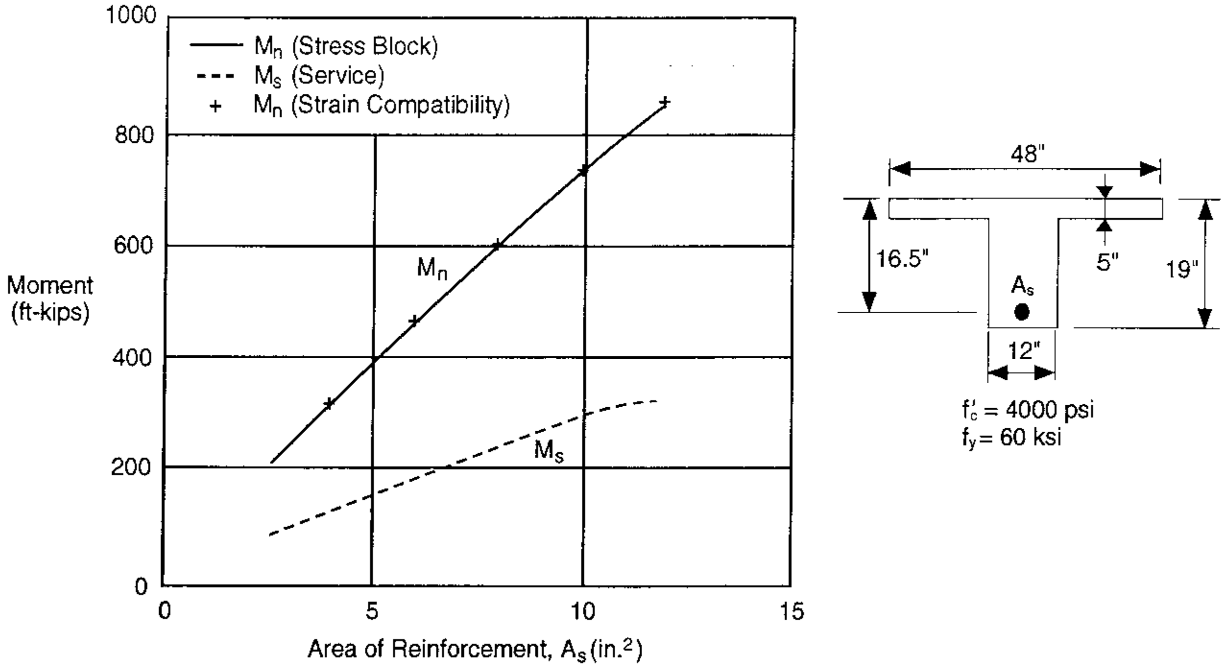
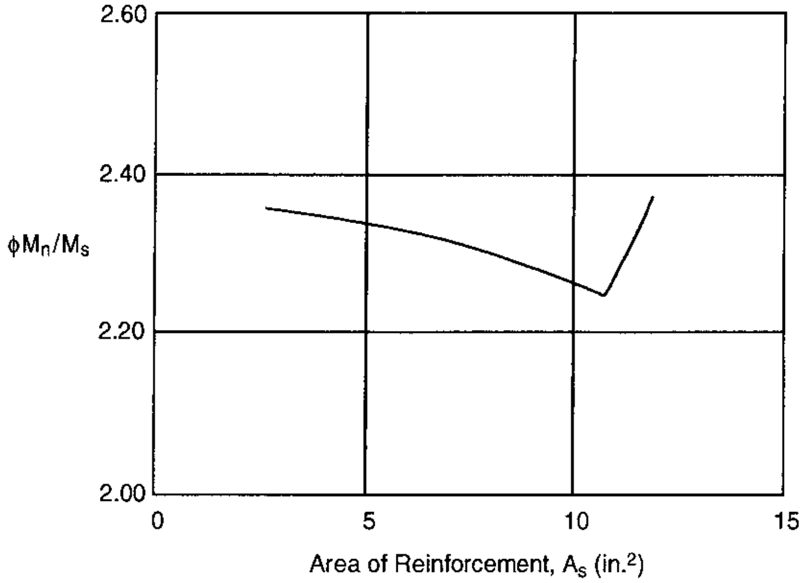


Figure 31-1 Rectangular section.





The following sections highlight provisions of Appendix A of the 1999 code.

SCOPE (A.1 OF '99 CODE)

The code specifies that any nonprestressed reinforced concrete member may be designed using the alternate design method of Appendix A. Prestressed concrete members are designed using a similar approach that is contained in code Chapter 18.

All other requirements of the code shall apply to members designed using the alternate design method, except the moment redistribution provisions of 8.4. This includes such items as distribution of flexural reinforcement and slenderness of compression members, as well as serviceability items such as control of deflections and crack control.

GENERAL (A.2 OF '99 CODE)

Load factors for all types of loads are taken to be unity for this design method. When wind and earthquake loads are combined with other loads, the member shall be designed to resist 75% of the total combined effect. This is similar to the provisions of the original working stress design method which allowed an overstress of one-third for load combinations including wind and earthquake.

When dead loads act to reduce the effects of other loads, 85% of the dead load may be used in computing load effects.

PERMISSIBLE SERVICE LOAD STRESSES (A.3 OF '99 CODE)

Concrete stresses at service loads must not exceed the following:

Flexure	Extreme fiber stress in compression	$0.45f'_c$
Bearing	On loaded area	$0.3f'_c$

Permissible concrete stresses for shear are also given in this section (A.3 of '99 Code) and in greater detail in A.7 of '99 Code.

Tensile stresses in reinforcement at service loads must not exceed the following:

Grade 40 and 50 reinforcement	20,000 psi
Grade 60 reinforcement or greater and welded wire fabric (plain or deformed)	24,000 psi

Permissible tensile stresses for a special case are also given in A.3.2(c).

FLEXURE (A.5 OF '99 CODE)

Members are designed for flexure using the following assumptions:

- Strains vary linearly as the distance from the neutral axis. A non-linear distribution of strain must be used for deep members (see 10.7).
- Under service load conditions, the stress-strain relationship of concrete in compression is linear for stresses not exceeding the permissible stress.
- In reinforced concrete members, concrete resists no tension.
- The modular ratio, $n = E_s/E_c$, may be taken as the nearest whole number, but not less than 6. Additional provisions are given for lightweight concrete.
- In members with compression reinforcement, an effective modular ratio of $2E_s/E_c$ must be used to transform the compression reinforcement for stress computations. The stress in the compression reinforcement must not exceed the permissible tensile stress.

DESIGN PROCEDURE FOR FLEXURE

The following equations are used in the alternate design method for the flexural design of a member with a rectangular cross section, reinforced with only tension reinforcement. They are based on the assumptions stated above and the notation defined in Fig. 31-3. See Refs. 33.2 and 33.3 or other texts on reinforced concrete design for derivation of these equations. Equations can also be developed for other cross sections, such as members with flanges or compression reinforcement.

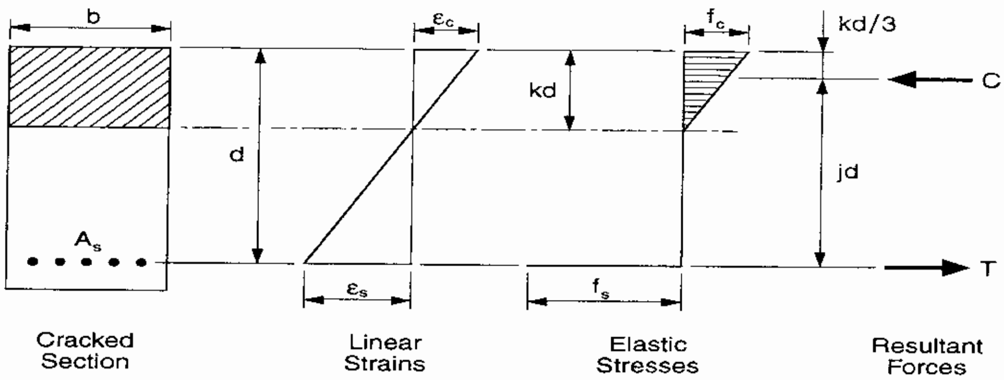


Figure 31-3 Assumptions for Alternate Design Method for Flexure

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

where

$$\rho = \frac{A_s}{bd}$$

$$n = \frac{E_s}{E_c} \geq 6$$

$$j = 1 - \left(\frac{k}{3}\right)$$

$$f_s = \frac{M_s}{A_s jd}$$

$$f_c = \frac{2M_s}{kjbd^2}$$

SHEAR AND TORSION (A.7 OF '99 CODE)

Shear and torsion design in Appendix A of ACI 318-99 is based on the strength design methods of code Chapter 11 ('99 code) with modified coefficients that allow use of the equations for unfactored loads at service load conditions.

A complete set of the modified equations is presented for shear design for the convenience of the user (A.7 of the '99 Code). Since the equations appear in the same form as in code Chapter 11, they will not be discussed here.

REFERENCES

33.1 Bentz, Evans C. and Collins, Michael P., "Response 2000 – Reinforced Concrete Sectional Analysis using the Modified Compression Field Theory." Downloadable at http://www.ecf.utoronto.ca/~bentz/r2k.htm

33.2 MacGregor, J.G., *Reinforced Concrete: Mechanics and Design*, 2nd Edition, Prentice Hall, Englewood Cliffs, NJ, 1997, 939 pp.

33.3 Leet, Kenneth, *Reinforced Concrete Design*, McGraw-Hill, New York, 1984, 544 pp.

Example 31.1—Design of Rectangular Beam with Tension Reinforcement Only

Given the rectangular beam of Example 7.1, modify the beam depth and/or required reinforcement to satisfy the permissible stresses of the alternate design method. The service load moments are: M_d = 56 ft-kips and $M\ell$ = 35 ft-kips.

f'_c = 4000 psi
f_y = 60,000 psi
A_s = 2.40 in.^{2}
b = 10 in.
h = 16 in.
d = 13.5 in.

Calculations and Discussion	Code Reference
1. To compare a design using the alternate design method to the load factor method of the code, check the service load stresses in concrete and steel in the design given in Example 10.1.	
$M_s = M_d + M_\ell = (56 + 35)\ (12) = 1092$ in.-kips	
$E_c = 57{,}000\sqrt{f'_c} = 57{,}000\sqrt{4000} = 3{,}605{,}000$ psi	*8.5.1*
$n = \dfrac{E_s}{E_c} = \dfrac{29{,}000{,}000}{3{,}605{,}000} = 8.04$ Use n = 8	*A.5.4*
$\rho = \dfrac{A_s}{bd} = \dfrac{2.40}{(10 \times 13.5)} = 0.0178$	
$\rho n = (0.0178)\ (8) = 0.142$	
$k = \sqrt{2\rho n + (\rho n)^2} - \rho n = \sqrt{2(0.142) + (0.142)^2} - 0.142 = 0.41$	
$j = 1 - \dfrac{k}{3} = 1 - \dfrac{0.41}{3} = 0.863$	
$f_s = \dfrac{M_s}{A_s jd} = \dfrac{1092}{[(2.40)\ (0.863)\ (13.5)]} = 39.05$ ksi > 24.0 ksi allowed N.G.	*A.3.2*
$f_c = \dfrac{2M_s}{kjbd^2} = \dfrac{2(1092)}{[(0.41)\ (0.863)\ (10)\ (13.5)^2]} = 3.39$ ksi > 0.45 (4.00) = 1.80 ksi allowed N.G.	*A.3.1*

Note: The above calculations are based on the assumption of linear-elastic material behavior. Since both f_c and f_s exceed the permissible stresses, increase the beam depth.

2. Check stresses in concrete and reinforcement with an increased member depth, with the same area of reinforcement.

h = 24 in. d = 21.5 in.

$$\rho = \frac{A_s}{bd} = \frac{2.40}{(10 \times 21.5)} = 0.0112$$

$\rho n = (0.0112)(8) = 0.0893$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n = \sqrt{2(0.0893) + (0.0893)^2} - 0.0893 = 0.343$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.343}{3} = 0.886$$

$$f_s = \frac{M_s}{A_s jd} = \frac{1092}{[(2.40)(0.886)(21.5)]} = 23.89 \text{ ksi} < 24.0 \text{ ksi allowed} \quad \text{O.K.}$$

$$f_c = \frac{2M_s}{kjbd^2} = \frac{2(1092)}{[(0.343)(0.886)(10)(21.5)^2]} = 1.55 \text{ ksi} < 0.45(4.0) = 1.80 \text{ ksi allowed} \quad \text{O.K.}$$

Note: It was necessary to increase the effective depth by nearly 60% in order to satisfy allowable stresses using the same quantity of reinforcement.

3. Compute the design moment strength, ϕM_n, of the modified member to determine the factor of safety (FS).

$$a = \frac{A_s f_y}{0.85bf'_c} = \frac{(2.40)(60)}{[(0.85)(10)(4.00)]} = 4.24 \text{ in.}$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = (2.40)(60)\left[21.5 - \left(\frac{4.24}{2}\right)\right] = 2791 \text{ in.-kips}$$

$\phi M_n = 0.9(2791) = 2512$ in.-kips

$$FS = \frac{\phi M_n}{M_s} = \frac{2512}{1092} = 2.30$$

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Alternative Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members

B.1 SCOPE

Section 8.1.2 allows the use of Appendix B to design reinforced and prestressed concrete flexural and compression members. The Appendix contains the provisions that were displaced from the main body of the code when the Unified Design Provisions (formerly Appendix B to the 1999 Code) were incorporated in the Code in 2002. Since it may be judged that an appendix is not an official part of a legal document unless it is specifically adopted, reference is made to Appendix B in the main body of the code in order to make it a legal part of the code.

Appendix B contains provisions for moment redistribution, design of flexural and compression members, and prestressed concrete that were in the main body of the code for many years prior to 2002. The use of these provisions is equally acceptable to those in the corresponding sections of the main body of the code.

Section B.1 contains the sections in Appendix B that replace those in the main body of the code when Appendix B is used in design. It must be emphasized that when any section of Appendix B is used, all sections of this appendix must be substituted in the main body of the code. All other sections in the body of the code are applicable.

According to RB.1, load factors and strength reduction factors of either Chapter 9 or new Appendix C (see Part 33) may be used. It is the intent that strength reduction factors given in Chapter 9 or Appendix C for tension-controlled sections be utilized for members subjected to bending only. Similarly, strength reduction factors for compression-controlled sections should be used for members subjected to flexure and axial load with ϕP_n greater than or equal to $0.10f'_cA_g$ or the balanced axial load ϕP_b, whichever is smaller (see 9.3.2.2 and C3.2.2). For other cases, ϕ can be increased linearly to 0.90 as ϕP_n decreases from $0.10f'_cA_g$ or ϕP_b, whichever is smaller, to zero (9.3.2.2 and C3.2.2).

B.8.4 REDISTRIBUTION OF NEGATIVE MOMENTS IN CONTINUOUS NONPRESTRESSED FLEXURAL MEMBERS

Section B.8.4 permits a redistribution of negative moments in continuous flexural members if reinforcement percentages do not exceed a specified amount.

A maximum 10 percent adjustment of negative moments was first permitted in the 1963 ACI Code (see Fig. RB.8.4). Experience with the use of that provision, though satisfactory, was still conservative. The 1971 code increased the maximum adjustment percentage to that shown in Fig. 32-1. The increase was justified by additional knowledge of ultimate and service load behavior obtained from tests and analytical studies. Appendix B retains the same adjustment percentage criteria.

A comparison between the permitted amount of redistribution according to 8.4 and B.8.4 as a function of the strain in the extreme tension steel ε_t is depicted in Figure 32-2.

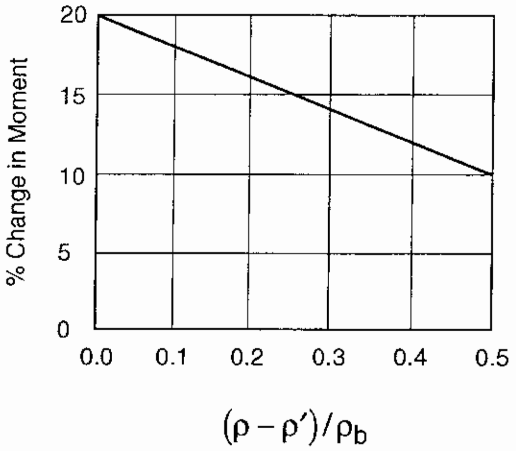

Figure 32-1 Permissible Moment Redistribution for Nonprestressed Members

Application of B.8.4 will permit, in many cases, substantial reduction in total reinforcement required without reducing safety, and reduce reinforcement congestion in negative moment regions.

According to 8.9, continuous members must be designed to resist more than one configuration of live loads. An elastic analysis is performed for each loading configuration, and an envelope moment value is obtained for the design of each section. Thus, for any of the loading conditions considered, certain sections in a given span will reach the ultimate moment, while others will have reserve capacity. Tests have shown that a structure can continue to carry additional loads if the sections that reached their moment capacities continue to rotate as plastic hinges and redistribute the moments to other sections until a collapse mechanism forms.

Recognition of this additional load capacity beyond the intended original design suggests the possibility of redesign with resulting savings in material. Section B.8.4 allows a redesign by decreasing or increasing the elastic negative moments for each loading condition (with the corresponding changes in positive moment required by statics). These moment changes may be such as to reduce both the maximum positive and negative moments in the final moment envelope. In order to ensure proper rotation capacity, the percentage of steel in the sections must conform to B.8.4, which is shown in Fig. 32-1.

In certain cases, the primary benefit to be derived from B.8.4 will be simply a reduction of negative moment at the supports, to avoid reinforcement congestion or reduce concrete dimensions. In this case, the steel percentage must still conform to Fig. 32-1.

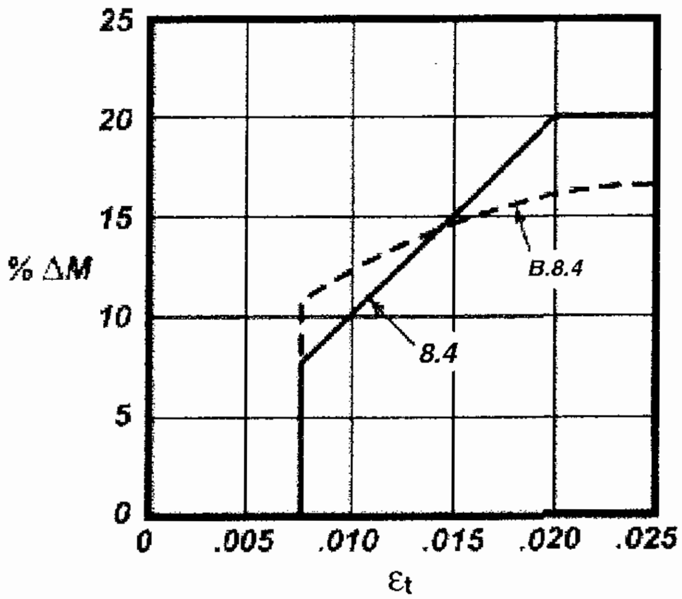

Figure 32-2 Comparison of Permissible Moment Redistribution for Nonprestressed Members

Limits of applicability of B.8.4 may be summarized as follows:

1. Provisions apply to continuous nonprestressed flexural members. Moment redistribution for prestressed members is addressed in B.18.10.4.

2. Provisions do not apply to members designed by the approximate moments of 8.3.3, or to slab systems designed by the Direct Design Method (see 13.6.1.7 and RB.8.4).

3. Bending moments must be determined by analytical methods, such as moment distribution, slope deflection, etc. Redistribution is not allowed for moments determined through approximate methods.

4. The reinforcement ratios ρ or $(\rho - \rho')$ at a cross-section where moment is to be adjusted must not exceed one-half of the balanced steel ratio, ρ_b, as defined by Eq. (B.8-1).

5. Maximum allowable percentage increase or decrease of negative moment is given by Eq. (B-1):

$$20\left(1-\frac{\rho-\rho'}{\rho_b}\right)$$

6. Adjustment of negative moments is made for each loading configuration considered. Members are then proportioned for the maximum adjusted moments resulting from all loading conditions.

7. Adjustment of negative support moments for any span requires adjustment of positive moments in the same span (B.8.4.2). A decrease of a negative support moment requires a corresponding increase in the positive span moment for equilibrium.

8. Static equilibrium must be maintained at all joints before and after moment redistribution.

9. In the case of unequal negative moments on the two sides of a fixed support (i.e., where adjacent spans are unequal), the difference between these two moments is taken into the support. Should either or both of these negative moments be adjusted, the resulting difference between the adjusted moments is taken into the support.

10. Moment redistribution may be carried out for as many cycles as deemed practical, provided that after each cycle of redistribution, a new allowable percentage increase or decrease in negative moment is calculated, based on the final steel ratios provided for the adjusted support moments from the previous cycle.

11. After the design is completed and the reinforcement is selected, the actual steel ratios provided must comply with Fig. 32-1 for the percent moment redistribution taken, to ensure that the requirements of B.8.4 are met.

Examples that illustrate these requirements can be found in Part 9 of Notes on ACI 318-99.

B.10.3 GENERAL PRINCIPLES AND REQUIREMENTS – NONPRESTRESSED MEMBERS

The flexural strength of a member is ultimately reached when the strain in the extreme compression fiber reaches the ultimate (crushing) strain of the concrete, ε_u. At that stage, the strain in the tension reinforcement could just reach the strain at first yield ($\varepsilon_s = \varepsilon_y = f_y / \varepsilon_u$), be less than the yield strain, or exceed the yield strain. Which steel strain condition exists at ultimate concrete strain depends on the relative proportion of reinforcement to concrete. If the steel amount is low enough, the strain in the tension steel will greatly exceed the yield strain ($\varepsilon_s >> \varepsilon_y$) when the concrete strain reaches ε_u, with large deflection and ample warning of impending failure (ductile failure condition). With a larger quantity of steel, the strain in the tension steel may not reach the yield strain ($\varepsilon_s < \varepsilon_y$) when the concrete strain reaches ε_u, which would mean small deflection and little warning of impending failure (brittle failure condition). For design it is desirable to restrict the ultimate strength condition so that a ductile failure mode would be expected.

The provisions of B.10.3.3 are intended to ensure a ductile mode of failure by limiting the amount of tension reinforcement to 75% of the balanced steel to ensure yielding of steel before crushing of concrete. The balanced steel will cause the strain in the tension steel to just reach yield strain when concrete reaches the crushing strain.

The maximum amount of reinforcement permitted in a rectangular section with tension reinforcement only is

$$\rho_{max} = 0.75\rho_b = 0.75\left[0.85\beta_1 \frac{f'_c}{f_y} \times \frac{87{,}000}{87{,}000+f_y}\right]$$

where ρ_b is the balanced reinforcement ratio for a rectangular section with tension reinforcement only.

The maximum amount of reinforcement permitted in a flanged section with tension reinforcement only is

$$\rho_{max} = 0.75\left[\frac{b_w}{b}(\rho_b + \rho_f)\right]$$

where b_w = width of the web
b = width of the effective flange (see 8.10)

$\rho_f = A_{sf} / b_w d$

h_f = thickness of the flange

A_{sf} = area of reinforcement required to equilibrate compressive strength of overhanging flanges (see Part 6)

The maximum amount of reinforcement permitted in a rectangular section with compression reinforcement is (B10.3.3)

$$\rho_{max} = 0.75\rho_b + \rho' \frac{f'_{sb}}{f_y}$$

where $\rho' = A'_s / bd$

A'_s = area of compression reinforcement

f'_{sb} = stress in compression reinforcement at balanced strain condition

$$= 87{,}000 - \frac{d'}{d}(87{,}000 + f_y) \le f_y$$

d' = distance from extreme compression fiber to centroid of compression reinforcement

Note that with compression reinforcement, the portion of ρ_b contributed by the compression reinforcement $(\rho' f'_{sb} / f_y)$ need not be reduced by the 0.75 factor. For ductile behavior of beams with compression reinforcement, only that portion of the total tension steel balanced by compression in the concrete (ρ_b) need be limited.

It should be realized that the limit on the amount of tension reinforcement for flexural members is a limitation for ductile behavior. Tests have shown that beams reinforced with the computed amount of balanced reinforcement actually behave in a ductile manner with gradually increasing deflections and cracking up to failure. Sudden compression failures do not occur unless the amount of reinforcement is considerably higher than the computed balanced amount.

One reason for the above is the limit on the ultimate concrete strain assumed at $\varepsilon_u = 0.003$ for design. The actual maximum strain based on physical testing may be much higher than this value. The 0.003 value serves as a lower bound on limiting strain. Unless unusual amounts of ductility are required, the $0.75\rho_b$ limitation will provide ample ductile behavior for most designs.

Comparison of the design using the unified design method and the provisions of B.10.3 for a rectangular beam with tension reinforcement only and for a rectangular beam with compression reinforcement can be found in Part 7 of Notes on ACI 318-99 (Examples 7.1 and 7.3).

B18.1 SCOPE–PRESTRESSED CONCRETE

This section contains a list of the provisions in the code that do not apply to prestressed concrete. Section RB.18.1.3 provides detailed commentary and specific reasons on why some sections are excluded.

B.18.8 LIMITS FOR REINFORCEMENT OF PRESTRESSED FLEXURAL MEMBERS

The requirements of B.18.8 for percentage of reinforcement are illustrated in Fig. 32-3. Note that reinforcement can be added to provide a reinforcement index higher than $0.36\beta_1$; however, this added reinforcement cannot be assumed to contribute to the moment strength.

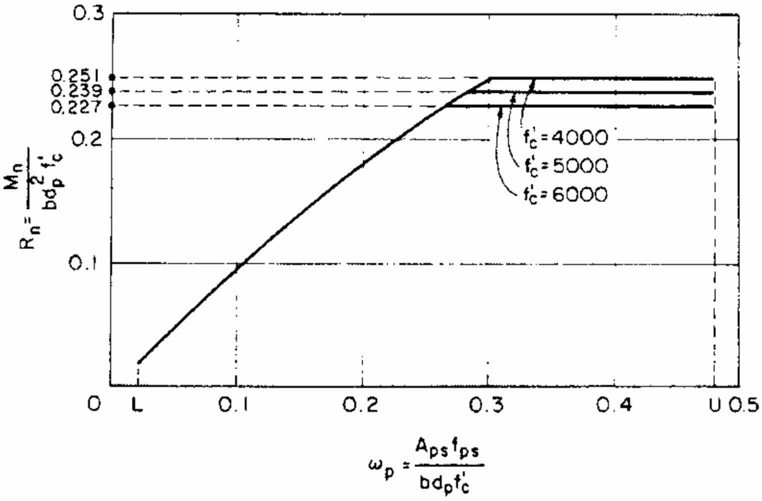

Figure 32-3 Permissible Limits of Prestressed Reinforcement and Influence on Moment Strength

Section B.18.8.3 requires the total amount of prestressed and nonprestressed reinforcement of flexural members to be adequate to develop a design moment strength at least equal to 1.2 times the cracking moment strength ($\phi M_n \geq 1.2M_{cr}$), where M_{cr} is computed by elastic theory using a modulus of rupture equal to $7.5\sqrt{f'_c}$ (see 9.5.2.3). The provisions of B.18.8.3 are analogous to 10.5 for nonprestressed members and are intended as a precaution against abrupt flexural failure resulting from rupture of the prestressing tendons immediately after cracking. The provision ensures that cracking will occur before flexural strength is reached, and by a large enough margin so that significant deflection will occur to warn that the ultimate capacity is being approached. The typical prestressed member will have a fairly large margin between cracking strength and flexural strength, but the designer must be certain by checking it.

The cracking moment M_{cr} for a prestressed member is determined by summing all the moments that will cause a stress in the bottom fiber equal to the modulus of rupture f_r. Refer to Part 24 for detailed equations to compute M_{cr} for prestressed members.

Note that an exception in B.18.8.3 waives the $1.2M_{cr}$ requirement for (a) two-way unbonded post-tensioned slabs, and (b) flexural members with shear and flexural strength at least twice that required by 9.2. See Part 24 for more information.

B.18.10.4 REDISTRIBUTION OF NEGATIVE MOMENTS IN CONTINUOUS PRESTRESSED FLEXURAL MEMBERS

Inelastic behavior at some sections of prestressed concrete beams and slabs can result in a redistribution of moments when member strength is approached. Recognition of this behavior can be advantageous in design under certain circumstances. Although a rigorous design method for moment redistribution is complex, a rational method can be realized by permitting a reasonable adjustment of the sum of the elastically calculated factored gravity load moments and the unfactored secondary moments due to prestress. The amount of adjustment should be kept within predetermined safe limits.

According to B.18.10.4.1, the maximum allowable percentage increase or decrease of negative moment in a continuous prestressed flexural member is

$$20\left[1-\frac{\omega_p+\frac{d}{d_p}(\omega-\omega')}{0.36\beta_1}\right]$$

Note that redistribution of negative moments is allowed only when bonded reinforcement is provided at the supports in accordance with 18.9. The bonded reinforcement ensures that beams and slabs with unbonded tendons act as flexural members after cracking and not as a series of tied arches.

Similar to nonprestressed members, adjustment of negative support moments for any span requires adjustment of positive moments in the same span (B.18.10.4.2). A decrease of a negative support moment requires a corresponding increase in the positive span moment for equilibrium.

The amount of allowable redistribution depends on the ability of the critical sections to deform inelastically by a sufficient amount. Sections with larger amounts of reinforcement will not be able to undergo sufficient amounts of inelastic deformations. Thus, redistribution of negative moments is allowed only when the section is designed so that the appropriate reinforcement index is less than $0.24\beta_1$ (see B.18.10.4.3). This requirement is in agreement with the requirements of B.8.4 for nonprestressed members. Note that each of the expressions in B.18.10.4.3 is equal to $0.85a/d_p$ where a is the depth of the equivalent rectangular stress distribution for the section under consideration (see 10.2.7.1).

A comparison between the permitted amount of redistribution according to 18.10.4 and B.18.10.4 of the 2005 code as a function of the strain in the extreme tension steel ε_t is depicted in Figure 32-4.

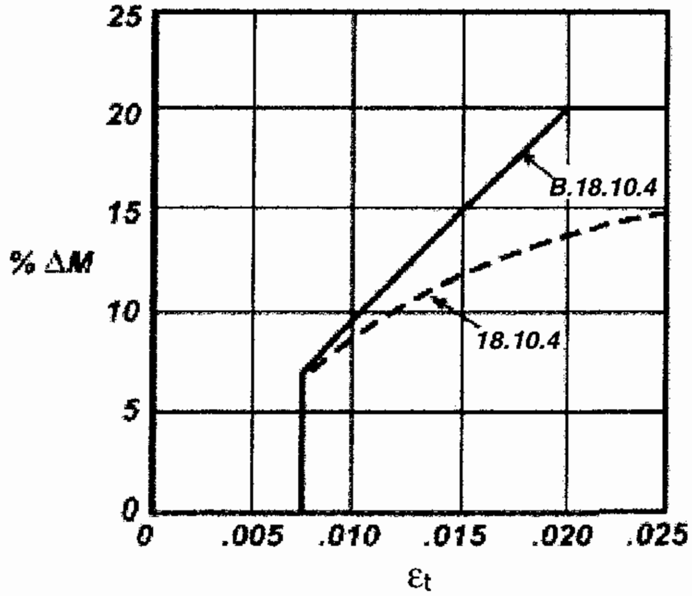

Figure 32-4 Comparison of Permissible Moment Redistribution for Prestressed Members

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Alternative Load and Strength Reduction Factors

C.1 GENERAL

Section 9.1.3 allows the use of load factor combinations and strength reduction factors of Appendix C to design structural members. Since it may be judged that an appendix is not an official part of a legal document unless it is specifically adopted, reference is made in 9.1.3 to Appendix C in the main body of the code in order to make it a legal part of the code. Appendix C contains revised versions of the load and strength reduction factors that were formerly in Chapter 9 of the 1999 Code and earlier editions.

The load and strength reduction factors in new Appendix C have evolved since the early 1960s when the strength design method was originally introduced in the code. Some of the factors have been changed from the values in the 1999 code for reasons stated below. In any case, these sets of factors are still considered to be reliable for the design of concrete structural members.

It is important to note that a consistent set of load and strength reduction factors must be utilized when designing members. It is not permissible to use the load factors of Chapter 9 in conjunction with the strength reduction factors of Appendix C.

C.2 REQUIRED STRENGTH

In general,

Design Strength ≥ Required Strength

or

Strength Reduction Factor × Nominal Strength ≥ Load Factor × Service Load Effects

Part 5 contains a comprehensive discussion on the philosophy of the strength design method, including the reasons why load factors and strength reduction factors are required.

Section C.2 prescribes load factors for specific combinations of loads. A list of these combinations is given in Table 33-1. The numerical value of the load factor assigned to each type of load is influenced by the degree of accuracy with which the load can usually be assessed, the variation which may be expected in the load during the lifetime of a structure, and the probability of simultaneous occurrence of different load types. Hence, dead loads, because they can usually be more accurately determined and are less variable, are assigned a lower load factor (1.4) as compared to live loads (1.7). Also, weight and pressure of liquids with well-defined densities and controllable maximum heights are assigned a reduced load factor of 1.4 due the lesser probability of overloading (see C.2.4). A higher load factor of 1.7 is required for earth and groundwater pressures due to considerable uncertainty of their magnitude and recurrence (see C.2.3). Note that while most usual combinations of loads are included, it should not be assumed that all cases are covered

Table 33-1 Required Strength for Different Load Combinations

Code Section	Loads†	Required Strength	Code Eq. No.
C.2.1	Dead (D) & Live (L)	U = 1.4D + 1.7L	C -1
C.2.2	Dead, Live & Wind (W)††	(i) U = 1.4D + 1.7L (ii) U = 0.75 (1.4D + 1.7L + 1.6W) (iii) U = 0.9D + 1.6W	C -1 C -2 C -3
C.2.2	Dead, Live & Earthquake (E)††	(i) U = 1.4D + 1.7L (ii) U = 0.75 (1.4D + 1.7L + 1.0E) (iii) U = 0.9D + 1.0E	C -1 C -2 C -3
C.2.3	Dead, Live & Earth and Groundwater Pressure (H)*	(i) U = 1.4D + 1.7L (ii) U = 1.4D + 1.7L + 1.7H (iii) U = 0.9D + 1.7H where D or L reduces H	C -1 C -4
C.2.4	Dead, Live & Fluid Pressure (F)**	(i) U = 1.4D + 1.7L (ii) U = 1.4D + 1.7L + 1.4F (iii) U = 0.9D + 1.4F where D or L reduces F	C -1
C.2.5	Impact (I)***	In all of the above equations, substitute (L+I) for L when impact must be considered.	
C.2.6	Dead, Live and Effects from Differential Settlement, Creep, Shrinkage, Expansion of Shrinkage-Compensating Concrete, or Temperature (T)	(i) U = 1.4D + 1.7L (ii) U = 0.75 (1.4D + 1.4T + 1.7L) (iii) U = 1.4 (D + T)	C -1 C -5 C -6

† *D, L, W, E, H, F, and T represent the designated service loads or their corresponding effects such as moments, shears, axial forces, torsion, etc. Note: E is a service-level earthquake force.*

†† *Where wind load W has not been reduced by a directionality factor, it is permitted to use 1.3W in place of 1.6W in Eq. (C-2) and (C-3). Where earthquake load E is based on service-load seismic forces, 1.4E shall be used in place of 1.0E in Eq. (C-2) and (C-3).*

* *Weight and pressure of soil and water in soil. (Groundwater pressure is to be considered part of earth pressure with a 1.7 load factor.)*

** *Weight and pressure of fluids with well-defined densities and controllable maximum heights*

*** *Impact factor is required for design of parking structures, loading docks, warehouse floors, elevator shafts, etc.*

The load factors for wind and earthquake forces have changed from those in Chapter 9 of the 1999 code. Since the wind load equation in ASCE 7-02 and IBC 2003 includes a factor for wind directionality that is equal to 0.85 for buildings, the corresponding load factor for wind in the load combination equations was increased accordingly (1.3/0.85 = 1.53, rounded up to 1.6). The code allows use of the previous wind load factor of 1.3 when the design wind load is obtained from other sources that do not include the wind directionality factor.

The most recent legacy model building codes and the 2003 IBC specify strength-level earthquake forces; thus, the earthquake load factor was reduced to 1.0. The code requires use of the previous load factor for earthquake loads, which is 1.4, when service-level earthquake forces from earlier editions of the model codes or other resource documents are used.

C.3 DESIGN STRENGTH

As noted above, the design strength of a member is the nominal strength of the member, which is determined in accordance with code requirements, multiplied by the appropriate strength reduction factor, ϕ. The purposes of the strength reduction factors are given in Part 5 and RC.3.

The ϕ-factors prescribed in C.3, which have changed from those given in Chapter 9 of the 1999 code, are contained in Table 33-2. Prior to the 2002 code, ϕ-factors were given in terms of the type of loading for members subjected to axial load, flexure, or combined flexure and axial load. Now, for these cases, the ϕ-factor is determined by the strain conditions at a cross-section at nominal strength. Figure RC3.2 shows the variation of ϕ with the net tensile strain ε_t for Grade 60 reinforcement and prestressing steel. The Unified Design Provisions are described in detail in Parts 5 and 6. As noted above, the ϕ-factors given in C.3 are consistent with the load factors given in C.2.

Table 33-2 Strength Reduction Factors ϕ in the Strength Design Method

Tension-controlled sections	0.90
Compression-controlled sections Members with spiral reinforcement conforming to 10.9.3 Other reinforced members	 0.75 0.70
Shear and torsion	0.85
Bearing on concrete (except for post-tensioned anchorage zones and strut-and-tie models)	0.70
Post-tensioned anchorage zones	0.85
Strut-and-tie models (Appendix A)	0.85

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34

Anchoring to Concrete

UPDATE FOR '05 CODE

ACI 318-05 incorporates the second edition of Appendix D, Anchoring to Concrete. Significant revisions were made primarily to clarify application of the provisions in special conditions. Several subscripts to the notations were revised to make each parameter used in the equations more meaningful and to reflect the intent of that parameter. Noteworthy changes include:

- Clarify how to determine the concrete breakout strength of post-installed mechanical anchors when the results of ACI 355.2 product evaluation indicate that values of the coefficient for basic concrete breakout strength, k_c, and the factor used to modify tensile strength of anchors based on presence or absence of cracks, $\psi_{c,N}$, are different than the default values provided in Appendix D (D.5.2.2, RD.5.2.2, D.5.2.6, and RD.6.2.6)
- Determine the value of h_{ef} to be used in computing the concrete breakout strength for anchors loaded in tension and located close to three or four edges, i.e. in narrow members (D.5.2.3 and RD.5.2.3)
- Clarify computation of eccentricities for tension, e'_N (D.5.2.4 and RD.5.2.4) and for shear e'_V (D.6.2.5 and RD.6.2.5)
- Introduce a new modifier $\psi_{cp,N}$ to the basic concrete breakout equations to account for uncracked concrete, use of post-installed anchors, and a free edge near the anchors (new D.5.2.7), and provide conservative default values for the critical edge distance c_{ac} used in determining $\psi_{cp,N}$ (new D.8.6)
- Require testing according to ACI 355.2 product evaluation report if the contribution of post-installed anchor sleeves to shear strength is considered [D.6.1.2(c)]
- Provide guidance for computing the nominal shear concrete breakout strength for anchor groups. [RD.6.2.1 and Fig. RD.6.2.1(b)] For anchors, excluding welded studs, the concrete breakout failure for shear must be determined for all potential concrete breakout failure surfaces. For welded studs, only the breakout failure surface originating from the anchors located farthest from the free edge needs to be considered.
- Clarify computation of shear breakout strength close to three or four edges, i.e. in narrow, thin members (D.6.2.4 and RD.6.2.4)
- Provide an equation to compute the nominal concrete pryout strength of a group of anchors (D.4.1.2 and D.6.3)

INTRODUCTION

Appendix D, Anchoring to Concrete, was introduced in ACI 318-02. It provides requirements for the design of anchorages to concrete using both cast-in-place and post-installed mechanical anchors. The following presents an overview regarding the development and publication of ACI 318 Appendix D. As of the late 1990's, ACI 318 and the American Institute of Steel Construction LRFD and ASD Specifications were silent regarding the design of anchorage to concrete. ACI 349-85 Appendix B and the Fifth Edition of PCI Design Handbook provided the primary sources of design information for connections to concrete using cast-in-place anchors. The design of connections to concrete using post-installed anchors has typically been based on information provided by individual anchor manufacturers.

During the 1990's, ACI Committee 318 took the lead in developing building code provisions for the design of anchorages to concrete using both cast-in-place and post-installed mechanical anchors. Committee 318 received support from ACI Committee 355 (ACI 355), Anchorage to Concrete, and ACI Committee 349, Concrete Nuclear Structures. Concurrent with the ACI 318 effort to develop design provisions, ACI 355 was involved with developing a test method for evaluating the performance of post-installed mechanical anchors in concrete. During the code cycle leading to ACI 318-99, a proposed Appendix D to ACI 318 dealing with the design of anchorages to concrete using both cast-in-place and post-installed mechanical anchors was approved by ACI 318. Final adoption of the proposed appendix awaited ACI 355 approval of a test method for evaluating the performance of post-installed mechanical anchors in concrete under the ACI consensus process.

Since ACI 355 was not able to complete the test method for evaluating post-installed mechanical anchors on time to meet the publication deadlines for the ACI 318-99 code, an attempt was made to process an ACI 318 Appendix D reduced in scope to only cast-in-place anchors (i.e., without post-installed mechanical anchors). However, there was not sufficient time to meet the deadlines established by the International Code Council for submittal of the published ACI 318-99 standard to be referenced in the International Building Code (IBC 2000). As a result, the anchorage to concrete provisions originally intended for ACI 318-99 Appendix D (excluding provisions for post-installed mechanical anchors) were submitted and approved for incorporation into Section 1913 of IBC 2000.

At the end of 2001, ACI Committee 355 completed ACI 355.2-01 titled "Evaluating the Performance of Post-Installed Mechanical Anchors." Availability of ACI 355.2 led the way to incorporating into ACI 318-02 a new Appendix D, Anchoring to Concrete, which provided design requirements for both cast-in-place and post-installed mechanical anchors. As a result, Section 1913 of IBC 2003 references ACI 318 Appendix D. It is anticipated that IBC 2006 Section 1913 will reference ACI 318-05 Appendix D, which in turn adopts ACI 355.2-04 "Qualification of Post-Installed Mechanical Anchors in Concrete" by reference.

It should be noted that ACI 318-05 Appendix D does not address adhesive and grouted anchors. It is anticipated that design provisions for adhesive and grouted anchors will be incorporated into ACI 318-08.

HISTORICAL BACKGROUND OF DESIGN METHODS

The 45-degree cone method used in ACI 349-85 Appendix B and the PCI Design Handbook, Fifth Edition, was developed in the mid 1970's. In the 1980's, comprehensive tests of different types of anchors with various embedment lengths, edge distances, and group effects were performed at the University of Stuttgart on both uncracked and cracked concrete. The Stuttgart test results led to the development of the Kappa (K) method that was introduced in ACI 349 and ACI 355 in the late 1980's. In the early 1990's, the K method was improved, and made user-friendlier at the University of Texas at Austin. This effort resulted in the Concrete Capacity Design (CCD) method. During this same period, an international database was assembled. During the mid 1990's, the majority of the work of ACI Committees 349 and 355 was to evaluate both the CCD method and the 45-degree cone method using the international database of test results. As a result of this evaluation, ACI Committees 318, 349, and 355 proceeded with implementation of the CCD method. The design provisions of ACI 318 Appendix D and ACI 349-01 Appendix B are based on the CCD method. Differences between the CCD method and the 45-degree cone method are discussed below.

GENERAL CONSIDERATIONS

The design of anchorages to concrete must address both strength of the anchor steel and that associated with the embedded portion of the anchors. The lesser of these two strengths will control the design.

The strength of the anchor steel depends on the steel properties and size of the anchor. The strength of the embedded portion of the anchorage depends on its embedment length, strength of the concrete, proximity to other anchors, distance to free edges, and the characteristics of the embedded end of the anchor (headed, hooked, expansion, undercut, etc.).

The primary difference between the ACI 318 Appendix D provisions and those of the 45-degree cone method lies in the calculation of the embedment capacity for concrete breakout (i.e., a concrete cone failure). In the 45-degree cone method, the calculation of breakout capacity is based on a 45-degree concrete cone failure model that results in an equation based on the embedment length squared (h_{ef}^2). The ACI 318 Appendix D provisions account for fracture mechanics and result in an equation for concrete breakout that is based on the embedment length to the 1.5 power ($h_{ef}^{1.5}$). Although the 45-degree concrete cone failure model gives conservative results for anchors with $h_{ef} \leq 6$ in., the ACI 318 Appendix D provisions have been shown to give a better prediction of embedment strength for both single anchors and for anchors influenced by edge and group effects.

In addition to better prediction of concrete breakout strength, the ACI 318 Appendix D provisions simplify the calculation of the effects of anchor groups and edges by using a rectangular area bounded by $1.5h_{ef}$ from each anchor and free edges rather than the overlapping circular cone areas typically used in the 45-degree cone method.

DISCUSSION OF DESIGN PROVISIONS

The following provides a section-by-section discussion of the design provisions of ACI 318-05 Appendix D. Section, equation, and figure numbers in the following discussion and examples refer to those used in ACI 318-05 Appendix D. Note that notation for Appendix D is presented in 2.1 of ACI 318.

D.1 DEFINITIONS

The definitions presented are generally self-explanatory and are further explained in the text and figures of Appendix D. The following tables are provided as an aid to the designer in determining values for many of the variables:

Table 34-1: This table provides information on the types of materials typically specified for cast-in-place anchor applications. The table provides values for specified tensile strength f_{uta} and specified yield strength f_{ya} as well as the elongation and reduction in area requirements necessary to determine if a material should be considered as a brittle or ductile steel element. As shown in Table 34-1, all typical anchor materials satisfy the ductile steel element requirements of D.1. When using cast-in-place anchor materials not given in Table 34-1, the designer should refer to the appropriate material specification to be sure the material falls within the ductile steel element definition. Some high strength materials may not meet these requirements and must be considered as brittle steel elements.

Table 34-2: This table provides information on the effective cross-sectional area A_{se} and bearing area A_{brg} for threaded cast-in-place anchors up to 2 in. in diameter.

Table 34-3: This table provides a fictitious sample information table for post-installed mechanical anchors that have been tested in accordance with ACI 355.2. This type of table will be available from manufactures that have tested their products in accordance with ACI 355.2. The table provides all of the values necessary for design of a particular post-installed mechanical anchor. The design of post-installed mechanical anchors must be based on this type of table unless values assumed in the design are specified in the project specifications (e.g., the pullout strength N_p).

As a further commentary on the five percent fractile in D.1 – Definitions, the five percent fractile is used to determine the nominal embedment strength of the anchor. It represents a value such that if 100 anchors are tested there is a 90% confidence that 95 of the anchors will exhibit strengths higher than the five percent fractile value. The five percent fractile is analogous to the use of f'_c for concrete strength and f_{ya} for steel strength in the nominal strength calculations in other parts of the ACI 318 code. For example, ACI 318 Section 5.3 requires that the required average compressive strength of the concrete f'_{cr} be statistically greater than the specified value of f'_c used in design calculations. For steel, f_{ya} represents the specified yield strength of the material. Since ASTM specifications give the minimum specified yield strength, the value of f_{ya} used in design is in effect a zero percent fractile (i.e., the designer is ensured that the actual steel used will have a yield value higher than the

Table 34-1 Properties of Cast-in-Place Anchor Materials

Material specification[1]	Grade or type	Diameter (in.)	Tensile strength, for design f_{uta} (ksi)	Tensile strength, min. (ksi)	Yield strength, min.		Elongation, min.		Reduction of area, min., (%)
					ksi	method	%	length	
AWS D1.1[2]	B	1/2 – 1	65	65	51	0.2%	20	2"	50
ASTM A 307[3]	A	≤ 4	60	60	—	—	18	2"	—
	C	≤ 4	58	58-80	36	—	23	2"	—
ASTM A 354[4]	BC	≤ 4	125	125	109	0.2%	16	2"	50
	BD	≤ 4	125	150	130	0.2%	14	2"	40
ASTM A 449[5]	1	≤ 1	120	120	92	0.2%	14	4D	35
		1 – 1-1/2	105	105	81	0.2%	14	4D	35
		> 1-1/2	90	90	58	0.2%	14	4D	35
ASTM F 1554[6]	36	≤ 2	58	58-80	36	0.2%	23	2"	40
	55	≤ 2	75	75-95	55	0.2%	21	2"	30
	105	≤ 2	125	125-150	105	0.2%	15	2"	45

Notes:

1. The materials listed are commonly used for concrete anchors. Although other materials may be used (e.g., ASTM A 193 for high temperature applications, ASTM A 320 for low temperature applications), those listed are preferred for normal use. Structural steel bolting materials such as ASTM A 325 and ASTM A 490 are not typically available in the lengths needed for concrete anchorage applications.
2. *AWS D1.1-04 Structural Welding Code - Steel* - This specification covers welded headed studs or welded hooked studs (unthreaded). None of the other listed specifications cover welded studs.
3. *ASTM A 307-04 Standard Specification for Carbon Steel Bolts and Studs, 60,000 psi Tensile Strength* - This material is commonly used for concrete anchorage applications. Grade A is headed bolts and studs. Grade C is nonheaded bolts (studs), either straight or bent, and is equivalent to ASTM A 36 steel. Note that although a reduction in area requirement is not provided, A 307 may be considered a ductile steel element. Under the definition of "Ductile steel element" in D.1, the code states: "A steel element meeting the requirements of ASTM A 307 shall be considered ductile."
4. *ASTM A 354-04 Standard Specification for Quenched and Tempered Alloy Steel Bolts, Studs, and Other Externally Threaded Fasteners* - The strength of Grade BD is equivalent to ASTM A 490.
5. *ASTM A 449-04b Standard Specification for Quenched and Tempered Steel Bolts and Studs* - This specification is referenced by ASTM A 325 for "equivalent" anchor bolts.
6. *ASTM F 1554-04 Standard Specification for Anchor Bolts* - This specification covers straight and bent, headed and headless, anchor bolts in three strength grades. Anchors are available in diameters ≤ 4 in. but reduction in area requirements vary for anchors > 2 in.

Table 34-2 Dimensional Properties of Threaded Cast-in-Place Anchors

Anchor Diameter (d_o) (in.)	Gross Area of Anchor (in.[2])	Effective Area of Anchor (A_{se}) (in.[2])	Bearing Area of Heads and Nuts (A_{brg}) (in.[2])			
			Square	Heavy Square	Hex	Heavy Hex
0.250	0.049	0.032	0.142	0.201	0.117	0.167
0.375	0.110	0.078	0.280	0.362	0.164	0.299
0.500	0.196	0.142	0.464	0.569	0.291	0.467
0.625	0.307	0.226	0.693	0.822	0.454	0.671
0.750	0.442	0.334	0.824	1.121	0.654	0.911
0.875	0.601	0.462	1.121	1.465	0.891	1.188
1.000	0.785	0.606	1.465	1.855	1.163	1.501
1.125	0.994	0.763	1.854	2.291	1.472	1.851
1.250	1.227	0.969	2.228	2.773	1.817	2.237
1.375	1.485	1.160	2.769	3.300	2.199	2.659
1.500	1.767	1.410	3.295	3.873	2.617	3.118
1.750	2.405	1.900	—	—	—	4.144
2.000	3.142	2.500	—	—	—	5.316

Table 34-3 Sample Table of Anchor Data for a Fictitious Post-Installed Torque-Controlled Mechanical Expansion Anchor as Presumed Developed from Qualification Testing in Accordance with ACI 355.2-04.

(Note: Fictitious data for example purposes only – data are not from a real anchor)

Anchor system is qualified for use in both cracked and uncracked concrete in accordance with test program of Table 4.2 of *ACI 355.2-04*. The material, ASTM F1554 grade 55, meets the ductile steel element requirements of *ACI 318-05 Appendix D* (tensile test elongation of at least 14 percent and reduction in area of at least 30 percent).

Characteristic	Symbol	Units	Nominal anchor diameter							
Installation information										
Outside diameter	d_o	*in.*	3/8		_		5/8		_	
Effective embedment depth	h_{ef}	*in.*	1.75		2.5		3		3.5	
			2.75		3.5		4.5		5	
			4.5		5.5		6.5		8	
Installation torque	T_{inst}	*ft-lb*	30		65		100		175	
Minimum edge distance	c_{min}	*in.*	1.75		2.5		3		3.5	
Minimum spacing	s_{min}	*in.*	1.75		2.5		3		3.5	
Minimum concrete thickness	h_{min}	*in.*	$1.5h_{ef}$		$1.5h_{ef}$		$1.5h_{ef}$		$1.5h_{ef}$	
Critical edge distance @ h_{min}	c_{ac}	*in.*	2.1		3.0		3.6		4.0	
Anchor data										
Anchor material	ASTM F 1554 Grade 55 (meets ductile steel element requirements)									
Category number	1, 2, or 3	—	2		2		1		1	
Yield strength of anchor steel	f_{ya}	*psi*	55,000		55,000		55,000		55,000	
Ultimate strength of anchor steel	f_{uta}	*psi*	75,000		75,000		75,000		75,000	
Effective tensile stress area	A_{se}	*in.*^{2}	0.0775		0.142		0.226		0.334	
Effective shear stress area	A_{se}	*in.*^{2}	0.0775		0.142		0.226		0.334	
Effectiveness factor for uncracked concrete	k_{uncr}	—	24		24		24		24	
Effectiveness factor for cracked concrete used for ACI 318 design	k_c*	—	17		17		17		17	
$\psi_{c,N}$ for ACI 318 design in cracked concrete	$\psi_{c,N}$*	—	1.0		1.0		1.0		1,0	
$\psi_{c,N} = k_{uncr}/k_{cr}$ for ACI 318 design in uncracked concrete	$\psi_{c,N}$*	—	1.4		1.4		1.4		1.4	
Pullout or pull-through resistance from tests	N_p	*lb*	h_{ef}	N_p	h_{ef}	N_p	h_{ef}	N_p	h_{ef}	N_p
			1.75	1,354	2.5	2,312	3	4,469	3.5	5,632
			2.75	2,667	3.5	3,830	4.5	8,211	5	9,617
			4.5	5,583	5.5	7,544	6.5	14,254	8	19,463
Tension resistance of single anchor for seismic loads	N_{eq}	*lb*	1.75	903	2.5	1,541	3	2,979	3.5	3,755
			4.5	3,722	5.5	5,029	6.5	9,503	8	12,975
Shear resistance of single anchor for seismic loads	V_{eq}	*lb*	2,906		5,321		8,475		12,543	
Axial stiffness in service load range	β	*lb/in.*	55,000		57,600		59,200		62,000	
Coefficient of variation for axial stiffness in service load range.	ν	%	12		11		10		9	

*These are values used for k_c and $\psi_{c,N}$ in ACI 318 for anchors qualified for use only in both cracked and uncracked concrete.

minimum specified value). All embedment strength calculations in Appendix D are based on a nominal strength calculated using 5 percent fractile values (e.g., the k_c values used in calculating basic concrete breakout strength are based on the 5 percent fractile).

D.2 Scope

These provisions apply to cast-in-place and post-installed mechanical anchors (such as those illustrated in Fig. RD.1) that are used to transmit structural loads between structural elements and safety related attachments to structural elements. The type of anchors included are cast-in-place headed studs, headed bolts, hooked rods (J and L bolts), and post-installed mechanical anchors that have met the anchor assessment requirements of ACI 355.2. Other types of cast-in-place anchors (e.g., specialty inserts) and post-installed anchors (e.g., adhesive, grouted, and pneumatically actuated nails or bolts) are currently excluded from the scope of Appendix D as well as post-installed mechanical anchors that have not met the anchor assessment requirements of ACI 355.2. As noted in D.2.4, these design provisions do not apply to anchorages loaded with high cycle fatigue and impact loads.

D.3 GENERAL REQUIREMENTS

The analysis methods prescribed in D.3 to determine loads on individual anchors in multiple anchor applications depend on the type of loading, rigidity of the attachment base plate, and the embedment of the anchors.

For multiple-anchor connections loaded concentrically in pure tension, the applied tensile load may be assumed to be evenly distributed among the anchors if the base plate has been designed so as not to yield. Prevention of yielding in the base plate will ensure that prying action does not develop in the connection.

For multiple-anchor connections loaded with an eccentric tension load or moment, distribution of loads to individual anchors should be determined by elastic analysis unless calculations indicate that sufficient ductility exists in the embedment of the anchors to permit a redistribution of load among individual anchors. If sufficient ductility is provided, a plastic design approach may be used. The plastic design approach requires ductile steel anchors sufficiently embedded so that embedment failure will not occur prior to a ductile steel failure. The plastic design approach assumes that the tension load (either from eccentric tension or moment) is equally distributed among the tension anchors. For connections subjected to moment, the plastic design approach is analogous to multiple layers of flexural reinforcement in a reinforced concrete beam. If the multiple layers of steel are adequately embedded and are a sufficient distance from the neutral axis of the member, they may be considered to have reached yield.

For both the elastic and plastic analysis methods of multiple-anchor connections subjected to moment, the exact location of the compressive resultant cannot be accurately determined by traditional concrete beam analysis methods. This is true for both the elastic linear stress-strain method (i.e., the transformed area method) and the ACI 318 stress block method since plane sections do not remain plane. For design purposes, the compression resultant from applied moment may be assumed to be located at the leading edge of the compression element of the attached member unless base plate stiffeners are provided. If base plate stiffeners are provided, the compressive resultant may be assumed to be located at the leading edge of the base plate.

Sections D.3.3.1 to D.3.3.5 provide special requirements for anchor design that includes seismic loads. Appendix D should not be used for the design of anchors in plastic hinge zones where high levels of cracking and spalling may be expected due to a seismic event. The Appendix D design provisions and the anchor evaluation criteria of ACI 355.2 are based on cracks that might occur normally in concrete (the cracked concrete tests and simulated seismic tests in ACI 355.2 are based on anchor performance in cracks from 0.012 in. to 0.020 in.). In regions of moderate or high seismic risk, or for structures assigned to intermediate or high seismic performance or design categories (see Table 1-3 in Part 1 for equivalent terminology used in building codes) all values for ϕN_n and ϕV_n must be reduced by multiplying by an additional factor of 0.75. Further, the strength of the connection must be controlled by the strength of ductile steel elements and not the embedment strength or the strength of brittle steel elements unless the structural attachment has been designed to yield at a load no greater than the design strength of the anchors, reduced by the factor of 0.75. Section RD.3.3 provides a detailed discussion of these requirements.

D.4 GENERAL REQUIREMENTS FOR STRENGTH OF ANCHORS

This section provides a general discussion of the failure modes that must be considered in the design of anchorages to concrete. The section also provides strength reduction factors, ϕ, for each type of failure mode. The failure modes that must be considered include those related to the steel strength and those related to the strength of the embedment.

Failure modes related to steel strength are simply tensile failure [Fig. RD.4.1(a)(i)] and shear failure [Fig. RD.4.1(b)(i)] of the anchor steel. Anchor steel strength is relatively easy to compute but typically does not control the design of the connection unless there is a specific requirement that the steel strength of a ductile steel element must control the design.

Embedment failure modes that must be considered are illustrated in Appendix D Fig. RD.4.1. They include:

- concrete breakout - a concrete cone failure emanating from the embedded end of tension anchors [Fig. RD.4.1(a)(iii)] or from the entry point of shear anchors located near an edge [Fig. RD.4.1(b)(iii)]
- pullout - a straight pullout of the anchor such as might occur for an anchor with a small head [Fig. RD.4.1(a)(ii)]
- side-face blowout - a spalling at the embedded head of anchors located near a free edge [Fig. RD.4.1(a)(iv)]
- concrete pryout - a shear failure mode that can occur with a short anchor popping out a wedge of concrete on the back side of the anchor [Fig. RD.4.1(b)(ii)]
- splitting - a tensile failure mode related to anchors placed in relatively thin concrete members [Fig. RD.4.1(a)(v)]

As noted in D.4.2, the use of any design model that results in predictions of strength that are in substantial agreement with test results is also permitted by the general requirements section. If the designer feels that the 45-degree cone method, or any other method satisfy this requirement he or she is permitted to use them. If not, the design provisions of the remaining sections of Appendix D should be used provided the anchor diameter does not exceed 2 in. and the embedment length does not exceed 25 in. These restrictions represent the upper limits of the database that the Appendix D design provisions are based on.

In the selection of the appropriate ϕ related to embedment failure modes, the presence of supplementary reinforcement designed to tie a potential failure prism to the structural member determines whether the ϕ for Condition A or Condition B applies. For the case of cast-in-place anchors loaded in shear directed toward a free edge, the supplementary reinforcement required for Condition A might be achieved by the use of hairpin reinforcement. It should be noted that for determining pullout strength for a single anchor, N_{pn}, and pryout strengths for a single anchor in shear, V_{cp}, or a group V_{cpg}, D.4.4(c) indicates that Condition B applies in all cases regardless of whether supplementary reinforcement is provided or not. In the case of post-installed anchors it is doubtful that this type of reinforcement will have been provided and Condition B will normally apply. The selection of ϕ for post-installed anchors also depends on the anchor category determined from the ACI 355.2 product evaluation tests. As part of the ACI 355.2 product evaluation tests, product reliability tests (i.e., sensitivity to installation variables) are performed and the results used to establish the appropriate category for the anchor. Since each post-installed mechanical anchor may be assigned a different category, product data tables resulting from ACI 355.2 testing should be referred to. Example data are shown in Table 34-3.

Table 34-4 summarizes the strength reduction factors, ϕ, to be used with the various governing conditions depending upon whether the load combinations of 9.2 or Appendix C are used.

D.5 DESIGN REQUIREMENTS FOR TENSILE LOADING

Methods to determine the nominal tensile strength as controlled by steel strength and embedment strength are presented in the section on tensile loading. The nominal tensile strength of the steel is based on the specified tensile strength of the steel Eq. (D-3). The nominal tensile strength of the embedment is based on (1) concrete breakout strength, Eq. (D-4) for single anchors or Eq. (D-5) for groups of anchors, (2) pullout strength, Eq. (D-14), or (3) side-face blowout strength, Eq. (D-17) for single anchors or Eq. (D-18) for groups. When combined with the appropriate strength reduction factors from D.4.4 or D.4.5, the smallest of these strengths will control the design tensile strength of the anchorage.

D.5.1 Steel Strength of Anchor in Tension

The tensile strength of the steel, N_{sa}, is determined from Eq. (D-3) using the effective cross-sectional area of the anchor A_{se} and the specified tensile strength of the anchor steel f_{uta}.

For cast-in-place anchors (i.e., threaded anchors, headed studs and hooked bars), the effective cross-sectional area of the anchor A_{se} is the net tensile stress area for threaded anchors and the gross area for headed studs that are welded to a base plate. These areas are provided in Table 34-2. For anchors of unusual geometry, the nominal steel strength may be taken as the lower 5% fractile of test results. For post-installed mechanical anchors the effective cross-sectional area of the anchor A_{se} must be determined from the results of the ACI 355.2 product evaluation tests. Example data are shown in Table 34-3.

The value of f_{uta} used in Eq. (D-3) is limited to $1.9f_{ya}$ or 125,000 psi. The limit of $1.9f_{ya}$ is intended to ensure that the anchor does not yield under service loads and typically is applicable only to stainless steel materials. The limit of 125,000 psi is based on the database used in developing the Appendix D provisions. Table 34-1 provides values for f_{ya} and f_{uta} for typical anchor materials. Note that neither of the limits applies to the typical anchor materials given in Table 34-1. For anchors manufactured according to specifications having a range for specified tensile strength, f_{uta} (e.g., ASTM F 1554), the lower limit value should be used to calculate the design strength. For post-installed mechanical anchors, both f_{ya} and f_{uta} must be determined from the results of the ACI 355.2 product evaluation tests. Example data are shown in Table 34-3.

D.5.2 Concrete Breakout Strength of Anchor in Tension

Figure RD.4.1(a)(iii) shows a typical concrete breakout failure (i.e., concrete cone failure) for a single headed cast-in-place anchor loaded in tension. Eq. (D-4) gives the concrete breakout strength for a single anchor, N_{cb}, while Eq. (D-5) gives the concrete breakout strength for a group of anchors in tension, N_{cbg}.

The individual terms in Eq. (D-4) and Eq. (D-5) are discussed below:

N_b: The basic concrete breakout strength for a single anchor located away from edges and other anchors (N_b) is given by Eq. (D-7) or Eq. (D-8). As previously noted, the primary difference between these equations and those of the 45-degree concrete cone method is the use of $h_{ef}^{1.5}$ in Eq. (D-7) [or alternatively $h_{ef}^{5/3}$ for anchors with $h_{ef} \geq 11$ in. in Eq. (D-8)] rather than h_{ef}^2. The use of $h_{ef}^{1.5}$ accounts for fracture mechanics principles and can be thought of as follows:

$$N_b = \frac{k\sqrt{f'_c}\,h_{ef}^2}{h_{ef}^{0.5}}\left[\frac{\text{general 45° concrete cone equation}}{\text{modification factor for fracture mechanics}}\right]$$

Resulting in:

$$N_b = k_c\sqrt{f'_c}\,h_{ef}^{1.5} \qquad \textit{Eq. (D-7)}$$

The fracture mechanics approach accounts for the high tensile stresses that exist at the embedded head of the anchor while other approaches (such as the 45-degree concrete cone method) assume a uniform distribution of stresses over the assumed failure surface.

Table 34-4 Strength Reduction Factors for Use with Appendix D

Strength Governed by	Strength Reduction Factor, ϕ, for use with Load Combinations in Section 9.2		Appendix C	
Ductile steel element				
Tension, N_{sa}	0.75		0.80	
Shear, V_{sa}	0.65		0.75	
Brittle steel element				
Tension, N_{sa}	0.65		0.70	
Shear, V_{sa}	0.60		0.65	
Concrete	Condition A	Condition B	Condition A	Condition B
Shear				
Breakout, V_{cb} and V_{cbg}	0.75	0.70	0.85	0.75
Pryout, V_{cp} and V_{cpg}	0.70	0.70	0.75	0.75
Tension				
Cast-in headed studs, headed bolts, or hooked bolts				
Breakout and side face blowout, N_{cb}, N_{cbg}, N_{sb} and N_{sbg}	0.75	0.70	0.85	0.75
Pullout, N_{pn}	0.70	0.70	0.75	0.75
Post-installed anchors with category determined per ACI 355.2				
Category 1 (low sensitivity to installation and high reliability)				
Breakout and side face blowout, N_{cb}, N_{cbg}, N_{sb} and N_{sbg}	0.75	0.65	0.85	0.75
Pullout, N_{pn}	0.65	0.65	0.75	0.75
Category 2 (med. sensitivity to installation and med. reliability)				
Breakout and side face blowout, N_{cb}, N_{cbg}, N_{sb} and N_{sbg}	0.65	0.55	0.75	0.65
Pullout, N_{pn}	0.55	0.55	0.65	0.65
Category 3 (high sensitivity to installation and low reliability)				
Breakout and side face blowout, N_{cb}, N_{cbg}, N_{sb} and N_{sbg}	0.55	0.45	0.65	0.55
Pullout, N_{pn}	0.45	0.45	0.55	0.55

The numeric constant k_c of 24 in Eq. (D-7) [or k_c of 16 in Eq. (D-8) if $h_{ef} \geq 11$ in.] is based on the 5% fractile of test results on headed cast-in-place anchors in cracked concrete. These k_c values must be used unless higher values of k_c are justified by ACI 355.2 product-specific tests. The value of k_c must not exceed 24. Note that the crack width used in tests to establish these k_c values was 0.012 in. If larger crack widths are anticipated, confining reinforcement to control crack width to about 0.012 in. should be provided or special testing in larger cracks should be performed.

$\dfrac{A_{Nc}}{A_{Nco}}$: This factor accounts for adjacent anchors and/or free edges. For a single anchor located away from free edges, the A_{Nco} term is the projected area of a 35-degree failure plane, measured relative to the surface of the concrete, and defined by a square with the sides $1.5h_{ef}$ from the centerline of the anchor [Fig. RD.5.2.1(a)]. The A_{Nc} term is a rectilinear projected area of the 35-degree failure plane at the surface of the concrete with sides $1.5h_{ef}$ from the centerline of the anchor(s) as limited by adjacent anchors and/or free edges. The definition of A_{Nc} is shown in Fig. RD.5.2.1(b). For a single anchor located at least $1.5h_{ef}$ from the closest free edge and $3h_{ef}$ from other anchors, A_{Nc} equals A_{Nco}.

Where a plate or washer is used to increase the bearing area of the head of an anchor, $1.5h_{ef}$ can be measured from the effective perimeter of the plate or washer where the effective perimeter is defined in D.5.2.8. Where a plate or washer is used, the projected area A_{Nc} can be based on $1.5h_{ef}$ measured from the effective perimeter of the plate or washer where the effective perimeter is defined in D.5.2.8 and shown in Fig. 34-1.

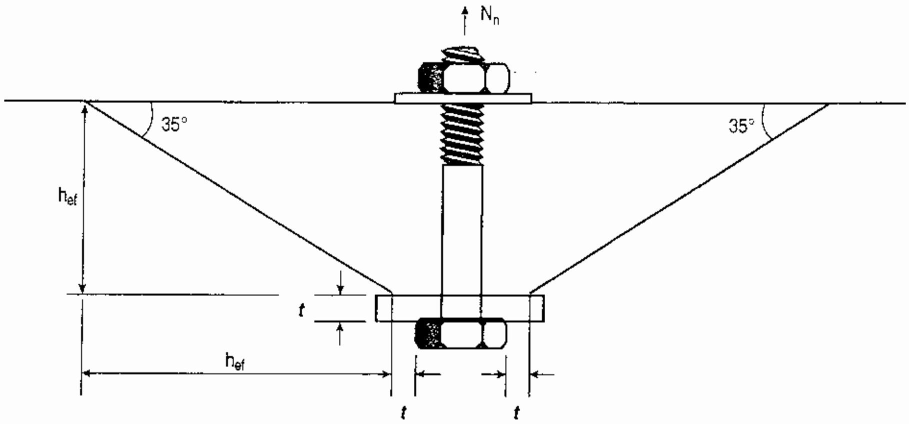


Figure 34-1 Effect of Washer Plate on Projected Area of Concrete Breakout

$\psi_{ec,N}$: This factor is applicable when multiple rows of tension anchors are present and the elastic design approach is used. In this case, the individual rows of tension anchors are assumed to carry different levels of load with the centerline of action of the applied tension load at an eccentricity (e'_N) from the centroid of the tension anchors. If the plastic design approach is used, all tension anchors are assumed to carry the same load and the eccentricity factor, $\psi_{ec,N}$, is taken as 1.0.

$\psi_{ed,N}$: This factor accounts for the non-uniform distribution of stresses when an anchor is located near a free edge of the concrete that are not accounted for by the $\frac{A_{Nc}}{A_{Nco}}$ term.

$\psi_{c,N}$: This factor is taken as 1.0 if cracks in the concrete are likely to occur at the location of the anchor(s). If calculations indicate that concrete cracking is not likely to occur under service loads (e.g., $f_t < f_r$), then $\psi_{e,N}$ may be taken as 1.25 for cast-in-place anchors or 1.4 for post-installed anchors.

$\psi_{cp,N}$: This factor is taken as 1.0 except when the design assumes uncracked concrete, uses post-installed anchors, and has a free edge near the anchors.

D.5.3 Pullout Strength of Anchor in Tension

A schematic of the pullout failure mode is shown in Fig. RD.4.1(a)(ii). The pullout strength of cast-in-place anchors is related to the bearing area at the embedded end of headed anchors, A_{brg}, and the properties of embedded hooks (e_h and d_o) for J-bolts and L-bolts. Obviously, if an anchor has no head or hook it will simply pull out of the concrete and not be able to achieve the concrete breakout strength associated with a full concrete cone failure (D.5.2). With an adequate head or hook size, pullout will not occur and the concrete breakout strength can be achieved. Eq. (D-14) provides the general requirement for pullout while Eq. (D-15) and Eq. (D-16) provide the specific requirements for headed and hooked anchors, respectively.

For headed anchors, the bearing area of the embedded head (A_{brg}) is the gross area of the head less the gross area of the anchor shaft (i.e., not the area of the embedded head). Washers or plates with an area larger than the head of an anchor can be used to increase the bearing area, A_{brg}, thus increasing the pullout strength (see D.5.2.8). In regions of moderate or high seismic risk, or for structures assigned to immediate or high seismic performance or design categories, where a headed bolt is being designed as a ductile steel element according to D.3.3.4, it may be necessary to use a bolt with a larger head or a washer in order to increase the design pullout strength, ϕN_{pn}, to assure that yielding of the steel takes place prior failure of the embedded portion of the anchor. Table 34-2 provides values for A_{brg} for standard bolt heads and nuts. Tables 34-5A, B and C can be used to quickly determine scenarios where the head of a bolt will not provide adequate pullout strength and will need to be increased in size.

For J-bolts and L-bolts, the minimum length of the hook measured from the inside surface of the shaft of the anchor is $3d_o$ while the maximum length for calculating pullout strength by Eq. (D-16) is $4.5d_o$. For other than high strength concrete, it is difficult to achieve design pullout strength of a hooked bolt that is equal to or greater than the design tensile strength of the steel. For example, a 1/2 in. diameter hooked bolt with the maximum hook length of $4.5d_o$ permitted in evaluating pullout strength in Eq. (D-16) requires that f'_c be at least 8700 psi to develop the design tensile strength of an ASTM A 307, Grade C, or ASTM F 1554, Grade 36 anchor (f_{uta} = 58,000 psi). This essentially prohibits the use of hooked bolts in many applications subject to seismic tensile loading due to the limitations of D.3.3.4 that the anchor strength must be governed by the ductile anchor steel.

For post-installed mechanical anchors, the value for the pullout strength, N_p, must be determined from the results of the ACI 355.2 product evaluation tests. Example data are shown in Table 34-3.

D.5.4 Concrete Side-Face Blowout Strength of Headed Anchor in Tension

The side-face blowout strength is associated with the lateral pressure that develops around the embedded end of headed anchors under load. Where the minimum edge distance for a single headed anchor is less than 0.4 h_{ef}, side-face blowout must be considered using Eq. (D-17). If an orthogonal free edge (i.e., an anchor in a corner) is located less than three times the distance from the anchor to the nearest edge) then an additional reduction factor of $[(1+c_{a2}/c_{a1})/4]$, where c_{a1} is the distance to the nearest edge and c_{a2} is the distance to the orthogonal edge, must be applied to Eq. (D-17).

For multiple anchor groups, the side-face blowout strength is the given by Eq. (D-18) provided the spacing between individual anchors parallel to a free edge is greater than or equal to six times the distance to the free edge. If the spacing of the anchors in the group is less than six times the distance to the free edge, Eq. (D-18) must be used.

D.6 DESIGN REQUIREMENTS FOR SHEAR LOADING

Methods to determine the nominal shear strength as controlled by steel strength and embedment strength are specified in D.6. The nominal shear strength of the steel is based on the specified tensile strength of the steel using Eq. (D-19) for headed studs, Eq. (D-20) for headed and hooked bolts, and for post-installed anchors. The nominal shear strength of the embedment is based on concrete breakout strength Eq. (D-21) for single anchors or Eq. (D-22) for groups of anchors, or pryout strength Eq. (D-29) for single anchors or Eq. (D-30) for groups. When combined with the appropriate strength reduction factors from D.4.4, the smaller of these strengths will control the design shear strength of the anchorage.

D.6.1 Steel Strength of Anchor in Shear

For cast-in-place anchors, the shear strength of the steel is determined from Eq. (D-19) for headed studs and Eq. (D-20) for headed and hooked bolts using the effective cross-sectional area of the anchor, A_{se}, and the specified tensile strength of the anchor steel, f_{uta}. For post-installed mechanical anchors, the shear strength of the steel is determined from Eq. (D-20) using the effective cross-sectional area of the anchor, A_{se}, and the specified tensile strength of the anchor steel, f_{uta}, unless the ACI 355.2 anchor qualification report provides a value for V_{sa}.

For cast-in-place anchors (i.e., headed anchors, headed studs and hooked bars), the effective cross-sectional area of the anchor (A_{se}) is the net tensile stress area for threaded anchors and the gross area for headed studs that are welded to a base plate. These areas are provided in Table 34-2. If the threads of headed anchors, L-, or J-bolts are located well above the shear plane (at least two diameters) the gross area of the anchor may be used for shear. For anchors of unusual geometry, the nominal steel strength may be taken as the lower 5% fractile of test results. For post-installed mechanical anchors the effective cross-sectional area of the anchor, A_{se}, or the nominal shear strength, V_{sa}, must be determined from the results of the ACI 355.2 product evaluation tests. Example data are shown in Table 34-3.

The value of f_{uta} used in Eq. (D-19) and Eq. (D-20) is limited to $1.9f_{ya}$ or 125,000 psi. The limit of $1.9f_{ya}$ is intended to ensure that the anchor does not yield under service loads and typically is applicable only to stainless steel materials. The limit of 125,000 psi is based on the database used in developing the Appendix D provisions. Table 34-1 provides values for f_{ya} and f_{uta} for typical anchor materials. Note that neither of the limits applies to the typical anchor materials given in Table 34-1. For anchors manufactured according to specifications having a range for specified tensile strength, f_{uta} (e.g., ASTM F 1554), the lower limit value should be used to calculate the design strength. For post-installed mechanical anchors, f_{ya} and f_{uta} must be determined from the results of the ACI 355.2 product evaluation tests. Example data are shown in Table 34-3.

When built-up grout pads are present, the nominal shear strength values given by Eq. (D-19) and Eq. (D-20) must be reduced by 20% to account for the flexural stresses developed in the anchor if the grout pad fractures upon application of the shear load.

D.6.2 Concrete Breakout Strength of Anchor in Shear

Fig. RD.4.1(b)(iii) shows typical concrete breakout failures for anchors loaded in shear directed toward a free edge. Eq. (D-21) gives the concrete breakout strength for a single anchor while Eq. (D-22) gives the concrete breakout strength for groups of anchors in shear. In cases where the shear is directed away from the free edge, the concrete breakout strength in shear need not be considered.

The individual terms in Eq. (D-21) and Eq. (D-22) are discussed below:

V_b : The basic concrete breakout strength for a single anchor in cracked concrete loaded in shear, directed toward a free edge (V_b) without any other adjacent free edges or limited concrete thickness is given by Eq. (D-24) for typical bolted connections and Eq. (D-25) for connections with welded studs or other anchors welded to the attached base plate. The primary difference between these equations and those using the 45-degree concrete cone method is the use of $c_{a1}{}^{1.5}$ rather than $c_{a1}{}^2$. The use of $c_{a1}{}^{1.5}$ accounts for fracture mechanics principles in the same way that $h_{ef}{}^{1.5}$ does for tension anchors. The fracture mechanics approach accounts for the high tensile stresses that exist in the concrete at the point where the anchor first enters the concrete.

ℓ_e, d_o : The terms involving ℓ_e and d_o in Eq. (D-24) and Eq. (D-25) relate to the shear stiffness of the anchor. A stiff anchor is able to distribute the applied shear load further into the concrete than a flexible anchor.

$\frac{A_{Vc}}{A_{Vco}}$: This factor accounts for adjacent anchors, concrete thickness, and free edges. For a single anchor in a thick concrete member with shear directed toward a free edge, the A_{Vco} term is the projected area on the side of the free edge of a 35-degree failure plane radiating from the point where the anchor first enters the concrete and directed toward the free edge [see Fig. RD.6.2.1(a)]. The A_{Vc} term is a rectilinear projected area of the 35-degree failure plane on the side of the free edge with sides 1.5 h_{ef} from the point where the anchor first enters the concrete as limited by adjacent anchors, concrete thickness and free edges. The definition of A_{Vc} is shown in Fig. RD.6.2.1(b).

$\psi_{ec,V}$: This factor applies when the applied shear load does not act through the centroid of the anchors loaded in shear [see Fig. RD.6.2.5]

$\psi_{ed,V}$: This factor accounts for the non-uniform distribution of stresses when an anchor is located in a corner that is not accounted for by the $\frac{A_{Vc}}{A_{Vco}}$ term [see Fig. RD.6.2.1(d)].

$\psi_{c,V}$: This factor is taken as 1.0 if cracks in the concrete are likely to occur at the location of the anchor(s) andno supplemental reinforcement has been provided. If calculations indicate that concrete cracking is not likely to occur (e.g., $f_t < f_r$ at service loads), then $\psi_{c,V}$ may be taken as 1.4. Values of $\psi_{c,V} > 1.0$ may be used if cracking at service loads is likely, provided No. 4 bar or greater edge reinforcement is provided (see D.6.2.7).

D.6.3 Concrete Pryout Strength of Anchor in Shear

The concrete pryout strength of an anchor in shear may control when an anchor is both short and relatively stiff. Fig. RD.4.1(b)(ii) shows this failure mode. As a mental exercise, this failure mode may be envisioned by thinking of a No. 8 bar embedded 2 in. in concrete with 3 ft. of the bar sticking out. A small push at the top of the bar will cause the bar to "pryout" of the concrete.

D.7 INTERACTION OF TENSILE AND SHEAR FORCES

The interaction requirements for tension and shear are based on a trilinear approximation to the following interaction equation (see Fig. RD.7):

$$\left[\frac{N_{ua}}{\phi N_n}\right]^{\frac{5}{3}} + \left[\frac{V_{ua}}{\phi V_n}\right]^{\frac{5}{3}} = 1$$

In the trilinear simplification, D.7.1 permits the full value of ϕN_n if $V_{ua} \leq 0.2\ \phi V_n$ and Section D.7.2 permits the full value of ϕV_n if $N_{ua} \leq 0.2\phi N_n$. If both of these conditions are not satisfied, the linear interaction of Eq. (D-31) must be used.

The most important aspect of the interaction provisions is that both ϕN_n and ϕV_n are the smaller of the anchor strengths as controlled by the anchor steel or the embedment. Tests have shown that the interaction relationship is valid whether steel strength or embedment strength controls for ϕN_n or ϕV_n.

D.8 REQUIRED EDGE DISTANCES, SPACINGS, AND THICKNESSES TO PRECLUDE SPLITTING FAILURE

Section D.8 provides minimum edge distance, spacing, and member thickness requirements to preclude a possible splitting failure of the structural member. For untorqued cast-in-place anchors (e.g., headed studs or headed bolts that are not highly preloaded after the attachment is installed), the minimum edge distance and member thickness is controlled by the cover requirements of 7.7 and the minimum anchor spacing is $4d_o$. For torqued cast-in-place anchors (e.g., headed bolts that are highly pre-loaded after the attachment is installed), the minimum edge distance and spacing is $6d_o$ and the member thickness is controlled by the cover requirements of 7.7.

Post-installed mechanical anchors can exert large lateral pressures at the embedded expansion device during installation that can lead to a splitting failure. Minimum spacing, edge distance, and member thickness requirements for post-installed anchors should be determined from the product-specific test results developed in the ACI 355.2 product evaluation testing. Example data are shown in Table 34-3. In the absence of the product-specific test results, the following should be used: a minimum anchor spacing of $6d_o$; a minimum edge distance of $6d_o$ for undercut anchors, $8d_o$ for torque-controlled anchors, and $10d_o$ for displacement controlled anchors; and a minimum member thickness of $1.5h_{ef}$ but need not exceed h_{ef} plus 4 in. Examples of each of these types of anchors are shown in ACI 355.2. In all cases, the minimum edge distance and member thickness should meet the minimum cover requirements of 7.7.

For untorqued anchors, D.8.4 provides a method to use a large diameter anchor nearer to an edge or with closer spacing than that required by D.8.1 to D.8.3. In this case, a fictitious anchor diameter d'_o is used in evaluating the strength of the anchor and in determining the minimum edge and spacing requirements.

For post-installed mechanical anchors, D.8.6 provides conservative default values for the critical edge distance c_{ac} used to determine $\psi_{cp,N}$. ACI 355.2 anchor qualification reports will provide values of c_{ac} associated with individual products (see sample Table 34-3.)

D.9 INSTALLATION OF ANCHORS

Cast-in-place anchors should be installed in accordance with construction documents. For threaded anchors, a metal or plywood template mounted above the surface of the concrete with nuts on each side of the template should be used to hold the anchors in a fixed position while the concrete is placed, consolidated, and hardens. Project specifications should require that post-installed anchors be installed in accordance with the manufacturer's installation instructions. As noted in RD.9, ACI 355.2 product evaluation testing is based on the manufacturer's installation instructions. As part of the ACI 355.2 product evaluation tests, product reliability tests (i.e., sensitivity to installation variables) are performed and the results are used to determine the category of the anchor to be used in the selection of the appropriate ϕ in D.4.4.

DESIGN TABLES FOR SINGLE CAST-IN ANCHORS

Tables have been provided to assist in the design of single anchors subject to tensile or shear loads. Tables 34-5A, B, and C provide design tensile strengths, ϕN_n, of single anchors in concrete with f'_c of 2500, 4000, and 6000 psi, respectively. Tables 34-6A, B, and C provide design shear strengths, ϕV_n, of single anchors in concrete with f'_c of 2500, 4000, and 6000 psi, respectively. A number of specified tensile strengths of steel, f_{uta}, are included to accommodate most anchor materials in use today. Notes accompany each group of tables that explain the assumptions used to develop the tables and how to adjust values for conditions that differ from those assumed.

According to D.8.2, minimum edge distances for cast-in headed anchors that will not be torqued must be based on minimum cover prescribed in 7.7. Thus, technically, concrete cover as low as 3/4 in. is permitted. If such a small cover is provided to the anchor shaft, the head of the anchor would end up having a cover smaller than 3/4 in. For corrosion protection, and in consideration of tolerances on placement (location and alignment) of anchors, it is recommended to provide a minimum concrete cover on cast-in anchors of 1-1/2 in. Tables 34-5 and 34-6 include design strengths for cast-in anchors with a minimum cover of 1-1/2 in.

NOTES FOR TENSION TABLES 34-5A, B AND C

NP – Not practical. Resulting edge distance, c_{a1}, yields less than 3/4 in. cover.

All Notation are identical to those used in 2.1 starting with ACI 318-05.

1. Design strengths in table are for single cast-in anchors near one edge only. The values do not apply where the distance between adjacent anchors is less than $3h_{ef}$, or where the perpendicular distance, c_{a2}, to the edge distance being considered, c_{a1}, is less than $1.5h_{ef}$.

2. In regions of moderate or high seismic risk (UBC Zone 2, 3 or 4), or in structures assigned to intermediate or high seismic performance or design categories (IBC Seismic Design Category C, D, E or F), the design strengths in the table must be reduced by 25%. In addition, the anchor must be designed so strength is governed by a ductile steel element, unless D.3.3.5 is satisfied. Therefore, the design strengths based on the three concrete failure modes, ϕN_{cb}, ϕN_{pn}, and ϕN_{sb}, must exceed the design strength of the steel in tension, ϕN_{sa}. This requirement effectively precludes the use of hooked anchor bolts in the seismic zones noted above.

3. For design purposes the tensile strength of the anchor steel, f_{uta}, must not exceed $1.9f_{ya}$ or 125,000 psi.

4. Design strengths in table are based on strength reduction factor, ϕ, of Section D.4.4. Factored tensile load N_{ua} must be computed from the load combinations of 9.2. Design strengths for concrete breakout, ϕN_{cb}, pullout, ϕN_{pn}, and sideface blowout, ϕN_{sb}, are based on Condition B. Where supplementary reinforcement is provided to satisfy Condition A, design strengths for ϕN_{cb} and ϕN_{sb} may be increased 7.1% to account for the increase in strength reduction factor from 0.70 to 0.75. This increase does not apply to pullout strength, ϕN_{pn}.

5. Design strengths for concrete breakout in tension, ϕN_{cb}, are based on N_b determined in accordance with Eq. (D-7) and apply to headed and hooked anchors. To determine the design strength of headed bolts with embedment depth, h_{ef}, greater than 11 in. in accordance with Eq. (D-8), multiply the table value by $[2(h_{ef}^{5/3})]/[3(h_{ef}^{1.5})]$.

6. Where analysis indicates that there will be no cracking at service load levels ($f_t < f_r$) in the region of the anchor, the design strengths for concrete breakout in tension, ϕN_{cb}, may be increased 25%.

7. The design strengths for pullout in tension, ϕN_{pn}, for headed bolts with diameter, d_o, less than 1-3/4 in. are based on bolts with regular hex heads. The design strengths for 1-3/4 and 2-in. bolts are based on heavy hex heads. For bolts with d_o less than 1-3/4 in. having heads with a larger bearing area, A_{brg}, than assumed, the design strengths may be increased by multiplying by the bearing area of the larger head and dividing by the bearing area of the regular hex head.

8. The design strengths for pullout in tension, ϕN_{pn}, for hooked bolts with hook-length, e_h, between 3 and 4.5 times diameter, d_o, may be determined by interpolation.

9. Where analysis indicates there will be no cracking at service load levels ($f_t < f_r$) in the region of the anchor, the design strengths for pullout in tension, ϕN_{pn}, may be increased 40%.

10. The design strengths for side-face blowout in tension, ϕN_{sb}, are applicable to headed bolts only and where edge distance, c_{a1}, is less than $0.4h_{ef}$. The values for $0.4h_{ef}$ are shown for interpolation purposes only. The design strengths for bolts with diameter, d_o, less than 1-3/4 in. are based on bolts with regular hex heads. The design strengths for 1-3/4 and 2 in. bolts are based on bolts with heavy hex heads. For bolts with d_o less than 1-3/4 in. having heads with a larger bearing area, A_{brg}, than assumed, the design strengths may be increased by multiplying by the square root of the quotient resulting from dividing the bearing area of the larger head by the bearing area of the regular hex head ($\sqrt{A_{brg2} / A_{brg1}}$).

NOTES FOR SHEAR TABLES 34-6A, B AND C

NP – Not practical. Resulting edge distance, c_{a1}, yields less than 3/4 in. cover.

All Notation are identical to those used in 2.1 starting with ACI 318-05.

1. Design strengths in table are for single cast-in anchors near one edge only. The values do not apply where the distance to an edge measured perpendicular to c_{a1} is less than $1.5c_{a1}$. See Note 9.

   The values do not apply where the distance between adjacent anchors is less than $3c_{a1}$, where c_{a1} is the distance from the center of the anchor to the edge in the direction of shear application.

2. In regions of moderate or high seismic risk (UBC Zone 2, 3 or 4), or in structures assigned to intermediate or high seismic performance or design categories (IBC Seismic Design Category C, D, E or F), the design strengths in the table must be reduced by 25%. In addition, the anchor must be designed so failure is initiated by a ductile steel element, unless D.3.3.5 is satisfied. This means that all the design strengths based on the two concrete failure modes, ϕV_{cb} and ϕV_{cp}, must equal or exceed the design strength of the steel in shear, ϕV_{sa}.

3. Concrete pryout strength, ϕV_{cp}, is to be taken equal to tension breakout strength, ϕN_{cb}, where h_{ef} is less than 2.5 in., and to be taken as twice ϕN_{cb} where h_{ef} is equal to or greater than 2.5 in. Condition B (see D.4.4) must be assumed even where supplementary reinforcement qualifying for Condition A is present (i.e., strength reduction factor, ϕ, must be taken equal to 0.70).

4. For design purposes the tensile strength of the anchor steel, f_{uta}, must not exceed $1.9f_{ya}$ or 125,000 psi.

5. Design strengths in table are based on strength reduction factor, ϕ, of Section D.4.4. Factored shear load V_{ua} must be computed from the load combinations of 9.2. Design strengths for concrete breakout, ϕV_{cb}, are based on Condition B. Where supplementary reinforcement is provided to satisfy Condition A, design strengths may be increased 7.1% to account for the increase in strength reduction factor from 0.70 to 0.75.

6. Where analysis indicates that there will be no cracking at service load levels ($f_t < f_r$) in the region of the anchor, the design strengths for concrete breakout in shear, ϕV_{cb}, may be increased 40%.

7. In regions of members where analysis indicates cracking at service level loads, the strengths in the table for concrete breakout, ϕV_{cb}, may be increased in accordance with the factors in D.6.2.7 if edge reinforcement is provided in accordance with that section.

8. The design strengths for concrete breakout, ϕV_{cb}, are based on the shear load being applied perpendicular to the edge. If the load is applied parallel to the edge, the strengths may be increased 100%.

9. Where the anchor is located near a corner with an edge distance perpendicular to direction of shear, c_{a2}, less than $1.5c_{a1}$, design strengths for concrete breakout, ϕV_{cb}, shall be reduced by multiplying by modification factor, $\psi_{ed,V}$, determined from Eq. (D-28). The calculated values in the table do not apply where two edge distances perpendicular to direction of shear, c_{a2}, are less than $1.5c_{a1}$. See D.6.2.4.

10. This value of thickness, h, is not practical since the head or hook would project below the bottom surface of the concrete. It was chosen to facilitate mental calculation of the actual edge distance, c_{a1}, since the variable used in the calculation c_{a1} is a function of embedment depth, h_{ef}.

11. Linear interpolation for intermediate values of edge distance, c_{a1}, is permissible. Linear interpolation for intermediate values of embedment depth, h_{ef}, is unconservative.

12. For 3/4 in. cover and for $c_{a1} = 0.25h_{ef}$ and $0.50h_{ef}$, see portion of table for $h = h_{ef}$.

13. For 3/4 in. cover and for $c_{a1} = 0.25h_{ef}$ and $0.50h_{ef}$, see portion of table for $h = h_{ef}$. For $c_{a1} = h_{ef}$, see portion of table for $h = 1.5h_{ef}$.

Table 34-5A. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 2500 psi)[1, 2, 4]
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_{sa} -Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{uta} - for design purposes[3] - psi							c_{a1} - edge distance in.					head[7]	"J" or "L" hook[8]		c_{a1} - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1/4	2	1,392	1,440	1,800	2,160	2,520	2,880	3,000	1,580	NP	NP	1,782	2,376	1,638	295	443	3,113	NP	NP
	3	1,392	1,440	1,800	2,160	2,520	2,880	3,000	2,401	NP	NP	3,274	4,365	1,638	295	443	3,113	NP	NP
	4	1,392	1,440	1,800	2,160	2,520	2,880	3,000	3,336	NP	3,584	5,040	6,720	1,638	295	443	3,113	NP	NP
	5	1,392	1,440	1,800	2,160	2,520	2,880	3,000	4,371	NP	5,009	7,044	9,391	1,638	295	443	3,113	NP	3,831
	6	1,392	1,440	1,800	2,160	2,520	2,880	3,000	5,496	NP	6,584	9,259	12,345	1,638	295	443	3,113	NP	4,597
3/8	2	3,393	3,510	4,388	5,265	6,143	7,020	7,313	1,613	NP	NP	1,782	2,376	2,296	664	997	3,827	NP	NP
	3	3,393	3,510	4,388	5,265	6,143	7,020	7,313	2,438	NP	NP	3,274	4,365	2,296	664	997	3,827	NP	NP
	4	3,393	3,510	4,388	5,265	6,143	7,020	7,313	3,377	NP	3,584	5,040	6,720	2,296	664	997	3,827	NP	NP
	5	3,393	3,510	4,388	5,265	6,143	7,020	7,313	4,415	NP	5,009	7,044	9,391	2,296	664	997	3,827	NP	4,536
	6	3,393	3,510	4,388	5,265	6,143	7,020	7,313	5,543	NP	6,584	9,259	12,345	2,296	664	997	3,827	NP	5,443
1/2	2	6,177	6,390	7,988	9,585	11,183	12,780	13,313	1,646	NP	NP	1,782	2,376	4,074	1,181	1,772	5,287	NP	NP
	3	6,177	6,390	7,988	9,585	11,183	12,780	13,313	2,475	NP	NP	3,274	4,365	4,074	1,181	1,772	5,287	NP	NP
	4	6,177	6,390	7,988	9,585	11,183	12,780	13,313	3,418	NP	3,584	5,040	6,720	4,074	1,181	1,772	5,287	NP	NP
	5	6,177	6,390	7,988	9,585	11,183	12,780	13,313	4,459	NP	5,009	7,044	9,391	4,074	1,181	1,772	5,287	NP	6,042
	6	6,177	6,390	7,988	9,585	11,183	12,780	13,313	5,591	NP	6,584	9,259	12,345	4,074	1,181	1,772	5,287	NP	7,250
	7	6,177	6,390	7,988	9,585	11,183	12,780	13,313	6,806	6,806	8,297	11,668	15,557	4,074	1,181	1,772	5,287	5,287	8,458
	8	6,177	6,390	7,988	9,585	11,183	12,780	13,313	8,099	8,316	10,137	14,255	19,007	4,074	1,181	1,772	5,287	6,042	9,667
5/8	3	9,831	10,170	12,713	15,255	17,798	20,340	21,188	2,513	NP	NP	3,274	4,365	6,356	1,846	2,769	6,839	NP	NP
	4	9,831	10,170	12,713	15,255	17,798	20,340	21,188	3,459	NP	3,584	5,040	6,720	6,356	1,846	2,769	6,839	NP	NP
	5	9,831	10,170	12,713	15,255	17,798	20,340	21,188	4,504	NP	5,009	7,044	9,391	6,356	1,846	2,769	6,839	NP	7,547
	6	9,831	10,170	12,713	15,255	17,798	20,340	21,188	5,639	NP	6,584	9,259	12,345	6,356	1,846	2,769	6,839	NP	9,056
	7	9,831	10,170	12,713	15,255	17,798	20,340	21,188	6,857	NP	8,297	11,668	15,557	6,356	1,846	2,769	6,839	NP	10,565
	8	9,831	10,170	12,713	15,255	17,798	20,340	21,188	8,153	8,316	10,137	14,255	19,007	6,356	1,846	2,769	6,839	7,547	12,074
	9	9,831	10,170	12,713	15,255	17,798	20,340	21,188	9,522	9,923	12,096	17,010	22,680	6,356	1,846	2,769	6,839	8,490	13,584
	10	9,831	10,170	12,713	15,255	17,798	20,340	21,188	10,960	11,621	14,167	19,922	26,563	6,356	1,846	2,769	6,839	9,433	15,093
3/4	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	3,500	NP	3,584	5,040	6,720	9,156	2,658	3,987	8,491	NP	NP
	5	14,529	15,030	18,788	22,545	26,303	30,060	31,313	4,549	NP	5,009	7,044	9,391	9,156	2,658	3,987	8,491	NP	9,057
	6	14,529	15,030	18,788	22,545	26,303	30,060	31,313	5,687	NP	6,584	9,259	12,345	9,156	2,658	3,987	8,491	NP	10,869
	7	14,529	15,030	18,788	22,545	26,303	30,060	31,313	6,908	NP	8,297	11,668	15,557	9,156	2,658	3,987	8,491	NP	12,680
	8	14,529	15,030	18,788	22,545	26,303	30,060	31,313	8,207	8,316	10,137	14,255	19,007	9,156	2,658	3,987	8,491	9,057	14,492
	9	14,529	15,030	18,788	22,545	26,303	30,060	31,313	9,579	9,923	12,096	17,010	22,680	9,156	2,658	3,987	8,491	10,190	16,303
	10	14,529	15,030	18,788	22,545	26,303	30,060	31,313	11,020	11,621	14,167	19,922	26,563	9,156	2,658	3,987	8,491	11,322	18,115
	12	14,529	15,030	18,788	22,545	26,303	30,060	31,313	14,097	15,277	18,623	26,189	34,918	9,156	2,658	3,987	8,491	13,586	21,738
7/8	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	3,500	NP	3,584	5,040	6,720	9,156	2,658	3,987	8,491	NP	NP
	6	20,097	20,790	25,988	31,185	36,383	41,580	43,313	5,736	NP	6,584	9,259	12,345	12,474	3,618	5,426	10,242	NP	12,686
	8	20,097	20,790	25,988	31,185	36,383	41,580	43,313	8,261	8,316	10,137	14,255	19,007	12,474	3,618	5,426	10,242	10,572	16,915
	12	20,097	20,790	25,988	31,185	36,383	41,580	43,313	14,161	15,277	18,623	26,189	34,918	12,474	3,618	5,426	10,242	15,858	25,373
	15	20,097	20,790	25,988	31,185	36,383	41,580	43,313	19,235	21,350	26,026	36,600	48,800	12,474	3,618	5,426	10,242	19,822	31,716
	18	20,097	20,790	25,988	31,185	36,383	41,580	43,313	24,803	28,065	34,213	48,112	64,149	12,474	3,618	5,426	10,242	23,787	38,059
	25	20,097	20,790	25,988	31,185	36,383	41,580	43,313	39,505	45,938	56,000	78,750	105,000	12,474	3,618	5,426	10,242	33,037	52,860

Table 34-5A. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 2500 psi)[1, 2, 4] (cont'd.)
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_{sa} - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{uta} - for design purposes[3] - psi							c_{a1} - edge distance in.					head[7]	"J" or "L" hook[8]		c_{a1} - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1	6	26,361	27,270	34,088	40,905	47,723	54,540	56,813	5,784	NP	6,584	9,259	12,345	16,282	4,725	7,088	12,078	NP	14,494
	9	26,361	27,270	34,088	40,905	47,723	54,540	56,813	9,693	9,923	12,096	17,010	22,680	16,282	4,725	7,088	12,078	13,588	21,741
	12	26,361	27,270	34,088	40,905	47,723	54,540	56,813	14,226	15,277	18,623	26,189	34,918	16,282	4,725	7,088	12,078	18,118	28,988
	15	26,361	27,270	34,088	40,905	47,723	54,540	56,813	19,307	21,350	26,026	36,600	48,800	16,282	4,725	7,088	12,078	22,647	36,235
	18	26,361	27,270	34,088	40,905	47,723	54,540	56,813	24,881	28,065	34,213	48,112	64,149	16,282	4,725	7,088	12,078	27,176	43,482
	21	26,361	27,270	34,088	40,905	47,723	54,540	56,813	30,908	35,366	43,113	60,627	80,837	16,282	4,725	7,088	12,078	31,706	50,729
	25	26,361	27,270	34,088	40,905	47,723	54,540	56,813	39,595	45,938	56,000	78,750	105,000	16,282	4,725	7,088	12,078	37,745	60,392
1-1/8	6	33,191	34,335	42,919	51,503	60,086	68,670	71,531	5,833	NP	6,584	9,259	12,345	20,608	5,980	8,970	14,013	NP	16,306
	9	33,191	34,335	42,919	51,503	60,086	68,670	71,531	9,750	9,923	12,096	17,010	22,680	20,608	5,980	8,970	14,013	15,287	24,459
	12	33,191	34,335	42,919	51,503	60,086	68,670	71,531	14,291	15,277	18,623	26,189	34,918	20,608	5,980	8,970	14,013	20,383	32,612
	15	33,191	34,335	42,919	51,503	60,086	68,670	71,531	19,378	21,350	26,026	36,600	48,800	20,608	5,980	8,970	14,013	25,478	40,766
	18	33,191	34,335	42,919	51,503	60,086	68,670	71,531	24,958	28,065	34,213	48,112	64,149	20,608	5,980	8,970	14,013	30,574	48,919
	21	33,191	34,335	42,919	51,503	60,086	68,670	71,531	30,991	35,366	43,113	60,627	80,837	20,608	5,980	8,970	14,013	35,670	57,072
	25	33,191	34,335	42,919	51,503	60,086	68,670	71,531	39,685	45,938	56,000	78,750	105,000	20,608	5,980	8,970	14,013	42,464	67,943
1-1/4	6	42,152	43,605	54,506	65,408	76,309	87,210	90,844	5,882	NP	6,584	9,259	12,345	25,438	7,383	11,074	16,041	NP	18,117
	9	42,152	43,605	54,506	65,408	76,309	87,210	90,844	9,807	9,923	12,096	17,010	22,680	25,438	7,383	11,074	16,041	16,984	27,175
	12	42,152	43,605	54,506	65,408	76,309	87,210	90,844	14,355	15,277	18,623	26,189	34,918	25,438	7,383	11,074	16,041	22,646	36,233
	15	42,152	43,605	54,506	65,408	76,309	87,210	90,844	19,450	21,350	26,026	36,600	48,800	25,438	7,383	11,074	16,041	28,307	45,292
	18	42,152	43,605	54,506	65,408	76,309	87,210	90,844	25,036	28,065	34,213	48,112	64,149	25,438	7,383	11,074	16,041	33,969	54,350
	21	42,152	43,605	54,506	65,408	76,309	87,210	90,844	31,075	35,366	43,113	60,627	80,837	25,438	7,383	11,074	16,041	39,630	63,408
	25	42,152	43,605	54,506	65,408	76,309	87,210	90,844	39,776	45,938	56,000	78,750	105,000	25,438	7,383	11,074	16,041	47,179	75,486
1-3/8	6	50,460	52,200	65,250	78,300	91,350	104,400	108,750	5,931	NP	6,584	9,259	12,345	30,786	8,933	13,400	18,166	NP	19,930
	9	50,460	52,200	65,250	78,300	91,350	104,400	108,750	9,865	9,923	12,096	17,010	22,680	30,786	8,933	13,400	18,166	18,685	29,895
	12	50,460	52,200	65,250	78,300	91,350	104,400	108,750	14,420	15,277	18,623	26,189	34,918	30,786	8,933	13,400	18,166	24,913	39,860
	15	50,460	52,200	65,250	78,300	91,350	104,400	108,750	19,521	21,350	26,026	36,600	48,800	30,786	8,933	13,400	18,166	31,141	49,826
	18	50,460	52,200	65,250	78,300	91,350	104,400	108,750	25,114	28,065	34,213	48,112	64,149	30,786	8,933	13,400	18,166	37,369	59,791
	21	50,460	52,200	65,250	78,300	91,350	104,400	108,750	31,158	35,366	43,113	60,627	80,837	30,786	8,933	13,400	18,166	43,597	69,756
	25	50,460	52,200	65,250	78,300	91,350	104,400	108,750	39,866	45,938	56,000	78,750	105,000	30,786	8,933	13,400	18,166	51,902	83,043
1-1/2	12	61,335	63,450	79,313	95,175	111,038	126,900	132,188	14,486	15,277	18,623	26,189	34,918	36,638	10,631	15,947	20,383	27,178	43,484
	15	61,335	63,450	79,313	95,175	111,038	126,900	132,188	19,593	21,350	26,026	36,600	48,800	36,638	10,631	15,947	20,383	33,972	54,355
	18	61,335	63,450	79,313	95,175	111,038	126,900	132,188	25,192	28,065	34,213	48,112	64,149	36,638	10,631	15,947	20,383	40,766	65,226
	21	61,335	63,450	79,313	95,175	111,038	126,900	132,188	31,242	35,366	43,113	60,627	80,837	36,638	10,631	15,947	20,383	47,561	76,097
	25	61,335	63,450	79,313	95,175	111,038	126,900	132,188	39,957	45,938	56,000	78,750	105,000	36,638	10,631	15,947	20,383	56,620	90,592
1-3/4	12	82,650	85,500	106,875	128,250	149,625	171,000	178,125	14,616	15,277	18,623	26,189	34,918	58,016	14,470	21,705	27,075	34,199	54,719
	15	82,650	85,500	106,875	128,250	149,625	171,000	178,125	19,737	21,350	26,026	36,600	48,800	58,016	14,470	21,705	27,075	42,749	68,399
	18	82,650	85,500	106,875	128,250	149,625	171,000	178,125	25,348	28,065	34,213	48,112	64,149	58,016	14,470	21,705	27,075	51,299	82,079
	21	82,650	85,500	106,875	128,250	149,625	171,000	178,125	31,409	35,366	43,113	60,627	80,837	58,016	14,470	21,705	27,075	59,849	95,758
	25	82,650	85,500	106,875	128,250	149,625	171,000	178,125	40,138	45,938	56,000	78,750	105,000	58,016	14,470	21,705	27,075	71,249	113,998
2	12	108,750	112,500	140,625	168,750	196,875	225,000	234,375	14,747	15,277	18,623	26,189	34,918	74,424	18,900	28,350	32,279	38,735	61,976
	15	108,750	112,500	140,625	168,750	196,875	225,000	234,375	19,881	21,350	26,026	36,600	48,800	74,424	18,900	28,350	32,279	48,419	77,470
	18	108,750	112,500	140,625	168,750	196,875	225,000	234,375	25,504	28,065	34,213	48,112	64,149	74,424	18,900	28,350	32,279	58,102	92,964
	21	108,750	112,500	140,625	168,750	196,875	225,000	234,375	31,577	35,366	43,113	60,627	80,837	74,424	18,900	28,350	32,279	67,786	108,458
	25	108,750	112,500	140,625	168,750	196,875	225,000	234,375	40,320	45,938	56,000	78,750	105,000	74,424	18,900	28,350	32,279	80,698	129,116

Table 34-5B. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 4000 psi)[1, 2, 4]
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_{sa} - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{uta} - for design purposes[3] - psi							c_{a1} - edge distance in.					head[7]	"J" or "L" hook[8]		c_{a1} - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1/4	2	1,392	1,440	1,800	2,160	2,520	2,880	3,000	1,998	NP	NP	2,254	3,005	2,621	473	709	3,937	NP	NP
	3	1,392	1,440	1,800	2,160	2,520	2,880	3,000	3,037	NP	NP	4,141	5,521	2,621	473	709	3,937	NP	NP
	4	1,392	1,440	1,800	2,160	2,520	2,880	3,000	4,220	NP	4,533	6,375	8,500	2,621	473	709	3,937	NP	NP
	5	1,392	1,440	1,800	2,160	2,520	2,880	3,000	5,528	NP	6,336	8,910	11,879	2,621	473	709	3,937	NP	4,846
	6	1,392	1,440	1,800	2,160	2,520	2,880	3,000	6,952	NP	8,328	11,712	15,616	2,621	473	709	3,937	NP	5,815
3/8	2	3,393	3,510	4,388	5,265	6,143	7,020	7,313	2,040	NP	NP	2,254	3,005	3,674	1,063	1,595	4,841	NP	NP
	3	3,393	3,510	4,388	5,265	6,143	7,020	7,313	3,084	NP	NP	4,141	5,521	3,674	1,063	1,595	4,841	NP	NP
	4	3,393	3,510	4,388	5,265	6,143	7,020	7,313	4,271	NP	4,533	6,375	8,500	3,674	1,063	1,595	4,841	NP	NP
	5	3,393	3,510	4,388	5,265	6,143	7,020	7,313	5,584	NP	6,336	8,910	11,879	3,674	1,063	1,595	4,841	NP	5,737
	6	3,393	3,510	4,388	5,265	6,143	7,020	7,313	7,012	NP	8,328	11,712	15,616	3,674	1,063	1,595	4,841	NP	6,885
1/2	2	6,177	6,390	7,988	9,585	11,183	12,780	13,313	2,082	NP	NP	2,254	3,005	6,518	1,890	2,835	6,687	NP	NP
	3	6,177	6,390	7,988	9,585	11,183	12,780	13,313	3,131	NP	NP	4,141	5,521	6,518	1,890	2,835	6,687	NP	NP
	4	6,177	6,390	7,988	9,585	11,183	12,780	13,313	4,323	NP	4,533	6,375	8,500	6,518	1,890	2,835	6,687	NP	NP
	5	6,177	6,390	7,988	9,585	11,183	12,780	13,313	5,641	NP	6,336	8,910	11,879	6,518	1,890	2,835	6,687	NP	7,642
	6	6,177	6,390	7,988	9,585	11,183	12,780	13,313	7,072	NP	8,328	11,712	15,616	6,518	1,890	2,835	6,687	NP	9,171
	7	6,177	6,390	7,988	9,585	11,183	12,780	13,313	8,609	8,609	10,495	14,759	19,678	6,518	1,890	2,835	6,687	6,687	10,699
	8	6,177	6,390	7,988	9,585	11,183	12,780	13,313	10,245	10,518	12,823	18,032	24,042	6,518	1,890	2,835	6,687	7,642	12,228
5/8	3	9,831	10,170	12,713	15,255	17,798	20,340	21,188	3,179	NP	NP	4,141	5,521	10,170	2,953	4,430	8,651	NP	NP
	4	9,831	10,170	12,713	15,255	17,798	20,340	21,188	4,375	NP	4,533	6,375	8,500	10,170	2,953	4,430	8,651	NP	NP
	5	9,831	10,170	12,713	15,255	17,798	20,340	21,188	5,697	NP	6,336	8,910	11,879	10,170	2,953	4,430	8,651	NP	9,546
	6	9,831	10,170	12,713	15,255	17,798	20,340	21,188	7,133	NP	8,328	11,712	15,616	10,170	2,953	4,430	8,651	NP	11,455
	7	9,831	10,170	12,713	15,255	17,798	20,340	21,188	8,674	NP	10,495	14,759	19,678	10,170	2,953	4,430	8,651	NP	13,364
	8	9,831	10,170	12,713	15,255	17,798	20,340	21,188	10,313	10,518	12,823	18,032	24,042	10,170	2,953	4,430	8,651	9,546	15,273
	9	9,831	10,170	12,713	15,255	17,798	20,340	21,188	12,044	12,551	15,300	21,516	28,688	10,170	2,953	4,430	8,651	10,739	17,182
	10	9,831	10,170	12,713	15,255	17,798	20,340	21,188	13,864	14,700	17,920	25,200	33,600	10,170	2,953	4,430	8,651	11,932	19,091
3/4	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	4,428	NP	4,533	6,375	8,500	14,650	4,253	6,379	10,741	NP	NP
	5	14,529	15,030	18,788	22,545	26,303	30,060	31,313	5,754	NP	6,336	8,910	11,879	14,650	4,253	6,379	10,741	NP	11,457
	6	14,529	15,030	18,788	22,545	26,303	30,060	31,313	7,194	NP	8,328	11,712	15,616	14,650	4,253	6,379	10,741	NP	13,748
	7	14,529	15,030	18,788	22,545	26,303	30,060	31,313	8,738	NP	10,495	14,759	19,678	14,650	4,253	6,379	10,741	NP	16,040
	8	14,529	15,030	18,788	22,545	26,303	30,060	31,313	10,381	10,518	12,823	18,032	24,042	14,650	4,253	6,379	10,741	11,457	18,331
	9	14,529	15,030	18,788	22,545	26,303	30,060	31,313	12,116	12,551	15,300	21,516	28,688	14,650	4,253	6,379	10,741	12,889	20,622
	10	14,529	15,030	18,788	22,545	26,303	30,060	31,313	13,939	14,700	17,920	25,200	33,600	14,650	4,253	6,379	10,741	14,321	22,914
	12	14,529	15,030	18,788	22,545	26,303	30,060	31,313	17,831	19,324	23,556	33,126	44,168	14,650	4,253	6,379	10,741	17,185	27,497
7/8	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	4,428	NP	4,533	6,375	8,500	14,650	4,253	6,379	10,741	NP	NP
	6	20,097	20,790	25,988	31,185	36,383	41,580	43,313	7,255	NP	8,328	11,712	15,616	19,958	5,788	8,682	12,955	NP	16,047
	8	20,097	20,790	25,988	31,185	36,383	41,580	43,313	10,450	10,518	12,823	18,032	24,042	19,958	5,788	8,682	12,955	13,373	21,396
	12	20,097	20,790	25,988	31,185	36,383	41,580	43,313	17,913	19,324	23,556	33,126	44,168	19,958	5,788	8,682	12,955	20,059	32,094
	15	20,097	20,790	25,988	31,185	36,383	41,580	43,313	24,331	27,006	32,921	46,295	61,727	19,958	5,788	8,682	12,955	25,074	40,118
	18	20,097	20,790	25,988	31,185	36,383	41,580	43,313	31,374	35,500	43,276	60,857	81,142	19,958	5,788	8,682	12,955	30,088	48,141
	25	20,097	20,790	25,988	31,185	36,383	41,580	43,313	49,970	58,107	70,835	99,612	132,816	19,958	5,788	8,682	12,955	41,789	66,863

Table 34-5B. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 4000 psi)[1, 2, 4] (cont'd.)

Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_{sa} - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{uta} - for design purposes[3] - psi							c_{a1} - edge distance in.					head[7]	"J" or "L" hook[8]		c_{a1} - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
	6	26,361	27,270	34,088	40,905	47,723	54,540	56,813	7,316	NP	8,328	11,712	15,616	26,051	7,560	11,340	15,278	NP	18,334
	9	26,361	27,270	34,088	40,905	47,723	54,540	56,813	12,260	12,551	15,300	21,516	28,688	26,051	7,560	11,340	15,278	17,188	27,500
	12	26,361	27,270	34,088	40,905	47,723	54,540	56,813	17,995	19,324	23,556	33,126	44,168	26,051	7,560	11,340	15,278	22,917	36,667
1	15	26,361	27,270	34,088	40,905	47,723	54,540	56,813	24,421	27,006	32,921	46,295	61,727	26,051	7,560	11,340	15,278	28,646	45,834
	18	26,361	27,270	34,088	40,905	47,723	54,540	56,813	31,472	35,500	43,276	60,857	81,142	26,051	7,560	11,340	15,278	34,376	55,001
	21	26,361	27,270	34,088	40,905	47,723	54,540	56,813	39,096	44,735	54,534	76,688	102,251	26,051	7,560	11,340	15,278	40,105	64,168
	25	26,361	27,270	34,088	40,905	47,723	54,540	56,813	50,084	58,107	70,835	99,612	132,816	26,051	7,560	11,340	15,278	47,744	76,390
	6	33,191	34,335	42,919	51,503	60,086	68,670	71,531	7,378	NP	8,328	11,712	15,616	32,973	9,568	14,352	17,725	NP	20,626
	9	33,191	34,335	42,919	51,503	60,086	68,670	71,531	12,333	12,551	15,300	21,516	28,688	32,973	9,568	14,352	17,725	19,337	30,939
	12	33,191	34,335	42,919	51,503	60,086	68,670	71,531	18,076	19,324	23,556	33,126	44,168	32,973	9,568	14,352	17,725	25,782	41,252
1-1/8	15	33,191	34,335	42,919	51,503	60,086	68,670	71,531	24,511	27,006	32,921	46,295	61,727	32,973	9,568	14,352	17,725	32,228	51,565
	18	33,191	34,335	42,919	51,503	60,086	68,670	71,531	31,570	35,500	43,276	60,857	81,142	32,973	9,568	14,352	17,725	38,674	61,878
	21	33,191	34,335	42,919	51,503	60,086	68,670	71,531	39,201	44,735	54,534	76,688	102,251	32,973	9,568	14,352	17,725	45,119	72,191
	25	33,191	34,335	42,919	51,503	60,086	68,670	71,531	50,198	58,107	70,835	99,612	132,816	32,973	9,568	14,352	17,725	53,713	85,941
	6	42,152	43,605	54,506	65,408	76,309	87,210	90,844	7,440	NP	8,328	11,712	15,616	40,701	11,813	17,719	20,290	NP	22,916
	9	42,152	43,605	54,506	65,408	76,309	87,210	90,844	12,405	12,551	15,300	21,516	28,688	40,701	11,813	17,719	20,290	21,484	34,374
	12	42,152	43,605	54,506	65,408	76,309	87,210	90,844	18,158	19,324	23,556	33,126	44,168	40,701	11,813	17,719	20,290	28,645	45,832
1-1/4	15	42,152	43,605	54,506	65,408	76,309	87,210	90,844	24,602	27,006	32,921	46,295	61,727	40,701	11,813	17,719	20,290	35,806	57,290
	18	42,152	43,605	54,506	65,408	76,309	87,210	90,844	31,668	35,500	43,276	60,857	81,142	40,701	11,813	17,719	20,290	42,967	68,748
	21	42,152	43,605	54,506	65,408	76,309	87,210	90,844	39,307	44,735	54,534	76,688	102,251	40,701	11,813	17,719	20,290	50,129	80,206
	25	42,152	43,605	54,506	65,408	76,309	87,210	90,844	50,313	58,107	70,835	99,612	132,816	40,701	11,813	17,719	20,290	59,677	95,483
	6	50,460	52,200	65,250	78,300	91,350	104,400	108,750	7,502	NP	8,328	11,712	15,616	49,258	14,293	21,440	22,978	NP	25,210
	9	50,460	52,200	65,250	78,300	91,350	104,400	108,750	12,478	12,551	15,300	21,516	28,688	49,258	14,293	21,440	22,978	23,634	37,815
	12	50,460	52,200	65,250	78,300	91,350	104,400	108,750	18,241	19,324	23,556	33,126	44,168	49,258	14,293	21,440	22,978	31,512	50,420
1-3/8	15	50,460	52,200	65,250	78,300	91,350	104,400	108,750	24,693	27,006	32,921	46,295	61,727	49,258	14,293	21,440	22,978	39,391	63,025
	18	50,460	52,200	65,250	78,300	91,350	104,400	108,750	31,767	35,500	43,276	60,857	81,142	49,258	14,293	21,440	22,978	47,269	75,630
	21	50,460	52,200	65,250	78,300	91,350	104,400	108,750	39,412	44,735	54,534	76,688	102,251	49,258	14,293	21,440	22,978	55,147	88,235
	25	50,460	52,200	65,250	78,300	91,350	104,400	108,750	50,427	58,107	70,835	99,612	132,816	49,258	14,293	21,440	22,978	65,651	105,041
	12	61,335	63,450	79,313	95,175	111,038	126,900	132,188	18,323	19,324	23,556	33,126	44,168	58,621	17,010	25,515	25,783	34,377	55,004
	15	61,335	63,450	79,313	95,175	111,038	126,900	132,188	24,783	27,006	32,921	46,295	61,727	58,621	17,010	25,515	25,783	42,972	68,755
1-1/2	18	61,335	63,450	79,313	95,175	111,038	126,900	132,188	31,865	35,500	43,276	60,857	81,142	58,621	17,010	25,515	25,783	51,566	82,505
	21	61,335	63,450	79,313	95,175	111,038	126,900	132,188	39,518	44,735	54,534	76,688	102,251	58,621	17,010	25,515	25,783	60,160	96,256
	25	61,335	63,450	79,313	95,175	111,038	126,900	132,188	50,542	58,107	70,835	99,612	132,816	58,621	17,010	25,515	25,783	71,619	114,591
	12	82,650	85,500	106,875	128,250	149,625	171,000	178,125	18,488	19,324	23,556	33,126	44,168	92,826	23,153	34,729	34,247	43,259	69,215
	15	82,650	85,500	106,875	128,250	149,625	171,000	178,125	24,965	27,006	32,921	46,295	61,727	92,826	23,153	34,729	34,247	54,074	86,519
1-3/4	18	82,650	85,500	106,875	128,250	149,625	171,000	178,125	32,063	35,500	43,276	60,857	81,142	92,826	23,153	34,729	34,247	64,889	103,822
	21	82,650	85,500	106,875	128,250	149,625	171,000	178,125	39,730	44,735	54,534	76,688	102,251	92,826	23,153	34,729	34,247	75,704	121,126
	25	82,650	85,500	106,875	128,250	149,625	171,000	178,125	50,771	58,107	70,835	99,612	132,816	92,826	23,153	34,729	34,247	90,123	144,198
	12	108,750	112,500	140,625	168,750	196,875	225,000	234,375	18,654	19,324	23,556	33,126	44,168	119,078	30,240	45,360	40,830	48,996	78,394
	15	108,750	112,500	140,625	168,750	196,875	225,000	234,375	25,148	27,006	32,921	46,295	61,727	119,078	30,240	45,360	40,830	61,245	97,992
2	18	108,750	112,500	140,625	168,750	196,875	225,000	234,375	32,261	35,500	43,276	60,857	81,142	119,078	30,240	45,360	40,830	73,494	117,591
	21	108,750	112,500	140,625	168,750	196,875	225,000	234,375	39,942	44,735	54,534	76,688	102,251	119,078	30,240	45,360	40,830	85,743	137,189
	25	108,750	112,500	140,625	168,750	196,875	225,000	234,375	51,001	58,107	70,835	99,612	132,816	119,078	30,240	45,360	40,830	102,075	163,320

Table 34-5C. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 6000 psi)[1, 2, 4]
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_{sa} - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4,5,6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4,10]		
		f_{uta} - for design purposes[3] - psi							c_{a1} - edge distance in.					head[7]	"J" or "L" hook[8]		c_{a1} - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1/4	2	1,392	1,440	1,800	2,160	2,520	2,880	3,000	2,447	NP	NP	2,761	3,681	3,931	709	1,063	4,822	NP	NP
	3	1,392	1,440	1,800	2,160	2,520	2,880	3,000	3,720	NP	NP	5,071	6,762	3,931	709	1,063	4,822	NP	NP
	4	1,392	1,440	1,800	2,160	2,520	2,880	3,000	5,168	NP	5,552	7,808	10,411	3,931	709	1,063	4,822	NP	NP
	5	1,392	1,440	1,800	2,160	2,520	2,880	3,000	6,771	NP	7,760	10,912	14,549	3,931	709	1,063	4,822	NP	5,935
	6	1,392	1,440	1,800	2,160	2,520	2,880	3,000	8,514	NP	10,200	14,344	19,125	3,931	709	1,063	4,822	NP	7,122
3/8	2	3,393	3,510	4,388	5,265	6,143	7,020	7,313	2,498	NP	NP	2,761	3,681	5,510	1,595	2,392	5,929	NP	NP
	3	3,393	3,510	4,388	5,265	6,143	7,020	7,313	3,777	NP	NP	5,071	6,762	5,510	1,595	2,392	5,929	NP	NP
	4	3,393	3,510	4,388	5,265	6,143	7,020	7,313	5,231	NP	5,552	7,808	10,411	5,510	1,595	2,392	5,929	NP	NP
	5	3,393	3,510	4,388	5,265	6,143	7,020	7,313	6,840	NP	7,760	10,912	14,549	5,510	1,595	2,392	5,929	NP	7,027
	6	3,393	3,510	4,388	5,265	6,143	7,020	7,313	8,588	NP	10,200	14,344	19,125	5,510	1,595	2,392	5,929	NP	8,432
1/2	2	6,177	6,390	7,988	9,585	11,183	12,780	13,313	2,550	NP	NP	2,761	3,681	9,778	2,835	4,253	8,190	NP	NP
	3	6,177	6,390	7,988	9,585	11,183	12,780	13,313	3,835	NP	NP	5,071	6,762	9,778	2,835	4,253	8,190	NP	NP
	4	6,177	6,390	7,988	9,585	11,183	12,780	13,313	5,295	NP	5,552	7,808	10,411	9,778	2,835	4,253	8,190	NP	NP
	5	6,177	6,390	7,988	9,585	11,183	12,780	13,313	6,908	NP	7,760	10,912	14,549	9,778	2,835	4,253	8,190	NP	9,360
	6	6,177	6,390	7,988	9,585	11,183	12,780	13,313	8,662	NP	10,200	14,344	19,125	9,778	2,835	4,253	8,190	NP	11,232
	7	6,177	6,390	7,988	9,585	11,183	12,780	13,313	10,544	10,544	12,854	18,076	24,101	9,778	2,835	4,253	8,190	8,190	13,104
	8	6,177	6,390	7,988	9,585	11,183	12,780	13,313	12,547	12,882	15,704	22,084	29,446	9,778	2,835	4,253	8,190	9,360	14,976
5/8	3	9,831	10,170	12,713	15,255	17,798	20,340	21,188	3,893	NP	NP	5,071	6,762	15,254	4,430	6,645	10,595	NP	NP
	4	9,831	10,170	12,713	15,255	17,798	20,340	21,188	5,359	NP	5,552	7,808	10,411	15,254	4,430	6,645	10,595	NP	NP
	5	9,831	10,170	12,713	15,255	17,798	20,340	21,188	6,978	NP	7,760	10,912	14,549	15,254	4,430	6,645	10,595	NP	11,691
	6	9,831	10,170	12,713	15,255	17,798	20,340	21,188	8,736	NP	10,200	14,344	19,125	15,254	4,430	6,645	10,595	NP	14,029
	7	9,831	10,170	12,713	15,255	17,798	20,340	21,188	10,623	NP	12,854	18,076	24,101	15,254	4,430	6,645	10,595	NP	16,367
	8	9,831	10,170	12,713	15,255	17,798	20,340	21,188	12,630	12,882	15,704	22,084	29,446	15,254	4,430	6,645	10,595	11,691	18,706
	9	9,831	10,170	12,713	15,255	17,798	20,340	21,188	14,751	15,372	18,739	26,352	35,136	15,254	4,430	6,645	10,595	13,152	21,044
	10	9,831	10,170	12,713	15,255	17,798	20,340	21,188	16,979	18,004	21,947	30,864	41,151	15,254	4,430	6,645	10,595	14,614	23,382
3/4	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	5,423	NP	5,552	7,808	10,411	21,974	6,379	9,568	13,155	NP	NP
	5	14,529	15,030	18,788	22,545	26,303	30,060	31,313	7,047	NP	7,760	10,912	14,549	21,974	6,379	9,568	13,155	NP	14,032
	6	14,529	15,030	18,788	22,545	26,303	30,060	31,313	8,811	NP	10,200	14,344	19,125	21,974	6,379	9,568	13,155	NP	16,838
	7	14,529	15,030	18,788	22,545	26,303	30,060	31,313	10,702	NP	12,854	18,076	24,101	21,974	6,379	9,568	13,155	NP	19,644
	8	14,529	15,030	18,788	22,545	26,303	30,060	31,313	12,714	12,882	15,704	22,084	29,446	21,974	6,379	9,568	13,155	14,032	22,451
	9	14,529	15,030	18,788	22,545	26,303	30,060	31,313	14,839	15,372	18,739	26,352	35,136	21,974	6,379	9,568	13,155	15,786	25,257
	10	14,529	15,030	18,788	22,545	26,303	30,060	31,313	17,071	18,004	21,947	30,864	41,151	21,974	6,379	9,568	13,155	17,540	28,064
	12	14,529	15,030	18,788	22,545	26,303	30,060	31,313	21,839	23,667	28,851	40,571	54,095	21,974	6,379	9,568	13,155	21,048	33,676
7/8	4	14,529	15,030	18,788	22,545	26,303	30,060	31,313	5,423	NP	5,552	7,808	10,411	21,974	6,379	9,568	13,155	NP	NP
	6	20,097	20,790	25,988	31,185	36,383	41,580	43,313	8,886	NP	10,200	14,344	19,125	29,938	8,682	13,023	15,866	NP	19,654
	8	20,097	20,790	25,988	31,185	36,383	41,580	43,313	12,798	12,882	15,704	22,084	29,446	29,938	8,682	13,023	15,866	16,378	26,205
	12	20,097	20,790	25,988	31,185	36,383	41,580	43,313	21,939	23,667	28,851	40,571	54,095	29,938	8,682	13,023	15,866	24,567	39,307
	15	20,097	20,790	25,988	31,185	36,383	41,580	43,313	29,799	33,075	40,320	56,700	75,600	29,938	8,682	13,023	15,866	30,709	49,134
	18	20,097	20,790	25,988	31,185	36,383	41,580	43,313	38,425	43,478	53,002	74,534	99,379	29,938	8,682	13,023	15,866	36,851	58,961
	25	20,097	20,790	25,988	31,185	36,383	41,580	43,313	61,200	71,166	86,755	121,999	162,665	29,938	8,682	13,023	15,866	51,181	81,890

Table 34-5C. Design Strengths for Single Cast-In Anchors Subject to Tensile Loads (f'_c = 6000 psi)[1, 2, 4] (cont'd.)
Notes pertaining to this table are given on Page 34-14

d_o in.	h_{ef} in.	ϕN_{sa} - Tension Strength of Anchor							ϕN_{cb} - Tension Breakout[4, 5, 6]					ϕN_{pn} - Pullout[9]			ϕN_{sb} - Sideface Blowout[4, 10]		
		f_{uta} - for design purposes[3] - psi							c_{a1} - edge distance in.					head[7]	"J" or "L" hook[8]		c_{a1} - edge distance in.		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$\geq 1.5h_{ef}$		$e_h = 3d_o$	$e_h = 4.5d_o$	1-1/2-in. cover	$0.25h_{ef}$	$0.4h_{ef}$
1	6	26,361	27,270	34,088	40,905	47,723	54,540	56,813	8,961	NP	10,200	14,344	19,125	39,077	11,340	17,010	18,712	NP	22,454
	9	26,361	27,270	34,088	40,905	47,723	54,540	56,813	15,016	15,372	18,739	26,352	35,136	39,077	11,340	17,010	18,712	21,051	33,681
	12	26,361	27,270	34,088	40,905	47,723	54,540	56,813	22,039	23,667	28,851	40,571	54,095	39,077	11,340	17,010	18,712	28,068	44,908
	15	26,361	27,270	34,088	40,905	47,723	54,540	56,813	29,910	33,075	40,320	56,700	75,600	39,077	11,340	17,010	18,712	35,084	56,135
	18	26,361	27,270	34,088	40,905	47,723	54,540	56,813	38,545	43,478	53,002	74,534	99,379	39,077	11,340	17,010	18,712	42,101	67,362
	21	26,361	27,270	34,088	40,905	47,723	54,540	56,813	47,882	54,789	66,790	93,924	125,232	39,077	11,340	17,010	18,712	49,118	78,589
	25	26,361	27,270	34,088	40,905	47,723	54,540	56,813	61,340	71,166	86,755	121,999	162,665	39,077	11,340	17,010	18,712	58,474	93,559
1-1/8	6	33,191	34,335	42,919	51,503	60,086	68,670	71,531	9,036	NP	10,200	14,344	19,125	49,459	14,352	21,528	21,709	NP	25,261
	9	33,191	34,335	42,919	51,503	60,086	68,670	71,531	15,104	15,372	18,739	26,352	35,136	49,459	14,352	21,528	21,709	23,683	37,892
	12	33,191	34,335	42,919	51,503	60,086	68,670	71,531	22,139	23,667	28,851	40,571	54,095	49,459	14,352	21,528	21,709	31,577	50,523
	15	33,191	34,335	42,919	51,503	60,086	68,670	71,531	30,020	33,075	40,320	56,700	75,600	49,459	14,352	21,528	21,709	39,471	63,154
	18	33,191	34,335	42,919	51,503	60,086	68,670	71,531	38,665	43,478	53,002	74,534	99,379	49,459	14,352	21,528	21,709	47,365	75,784
	21	33,191	34,335	42,919	51,503	60,086	68,670	71,531	48,011	54,789	66,790	93,924	125,232	49,459	14,352	21,528	21,709	55,259	88,415
	25	33,191	34,335	42,919	51,503	60,086	68,670	71,531	61,480	71,166	86,755	121,999	162,665	49,459	14,352	21,528	21,709	65,785	105,256
1-1/4	6	42,152	43,605	54,506	65,408	76,309	87,210	90,844	9,112	NP	10,200	14,344	19,125	61,051	17,719	26,578	24,850	NP	28,066
	9	42,152	43,605	54,506	65,408	76,309	87,210	90,844	15,193	15,372	18,739	26,352	35,136	61,051	17,719	26,578	24,850	26,312	42,099
	12	42,152	43,605	54,506	65,408	76,309	87,210	90,844	22,239	23,667	28,851	40,571	54,095	61,051	17,719	26,578	24,850	35,083	56,132
	15	42,152	43,605	54,506	65,408	76,309	87,210	90,844	30,131	33,075	40,320	56,700	75,600	61,051	17,719	26,578	24,850	43,853	70,165
	18	42,152	43,605	54,506	65,408	76,309	87,210	90,844	38,786	43,478	53,002	74,534	99,379	61,051	17,719	26,578	24,850	52,624	84,198
	21	42,152	43,605	54,506	65,408	76,309	87,210	90,844	48,141	54,789	66,790	93,924	125,232	61,051	17,719	26,578	24,850	61,395	98,231
	25	42,152	43,605	54,506	65,408	76,309	87,210	90,844	61,620	71,166	86,755	121,999	162,665	61,051	17,719	26,578	24,850	73,089	116,942
1-3/8	6	50,460	52,200	65,250	78,300	91,350	104,400	108,750	9,188	NP	10,200	14,344	19,125	73,886	21,440	32,160	28,142	NP	30,876
	9	50,460	52,200	65,250	78,300	91,350	104,400	108,750	15,283	15,372	18,739	26,352	35,136	73,886	21,440	32,160	28,142	28,946	46,314
	12	50,460	52,200	65,250	78,300	91,350	104,400	108,750	22,340	23,667	28,851	40,571	54,095	73,886	21,440	32,160	28,142	38,595	61,751
	15	50,460	52,200	65,250	78,300	91,350	104,400	108,750	30,242	33,075	40,320	56,700	75,600	73,886	21,440	32,160	28,142	48,243	77,189
	18	50,460	52,200	65,250	78,300	91,350	104,400	108,750	38,906	43,478	53,002	74,534	99,379	73,886	21,440	32,160	28,142	57,892	92,627
	21	50,460	52,200	65,250	78,300	91,350	104,400	108,750	48,270	54,789	66,790	93,924	125,232	73,886	21,440	32,160	28,142	67,541	108,065
	25	50,460	52,200	65,250	78,300	91,350	104,400	108,750	61,760	71,166	86,755	121,999	162,665	73,886	21,440	32,160	28,142	80,406	128,649
1-1/2	12	61,335	63,450	79,313	95,175	111,038	126,900	132,188	22,441	23,667	28,851	40,571	54,095	87,931	25,515	38,273	31,578	42,103	67,365
	15	61,335	63,450	79,313	95,175	111,038	126,900	132,188	30,353	33,075	40,320	56,700	75,600	87,931	25,515	38,273	31,578	52,629	84,207
	18	61,335	63,450	79,313	95,175	111,038	126,900	132,188	39,027	43,478	53,002	74,534	99,379	87,931	25,515	38,273	31,578	63,155	101,048
	21	61,335	63,450	79,313	95,175	111,038	126,900	132,188	48,399	54,789	66,790	93,924	125,232	87,931	25,515	38,273	31,578	73,681	117,889
	25	61,335	63,450	79,313	95,175	111,038	126,900	132,188	61,901	71,166	86,755	121,999	162,665	87,931	25,515	38,273	31,578	87,715	140,345
1-3/4	12	82,650	85,500	106,875	128,250	149,625	171,000	178,125	22,643	23,667	28,851	40,571	54,095	139,238	34,729	52,093	41,944	52,982	84,771
	15	82,650	85,500	106,875	128,250	149,625	171,000	178,125	30,576	33,075	40,320	56,700	75,600	139,238	34,729	52,093	41,944	66,227	105,963
	18	82,650	85,500	106,875	128,250	149,625	171,000	178,125	39,269	43,478	53,002	74,534	99,379	139,238	34,729	52,093	41,944	79,472	127,156
	21	82,650	85,500	106,875	128,250	149,625	171,000	178,125	48,659	54,789	66,790	93,924	125,232	139,238	34,729	52,093	41,944	92,718	148,348
	25	82,650	85,500	106,875	128,250	149,625	171,000	178,125	62,182	71,166	86,755	121,999	162,665	139,238	34,729	52,093	41,944	110,378	176,605
2	12	108,750	112,500	140,625	168,750	196,875	225,000	234,375	22,846	23,667	28,851	40,571	54,095	178,618	45,360	68,040	50,006	60,008	96,012
	15	108,750	112,500	140,625	168,750	196,875	225,000	234,375	30,800	33,075	40,320	56,700	75,600	178,618	45,360	68,040	50,006	75,010	120,016
	18	108,750	112,500	140,625	168,750	196,875	225,000	234,375	39,511	43,478	53,002	74,534	99,379	178,618	45,360	68,040	50,006	90,012	144,019
	21	108,750	112,500	140,625	168,750	196,875	225,000	234,375	48,919	54,789	66,790	93,924	125,232	178,618	45,360	68,040	50,006	105,014	168,022
	25	108,750	112,500	140,625	168,750	196,875	225,000	234,375	62,463	71,166	86,755	121,999	162,665	178,618	45,360	68,040	50,006	125,016	200,026

Table 34-6A. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 2500 psi)[1, 2, 3, 5]
Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_{sa} - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]											
		f_{uta} - for design purposes [4] - psi							$h = h_{ef}$ [10] and c_{a1} = [11]						$h = 1.5h_{ef}$ and c_{a1} = [11, 12]			$h = 2.25h_{ef}$ and c_{a1} = [11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1/4	2	724	749	936	1,123	1,310	1,498	1,560	316	NP	NP	350	429	606	525	643	910	965	1,114	1,364
	3	724	749	936	1,123	1,310	1,498	1,560	385	NP	NP	643	788	1,114	965	1,182	1,671	1,772	2,047	2,507
	4	724	749	936	1,123	1,310	1,498	1,560	385	NP	525	990	1,213	1,715	1,486	1,819	2,573	2,729	3,151	3,859
	5	724	749	936	1,123	1,310	1,498	1,560	385	NP	734	1,384	1,695	2,397	2,076	2,542	3,596	3,814	4,404	5,393
	6	724	749	936	1,123	1,310	1,498	1,560	385	NP	965	1,819	2,228	3,151	2,729	3,342	4,727	5,013	5,789	7,090
3/8	2	1,764	1,825	2,282	2,738	3,194	3,650	3,803	363	NP	NP	395	484	685	593	726	1,027	1,090	1,258	1,541
	3	1,764	1,825	2,282	2,738	3,194	3,650	3,803	499	NP	NP	788	965	1,364	1,182	1,447	2,047	2,171	2,507	3,070
	4	1,764	1,825	2,282	2,738	3,194	3,650	3,803	499	NP	643	1,213	1,485	2,101	1,819	2,228	3,151	3,342	3,859	4,727
	5	1,764	1,825	2,282	2,738	3,194	3,650	3,803	499	NP	899	1,695	2,076	2,936	2,542	3,114	4,404	4,671	5,393	6,606
	6	1,764	1,825	2,282	2,738	3,194	3,650	3,803	499	NP	1,182	2,228	2,729	3,859	3,342	4,093	5,789	6,140	7,090	8,683
1/2	2	3,212	3,323	4,154	4,984	5,815	6,646	6,923	403	NP	NP	431	528	747	647	792	1,120	1,188	1,372	1,680
	3	3,212	3,323	4,154	4,984	5,815	6,646	6,923	574	NP	NP	859	1,052	1,487	1,288	1,578	2,231	2,366	2,733	3,347
	4	3,212	3,323	4,154	4,984	5,815	6,646	6,923	608	NP	743	1,400	1,715	2,426	2,101	2,573	3,638	3,859	4,456	5,458
	5	3,212	3,323	4,154	4,984	5,815	6,646	6,923	608	NP	1,038	1,957	2,397	3,390	2,936	3,596	5,085	5,393	6,228	7,627
	6	3,212	3,323	4,154	4,984	5,815	6,646	6,923	608	NP	1,364	2,573	3,151	4,456	3,859	4,727	6,684	7,090	8,187	10,026
	7	3,212	3,323	4,154	4,984	5,815	6,646	6,923	608	608	1,719	3,242	3,971	5,615	4,863	5,956	8,423	8,934	10,316	12,635
	8	3,212	3,323	4,154	4,984	5,815	6,646	6,923	608	743	2,101	3,961	4,851	6,861	5,942	7,277	10,291	10,915	12,604	15,437
5/8	3	5,112	5,288	6,611	7,933	9,255	10,577	11,018	647	NP	NP	918	1,125	1,590	1,377	1,687	2,386	2,530	2,922	3,578
	4	5,112	5,288	6,611	7,933	9,255	10,577	11,018	685	NP	794	1,497	1,834	2,594	2,246	2,751	3,890	4,126	4,765	5,836
	5	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	NP	1,160	2,188	2,680	3,790	3,282	4,020	5,685	6,030	6,963	8,528
	6	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	NP	1,525	2,876	3,523	4,982	4,315	5,284	7,473	7,927	9,153	11,210
	7	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	NP	1,922	3,625	4,439	6,278	5,437	6,659	9,417	9,989	11,534	14,126
	8	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	830	2,349	4,429	5,424	7,671	6,643	8,136	11,506	12,204	14,092	17,259
	9	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	991	2,802	5,284	6,472	9,153	7,927	9,708	13,729	14,562	16,815	20,594
	10	5,112	5,288	6,611	7,933	9,255	10,577	11,018	716	1,160	3,282	6,189	7,580	10,720	9,284	11,370	16,080	17,055	19,694	24,120
3/4	4	7,555	7,816	9,770	11,723	13,677	15,631	16,283	761	NP	839	1,582	1,937	2,739	2,372	2,906	4,109	4,358	5,033	6,164
	5	7,555	7,816	9,770	11,723	13,677	15,631	16,283	796	NP	1,226	2,311	2,831	4,003	3,467	4,246	6,005	6,369	7,354	9,007
	6	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	NP	1,671	3,151	3,859	5,458	4,727	5,789	8,187	8,683	10,026	12,280
	7	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	NP	2,106	3,971	4,863	6,878	5,956	7,295	10,316	10,942	12,635	15,474
	8	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	910	2,573	4,851	5,942	8,403	7,277	8,912	12,604	13,369	15,437	18,906
	9	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	1,085	3,070	5,789	7,090	10,026	8,683	10,635	15,040	15,952	18,420	22,560
	10	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	1,271	3,596	6,780	8,304	11,743	10,170	12,455	17,615	18,683	21,574	26,422
	12	7,555	7,816	9,770	11,723	13,677	15,631	16,283	826	1,671	4,727	8,912	10,915	15,437	13,369	16,373	23,155	24,560	28,359	34,733
7/8	4	10,450	10,811	13,514	16,216	18,919	21,622	22,523	838	NP	878	1,656	2,029	2,869	2,485	3,043	4,304	4,565	5,271	6,455
	6	10,450	10,811	13,514	16,216	18,919	21,622	22,523	908	NP	1,750	3,300	4,042	5,716	4,950	6,063	8,574	9,094	10,501	12,861
	8	10,450	10,811	13,514	16,216	18,919	21,622	22,523	937	983	2,779	5,240	6,418	9,076	7,860	9,627	13,614	14,440	16,674	20,421
	12	10,450	10,811	13,514	16,216	18,919	21,622	22,523	937	1,805	5,105	9,627	11,790	16,674	14,440	17,685	25,010	26,528	30,631	37,516
	15	10,450	10,811	13,514	16,216	18,919	21,622	22,523	937	2,523	7,135	13,453	16,477	23,302	20,180	24,716	34,953	37,073	42,809	52,430
	18	10,450	10,811	13,514	16,216	18,919	21,622	22,523	937	3,316	9,379	17,685	21,660	30,631	26,528	32,489	45,947	48,734	56,273	68,921
	25	10,450	10,811	13,514	16,216	18,919	21,622	22,523	937	5,428	15,352	28,947	35,453	50,138	43,421	53,179	75,207	79,769	92,109	112,811

Table 34-6A. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 2500 psi)[1, 2, 3, 5] (cont'd.)
Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_{sa} - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]											
		f_{uta} - for design purposes [4] - psi							h = h_{ef}[10] and c_{a1} =[11]						h = 1.5h_{ef} and c_{a1} =[11, 12]			h = 2.25h_{ef} and c_{a1} =[11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	0.25h_{ef}	0.5h_{ef}	h_{ef}	1.5h_{ef}	3h_{ef}	h_{ef}	1.5h_{ef}	3h_{ef}	1.5h_{ef}	2h_{ef}	3h_{ef}
1	6	13,708	14,180	17,726	21,271	24,816	28,361	29,543	992	NP	1,822	3,435	4,207	5,950	5,153	6,311	8,924	9,466	10,930	13,387
	9	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	1,253	3,545	6,684	8,187	11,578	10,026	12,280	17,366	18,420	21,269	26,050
	12	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	1,930	5,458	10,291	12,604	17,825	15,437	18,906	26,737	28,359	32,746	40,106
	15	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	2,697	7,627	14,382	17,615	24,911	21,574	26,422	37,366	39,633	45,764	56,050
	18	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	3,545	10,026	18,906	23,155	32,746	28,359	34,733	49,119	52,099	60,159	73,679
	21	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	4,467	12,635	23,824	29,179	41,265	35,737	43,768	61,898	65,652	75,809	92,846
	25	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,050	5,802	16,412	30,946	37,901	53,600	46,419	56,851	80,400	85,277	98,469	120,600
1-1/8	6	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,076	NP	1,887	3,559	4,358	6,164	5,338	6,538	9,245	9,806	11,323	13,868
	9	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	1,329	3,760	7,090	8,683	12,280	10,635	13,025	18,420	19,537	22,560	27,630
	12	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	2,047	5,789	10,915	13,369	18,906	16,373	20,053	28,359	30,079	34,733	42,539
	15	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	2,860	8,090	15,255	18,683	26,422	22,882	28,025	39,633	42,037	48,540	59,450
	18	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	3,760	10,635	20,053	24,560	34,733	30,079	36,840	52,099	55,259	63,808	78,149
	21	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	4,738	13,401	25,270	30,949	43,768	37,904	46,423	65,652	69,635	80,407	98,478
	25	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,167	6,154	17,407	32,823	40,200	56,851	49,235	60,300	85,277	90,450	104,442	127,915
1-1/4	6	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,161	NP	1,948	3,673	4,498	6,362	5,509	6,747	9,542	10,121	11,687	14,314
	9	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,259	1,372	3,881	7,317	8,962	12,674	10,976	13,443	19,011	20,165	23,284	28,517
	12	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,286	2,157	6,102	11,506	14,092	19,929	17,259	21,138	29,893	31,706	36,611	44,840
	15	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,286	3,015	8,528	16,080	19,694	27,851	24,120	29,541	41,777	44,311	51,166	62,665
	18	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,286	3,963	11,210	21,138	25,888	36,611	31,706	38,832	54,917	58,248	67,260	82,376
	21	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,286	4,994	14,126	26,636	32,623	46,136	39,955	48,934	69,204	73,401	84,757	103,805
	25	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,286	6,487	18,349	34,599	42,374	59,926	51,898	63,562	89,890	95,342	110,092	134,834
1-3/8	6	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,248	NP	2,004	3,779	4,629	6,546	5,669	6,943	9,819	10,415	12,026	14,729
	9	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,353	1,412	3,993	7,530	9,222	13,042	11,295	13,833	19,563	20,749	23,959	29,344
	12	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,409	2,263	6,400	12,067	14,780	20,901	18,101	22,169	31,352	33,254	38,398	47,028
	15	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,409	3,162	8,944	16,865	20,655	29,211	25,297	30,983	43,816	46,474	53,663	65,724
	18	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,409	4,157	11,757	22,169	27,152	38,398	33,254	40,728	57,598	61,092	70,542	86,396
	21	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,409	5,238	14,816	27,937	34,215	48,388	41,905	51,323	72,581	76,984	88,894	108,872
	25	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,409	6,804	19,244	36,287	44,443	62,851	54,431	66,664	94,277	99,996	115,465	141,416
1-1/2	12	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,535	2,363	6,684	12,604	15,437	21,831	18,906	23,155	32,746	34,733	40,106	49,119
	15	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,535	3,303	9,342	17,615	21,574	30,510	26,422	32,360	45,764	48,540	56,050	68,647
	18	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,535	4,342	12,280	23,155	28,359	40,106	34,733	42,539	60,159	63,808	73,679	90,238
	21	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,535	5,471	15,474	29,179	35,737	50,539	43,768	53,605	75,809	80,407	92,846	113,713
	25	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,535	7,106	20,100	37,901	46,419	65,646	56,851	69,628	98,469	104,442	120,600	147,704
1-3/4	12	42,978	44,460	55,575	66,690	77,805	88,920	92,625	1,743	2,475	7,001	13,201	16,167	22,864	19,801	24,251	34,296	36,377	42,004	51,444
	15	42,978	44,460	55,575	66,690	77,805	88,920	92,625	1,798	3,567	10,090	19,026	23,302	32,954	28,539	34,953	49,431	52,430	60,541	74,147
	18	42,978	44,460	55,575	66,690	77,805	88,920	92,625	1,798	4,689	13,264	25,010	30,631	43,319	37,516	45,947	64,979	68,921	79,583	97,468
	21	42,978	44,460	55,575	66,690	77,805	88,920	92,625	1,798	5,909	16,714	31,517	38,600	54,589	47,275	57,900	81,883	86,850	100,286	122,824
	25	42,978	44,460	55,575	66,690	77,805	88,920	92,625	1,798	7,676	21,710	40,938	50,138	70,906	61,406	75,207	106,359	112,811	130,262	159,538
2	12	56,550	58,500	73,125	87,750	102,375	117,000	121,875	1,960	2,576	7,287	13,740	16,828	23,799	20,610	25,242	35,698	37,863	43,721	53,547
	15	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,049	3,765	10,648	20,079	24,591	34,778	30,118	36,887	52,166	55,331	63,891	78,250
	18	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,076	5,013	14,180	26,737	32,746	46,310	40,106	49,119	69,465	73,679	85,077	104,198
	21	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,076	6,317	17,868	33,693	41,265	58,358	50,539	61,898	87,536	92,846	107,210	131,305
	25	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,076	8,206	23,209	43,764	53,600	75,802	65,646	80,400	113,702	120,600	139,256	170,554

Table 34-6B. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 4000 psi)[1, 2, 3, 5]

Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_{sa} - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]											
		f_{uta} - for design purposes [4] - psi							$h = h_{ef}$ [10] and c_{a1} = [11]						$h = 1.5h_{ef}$ and c_{a1} = [11, 12]			$h = 2.25h_{ef}$ and c_{a1} = [11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1/4	2	724	749	936	1,123	1,310	1,498	1,560	399	NP	NP	443	542	767	664	814	1,151	1,220	1,409	1,726
	3	724	749	936	1,123	1,310	1,498	1,560	487	NP	NP	814	996	1,409	1,220	1,495	2,114	2,242	2,589	3,171
	4	724	749	936	1,123	1,310	1,498	1,560	487	NP	664	1,253	1,534	2,170	1,879	2,301	3,254	3,452	3,986	4,882
	5	724	749	936	1,123	1,310	1,498	1,560	487	NP	928	1,751	2,144	3,032	2,626	3,216	4,548	4,824	5,570	6,822
	6	724	749	936	1,123	1,310	1,498	1,560	487	NP	1,220	2,301	2,818	3,986	3,452	4,228	5,979	6,341	7,322	8,968
3/8	2	1,764	1,825	2,282	2,738	3,194	3,650	3,803	459	NP	NP	500	613	866	750	919	1,299	1,378	1,591	1,949
	3	1,764	1,825	2,282	2,738	3,194	3,650	3,803	631	NP	NP	996	1,220	1,726	1,495	1,831	2,589	2,746	3,171	3,883
	4	1,764	1,825	2,282	2,738	3,194	3,650	3,803	631	NP	814	1,534	1,879	2,657	2,301	2,818	3,986	4,228	4,882	5,979
	5	1,764	1,825	2,282	2,738	3,194	3,650	3,803	631	NP	1,137	2,144	2,626	3,714	3,216	3,939	5,570	5,908	6,822	8,355
	6	1,764	1,825	2,282	2,738	3,194	3,650	3,803	631	NP	1,495	2,818	3,452	4,882	4,228	5,178	7,322	7,766	8,968	10,983
1/2	2	3,212	3,323	4,154	4,984	5,815	6,646	6,923	510	NP	NP	545	668	944	818	1,002	1,417	1,502	1,735	2,125
	3	3,212	3,323	4,154	4,984	5,815	6,646	6,923	726	NP	NP	1,086	1,330	1,881	1,629	1,996	2,822	2,993	3,456	4,233
	4	3,212	3,323	4,154	4,984	5,815	6,646	6,923	769	NP	939	1,771	2,170	3,068	2,657	3,254	4,602	4,882	5,637	6,904
	5	3,212	3,323	4,154	4,984	5,815	6,646	6,923	769	NP	1,313	2,476	3,032	4,288	3,714	4,548	6,432	6,822	7,878	9,648
	6	3,212	3,323	4,154	4,984	5,815	6,646	6,923	769	NP	1,726	3,254	3,986	5,637	4,882	5,979	8,455	8,968	10,355	12,683
	7	3,212	3,323	4,154	4,984	5,815	6,646	6,923	769	769	2,175	4,101	5,023	7,103	6,151	7,534	10,655	11,301	13,049	15,982
	8	3,212	3,323	4,154	4,984	5,815	6,646	6,923	769	939	2,657	5,010	6,136	8,678	7,516	9,205	13,017	13,807	15,943	19,526
5/8	3	5,112	5,288	6,611	7,933	9,255	10,577	11,018	818	NP	NP	1,161	1,422	2,012	1,742	2,134	3,018	3,201	3,696	4,526
	4	5,112	5,288	6,611	7,933	9,255	10,577	11,018	867	NP	1,004	1,894	2,320	3,281	2,841	3,480	4,921	5,220	6,027	7,381
	5	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	NP	1,468	2,768	3,390	4,794	4,152	5,085	7,191	7,627	8,807	10,787
	6	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	NP	1,930	3,638	4,456	6,302	5,458	6,684	9,453	10,026	11,578	14,180
	7	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	NP	2,432	4,585	5,615	7,941	6,878	8,423	11,912	12,636	14,589	17,868
	8	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	1,050	2,971	5,602	6,861	9,703	8,403	10,291	14,554	15,437	17,825	21,831
	9	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	1,253	3,545	6,684	8,187	11,578	10,026	12,280	17,366	18,420	21,269	26,050
	10	5,112	5,288	6,611	7,933	9,255	10,577	11,018	906	1,468	4,152	7,829	9,588	13,560	11,743	14,382	20,340	21,574	24,911	30,510
3/4	4	7,555	7,816	9,770	11,723	13,677	15,631	16,283	963	NP	1,061	2,001	2,450	3,465	3,001	3,675	5,198	5,513	6,366	7,796
	5	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,007	NP	1,550	2,923	3,581	5,064	4,385	5,371	7,595	8,056	9,303	11,393
	6	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	NP	2,114	3,986	4,882	6,904	5,979	7,322	10,355	10,983	12,683	15,533
	7	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	NP	2,664	5,023	6,151	8,699	7,534	9,227	13,049	13,841	15,982	19,574
	8	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	1,151	3,254	6,136	7,516	10,629	9,205	11,273	15,943	16,910	19,526	23,915
	9	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	1,373	3,883	7,322	8,968	12,683	10,983	13,452	19,024	20,178	23,299	28,536
	10	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	1,608	4,548	8,576	10,503	14,854	12,864	15,755	22,281	23,633	27,289	33,422
	12	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,044	2,114	5,979	11,273	13,807	19,526	16,910	20,711	29,289	31,066	35,872	43,934
7/8	4	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,060	NP	1,111	2,095	2,566	3,629	3,143	3,849	5,444	5,774	6,667	8,165
	6	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,149	NP	2,214	4,174	5,113	7,230	6,262	7,669	10,845	11,503	13,283	16,268
	8	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,185	1,243	3,515	6,628	8,118	11,480	9,942	12,177	17,220	18,265	21,091	25,831
	12	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,185	2,283	6,458	12,177	14,913	21,091	18,265	22,370	31,636	33,555	38,746	47,454
	15	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,185	3,191	9,025	17,017	20,842	29,475	25,526	31,263	44,213	46,894	54,149	66,319
	18	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,185	4,194	11,863	22,370	27,398	38,746	33,555	41,096	58,119	61,644	71,181	87,178
	25	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,185	6,865	19,418	36,616	44,845	63,420	54,923	67,267	95,130	100,901	116,510	142,695

Table 34-6B. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 4000 psi)[1, 2, 3, 5] (cont'd.)

Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_{sa} - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]											
		f_{uta} - for design purposes [4] - psi							$h = h_{ef}$ [10] and c_{a1} = [11]						$h = 1.5h_{ef}$ and c_{a1} = [11, 12]			$h = 2.25h_{ef}$ and c_{a1} = [11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1	6	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,254	NP	2,304	4,345	5,322	7,526	6,518	7,982	11,289	11,973	13,826	16,933
	9	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	1,585	4,484	8,455	10,355	14,645	12,683	15,533	21,967	23,299	26,904	32,950
	12	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	2,441	6,904	13,017	15,943	22,547	19,526	23,915	33,820	35,872	41,421	50,730
	15	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	3,411	9,648	18,192	22,281	31,510	27,289	33,422	47,265	50,132	57,888	70,898
	18	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	4,484	12,683	23,915	29,289	41,421	35,872	43,934	62,132	65,901	76,096	93,198
	21	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	5,650	15,982	30,136	36,909	52,197	45,204	55,363	78,295	83,044	95,891	117,442
	25	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,329	7,339	20,759	39,144	47,941	67,799	58,716	71,912	101,699	107,868	124,555	152,548
1-1/8	6	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,361	NP	2,387	4,501	5,513	7,796	6,752	8,269	11,695	12,404	14,323	17,542
	9	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	1,681	4,756	8,968	10,983	15,533	13,452	16,475	23,299	24,713	28,536	34,949
	12	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	2,589	7,322	13,807	16,910	23,915	20,711	25,365	35,872	38,048	43,934	53,808
	15	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	3,618	10,233	19,296	23,633	33,422	28,944	35,449	50,132	53,173	61,399	75,198
	18	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	4,756	13,452	25,365	31,066	43,934	38,048	46,599	65,901	69,898	80,711	98,851
	21	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	5,993	16,951	31,964	39,147	55,363	47,946	58,721	83,044	88,082	101,708	124,566
	25	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,476	7,785	22,018	41,518	50,849	71,912	62,277	76,274	107,868	114,411	132,110	161,801
1-1/4	6	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,469	NP	2,464	4,646	5,690	8,047	6,969	8,535	12,070	12,802	14,783	18,105
	9	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,593	1,735	4,909	9,256	11,336	16,032	13,884	17,004	24,048	25,506	29,452	36,071
	12	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,627	2,729	7,718	14,554	17,825	25,208	21,831	26,737	37,812	40,106	46,310	56,718
	15	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,627	3,814	10,787	20,340	24,911	35,229	30,510	37,366	52,844	56,050	64,721	79,266
	18	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,627	5,013	14,180	26,737	32,746	46,310	40,106	49,119	69,465	73,679	85,077	104,198
	21	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,627	6,317	17,868	33,693	41,265	58,358	50,539	61,898	87,536	92,846	107,210	131,305
	25	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,627	8,206	23,209	43,764	53,600	75,802	65,646	80,400	113,702	120,600	139,256	170,554
1-3/8	6	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,579	NP	2,535	4,781	5,855	8,280	7,171	8,783	12,420	13,174	15,212	18,631
	9	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,712	1,786	5,051	9,524	11,665	16,497	14,287	17,497	24,745	26,246	30,306	37,118
	12	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,782	2,862	8,095	15,264	18,695	26,438	22,896	28,042	39,658	42,063	48,571	59,487
	15	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,782	4,000	11,313	21,332	26,127	36,949	31,999	39,190	55,423	58,785	67,879	83,135
	18	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,782	5,258	14,872	28,042	34,345	48,571	42,063	51,517	72,856	77,275	89,230	109,284
	21	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,782	6,626	18,740	35,337	43,279	61,206	53,006	64,919	91,809	97,378	112,443	137,713
	25	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,782	8,606	24,342	45,900	56,216	79,501	68,850	84,324	119,252	126,486	146,053	178,878
1-1/2	12	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,942	2,989	8,455	15,943	19,526	27,614	23,915	29,289	41,421	43,934	50,730	62,132
	15	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,942	4,178	11,816	22,281	27,289	38,592	33,422	40,933	57,888	61,399	70,898	86,832
	18	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,942	5,492	15,533	29,289	35,872	50,730	43,934	53,808	76,096	80,711	93,198	114,143
	21	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,942	6,920	19,574	36,909	45,204	63,928	55,363	67,805	95,891	101,708	117,442	143,837
	25	31,894	32,994	41,243	49,491	57,740	65,988	68,738	1,942	8,989	25,425	47,941	58,716	83,037	71,912	88,074	124,555	132,110	152,548	186,832
1-3/4	12	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,205	3,131	8,855	16,698	20,450	28,921	25,046	30,675	43,382	46,013	53,131	65,073
	15	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,274	4,512	12,763	24,066	29,475	41,684	36,099	44,213	62,526	66,319	76,578	93,789
	18	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,274	5,932	16,777	31,636	38,746	54,795	47,454	58,119	82,193	87,178	100,665	123,289
	21	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,274	7,475	21,142	39,866	48,825	69,050	59,799	73,238	103,574	109,857	126,852	155,362
	25	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,274	9,709	27,462	51,782	63,420	89,690	77,674	95,130	134,535	142,695	164,770	201,802
2	12	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,479	3,259	9,217	17,380	21,286	30,103	26,070	31,929	45,155	47,894	55,303	67,732
	15	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,592	4,762	13,469	25,398	31,106	43,991	38,097	46,659	65,986	69,989	80,816	98,979
	18	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,626	6,341	17,936	33,820	41,421	58,578	50,730	62,132	87,868	93,198	107,615	131,801
	21	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,626	7,991	22,602	42,618	52,197	73,817	63,928	78,295	110,726	117,442	135,611	166,089
	25	56,550	58,500	73,125	87,750	102,375	117,000	121,875	2,626	10,380	29,358	55,358	67,799	95,882	83,037	101,699	143,823	152,546	176,147	215,735

Table 34-6C. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 6000 psi)[1, 2, 3, 5]
Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_{sa} - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]											
		f_{uta} - for design purposes [4] - psi								$h = h_{ef}$ [10] and c_{a1} = [11]					$h = 1.5h_{ef}$ and c_{a1} = [11, 12]			$h = 2.25h_{ef}$ and c_{a1} = [11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1/4	2	724	749	936	1,123	1,310	1,498	1,560	489	NP	NP	542	664	939	814	996	1,409	1,495	1,726	2,114
	3	724	749	936	1,123	1,310	1,498	1,560	596	NP	NP	996	1,220	1,726	1,495	1,831	2,589	2,746	3,171	3,883
	4	724	749	936	1,123	1,310	1,498	1,560	596	NP	814	1,534	1,879	2,657	2,301	2,818	3,986	4,228	4,882	5,979
	5	724	749	936	1,123	1,310	1,498	1,560	596	NP	1,137	2,144	2,626	3,714	3,216	3,939	5,570	5,908	6,822	8,355
	6	724	749	936	1,123	1,310	1,498	1,560	596	NP	1,495	2,818	3,452	4,882	4,228	5,178	7,322	7,766	8,968	10,983
3/8	2	1,764	1,825	2,282	2,738	3,194	3,650	3,803	563	NP	NP	613	750	1,061	919	1,125	1,591	1,688	1,949	2,387
	3	1,764	1,825	2,282	2,738	3,194	3,650	3,803	772	NP	NP	1,220	1,495	2,114	1,831	2,242	3,171	3,363	3,883	4,756
	4	1,764	1,825	2,282	2,738	3,194	3,650	3,803	772	NP	996	1,879	2,301	3,254	2,818	3,452	4,882	5,178	5,979	7,322
	5	1,764	1,825	2,282	2,738	3,194	3,650	3,803	772	NP	1,393	2,626	3,216	4,548	3,939	4,824	6,822	7,236	8,355	10,233
	6	1,764	1,825	2,282	2,738	3,194	3,650	3,803	772	NP	1,831	3,452	4,228	5,979	5,178	6,341	8,968	9,512	10,983	13,452
1/2	2	3,212	3,323	4,154	4,984	5,815	6,646	6,923	625	NP	NP	668	818	1,157	1,002	1,227	1,735	1,840	2,125	2,602
	3	3,212	3,323	4,154	4,984	5,815	6,646	6,923	889	NP	NP	1,330	1,629	2,304	1,996	2,444	3,456	3,666	4,233	5,185
	4	3,212	3,323	4,154	4,984	5,815	6,646	6,923	942	NP	1,151	2,170	2,657	3,758	3,254	3,986	5,637	5,979	6,904	8,455
	5	3,212	3,323	4,154	4,984	5,815	6,646	6,923	942	NP	1,608	3,032	3,714	5,252	4,548	5,570	7,878	8,355	9,648	11,816
	6	3,212	3,323	4,154	4,984	5,815	6,646	6,923	942	NP	2,114	3,986	4,882	6,904	5,979	7,322	10,355	10,983	12,683	15,533
	7	3,212	3,323	4,154	4,984	5,815	6,646	6,923	942	942	2,664	5,023	6,151	8,699	7,534	9,227	13,049	13,841	15,982	19,574
	8	3,212	3,323	4,154	4,984	5,815	6,646	6,923	942	1,151	3,254	6,136	7,516	10,629	9,205	11,273	15,943	16,910	19,526	23,915
5/8	3	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,002	NP	NP	1,422	1,742	2,464	2,134	2,613	3,696	3,920	4,526	5,544
	4	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,061	NP	1,230	2,320	2,841	4,018	3,480	4,262	6,027	6,393	7,381	9,040
	5	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	NP	1,798	3,390	4,152	5,872	5,085	6,228	8,807	9,342	10,787	13,211
	6	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	NP	2,363	4,456	5,458	7,718	6,684	8,187	11,578	12,280	14,180	17,366
	7	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	NP	2,978	5,615	6,878	9,726	8,423	10,316	14,589	15,474	17,868	21,884
	8	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	1,286	3,638	6,861	8,403	11,883	10,291	12,604	17,825	18,906	21,831	26,737
	9	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	1,535	4,342	8,187	10,026	14,180	12,280	15,040	21,269	22,560	26,050	31,904
	10	5,112	5,288	6,611	7,933	9,255	10,577	11,018	1,110	1,798	5,085	9,588	11,743	16,607	14,382	17,615	24,911	26,422	30,510	37,366
3/4	4	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,180	NP	1,299	2,450	3,001	4,244	3,675	4,501	6,366	6,752	7,796	9,549
	5	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,233	NP	1,899	3,581	4,385	6,202	5,371	6,578	9,303	9,867	11,393	13,954
	6	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	NP	2,589	4,882	5,979	8,455	7,322	8,968	12,683	13,452	15,533	19,024
	7	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	NP	3,262	6,151	7,534	10,655	9,227	11,301	15,982	16,951	19,574	23,973
	8	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	1,409	3,986	7,516	9,205	13,017	11,273	13,807	19,526	20,711	23,915	29,289
	9	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	1,681	4,756	8,968	10,983	15,533	13,452	16,475	23,299	24,713	28,536	34,949
	10	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	1,969	5,570	10,503	12,864	18,192	15,755	19,296	27,289	28,944	33,422	40,933
	12	7,555	7,816	9,770	11,723	13,677	15,631	16,283	1,279	2,589	7,322	13,807	16,910	23,915	20,711	25,365	35,872	38,048	43,934	53,808
7/8	4	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,298	NP	1,361	2,566	3,143	4,445	3,849	4,714	6,667	7,072	8,165	10,001
	6	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,407	NP	2,711	5,113	6,262	8,855	7,669	9,392	13,283	14,089	16,268	19,924
	8	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,451	1,522	4,305	8,118	9,942	14,060	12,177	14,913	21,091	22,370	25,831	31,636
	12	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,451	2,796	7,909	14,913	18,265	25,831	22,370	27,398	38,746	41,096	47,454	58,119
	15	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,451	3,908	11,053	20,842	25,526	36,099	31,263	38,289	54,149	57,434	66,319	81,224
	18	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,451	5,137	14,530	27,398	33,555	47,454	41,096	50,332	71,181	75,499	87,178	106,771
	25	10,450	10,811	13,514	16,216	18,919	21,622	22,523	1,451	8,408	23,783	44,845	54,923	77,674	67,267	82,385	116,510	123,578	142,695	174,765

Table 34-6C. Design Strengths for Single Cast-In Anchors Subject to Shear Loads (f'_c = 6000 psi)[1, 2, 3, 5] (cont'd.)

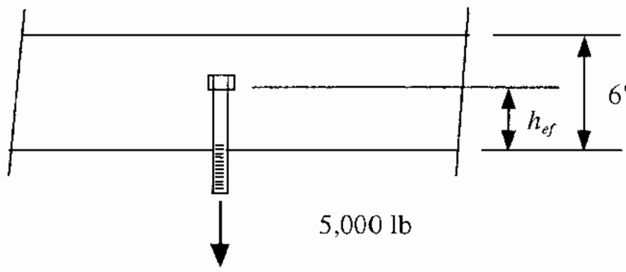
Notes pertaining to this table are given on Page 34-15

d_o in.	h_{ef} in.	ϕV_{sa} - Shear Strength of Anchor							ϕV_{cb} - Shear Breakout [5, 6, 7, 8, 9]											
		f_{uta} - for design purposes [4] - psi								$h = h_{ef}$[10] and c_{a1} =[11]					$h = 1.5h_{ef}$ and c_{a1} =[11, 12]			$h = 2.25h_{ef}$ and c_{a1} =[11, 13]		
		58,000	60,000	75,000	90,000	105,000	120,000	125,000	1-1/2-in. cover	$0.25h_{ef}$	$0.5h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	h_{ef}	$1.5h_{ef}$	$3h_{ef}$	$1.5h_{ef}$	$2h_{ef}$	$3h_{ef}$
1	6	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,536	NP	2,822	5,322	6,518	9,217	7,982	9,776	13,826	14,664	16,933	20,739
	9	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	1,942	5,492	10,355	12,683	17,936	15,533	19,024	26,904	28,536	32,950	40,356
	12	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	2,989	8,455	15,943	19,526	27,614	23,915	29,289	41,421	43,934	50,730	62,132
	15	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	4,178	11,816	22,281	27,289	38,592	33,422	40,933	57,888	61,399	70,898	86,832
	18	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	5,492	15,533	29,289	35,872	50,730	43,934	53,808	76,096	80,711	93,198	114,143
	21	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	6,920	19,574	36,909	45,204	63,928	55,363	67,805	95,891	101,708	117,442	143,837
	25	13,708	14,180	17,726	21,271	24,816	28,361	29,543	1,627	8,989	25,425	47,941	58,716	83,037	71,912	88,074	124,555	132,110	152,548	186,832
1-1/8	6	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,667	NP	2,924	5,513	6,752	9,549	8,269	10,128	14,323	15,192	17,542	21,485
	9	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	2,059	5,825	10,983	13,452	19,024	16,475	20,178	28,536	30,267	34,949	42,804
	12	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	3,171	8,968	16,910	20,711	29,289	25,365	31,066	43,934	46,599	53,808	65,901
	15	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	4,431	12,533	23,633	28,944	40,933	35,449	43,416	61,399	65,124	75,198	92,099
	18	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	5,825	16,475	31,066	38,048	53,808	46,599	57,072	80,711	85,607	98,851	121,067
	21	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	7,340	20,761	39,147	47,946	67,805	58,721	71,918	101,708	107,878	124,566	152,562
	25	17,259	17,854	22,318	26,781	31,245	35,708	37,196	1,807	9,534	26,967	50,849	62,277	88,074	76,274	93,416	132,110	140,124	161,801	198,165
1-1/4	6	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,799	NP	3,018	5,690	6,969	9,855	8,535	10,453	14,783	15,680	18,105	22,174
	9	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,951	2,126	6,012	11,336	13,884	19,635	17,004	20,826	29,452	31,239	36,071	44,178
	12	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,992	3,342	9,453	17,825	21,831	30,874	26,737	32,746	46,310	49,119	56,718	69,465
	15	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,992	4,671	13,211	24,911	30,510	43,147	37,366	45,764	64,721	68,647	79,266	97,081
	18	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,992	6,140	17,366	32,746	40,106	56,718	49,119	60,159	85,077	90,238	104,198	127,616
	21	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,992	7,737	21,884	41,265	50,539	71,473	61,898	75,809	107,210	113,713	131,305	160,815
	25	21,919	22,675	28,343	34,012	39,681	45,349	47,239	1,992	10,050	28,426	53,600	65,646	92,838	80,400	98,469	139,256	147,704	170,554	208,885
1-3/8	6	26,239	27,144	33,930	40,716	47,502	54,288	56,550	1,933	NP	3,105	5,855	7,171	10,141	8,783	10,756	15,212	16,135	18,631	22,818
	9	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,097	2,187	6,186	11,665	14,287	20,204	17,497	21,430	30,306	32,145	37,118	45,460
	12	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,183	3,505	9,914	18,695	22,896	32,380	28,042	34,345	48,571	51,517	59,487	72,856
	15	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,183	4,899	13,856	26,127	31,999	45,253	39,190	47,998	67,879	71,997	83,135	101,819
	18	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,183	6,440	18,214	34,345	42,063	59,487	51,517	63,095	89,230	94,643	109,284	133,845
	21	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,183	8,115	22,952	43,279	53,006	74,962	64,919	79,509	112,443	119,263	137,713	168,664
	25	26,239	27,144	33,930	40,716	47,502	54,288	56,550	2,183	10,540	29,813	56,216	68,850	97,369	84,324	103,275	146,053	154,913	178,878	219,080
1-1/2	12	31,894	32,994	41,243	49,491	57,740	65,988	68,738	2,378	3,661	10,355	19,526	23,915	33,820	29,289	35,872	50,730	53,808	62,132	76,096
	15	31,894	32,994	41,243	49,491	57,740	65,988	68,738	2,378	5,117	14,472	27,289	33,422	47,265	40,933	50,132	70,898	75,198	86,832	106,347
	18	31,894	32,994	41,243	49,491	57,740	65,988	68,738	2,378	6,726	19,024	35,872	43,934	62,132	53,808	65,901	93,198	98,851	114,143	139,796
	21	31,894	32,994	41,243	49,491	57,740	65,988	68,738	2,378	8,476	23,973	45,204	55,363	78,295	67,805	83,044	117,442	124,566	143,837	176,164
	25	31,894	32,994	41,243	49,491	57,740	65,988	68,738	2,378	11,009	31,139	58,716	71,912	101,699	88,074	107,868	152,548	161,801	186,832	228,822
1-3/4	12	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,701	3,834	10,845	20,450	25,046	35,421	30,675	37,570	53,131	56,354	65,073	79,697
	15	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,786	5,527	15,631	29,475	36,099	51,052	44,213	54,149	76,578	81,224	93,789	114,868
	18	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,786	7,265	20,548	38,746	47,454	67,110	58,119	71,181	100,665	106,771	123,289	150,997
	21	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,786	9,155	25,894	48,825	59,799	84,568	73,238	89,698	126,852	134,547	155,362	190,278
	25	42,978	44,460	55,575	66,690	77,805	88,920	92,625	2,786	11,891	33,634	63,420	77,674	109,847	95,130	116,510	164,770	174,765	201,802	247,156
2	12	56,550	58,500	73,125	87,750	102,375	117,000	121,875	3,036	3,991	11,289	21,286	26,070	36,869	31,929	39,105	55,303	58,658	67,732	82,955
	15	56,550	58,500	73,125	87,750	102,375	117,000	121,875	3,175	5,832	16,496	31,106	38,097	53,877	46,659	57,145	80,816	85,718	98,979	121,224
	18	56,550	58,500	73,125	87,750	102,375	117,000	121,875	3,216	7,766	21,967	41,421	50,730	71,744	62,132	76,096	107,615	114,143	131,801	161,423
	21	56,550	58,500	73,125	87,750	102,375	117,000	121,875	3,216	9,787	27,681	52,197	63,928	90,407	78,295	95,891	135,611	143,837	166,089	203,416
	25	56,550	58,500	73,125	87,750	102,375	117,000	121,875	3,216	12,712	35,956	67,799	83,037	117,431	101,699	124,555	176,147	186,832	215,735	264,221

Example 34.1—Single Headed Bolt in Tension Away from Edges

Design a single headed bolt installed in the bottom of a 6 in. slab to support a 5000 lb service dead load.

f'_c = 4000 psi



Calculations and Discussion	Code Reference
1. Determine factored design load (only dead load is present)	*9.2*
$N_u = 1.4\ (5000) = 7000$ lb	*Eq. (9-1)*
2. Determine anchor diameter and material	*D.5.1*
The strength of most anchors is likely to be controlled by the embedment strength rather than the steel strength. As a result, it is usually economical to design the anchor using a mild steel rather than a high strength steel. ASTM F 1554 "Standard Specification for Anchor Bolts, Steel, 36, 55, and 105-ksi Yield Strength," covers straight and bent, headed and headless, anchors in three strength grades.	
Assume an ASTM F 1554 Grade 36 headed anchor for this example.	
The basic requirement for the anchor steel is:	
$\phi N_{sa} \geq N_{ua}$	*Eq. (D-1)* *D.4.1.2*
where:	
$\phi = 0.75$	*D.4.4(a)*
Per the Ductile Steel Element definition in D.1, ASTM F 1554 Grade 36 steel qualifies as a ductile steel element (23% minimum elongation in 2 in. which is greater than the 14% required and a minimum reduction in area of 40% that is greater than the 30% required, see Table 34.1). This results in $\phi = 0.75$ rather than $\phi = 0.65$ if the steel had not met the ductile steel element requirements.	
$N_{sa} = nA_{se}f_{uta}$	*Eq. (D-3)*
For design purposes, Eq. (D-1) with Eq. (D-3) may be rearranged as:	

$$A_{se} \geq \frac{N_{ua}}{\phi\ n\ f_{uta}}$$

where:

$N_{ua} = 7000$ lbs
$\phi = 0.75$
$n = 1$
$f_{uta} = 58{,}000$ psi

Per ASTM F 1554, Grade 36 has a specified minimum yield strength of 36 ksi and a specified tensile strength of 58-80 ksi (see Table 34-1). For design purposes, the minimum tensile strength of 58 ksi should be used.

Note: Per D.5.1.2, f_{uta} shall not be taken greater than $1.9f_{ya}$ or 125,000 psi. For ASTM F 1554 Grade 36, $1.9f_{ya} = 1.9(36{,}000) = 68{,}400$ psi, therefore use the specified minimum f_{uta} of 58,000 psi. *D.5.1.2*

Substituting:

$$A_{se} = \frac{7000}{0.75(1)(58{,}000)} = 0.161 \text{ in.}^2$$

Per Table 34-2, a 5/8 in. diameter threaded anchor will satisfy this requirement ($A_{se} = 0.226$ in.^{2}).

3. Determine the required embedment length (h_{ef}) based on concrete breakout *D.5.2*

The basic requirement for the single anchor embedment is:

$$\phi N_{cb} \geq N_{ua}$$ *Eq. (D-1)* *D.4.1.2*

where:

$\phi = 0.70$ *D.4.4(c)*

Condition B applies, no supplementary reinforcement has been provided to tie the failure prism associated with the concrete breakout failure mode of the anchor to the supporting structural member. This is likely to be the case for anchors loaded in tension attached to a slab. Condition A (with $\phi = 0.75$) may apply when the anchoring is attached to a deeper member (such as a pedestal or beam) where there is space available to install supplementary reinforcement across the failure prism.

$$N_{cb} = \frac{A_{Nc}}{A_{Nco}} \psi_{ed,N} \psi_{c,N}, \psi_{cp,} N_b$$ *Eq. (D-4)*

where:

$\frac{A_{Nc}}{A_{Nco}}$ and $\psi_{ed,N}$ terms are 1.0 for single anchors away from edges

For cast-in anchors $\psi_{cp,N} = 1.0$

$$N_b = 24\sqrt{f'_c}\, h_{ef}^{1.5}$$ *Eq. (D-7)*

For design of a single anchor away from edges, Eq. (D-1) with Eq. (D-4) and Eq. (D-7) may be rearranged as:

$$h_{ef} = \left(\frac{N_{ua}}{\phi\, \psi_{c,N}\, 24\sqrt{f'_c}} \right)^{\frac{2}{3}}$$

where:

$\psi_{c,N}$ = 1.0 for locations where concrete cracking is likely to occur (i.e., the bottom of the slab) *D.5.2.6*

Substituting:

$$h_{ef} = \left(\frac{7000}{0.70(1.0)24\sqrt{4000}} \right)^{\frac{2}{3}} = 3.51 \text{ in.}$$

Select 4 in. embedment for this anchor.

Note: The case of a single anchor away from an edge is essentially the only case where h_{ef} can be solved for directly. Whenever edges or adjacent anchors are present, the solution for h_{ef} is iterative.

4. Determine the required head size for the anchor *D.5.3*

The basic requirement for pullout strength (i.e., the strength of the anchor related to the embedded anchor having insufficient bearing area so that the anchor pulls out without a concrete breakout failure) is:

$\phi N_{pn} \geq N_{ua}$ *Eq. (D-1)* *D.4.1.2*

where:

$\phi = 0.70$ *D.4.4(c)*

Condition B applies for pullout strength in all cases

$N_{pn} = \psi_{c,P}\, N_p$ *Eq. (D-14)*

where:

$N_p = A_{brg}\, 8\, f'_c$ *Eq. (D-15)*

$\psi_{c,P}$ = 1.0 for locations where concrete cracking is likely to occur (i.e., the bottom of the slab) *D.5.3.6*

For design purposes Eq. (D-1) with Eq. (D-14) and Eq. (D-15) may be rearranged as:

$$A_{brg} = \frac{N_{ua}}{\phi \psi_{c,P}\, 8\, f'_c}$$

Substituting:

$$A_{brg} = \frac{7000}{0.70(1.0)(8)(4000)} = 0.313 \text{ in}^2$$

As shown in Table 34-2, any type of standard head (square, heavy square, hex, or heavy hex) is acceptable for this 5/8 in. diameter anchor. ASTM F 1554 specifies a hex head for Grade 36 bolts less than 1-1/2 in. in diameter.

5. Evaluate side-face blowout *D.5.4*

Since this anchor is located far from a free edge of concrete ($c_{a1} \geq 0.4\ h_{ef}$) this type of failure mode is not applicable.

6. Required edge distances, spacings, and thicknesses to preclude splitting failure *D.8*

Since this is a cast-in-place anchor and is located far from a free edge of concrete, the only requirement is that the minimum cover requirements of Section 7.7 should be met. Assuming this is an interior slab, the requirements of Section 7.7 will be met with the 4 in. embedment length plus the head thickness. The head thickness for square, hex, and heavy hex heads and nuts are at most equal to the anchor diameter (refer to ANSI B.18.2.1 and ANSI B.18.2.2 for exact dimensions). This results in ~1-3/8 in. cover from the top of the anchor head to the top of the slab.

7. Summary:

Use an ASTM F 1554 Grade 36, 5/8 in. diameter headed anchor with a 4 in. embedment.

Alternate design using Table 34-5B

Note: Step numbers correspond to those in the main example above, but prefaced with "A".

Table 34-5B has been selected because it contains design tension strength values based on concrete with f'_c = 4000 psi. Table Note 4 indicates that the values in the table are based on Condition B (no supplementary reinforcement), and Notes 6 and 10 indicate that cracked concrete was assumed.

A2. Determine anchor diameter and material. *D.5.1*
Eq. (D-3)

Tentatively try a bolt complying with ASTM F 1554 Grade 36 with a f_{uta} of 58,000 psi. Using the factored tensile load from Step 1 above (7000 lb), and 58,000 psi as the value of f_{uta}, go down the column for 58,000 until an anchor size has a design tensile strength, ϕN_{sa}, equal to or greater than 7000 lb. Table 34-6B shows that a 5/8 in. diameter bolt has a design tensile strength equal to

$\phi N_{sa} = 9831 \text{ lb} > N_u = 7000 \text{ lb}$ O.K.

Since this is greater than the required strength, tentatively use a 5/8 in. headed bolt.

A3. Determine the required embedment length (h_{ef}) based on concrete breakout strength (ϕN_{cb}) — *D.5.2, Eq. (D-4)*

Since the anchor will be far from an edge, use the column labeled "$>1.5h_{ef}$." In this case "far from an edge" means that the edge distance, c_{a1}, must equal or exceed $1.5h_{ef}$. A 5/8 in. bolt with 3 in. embedment has a design tension breakout strength,

$\phi N_{cb} = 5521 \text{ lb} < N_{ua} = 7000 \text{ lb}$

A 5/8-in. bolt with 4 in. embedment has

$\phi N_{cb} = 8500 \text{ lb} > N_{ua} = 7000 \text{ lb}$ O.K.

Tentatively use 5/8 in. bolt with embedment depth of 4 in.

A4. Determine if the bearing area of the head of the 5/8-in. bolt, A_{brg}, is large enough to prevent anchor pullout (ϕN_{pn}). — *D.5.3, Eq. (D-14)*

Values for design tension pullout strength, ϕN_{pn}, in Table 34-5B for headed bolts with a diameter of less than 1-3/4 in. are based on a regular hex head (Table Note 7). Under the column labeled "head w/o washer," a 5/8 in. bolt has a design pullout strength,

$\phi N_{pn} = 10{,}170 \text{ lb} > N_{ua} = 7000 \text{ lb}$ O.K.

A5. Determine if the anchor has enough edge distance, c_{a1}, to prevent side-face blowout (ϕN_{sb}). — *D.5.4.1*

Since the anchor is farther from an edge than $0.4h_{ef}$ (0.4 x 4 in. = 1.6 in.), side-face blowout does not need to be considered.

A6. Required edge distances, spacings, and thicknesses to preclude splitting failure.

See Step 6 above.

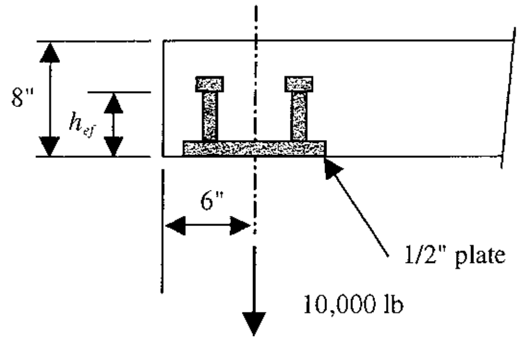
A7. Summary:

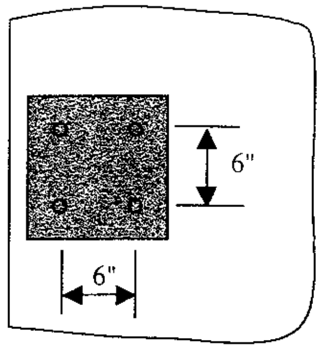
Use an ASTM F 1554 Grade 36, 5/8 in. diameter headed bolt with a 4 in. embedment.

Example 34.2—Group of Headed Studs in Tension Near an Edge

Design a group of four welded, headed studs spaced 6 in. on center each way and concentrically loaded with a 10,000 lb service dead load. The anchor group is to be installed in the bottom of an 8 in. thick slab with the centerline of the connection 6 in. from a free edge of the slab.

$f'_c = 4000$ psi





Calculations and Discussion	Code Reference
1. Determine factored design load	*9.2*
$N_{ua} = 1.4\ (10{,}000) = 14{,}000$ lb	*Eq. (9-1)*
2. Determine anchor diameter	*D.5.1*
Assume AWS D1.1 Type B welded, headed studs.	
The basic requirement for the anchor steel is:	
$\phi N_{sa} \geq N_{ua}$	*Eq. (D-1)* *D.4.1.2*
where:	
$\phi = 0.75$	*D.4.4(a)i*
Per the Ductile Steel Element definition in D.1, AWS D1.1 Type B studs qualify as a ductile steel element (20% minimum elongation in 2 in. which is greater than the 14% required and a minimum reduction in area of 50% that is greater than the 30% required, see Table 34-1).	
$N_{sa} = n\ A_{se}\ f_{uta}$	*Eq. (D-3)*
For design purposes, Eq. (D-1) with Eq. (D-3) may be rearranged as:	
$A_{se} = \dfrac{N_{ua}}{\phi\ n\ f_{uta}}$	

where:

N_{ua} = 14,000 lbs
ϕ = 0.75
n = 4
f_{uta} = 60,000 psi

Note: Per D5.1.2, f_{uta} shall not be taken greater than $1.9f_{ya}$ or 125,000 psi. For AWS D1.1 headed studs, $1.9f_{ya}$ = 1.9(50,000) = 95,000 psi, therefore use the specified minimum f_{uta} of 60,000 psi. *D.5.1.2*

Substituting:

$$A_{se} = \frac{14{,}000}{0.75(4)(60{,}000)} = 0.078 \text{ in.}^2$$

Per Table 34-2, 1/2 in. diameter welded, headed studs will satisfy this requirement (A_{se} = 0.196 in.^{2}).

Note: Per AWS D1.1 Table 7.1, Type B welded studs are 1/2 in., 5/8 in., 3/4 in., 7/8 in., and 1 in. diameters. Although individual manufacturers may list smaller diameters they are not explicitly covered by AWS D1.1

3. Determine the required embedment length (h_{ef}) based on concrete breakout *D.5.2*

Two different equations are given for calculating concrete breakout strength; for single anchors Eq. (D-4) applies, and for anchor groups Eq. (D-5) applies. An "anchor group" is defined as:

"A number of anchors of approximately equal effective embedment depth with each anchor spaced at less than three times its embedment depth from one or more adjacent anchors." *D.1*

Since the spacing between anchors is 6 in., they must be treated as a group if the embedment depth exceeds 2 in. Although the embedment depth is unknown, at this point it will be assumed that the provisions for an anchor group will apply.

The basic requirement for embedment of a group of anchors is:

$\phi N_{cbg} \geq N_{ua}$ *Eq. (D-1)* *D.4.1.2*

where:

ϕ = 0.70 *D.4.4(c)ii*

Condition B applies since no supplementary reinforcement has been provided (e.g., hairpin type reinforcement surrounding the anchors and anchored into the concrete).

$$N_{cbg} = \frac{A_{Nc}}{A_{Nco}} \psi_{ec,N} \psi_{ed,N} \psi_{c,N} \psi_{cp,N} N_b$$ Eq. (D-5)

Since this connection is likely to be affected by both group effects and edge effects, the embedment length h_{ef} cannot be determined from a closed form solution. Therefore, an embedment length must be assumed. The strength of the connection is then determined and compared with the required strength.

Note: Welded studs are generally available in fixed lengths. Available lengths may be determined from manufacturers' catalogs. For example, the Nelson Stud http://www.nelsonstudwelding,com/ has an effective embedment of 4 in. for a standard 1/2 in. concrete anchor stud.

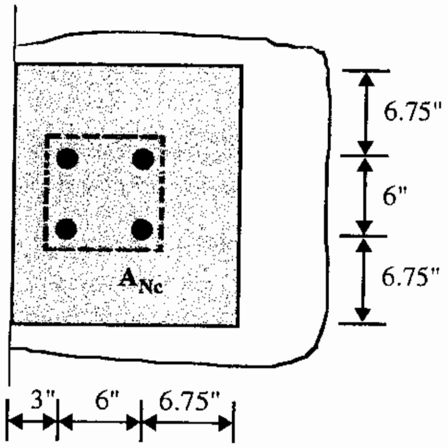
Assume an effective embedment length of $h_{ef} = 4.5$ in.

Note: The effective embedment length h_{ef} for the welded stud anchor is the effective embedment length of the stud plus the thickness of the embedded plate.

Evaluate the terms in Eq. (D-5) with $h_{ef} = 4.5$ in.

Determine A_{Nc} and A_{Nco} for the anchorage: D.5.2.1

A_{Nc} is the projected area of the failure surface as approximated by a rectangle with edges bounded by 1.5 h_{ef} (1.5 x 4.5 = 6.75 in. in this case) and free edges of the concrete from the centerlines of the anchors.



$A_{Nc} = (3+6+6.75)(6.75+6+6.75) = 307$ in.^{2}

$A_{Nco} = 9\ h_{ef}^2 = 9\ (4.5)^2 = 182$ in.^{2} Eq. (D-6)

Check: $A_{Nc} \le nA_{Nco}$ $307 < 4(182)$ O.K. D.5.2.1

Determine $\psi_{ec,N}$: D.5.2.4

$\psi_{ec,N} = 1.0$ (no eccentricity in the connection)

Determine $\psi_{ed,N}$ since $c_{a1} < 1.5\,h_{ef}$ — D.5.2.5

$$\psi_{ed,N} = 0.7 + 0.3\frac{c_{a,min}}{1.5h_{ef}}$$ Eq. (D-11)

$$\psi_{ed,N} = 0.7 + 0.3\frac{3.0}{1.5(4.5)} = 0.83$$

Determine $\psi_{c,N}$: — D.5.2.6

$\psi_{c,N} = 1.0$ for locations where concrete cracking is likely to occur (i.e., the bottom of the slab)

For cast-in anchors $\psi_{cp,N} = 1.0$ — D.5.2.6

Determine N_b: — D.5.2.2

$$N_b = 24\sqrt{f'_c}\,h_{ef}^{1.5} = 24\sqrt{4000}\,(4.5)^{1.5} = 14{,}490 \text{ lb}$$ Eq. (D-7)

Substituting into Eq. (D-5):

$$N_{cbg} = \left[\frac{307}{182}\right](1.0)(0.83)(1.0)(14{,}490) = 20{,}287 \text{ lb}$$

The final check on the assumption of $h_{ef} = 4.5$ in. is satisfied by meeting the requirements of Eq. (D-1):

$(0.70)\,(20{,}287) \geq 14{,}000$

$14{,}201 > 14{,}000$ O.K.

Specify a 4 in. length for the welded, headed studs with the 1/2 in.-thick base plate.

4. Determine if welded stud head size is adequate for pullout — D.5.3

$\phi\, N_{pn} \geq N_{ua}$ — Eq. (D-1), D.4.1.2

where:

$\phi = 0.70$ — D.4.4(c)ii

Condition B applies for pullout strength in all cases.

$N_{pn} = \psi_{c,P}N_p$ — Eq. (D-14)

where:

$N_p = A_{brg}\, 8\, f'_c$ — Eq. (D-15)

$\psi_{c,P}$ = 1.0 for locations where concrete cracking is likely to occur (i.e., the bottom of the slab) *D.5.3.6*

For design purposes Eq. (D-1) with Eq. (D-14) and Eq. (D-15) may be rearranged as:

$$A_{brg} = \frac{N_{ua}}{\phi\, \psi_{c,P}\, 8\, f'_c}$$

For the group of four studs the individual factored tension load N_u on each stud is:

$$N_{ua} = \frac{14{,}000}{4} = 3500 \text{ lb}$$

Substituting:

$$A_{brg} = \frac{3500}{0.70\,(1.0)\,(8)\,(4000)} = 0.156 \text{ in.}^2$$

The bearing area of welded, headed studs should be determined from manufacturers' catalogs. As shown on the Nelson Stud web page the diameter of the head for a 1/2 in. diameter stud is 1 in.

$$A_{brg,\,provided} = \frac{\pi}{4}(1.0^2 - 0.5^2) = 0.589 \text{ in.}^2 > 0.156 \text{ in.}^2 \quad \text{O.K.}$$

5. Evaluate side-face blowout *D.5.4*

Side-face blowout needs to be considered when the edge distance from the centerline of the anchor to the nearest free edge is less than $0.4h_{ef}$. For this example:

$0.4h_{ef} = 0.4\,(4.5) = 1.8$ in. < 3 in. actual edge distance O.K.

The side-face blowout failure mode is not applicable.

6. Required edge distances, spacings, and thickness to preclude splitting failure *D.8*

Since a welded, headed anchor is not torqued the minimum cover requirements of 7.7 apply.

Per 7.7 the minimum clear cover for a 1/2 in. bar not exposed to earth or weather is 3/4 in. which is less than the 2-3/4 in. provided (3 - 1/4 = 2-3/4 in.) O.K.

7. Summary:

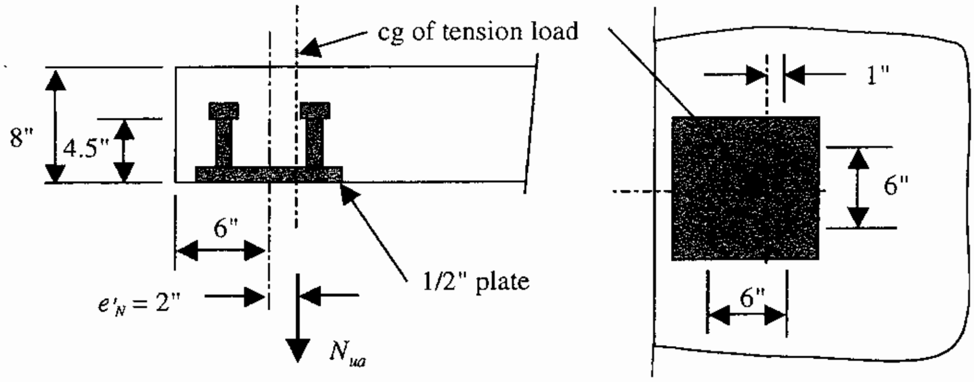
Use ASW D1.1 Type B 1/2 in. diameter welded studs with an effective embedment of 4.5 in. (4 in. from the stud plus 1/2 in. from the embedded plate).

Example 34.3—Group of Headed Studs in Tension Near an Edge with Eccentricity

Determine the factored tension load capacity (N_{ua}) for a group of four 1/2 in.×4 in. AWS D1.1 Type B headed studs spaced 6 in. on center each way and welded to a 1/2-in.-thick base plate. The centerline of the structural attachment to the base plate is located 2 in. off of the centerline of the base plate resulting in an eccentricity of the tension load of 2 in. The fastener group is installed in the bottom of an 8 in.-thick slab with the centerline of the connection 6 in. from a free edge of the slab.

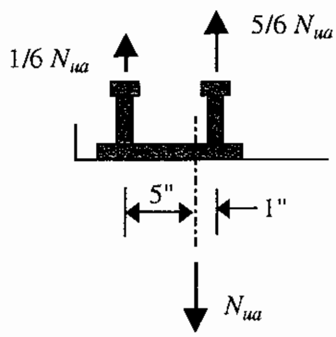
Note: This is the configuration chosen as a solution for Example 34.2 to support a 14,000 lb factored tension load centered on the connection. The only difference is the eccentricity of the tension load. From Example 34.2, the spacing between anchors dictates that they be designed as an anchor group.

$f'_c = 4000$ psi



Calculations and Discussion	Code Reference
1. Determine distribution of loads to the anchors	*D.3.1*

Assuming an elastic distribution of loads to the anchors, the eccentricity of the tension load will result in a higher force on the interior row of fasteners. Although the studs are welded to the base plate, their flexural stiffness at the joint with the base plate is minimal compared to that of the base plate. Therefore, assume a simple support condition for the base plate:



The two interior studs will control the strength related to the steel ϕN_{sa} and the pullout strength ϕN_{pn} (i.e., 5/6 N_{ua} must be less than or equal to $\phi N_{sa,\ 2\ studs}$ and $\phi N_{pn,\ 2\ studs}$). Rearranged:

$N_{ua} \leq 6/5\ \phi N_{sa,\ 2\ studs}$ and $N_{ua} \leq 6/5\ \phi N_{pn,\ 2\ studs}$)

Example 34.3 (cont'd)	Calculations and Discussion	Code Reference

2. Determine the design steel strength as controlled by the two anchors with the highest tensile load (ϕN_{sa}) *D.5.1*

$\phi N_{sa} = \phi\ n\ A_{se}\ f_{uta}$ *Eq. (D-3)*

where:

$\phi = 0.75$ *D.4.4(a)i*

Per the Ductile Steel Element definition in D.1, AWS D1.1 Type B studs qualify as a ductile steel element [20% minimum elongation in 2 in. which is greater than the 14% required and a minimum reduction in area of 50% that is greater than the 30% required, see Table 34-1)].

n = 2 (for the two inner studs with the highest tension load)

$A_{se} = 0.196$ in.^{2} (see Table 34-2)

$f_{uta} = 65{,}000$ (see Table 34-1)

Substituting:

$\phi N_{sa,\ 2studs} = 0.75\ (2)\ (0.196)\ (65{,}000) = 19{,}110$ lb

Therefore, the maximum N_{ua} as controlled by the anchor steel is:

$\phi N_{sa} = 6/5\ \phi N_{sa,2\,studs} = 6/5\ (19{,}110) = 22{,}932$ lb

3. Determine design breakout strength (ϕN_{cbg}) *D.5.2*

The only difference between concrete breakout strength in this example and Example 34.2 is the introduction of the eccentricity factor $\psi_{ec,N}$.

From Example 34.2 with $\psi_{ec,N} = 1.0$:

$$N_{cbg} = \frac{A_{Nc}}{A_{Nco}}\ \Psi_{ec,N}\ \Psi_{ed,N}\ \Psi_{c,N}\ \Psi_{cp,N}\ N_b = 14{,}201 \text{ lb}$$ *Eq. (D-5)*

Determine $\psi_{ec,N}$ for this example ($e_N = e'_N = 2$ in $< s/2 = 3$ in.): *D.5.2.4*

$$\Psi_{ec,N} = \frac{1}{\left(1+\dfrac{2e'_N}{3h_{ef}}\right)}$$ *Eq. (D-9)*

where:

$e'_N = 2$ in. (distance between centroid of anchor group and tension force)

$h_{ef} = 4.5$ in. (1/2 in. plate plus 4 in. embedment of headed stud)

Substituting:

$$\psi_{ec,N} = \frac{1}{\left(1+\frac{2(2)}{3(4.5)}\right)} = 0.77$$

Therefore:

$\phi N_{cbg} = (0.77)\ (14{,}201) = 10{,}935$ lb

4. Determine the design pullout strength as controlled by the two anchors with the highest tensile load (ϕN_{pn}) *D.5.3*

$\phi N_{pn,\ 1\ stud} = \phi\ \psi_{c,P}\ N_p = \phi\ \psi_{c,P}\ A_{brg}\ 8\ f'_c$ *Eq. (D-14)* *Eq. (D-15)*

where:

$\phi = 0.70$ – Condition B applies for pullout. *D.4.4(c)ii*

$\psi_{c,P} = 1.0$ for locations where concrete cracking is likely to occur (i.e., the bottom of the slab) *D.5.3.6*

$A_{brg} = 0.589$ in.^{2} (see Step 4 of Example 35.2)

Substituting:

$\phi N_{pn,\ 1\ stud} = (0.70)\ (1.0)\ (0.589)\ (8.0)\ (4000) = 13{,}194$ lb

For the two equally loaded inner studs:

$\phi N_{pn,2\ studs} = 2\ (13{,}194) = 26{,}387$ lb

Therefore, the maximum N_{ua} as controlled by pullout is:

$\phi N_{pn} = 6/5\ \phi N_{pn,\ 2\ studs} = 6/5\ (26{,}387) = 31{,}664$ lb

5. Evaluate side-face blowout *D.5.4*

Side-face blowout needs to be considered when the edge distance from the centerline of the anchor to the nearest free edge is less than 0.4 h_{ef}. For this example:

$0.4\ h_{ef} = 0.4\ (4.5) = 1.8$ in. < 3 in. actual edge distance O.K.

The side-face blowout failure mode is not applicable.

6. Required edge distances, spacings, and thickness to preclude splitting failure *D.8*

Since a welded, headed anchor is not torqued the minimum cover requirements of 7.7 apply.

Per 7.7 the minimum clear cover for a 1/2 in. bar not exposed to earth or weather is 3/4 in. which is less than the 2-3/4 in. provided (3 in. - 1/4 in. = 2-3/4 in.) O.K. *7.7.1(c)*

7. Summary:

Steel strength, (ϕN_{sa}):	22,932 lb
Embedment strength – concrete breakout, (ϕN_{cbg}):	10,935 lb ← controls
Embedment strength – pullout, (ϕN_{pn}):	31,664 lb
Embedment strength – side-face blowout, (ϕN_{sb}):	N/A

The maximum factored tension load N_{ua} for this anchorage is 10,935 lb

Note: Example 34.2 with the same connection but without an eccentricity was also controlled by concrete breakout strength but had a factored load capacity of 14,201 lb (see Step 3 of Example 34.2).

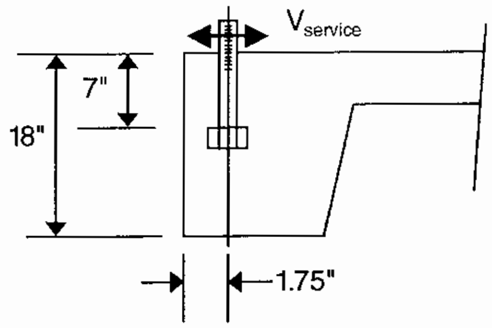
Example 34.4—Single Headed Bolt in Shear Near an Edge

Determine the reversible service wind load shear capacity for a single 1/2 in. diameter headed anchor with a 7 in. embedment installed with its centerline 1-3/4 in. from the edge of a concrete foundation.

Note: This is the minimum anchorage requirement at the foundation required by IBC 2000 Section 2308.6 for conventional light-frame wood construction. The 1-3/4 in. edge distance represents a typical connection at the base of wood framed walls using 2×4 members.

$f'_c = 4000$ psi

ASTM F 1554 Grade 36



Calculations and Discussion	Code Reference

1. This problem provides the anchor diameter, embedment length, and material properties, and requires computing the maximum service shear load capacity to resist wind load. In this case, it is best to first determine the controlling factored shear load, V_{ua}, based on the smaller of the steel strength and embedment strength then as a last step determine the maximum service load. Step 6 of this example provides the conversion of the controlling factored shear load V_{ua} to a service load due to wind.

2. Determine V_{ua} as controlled by the anchor steel — *D.6.1*

   $\phi V_{sa} \geq V_{ua}$ — *Eq. (D-2)*, *D.4.1.2*

   where:

   $\phi = 0.65$ — *D.4.4(a)i*
   Per the Ductile Steel Element definition in D.1, ASTM F 1554 Grade 36 steel qualifies as a ductile steel element.

   $V_{sa} = n\ 0.6\ A_{se}\ f_{uta}$ — *Eq. (D-20)*

   To determine V_{ua} for the steel strength Eq. (D-2) can be combined with Eq. (D-20) to give:

   $V_{ua} = \phi V_{sa} = \phi\ n\ 0.6\ A_{se}\ f_{uta}$

where:

$\phi = 0.65$
$n = 1$
$A_{se} = 0.142$ in.^{2} for the 1/2 in. threaded bolt (Table 34-2)
$f_{uta} = 58{,}000$ psi

Per ASTM F 1554 Grade 36 has a specified minimum yield strength of 36 ksi and a specified tensile strength of 58-80 ksi (see Table 34-1). For design purposes, the minimum tensile strength of 58 ksi should be used.

Note: Per D6.1.2, f_{uta} shall not be taken greater than $1.9f_{ya}$ or 125,000 psi. For ASTM F 1554 Grade 36, $1.9f_{ya} = 1.9(36{,}000) = 68{,}400$ psi. Therefore, use the specified minimum f_{uta} of 58,000 psi.

Substituting, V_{ua} as controlled by steel strength is:

$V_{ua} = \phi V_{sa} = 0.65\,(1)(0.6)(0.142)(58{,}000) = 3212$ lb

3. Determine V_{ua} for embedment strength governed by concrete breakout strength with shear directed toward a free edge — *D.6.2*

$\phi V_{cb} \geq V_{ua}$ — *Eq. (D-2)*, *D.4.1.2*

where:

$\phi = 0.70$ — *D.4.4(c)i*

No supplementary reinforcement has been provided (i.e., hairpin type reinforcement surrounding the anchor and anchored into the concrete).

$$V_{cb} = \frac{A_{Vc}}{A_{Vco}}\,\psi_{ed,V}\,\psi_{c,V}\,V_b$$ *Eq. (D-21)*

where:

$\frac{A_{Vc}}{A_{Vco}}$ and $\psi_{ed,V}$ terms are 1.0 for single shear anchors not influenced by more than one free edge (i.e., the member thickness is greater than $1.5c_{a1}$ and the distance to an orthogonal edge c_{a2} is greater than $1.5c_{a1}$)

$\psi_{c,V} = 1.0$ for locations where concrete cracking is likely to occur (i.e., the edge of the foundation is susceptible to cracks) — *D.6.2.7*

$$V_b = 7\left(\frac{\ell_e}{d_o}\right)^{0.2}\sqrt{d_o}\,\sqrt{f'_c}\,c_{a1}^{1.5}$$ *Eq. (D-24)*

where:

ℓ_e = load bearing length of the anchor for shear, not to exceed $8d_o$ *2.1* *D.6.2.2*

For this problem $8d_o$ will control since the embedment depth h_{ef} is 7 in.

$\ell_e = 8d_o = 8\ (0.5) = 4.0$ in.

To determine V_{ua} for the embedment strength governed by concrete breakout strength Eq. (D-2) can be combined with Eq. (D-21) and Eq. (D-24) to give:

$$V_{ua} = \phi V_{cb} = \phi\,\frac{A_{Vc}}{A_{Vco}}\,\psi_{ed,V}\,\psi_{c,V}\,7\left(\frac{\ell_e}{d_o}\right)^{0.2}\sqrt{d_o}\,\sqrt{f'_c}\,c_{a1}^{1.5}$$ *Eq. (D-21)* *Eq. (D-24)*

Substituting, V_u for the embedment strength as controlled by concrete breakout strength is:

$$V_{ua} = \phi V_{cb} = 0.70\,(1.0)\,(1.0)\,(1.0)\,(7)\left(\frac{8\,(0.5)}{(0.5)}\right)^{0.2}\sqrt{0.5}\,\sqrt{4000}\,(1.75)^{1.5} = 769 \text{ lb}$$

4. Determine V_{ua} for embedment strength governed by concrete pryout strength *D.6.3*

Note: The pryout failure mode is normally only a concern for shallow, stiff anchors. Since this example problem addresses both shear directed toward the free edge and shear directed inward from the free edge, the pryout strength will be evaluated.

$\phi V_{cp} \geq V_{ua}$ *Eq. (D-2)* *D.4.1.2*

where:

$\phi = 0.70$ – Condition B applies for pryout strength in all cases *D.4.4(c)i*

$V_{cp} = k_{cp}\,N_{cb}$ *Eq. (D-29)*

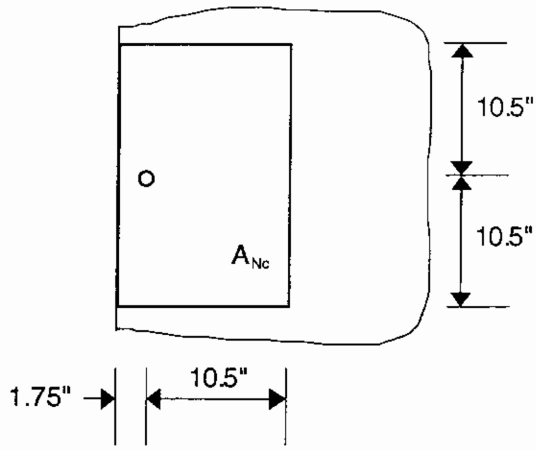
where:

$k_{cp} = 2.0$ for $h_{ef} \geq 2.5$ in.

$$N_{cb} = \frac{A_{Nc}}{A_{Nco}}\,\psi_{ed,N}\,\psi_{c,N}\,\psi_{cp,N}\,N_b$$ *Eq. (D-4)*

Evaluate the terms of Eq. (D-4) for this problem:

A_{Nc} is the projected area of the tensile failure surface as approximated by a rectangle with edges bounded by 1.5 h_{ef} (1.5 × 7 = 10.5 in.) and free edges of the concrete from the centerline of the anchor.



A_{Nc} = (1.75 + 10.5) (10.5 + 10.5) = 257 in.^{2}

$A_{Nco} = 9\ h_{ef}^2 = 9\ (7.0)^2 = 441$ in.^{2} *Eq. (D-6)*

Determine $\psi_{ed,N}$: *D.5.2.5*

$$\psi_{ed,N} = 0.7 + 0.3\frac{c_{a,min}}{1.5h_{ef}}$$ *Eq. (D-11)*

$$\psi_{ed,N} = 0.7 + 0.3\frac{1.75}{1.5(7.0)} = 0.75$$

Determine $\psi_{c,N}$: *D.5.2.6*

$\psi_{c,N}$ = 1.0 for locations where concrete cracking is likely to occur (i.e., the edge of the foundation is susceptible to cracks)

Determine N_b for the fastening: *D.5.2.2*

$$N_b = 24\sqrt{f'_c}\ h_{ef}^{1.5} = 24\sqrt{4000}\left(7.0\right)^{1.5} = 28{,}112 \text{ lb}$$ *Eq. (D-7)*

Substituting into Eq. (D-4):

$$N_{cb} = \left[\frac{257}{441}\right](0.75)\ (1.0)\ (28{,}112) = 12{,}287 \text{ lb}$$

To determine V_{ua} for the embedment strength governed by pryout strength Eq. (D-2) can be combined with Eq. (D-29) to give:

$V_{ua} = \phi V_{cp} = \phi k_{cp} N_{cb}$

Substituting, V_{ua} for the embedment strength governed by pryout is:

$V_{ua} = \phi V_{cp} = 0.70\,(2.0)\,(12{,}287) = 17{,}202$ lb

5. Required edge distances, spacings, and thickness to preclude splitting failure *D.8*

Since a headed anchor used to attach wood frame construction is not likely to be torqued significantly, the minimum cover requirements of 7.7 apply.

Per 7.7 the minimum clear cover for a 1/2 in. bar is 1-1/2 in. when exposed to earth or weather. The clear cover provided for the bolt is exactly 1-1/2 in. (1-3/4 in. to bolt centerline less one half bolt diameter). Note that the bolt head will have slightly less cover (1-3/16 in. for a hex head) say O.K. (note that this is within the minus 3/8 in. tolerance allowed for cover) *7.7* *7.5.2.1*

6. Summary:

The factored shear load ($V_{ua} = \phi V_n$) based on steel strength and embedment strength (concrete breakout and pryout) can be summarized as:

Steel strength, (ϕV_{sa}):	3212 lb
Embedment strength – concrete breakout, (ϕV_{cb}):	769 lb ← controls
Embedment strength – pryout, (ϕV_{cp}):	17,202 lb

In accordance with 9.2 the load factor for wind load is 1.6:

$$V_{service} = \frac{V_{ua}}{1.6} = \frac{769}{1.6} = 481 \text{ lb}$$ *9.2*

The reversible service load shear strength from wind load of the IBC 2000 Section 2308.6 minimum foundation connection for conventional wood-frame construction (1/2 in. diameter bolt embedded 7 in.) is 481 lb per bolt. The strength of the attached member (i.e., the 2×4 sill plate) also needs to be evaluated.

Note that this embedment strength is only related to the anchor being installed in concrete with a specified compressive strength of 4000 psi. In many cases, concrete used in foundations such as this is specified at 2500 psi, the minimum strength permitted by the code. Since the concrete breakout strength controlled the strength of the connection, a revised strength based on using 2500 psi concrete rather than the 4000 psi concrete used in the example can be determined as follows:

$$V_{service@2500} = 481\frac{\sqrt{2500}}{\sqrt{4000}} = 380 \text{ lb}$$

Alternate design using Table 34-6B.

Note: Step numbers correspond to those in the main example above, but prefaced with "A".

Table 34-6B has been selected because it contains design shear strength values based on concrete with f'_c = 4000 psi. Table Note 5 indicates that the values in the table are based on Condition B (no supplementary reinforcement), and Note 6 indicates that cracked concrete was assumed.

A2. Determine V_{ua} as controlled by the anchor steel — *D.6.1 Eq. (D-20)*

From Step 2, use ASTM F 1554, Grade 36 headed bolt with f_{uta} = 58,000 psi. From Table 34-6B, for specified compressive strength of concrete, f'_c, = 4000 psi, determine the design shear strength, ϕV_{sa}, for a 1/2-in. bolt.

$\phi V_{sa} = 3212$ lb

A3. Determine V_{ua} for embedment strength governed by concrete breakout strength with shear directed toward a free edge — *D.6.2 Eq. (D-21)*

Determine the design concrete breakout strength in shear, ϕV_{cb}, based on 7-in. embedment, and an edge distance, c_{a1}, of 1-3/4 in. In the table c_{a1}, is a function of embedment depth, h_{ef}. Therefore, the edge distance is:

$c_{a1} = c_{a1}/h_{ef} = 1.75/7 = 0.25h_{ef}$

From table, the design concrete breakout strength in shear is,

$\phi V_{cb} = 769$ lb

A4. Determine V_{ua} for embedment strength governed by concrete pryout strength — *D.6.3*

Determine the design concrete pryout strength in shear, ϕV_{cp}, based on 7-in. embedment, and an edge distance of 1-3/4 in. This cannot be determined from Table 34-6B; however, since

$\phi V_{cp} = \phi k_{cp} N_{cb}$ — *Eq. (D-29)*

where $k_{cp} = 2$, since $h_{ef} > 2.5$ in., and N_{cb} can be determined from Table 34-5B.

Note that the values in Tables 34-5 and 34-6 are design strengths, thus they include the strength reduction factor, ϕ. Since Table 34-5B is based on Condition B (with no supplementary reinforcement), the ϕ-value used for the concrete tensile strength calculations was 0.70, which is the same as to be used to determine the concrete pryout strength in shear. Therefore, the design concrete breakout strength, ϕN_{cb}, value from Table 34-5B can be used above without adjustment. From Table 34-5B, for an edge distance, c, equal to $0.25h_{ef}$

$\phi N_{cb} = 8609$ lb

Substituting in Equation (D-29)

$\phi V_{cp} = k_{cp}\phi N_{cb} = (2)(8609) = 17{,}218$ lb

Note that the above value differs slightly from that obtained in Step #4 above. The table values are more precise due to rounding that occurred in the long-hand calculations.

A5. Required edge distances, spacings, and thickness to preclude splitting failure

See Step 5 above.

A6. Determine service wind shear load:

The factored shear load ($V_{ua} = \phi V_n$) based on steel strength and embedment strength (concrete breakout and pryout) can be summarized as:

Steel strength, (ϕV_{sa}):	3212 lb	
Embedment strength - concrete breakout, (ϕV_{cb}):	769 lb	← controls
Embedment strength - pryout, (ϕV_{cp}):	17,218 lb	

From this point, the allowable service wind load shear capacity of the 1/2 in. anchor is determined as in Step 6 above.

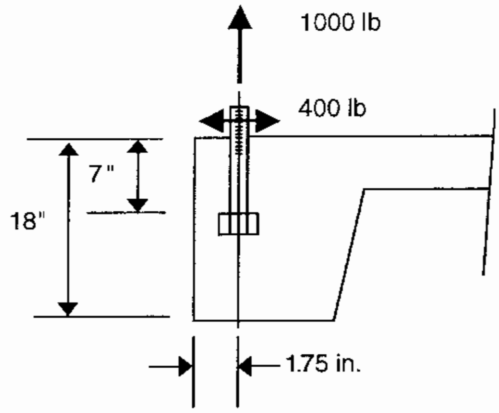
Example 34.5—Single Headed Bolt in Tension and Shear Near an Edge

Determine if a single 1/2 in. diameter hex headed anchor with a 7 in. embedment installed with its centerline 1-3/4 in. from the edge of a concrete foundation is adequate for a service tension load from wind of 1000 lb and reversible service shear load from wind of 400 lb.

Note: This is an extension of Example 34.4 that includes a tension load on the fastener as well as a shear load.

$f'_c = 4000$ psi

ASTM F 1554 Grade 36 hex head anchor



Calculations and Discussion	Code Reference
1. Determine the factored design loads	*9.2*
$N_{ua} = 1.6\ (1000) = 1600$ lb	
$V_{ua} = 1.6\ (400) = 640$ lb	
2. This is a tension/shear interaction problem where values for both the design tensile strength (ϕN_n) and design shear strength (ϕV_n) will need to be determined. ϕN_n is the smallest of the design tensile strengths as controlled by steel (ϕN_{sa}), concrete breakout (ϕN_{cb}), pullout (ϕN_{pn}), and side-face blowout (ϕN_{sb}). ϕV_n is the smallest of the design shear strengths as controlled by steel (ϕV_{sa}), concrete breakout (ϕV_{cb}), and pryout (ϕV_{cp}).	*D.7* *D.4.1.2*
3. Determine the design tensile strength (ϕN_n)	*D.5*
a. Steel strength, (ϕN_{sa}):	*D.5.1*
$\phi N_{sa} = \phi\ n\ A_{se}\ f_{uta}$	*Eq. (D-3)*
where:	
$\phi = 0.75$	*D.4.4(a)i*
Per the Ductile Steel Element definition in D.1, ASTM F 1554 Grade 36 steel qualifies as a ductile steel element.	

$A_{se} = 0.142$ in.^{2} (see Table 34-2)

$f_{uta} = 58{,}000$ psi (see Table 34-1)

Substituting:

$\phi N_{sa} = 0.75\ (1)\ (0.142)\ (58{,}000)) = 6177$ lb

b. Concrete breakout strength (ϕN_{cb}): *D.5.2*

Since no supplementary reinforcement has been provided, $\phi = 0.70$ *D.4.4(c)ii*

In the process of calculating the pryout strength for this fastener in Example 34.4 Step 4, N_{cb} for this fastener was found to be 12,287 lb

$\phi N_{cb} = 0.70\ (12{,}287) = 8601$ lb

c. Pullout strength (ϕN_{pn}) *D.5.3*

$\phi N_{pn} = \phi\ \psi_{c,P}\ N_p$ *Eq. (D-14)*

where:

$\phi = 0.70$ – Condition B applies for pullout strength in all cases *D.4.4(c)ii*

$\psi_{c,P} = 1.0$, cracking may occur at the edges of the foundation *D.5.3.6*

$N_p = A_{brg}\ 8\ f'_c$ *Eq. (D-15)*

$A_{brg} = 0.291$ in.^{2}, for 1/2 in. hex head bolt (see Table 34-2)

Pullout Strength (ϕN_{pn})

$\phi N_{pn} = 0.70\ (1.0)\ (0.291)\ (8)\ (4000) = 6518$ lb

d. Concrete side-face blowout strength (ϕN_{sb}) *D.5.4*

The side-face blowout failure mode must be investigated when the edge distance (c) is less than 0.4 h_{ef} *D.5.4.1*

$0.4\ h_{ef} = 0.4\ (7) = 2.80$ in. > 1.75 in.

Therefore, the side-face blowout strength must be determined

$$\phi N_{sb} = \phi \left(160\ c_{a1} \sqrt{A_{brg}}\ \sqrt{f'_c}\right)$$ *Eq. (D-17)*

where:

$\phi = 0.70$, no supplementary reinforcement has been provided — D.4.4(c)ii

$c_{a1} = 1.75$ in.

$A_{brg} = 0.291$ in.^{2}, for 1/2 in. hex head bolt (see Table 34-2)

Substituting:

$$\phi N_{sb} = 0.70\left(160\ (1.75)\ \sqrt{0.291}\ \sqrt{4000}\right) = 6687 \text{ lb}$$

Summary of steel strength, concrete breakout strength, pullout strength, and side-face blowout strength for tension:

Steel strength, (ϕN_{sa}):	6177 lb ← controls		D.5.1
Embedment strength – concrete breakout, (ϕN_{cb}):	8601 lb		D.5.2
Embedment strength – pullout, (ϕN_{pn}):	6518 lb		D.5.3
Embedment strength – side-face blowout, (ϕN_{sb}):	6687 lb		D.5.4

Check $\phi N_n \geq N_{ua}$ — Eq. (D-1)

6177 lb > 1600 lb O.K.

Therefore:

$\phi N_n = 6177$ lb

4. Determine the design shear strength (ϕV_n) — D.6

Summary of steel strength, concrete breakout strength, and pryout strength for shear from Example 35.4, Step 6:

Steel strength, (ϕV_{sa}):	3212 lb	D.6.1
Embedment strength – concrete breakout, (ϕV_{cb}):	769 lb ← controls	D.6.2
Embedment strength – pryout, (ϕV_{cp}):	17,202 lb	D.6.3

Check $\phi V_n \geq V_{ua}$ — Eq. (D-2)

769 lb > 640 lb O.K

Therefore:

$\phi V_n = 769$ lb

5. Check tension and shear interaction — D.7

If $V_{ua} \leq 0.2\phi V_n$ then the full tension design strength is permitted — D.7.1

$V_{ua} = 640$ lb

$0.2\phi V_n = 0.2\ (769) = 154 \text{ lb} < 640 \text{ lb}$

V_{ua} exceeds $0.2\phi V_n$, the full tension design strength is not permitted

If $N_{ua} \leq 0.2\ \phi N_n$ then the full shear design strength is permitted *D.7.2*

$N_{ua} = 1600$ lb

$0.2\phi N_n = 0.2\ (6177) = 1235$ lb < 1600 lb

N_{ua} exceeds $0.2\phi N_n$, the full shear design strength is not permitted

The interaction equation must be used *D.7.3*

$$\frac{N_{ua}}{\phi N_n} + \frac{V_{ua}}{\phi V_n} \leq 1.2 \qquad \textit{Eq. (D-29)}$$

$$\frac{1600}{6177} + \frac{640}{769} = 0.26 + 0.83 = 1.09 < 1.2 \quad \text{O.K.}$$

6. Required edge distances, spacings, and thickness to preclude splitting failure *D.8*

Since a headed anchor used to attach wood frame construction is not likely to be torqued significantly, the minimum cover requirements of 7.7 apply.

Per 7.7 the minimum clear cover for a 1/2 in. bar is 1-1/2 in. when exposed to earth or weather. The clear cover provided for the bolt is exactly 1-1/2 in. (1-3/4 in. to bolt centerline less one half bolt diameter). Note that the bolt head will have slightly less cover (1-3/16 in. for a hex head) say O.K. (note that this is within the minus 3/8 in. tolerance allowed for cover) *7.7* *7.5.2.1* *D.5*

7. Summary

Use a 1/2 in. diameter ASTM F 1554 Grade 36 hex headed anchor embedded 7 in.

Alternate design using Tables 34-5B and 34-6B

Note: Step numbers correspond to those in the main example above, but prefaced with "A".

Tables 34-5 and 34-6 have been selected because they contain design tension and shear values, respectively, based on concrete with $f'_c = 4000$ psi. Table Notes 4 and 5, respectively, indicate that the values in the tables are based on Condition B (no supplementary reinforcement). Cracked concrete is assumed in both tables (Table 34-5 Notes 6 and 10, and Table 34-6 Note 6).

A3. Determine the design tensile strength (ϕN_n): *D.5.1* *Eq. (D-3)*

A3a. Determine the design tensile strength of steel (ϕN_{sa}):

Based on Step 3a, assume an ASTM F 1554, Grade 36 bolt, with a $f_{uta} = 58{,}000$ psi.

Using Table 34-5B, under the column for 58,000 a 1/2-in. diameter bolt has a design tensile strength,

ϕN_{sa} = 6177 lb.

A3b. Determine design concrete breakout strength (ϕN_{cb}): *D.5.2 Eq. (D-4)*

Since breakout strength varies with edge distance for anchors close to an edge ($c_{a1} < 1.5h_{ef}$), determine the edge distance as a function of embedment depth. Since c = 1-3/4 in.

$c_{a1} = c_{a1}/h_{ef} = 1.75/7 = 0.25h_{ef}$

Under column labeled "$0.25h_{ef}$" for a 1/2 in. bolt with 7 in. embedment depth,

ϕN_{cb} = 8609 lb

Note that the above value differs slightly from that obtained in Step 3b above. The table values are more precise due to rounding that occurred in the long-hand calculations.

A3c. Determine design concrete pullout strength (ϕN_{pn}) *D.5.3 Eq. (D-14)*

From the table under the column labeled "head" for a 1/2 in. bolt

ϕN_{pn} = 6518 lb

A3d. Determine design concrete side-face blowout strength (ϕN_{sb}) *D.5.4*

Side face blowout is not applicable where the edge distance is equal to or greater than $0.4h_{ef}$. In this case edge distance, c_{a1}, as calculated above is $0.25h_{ef}$; therefore, it must be evaluated. From the table under the column labeled "$0.25h_{ef}$" for a 1/2 in. bolt with 7 in. embedment, *D.5.4.1*

ϕN_{sb} = 6687 lb

Summary of steel strength, concrete breakout strength, pullout strength, and side-face blowout strength for tension:

Steel strength, (ϕN_{sa}):	6177 lb ← controls
Embedment strength – concrete breakout, (ϕN_{cb}):	8609 lb
Embedment strength – pullout, (ϕN_{pn}):	6518 lb
Embedment strength – side-face blowout, (ϕN_{sb}):	6687 lb

Therefore:

ϕN_n = 6177 lb

A4. Determine the design shear strength (ϕV_n)

Summary of steel strength, concrete breakout strength, and pryout strength for shear from Step A6 of Example 34.4, alternate solution using Table 34-6B

Steel strength, (ϕV_{sa}):	3212 lb
Embedment strength - concrete breakout, (ϕV_{cb}):	769 lb ← controls
Embedment strength - pryout, (ϕV_{cp}):	17,218 lb

Therefore:

$\phi V_n = 769$ lb

A5. Check tension and shear interaction.

See Step 5 above.

A6. Required edge distances, spacings, and thickness to preclude splitting failure

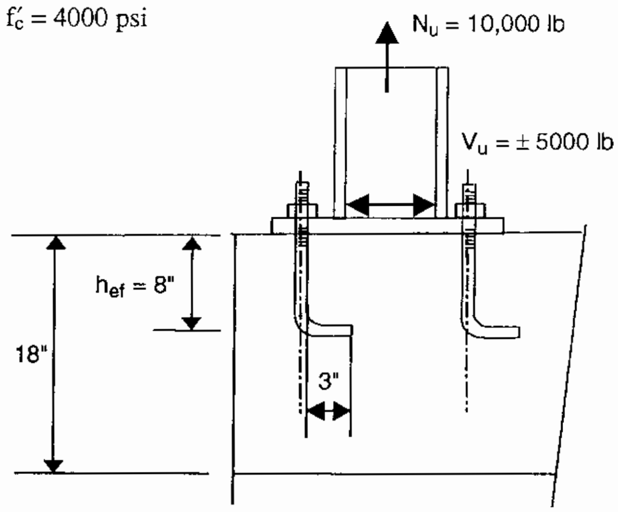
See Step 6 above.

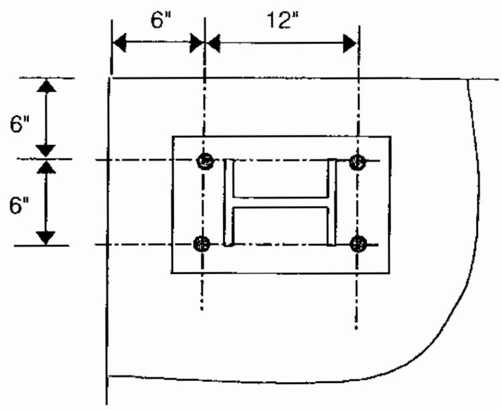
A7. Summary

Use a 1/2 in. diameter ASTM F 1554 Grade 36 hex headed bolt embedded 7 in.

Example 34.6—Group of L-Bolts in Tension and Shear Near Two Edges

Design a group of four L-bolts spaced as shown to support a 10,000 lb factored tension load and 5000 lb reversible factored shear load resulting from wind load. The connection is located at the base of a column in a corner of the building foundation.





Note: OSHA Standard 29 CFR Part 1926.755 requires that the column anchorage use a least four anchors and be able to sustain a minimum eccentric gravity load of 300 lb located 18 in. from the face of the extreme outer face of the column in each direction. The load is to be applied at the top of the column. The intent is that the column be able to sustain an iron worker hanging off the side of the top of the column.

Calculations and Discussion	Code Reference
1. The solution to this example is found by assuming the size of the anchors, then checking compliance with the design provisions. Try four 5/8 in. ASTM F 1554 Grade 36 L-bolts with h_{ef} = 8 in. and a 3 in. extension, e_h, as shown in the figure.	
2. This is a tension/shear interaction problem where values for both the design tensile strength (ϕN_n) and design shear strength (ϕV_n) will need to be determined. ϕN_n is the smallest of the design tensile strengths as controlled by steel (ϕN_{sa}), concrete breakout (ϕN_{cb}), pullout (ϕN_{pn}), and side-face blowout (ϕN_{sb}). ϕV_n is the smallest of the design shear strengths as controlled by steel (ϕV_{sa}), concrete breakout (ϕV_{cb}), and pryout (ϕV_{cp}).	*D.7* *D.4.1.2*
3. Determine the design tensile strength (ϕN_n)	*D.5*
a. Steel strength, (ϕN_{sa}):	*D.5.1*

$\phi N_{sa} = \phi\ n\ A_{se}\ f_{uta}$ *Eq. (D-3)*

where:

$\phi = 0.75$ *D.4.4(a)i*

Per Table 34-1, the ASTM F 1554 Grade 36 L-bolt meets the Ductile Steel Element definition of D.1.

$A_{se} = 0.226$ in.^{2} (see Table 34-2)

$f_{uta} = 58{,}000$ psi (see Table 34-1)

Substituting:

$\phi N_{sa} = 0.75\ (4)\ (0.226)\ (58{,}000) = 39{,}324$ lb

b. Concrete breakout strength (ϕN_{cbg}): *D.5.2*

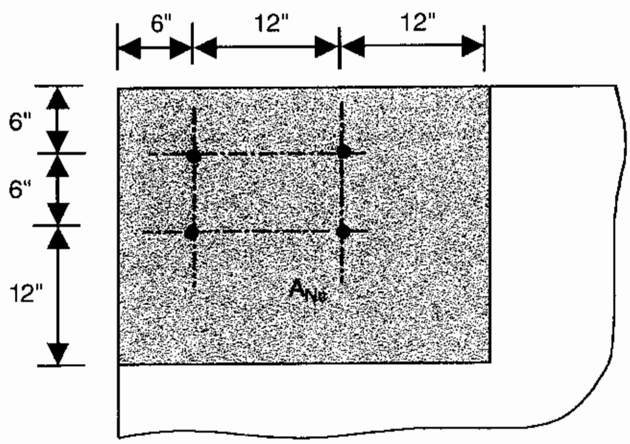
Since the spacing of the anchors is less than 3 times the effective embedment depth h_{ef} ($3 \times 8 = 24$), the anchors must be treated as an anchor group. *D.1*

$$\phi\ N_{cbg} = \phi\ \frac{A_{Nc}}{A_{Nco}}\ \psi_{ec,N}\ \psi_{ed,N}\ \psi_{c,N}\ \psi_{cp,N}\ N_b$$ *Eq. (D-5)*

Since no supplementary reinforcement has been provided, $\phi = 0.70$ *D.4.4(c)ii*

Determine A_{Nc} and A_{Nco}: *D.5.2.1*

A_{Nc} is the projected area of the failure surface as approximated by a rectangle with edges bounded by $1.5\ h_{ef}$ ($1.5 \times 8.0 = 12.0$ in. in this case) and free edges of the concrete from the centerlines of the anchors.



$A_{Nc} = (6 + 12 + 12)(6 + 6 + 12) = 720 \text{ in.}^2$

$A_{Nco} = 9\, h_{ef}^2 = 9\,(8)^2 = 576 \text{ in.}^2$ — *Eq. (D-6)*

Check: $A_{Nc} \le nA_{Nco}$ $720 < 4(576)$ O.K.

Determine $\psi_{ec,N}$: — *D.5.2.4*

$\psi_{ec,N} = 1.0$ (no eccentricity in the connection)

Determine $\psi_{ed,N}$ [$c_{a,min} < 1.5\, h_{ef}$, $6 < 1.5\,(8)$]: — *D.5.2.5*

$$\psi_{ed,N} = 0.7 + 0.3\frac{c_{a,min}}{1.5h_{ef}}$$ — *Eq. (D-11)*

$$\psi_{ed,N} = 0.7 + 0.3\frac{6.0}{1.5\,(8.0)} = 0.85$$

Determine $\psi_{c,N}$: — *D.5.2.6*

$\psi_{c,N} = 1.0$ for locations where concrete cracking is likely to occur (i.e., the edge of the foundation)

Determine $\psi_{cp,N}$: — *D.5.2.7*

For cast-in-place anchors, $\psi_{cp,N} = 1.0$

Determine N_b: — *D.5.2.2*

$$N_b = 24\sqrt{f'_c}\; h_{ef}^{1.5} = 24\sqrt{4000}\,(8.0)^{1.5} = 34{,}346 \text{ lb}$$ — *Eq. (D-7)*

Substituting into Eq. (D-5):

$$\phi N_{cbg} = 0.70\left[\frac{720}{576}\right](1.0)\,(0.85)\,(1.0)(1.0)\,(34{,}346) = 25{,}545 \text{ lb}$$

c. Pullout strength (ϕN_{pn}) — *D.5.3*

$\phi N_{pn} = \phi\, \psi_{c,P}\, N_p$ — *Eq. (D-14)*

where:

$\phi = 0.70$, Condition B always applies for pullout strength — *D.4.4(c)ii*

$\psi_{c,P} = 1.0$, cracking may occur at the edges of the foundation — *D.5.3.6*

N_p for the L-bolts:

$N_p = 0.9\, f'_c\, e_h\, d_o$ — *Eq. (D-16)*

e_h = maximum effective value of $4.5d_o = 4.5\ (0.625) = 2.81$ in.

$e_{h,provided} = 3$ in. > 2.81 in., therefore use $e_h = 4.5d_o = 2.81$ in. *D.5.3.5*

Substituting into Eq. (D-14) and Eq. (D-16) with 4 L-bolts (ϕN_{pn})

$\phi N_{pn} = 4\ (0.70)\ (1.0)\ [(0.9)\ (4000)\ (2.81)\ (0.625)] = 17{,}703$ lb

Note: If 5/8 in. hex head bolts where used ϕN_{pn} would be significantly increased as shown below:

N_p for the hex head bolts:

$N_p = A_{brg}\ 8\ f'_c$ *Eq. (D-15)*

$A_{brg} = 0.454$ in.^{2}, for 5/8 in. hex head bolt (see Table 34-2)

Substituting into Eq. (D-12) and Eq. (D-13) with 4 bolts (ϕN_{pn})

$\phi N_{pn} = 4\ (0.70)\ (1.0)\ (0.454)\ (8)\ (4000) = 40{,}678$ lb

The use of hex head bolts would increase the pullout capacity by a factor of 2.3 over that of the L-bolts.

d. Concrete side-face blowout strength (ϕN_{sb}) *D.5.4*

The side-face blowout failure mode must be investigated for headed anchors where the edge distance (c_{a1}) is less than 0.4 h_{ef}. Since L-bolts are used here the side face blowout failure is not applicable. The calculation below is simply to show that if headed anchors were used the anchors are far enough from the edge that the side-face blowout strength is not applicable. *D.5.4.1*

$0.4\ h_{ef} = 0.4\ (8) = 3.2$ in. < 6.0 in.

Therefore, the side-face blowout strength is not applicable (N/A).

Summary of design strengths based on steel strength, concrete breakout strength, pullout strength, and side-face blowout strength for tension:

Steel strength, (ϕN_{sa}):	39,324 lb	*D.5.1*
Embedment strength - concrete breakout, (ϕN_{cbg}):	25,545 lb	*D.5.2*
Embedment strength - pullout, (ϕN_{pn}):	17,703 lb ← controls	*D.5.3*
Embedment strength - side-face blowout, (ϕN_{sb}):	N/A	*D.5.4*

Therefore:

$\phi N_n = 17{,}703$ lb

Note: If hex head bolts were used the concrete breakout strength of 25,545 lb would control rather than the L-bolt pullout strength of 17,703 lb (i.e., 44% higher tensile capacity if hex head bolts were used).

4. Determine the design shear strength (ϕV_n) — *D.6*

a. Steel strength, (ϕV_{sa}): — *D.6.1*

$\phi V_{sa} = \phi \, n \, 0.6 \, A_{se} \, f_{uta}$ — *Eq. (D-19)*

where:

$\phi = 0.65$ — *D.4.4(a)ii*

Per Table 34-1, the ASTM F 1554 Grade 36 meets the Ductile Steel Element definition of Section D.1.

$A_{se} = 0.226 \text{ in.}^2$ (see Table 34-2)

$f_{uta} = 58{,}000$ psi (see Table 34-1)

Substituting:

$\phi V_{sa} = 0.65 \, (4) \, (0.6) \, (0.226) \, (58{,}000) = 20{,}448$ lb

b. Concrete breakout strength (ϕV_{cbg}): — *D.6.2*

Two potential concrete breakout failures need to be considered. The first is for the two anchors located near the free edge toward which the shear is directed (when the shear acts from right to left). For this potential breakout failure, these two anchors are assumed to carry one-half of the shear (see Fig. RD.6.2.1(b) upper right). For this condition, the total breakout strength for shear will be taken as twice the value calculated for these two anchors. The reason for this is that although the four-anchor group may be able to develop a higher breakout strength, the group will not have the opportunity to develop this strength if the two anchors nearest the edge fail first. The second potential concrete breakout failure is for the entire group transferring the total shear load. This condition also needs to be considered and may control when anchors are closely spaced or where the concrete member thickness is limited. For the case of welded studs, only the breakout strength of entire group for the total shear force needs to be considered (see Fig. RD.6.2.1(b) lower right), however this is not permitted for cast-in-place anchors that are installed through holes in the attached base plate.

$$\phi V_{cbg} = \phi \frac{A_{Vc}}{A_{Vco}} \psi_{ec,V} \, \psi_{ed,V} \, \psi_{c,V} \, V_b$$ — *Eq. (D-22)*

Determine the values of ϕ, $\psi_{ec,V}$, and $\psi_{c,V}$ (these are the same for both potential concrete breakout failures):

No supplementary reinforcement has been provided, $\phi = 0.70$ *D.4.4(c)i*

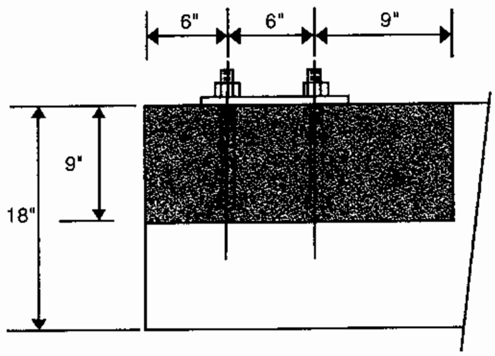
There is no eccentricity in the connection, $\psi_{ec,V} = 1.0$ *D.6.2.5*

For locations where concrete cracking is likely to occur (i.e., the edge of the foundation), $\psi_{c,V} = 1.0$ *D.6.2.6*

For concrete breakout failure of the two anchors located nearest the edge:

Determine A_{Vc} and A_{Vco}:

A_{Vc} is the projected area of the shear failure surface on the free edge toward which shear is directed. The projected area is determined by a rectangle with edges bounded by 1.5 c_{a1} (1.5 x 6.0 = 9.0 in. in this case) and free edges of the concrete from the centerlines of the anchors and the surface of the concrete. Although the 1.5 c_{a1} distance is not specified in D.6.2.1, it is shown in Fig. RD.6.2.1(b).



$A_{Vc} = (6+6+9)(9) = 189 \text{ in.}^2$

$A_{Vco} = 4.5\, c_{a1}^2 = 4.5\,(6)^2 = 162 \text{ in.}^2$ *Eq. (D-23)*

Check: $A_{Vc} \leq nA_{Vco}$ $189 < 2(162)$ O.K. *D.6.2.1*

Determine $\psi_{ed,V}$ [$c_{a2} < 1.5\, c_{a1}$, $6 < (1.5 \times 6)$]: *D.6.2.6*

$$\psi_{ed,V} = 0.7 + 0.3 \frac{c_{a2}}{1.5\, c_{a1}}$$ *Eq. (D-28)*

$$\psi_{ed,V} = 0.7 + 0.3 \frac{6.0}{1.5\,(6.0)} = 0.90$$

The single anchor shear strength, V_b:

$$V_b = 7\left(\frac{\ell_e}{d_o}\right)^{0.2} \sqrt{d_o}\, \sqrt{f'_c}\, c_{a1}^{1.5}$$ *Eq. (D-24)*

where:

ℓ_e = load bearing length of the anchor for shear, not to exceed $8d_o$

For this problem $8d_o$ will control:

Substituting into Eq. (D-24):

$$V_b = (7)\left(\frac{5.0}{0.625}\right)^{0.2}\sqrt{0.625}\,\sqrt{4000}\,6.0^{1.5} = 7797 \text{ lb}$$

Substituting into Eq. (D-22) the design breakout strength of the two anchors nearest the edge toward which the shear is directed is:

$$\phi V_{cbg} = 0.70\left(\frac{189}{162}\right)(1.0)\,(0.90)\,(1.0)\,(7797) = 5731 \text{ lb}$$

The total breakout shear strength of the four anchor group related to an initial concrete breakout failure of the two anchors located nearest the free egde is:

$$\phi V_{cbg} = 2\,(5731) = 11{,}462 \text{ lb}$$

For concrete breakout failure of the entire four anchor group:

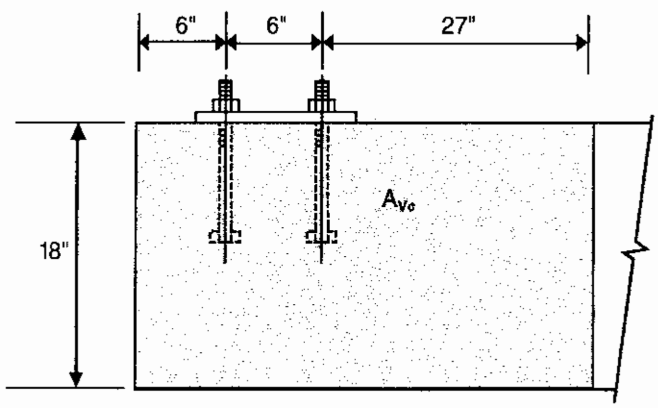
Determine A_{Vc} and A_{Vco}:

A_{Vc} is the projected area of the shear failure surface on the free edge toward which shear is directed. The projected area is determined by a rectangle with edges bounded by $1.5\,c_{a1}$ ($1.5 \times 18.0 = 27.0$ in. in this case) and free edges (side and bottom) of the concrete from the centerlines of the anchors and the surface of the concrete. Although the $1.5\,c_{a1}$ distance is not specified in Section D.6.2.1, it is shown in Commentary Figure RD.6.2.1(b).

Eq. (D-23)

D.6.2.6

Eq. (D-28)



$A_{Vc} = (6+6+27)\ (18) = 702\ \text{in.}^2$ *Eq. (D-24)*

$A_{Vco} = 4.5\ c_{a1}{}^2 = 4.5\ (18)^2 = 1458\ \text{in.}^2$

Check: $A_{Vc} \leq nA_{Vco}$ $702 < 2\ (1458)$ O.K.

Determine $\psi_{ed,V}$ [$c_{a2} < 1.5\ c_{a1}$, $6 < (1.5 \times 18)$]:

$$\psi_{ed,V} = 0.7 + 0.3\frac{c_{a2}}{1.5\ c_{a1}}$$

$$\psi_{ed,V} = 0.7 + 0.3\frac{6.0}{1.5\ (18.0)} = 0.77$$

The single anchor shear strength, V_b:

$$V_b = 7\left(\frac{\ell_e}{d_o}\right)^{0.2}\sqrt{d_o}\ \sqrt{f'_c}\ c_{a1}^{1.5}$$

where:

$\ell_e = 5.0$ in. (no change)

Substituting into Eq. (D-24):

$$V_b = (7)\left(\frac{5.0}{0.625}\right)^{0.2}\sqrt{0.625}\ \sqrt{4000}\ 18.0^{1.5} = 40{,}513\ \text{lb}$$

Substituting into Eq. (D-22) the design breakout strength of the four anchor group is:

$$\phi V_{cbg} = 0.70\left(\frac{702}{1458}\right)(1.0)\ (0.77)\ (1.0)\ (40{,}513) = 10{,}514\ \text{lb}$$

The concrete breakout shear strength of the four anchor group is controlled by the breakout of the full group.

$\phi V_{cbg} = 10{,}514$ lb

c. Pryout strength (ϕV_{cp}) *D.6.3*

Note: The pryout failure mode is normally only a concern for shallow, stiff anchors. Since this example problem addresses both shear directed toward the free edge and shear directed away from the free edge, the pryout strength will be evaluated.

$\phi V_{cpg} = \phi\, k_{cp}\, N_{cbg}$ *Eq. (D-30)*

where:

$\phi = 0.70$, Condition B always applies for pryout strength *D.4.4(c)i*

$k_{cp} = 2.0$ for $h_{ef} \geq 2.5$ in.

From Step 3(b) above

$$N_{cbg} = \left[\frac{720}{576}\right] (1.0)\ (0.85)\ (1.0)\ (1.0)\ (34{,}346) = 36{,}493 \text{ lb}$$

Substituting into Eq. (D-30):

$\phi V_{cpg} = 0.70\ (2.0)\ (36{,}493) = 51{,}090$ lb

Summary of design strengths based on steel strength, concrete breakout strength, and pryout strength for shear:

Steel strength, (ϕV_{sa}):	20,448 lb	*D.6.1*
Embedment strength - concrete breakout, (ϕV_{cbg}):	10,514 lb ← controls	*D.6.2*
Embedment strength - pryout, (ϕV_{cp}):	51,090 lb	*D.6.3*

Therefore:

$\phi V_n = 10{,}514$ lb

5. Check tension and shear interaction *D.7*

If $V_{ua} \leq 0.2\phi V_n$ then the full tension design strength is permitted *D.7.1*

$V_{ua} = 5000$ lb

$0.2\phi V_n = 0.2\ (10{,}514) = 2103 \text{ lb} < 5000$ lb

V_{ua} exceeds $0.2\phi V_n$, the full tension design strength is not permitted

If $N_{ua} \leq 0.2\phi N_n$ then the full shear design strength is permitted *D.7.2*

$N_{ua} = 10{,}000$ lb

$0.2\phi N_n = 0.2\ (17{,}703) = 3541\ \text{lb} < 10{,}000\ \text{lb}$

N_{ua} exceeds $0.2\phi N_n$, the full shear design strength is not permitted

The interaction equation must be used. *D.7.3*

$$\frac{N_{ua}}{\phi N_n} + \frac{V_{ua}}{\phi V_n} \leq 1.2 \qquad \textit{Eq. (D-29)}$$

$$\frac{10{,}000}{17{,}703} + \frac{5000}{10{,}514} = 0.56 + 0.48 = 1.04 < 1.2 \quad \text{O.K.}$$

6. Required edge distances, spacings, an thicknesses to preclude splitting failure *D.8*

   Since cast-in-place L-bolts are not likely to be highly torqued, the minimum cover requirements of 7.7 apply.

   Per 7.7 the minimum clear cover for a 5/8 in. bar is 1-1/2 in. when exposed to earth or weather. The clear cover provided for the bolt exceeds this requirement with the 6 in. edge distance to the bolt centerline – O.K.

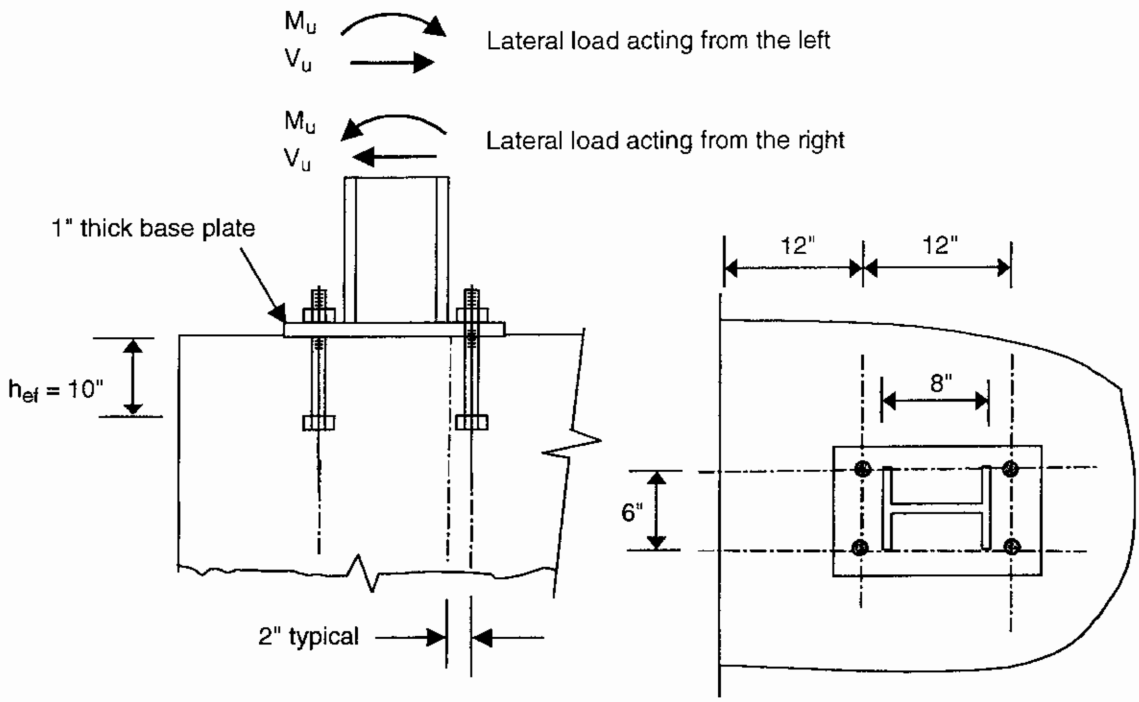
7. Summary

   Use 5/8 in. diameter ASTM F 1554 Grade 36 L-bolts with an embedment of 8 in. (measured to the upper surface of the L) and a 3 in. extension, e_h, as shown in the figure.

   Note: The use of hex head bolts rather than L-bolts would significantly increase the tensile strength of the connection. If hex head bolts were used, the design tensile strength would increase from 17,719 lb as controlled by the pullout strength of the L-bolts to 25,545 lb as controlled by concrete breakout for hex head bolts.

Example 34.7—Group of Headed Bolts in Moment and Shear Near an Edge in a Region of Moderate or High Seismic Risk

Design a group of four headed anchors spaced as shown for a reversible 18.0 k-ft factored moment and a 5.0 kip factored shear resulting from lateral seismic load in a region of moderate or high seismic risk. The connection is located at the base of an 8 in. steel column. $f'_c = 4000$ psi



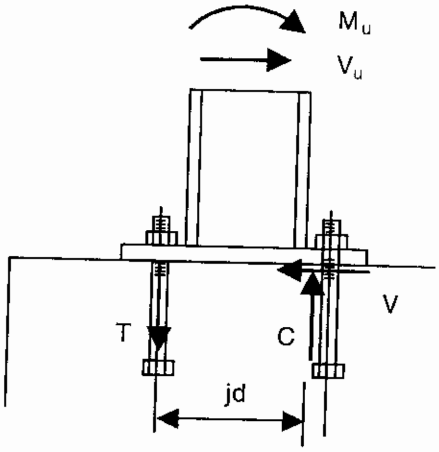
Calculations and Discussion	Code Reference

1. The solution to this example is found by assuming the size of the anchors, then checking for compliance with the design provisions for seismic loadings in regions of moderate or high seismic risk. For this example, assume four 3/4 in. ASTM F 1554 Grade 36 hex head anchors with $h_{ef} = 10$ in.

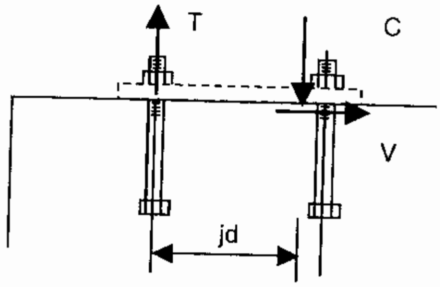
2. Since this connection is subjected to seismic load in a region of moderate or high seismic risk, the design tensile strength is $0.75\phi N_n$ and design shear strength is $0.75\phi V_n$. Unless the attachment has been designed to yield at a load lower than the design strength of the anchors (including the 0.75 factor), the strength of the anchors must be controlled by the tensile and shear strengths of ductile steel elements (D.3.3.3). To ensure ductile behavior, $0.75\phi N_{sa}$ must be larger than the concrete breakout (ϕN_{cb}), pullout (ϕN_{pn}), and side-face blowout (ϕN_{sb}). Further, $0.75\phi V_{sa}$ must be larger than concrete breakout (ϕV_{cb}), and pryout (ϕV_{cp}). *D.3.3*

3. This problem involves the design of the connection of the steel column to the foundation for lateral loads coming from either the left or the right of the structure as shown below:

Lateral load acting from the left:

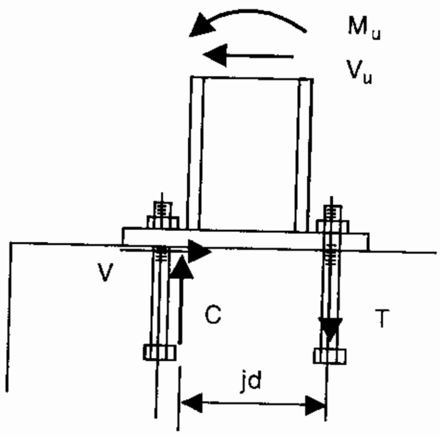


Resisting forces

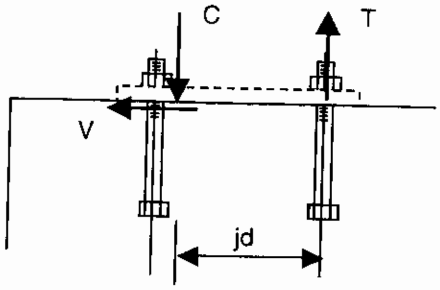


Forces on embedment

Lateral load acting from the right:



Resisting forces



Forces on embedment

As shown in the figures above, due to the free edge on the left, the critical case for tension on the anchors occurs when the lateral load is acting from the left while the critical case for shear occurs when the lateral load is acting from the right.

4. Distribution of the applied moment and shear loads to the anchors

Tension in the anchors resulting from the applied moment - The exact location of the compressive resultant from the applied moment cannot be accurately determined by traditional concrete beam methods. This is true for both the elastic linear stress-strain method (i.e., the transformed area method) and the ACI 318 stress block method since plane sections do not remain plane and different cross-sections and materials are utilized on each side of the connection. These methods require additional work that is simply not justified and in many cases can yield unconservative results for the location of the compressive resultant. The actual location of the compressive resultant is dependent on the stiffness of the base plate.

If the base plate rotates as a rigid body the compressive resultant will be at the leading edge of the base plate. For example, take a book, lay it on your desk and lift one end. The end opposite of the one being lifted is where the compressive resultant is located; this is rigid base plate behavior where the compressive resultant is located at the leading edge of the base plate. The assumption of rigid base plate behavior is conservative for determining base plate thickness but is unconservative for determining the tension force in the anchors since it provides a maximum distance (lever arm) between the tensile and compressive resultants from the applied moment.

If the base plate is flexible, the compressive resultant will be very near the edge of the attached structural member that is in compression from the applied moment. For example; take a piece of paper, lay it on your desk and lift one end. A portion of the paper opposite of the one being lifted will remain flat on the desktop. Since this portion of the paper remains flat, it has no curvature and therefore carries no moment. For this case, the compressive resultant must be located at the point where the piece of paper with one end lifted first contacts the desktop. References D.4 and D.5 of the ACI 318 Commentary show that the minimum distance between the edge of the attached structural member that is in compression from the applied moment and the compressive resultant from the applied moment is equal to the yield moment of the base plate divided by the compressive resultant from the applied moment. Since the determination of this distance adds unwarranted difficulty to the calculations, it is conservative to assume that the compressive resultant is located at the edge of the attached structural member that is in compression from the applied moment when determining the tensile resultant in the anchors from the applied moment.

For this example, the internal moment arm jd will be conservatively determined by assuming flexible base plate behavior with the compressive resultant located at the edge of the compression element of the attached member.

jd = 2+8 = 10 in.

By summing moments about the location of the compressive resultant (see figures in Step 3):

$M_u = T\,(jd)$

where:

M_u = 18.0 k-ft = 216,000 in.-lb
T = N_{ua} (i.e., the factored tensile load acting on the anchors in tension)
jd = 2+8 = 10 in.

Rearranging and substituting:

$$N_{ua} = \frac{M_u}{jd} = \frac{216{,}000}{10} = 21{,}600 \text{ lb}$$

Shear – Although the compressive resultant from the applied moment will allow for the development of a frictional shear resistance between the base plate and the concrete, the frictional resistance will be neglected for this example and the anchors on the compression side will be designed to transfer the entire shear. The assumption of the anchors on the compression side transferring the entire shear is supported by test results reported in Ref. D.4, D.5, and D.6. This assumption is permitted by D.3.1 which allows for plastic analysis where the nominal strength is controlled by ductile steel elements (as required by D.3.3.4).

References D.4, D.5, D.6 and ACI 349-01 *Code Requirements for Nuclear Safety Related Concrete Structures* B.6.1.4 provide information regarding the contribution of friction to the shear strength. As noted in these references, the coefficient of friction between the steel base plate and concrete may be assumed to be 0.40. For this example, the frictional shear resistance is likely to have the potential to transfer 8640 lbs (0.40 × 21,600). Although the potential frictional resistance between the base plate and the concrete will be neglected in this example, it does exist and will be located at the compressive reaction (i.e., near the anchors in the compression zone).

To summarize, the assumption of the entire shear being transferred by the anchors in the compression zone is permitted by D.3.1, represents a conservative condition for shear design, is supported by test results, and best represents where the shear will actually be transferred to the concrete if the friction force were considered.

V_u = 5000 lb on the two anchors on the compression side

5. Determine the design tensile strength for seismic load ($0.75\phi N_n$) *D.5*

   a. Steel strength, (ϕN_{sa}): *D.5.1*

   $\phi N_{sa} = \phi\, n\, A_{se}\, f_{uta}$ *Eq. (D-3)*

   where:

   ϕ = 0.75 *D.4.4(a)i*

Per Table 34-1, the ASTM F 1554 Grade 36 bolt meets the Ductile Steel Element definition of Section D.1.

$A_{se} = 0.334$ in.^{2} (see Table 34-2)

$f_{uta} = 58{,}000$ psi (see Table 34-1)

Substituting:

$\phi N_{sa} = 0.75\ (2)\ (0.334)\ (58{,}000) = 29{,}058$ lb

b. Concrete breakout strength (ϕN_{cbg}): *D.5.2*

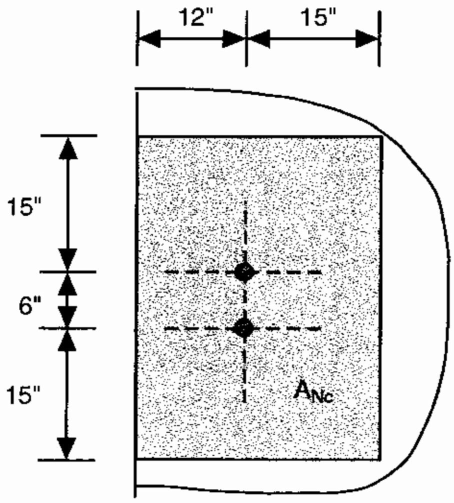
Since the spacing of the anchors is less than 3 times the effective embedment depth h_{ef} (3×10 in. = 30 in.), the anchors must be treated as an anchor group. *D.1*

$$\phi N_{cbg} = \phi \frac{A_{Nc}}{A_{Nco}} \psi_{ec,N} \psi_{ed,N} \psi_{c,N} \psi_{cp,N} N_b$$ *Eq. (D-5)*

Since no supplementary reinforcement has been provided, $\phi = 0.70$ *D.4.4(c)ii*

Determine A_{Nc} and A_{Nco}: *D.5.2.1*

A_{Nc} is the projected area of the failure surface as approximated by a rectangle with edges bounded by 1.5 h_{ef} ($1.5 \times 10.0 = 15.0$ in.) and free edges of the concrete from the centerlines of the anchors.



$A_{Nc} = (12 + 15)\ (15 + 6 + 15) = 972\ \text{in.}^2$

$A_{Nco} = 9\ h_{ef}^2 = 9\ (10)^2 = 900\ \text{in.}^2$ — Eq. (D-6)

Check: $A_{Nc} \leq nA_{Nco}$ $\quad 972 < 2(900) \quad$ O.K.

Determine $\psi_{ec,N}$: — D.5.2.4

$\psi_{ec,N} = 1.0$ (no eccentricity in the connection)

Determine $\psi_{ed,N}$: — D.5.2.5

$$\psi_{ed,N} = 0.7 + 0.3 \frac{c_{a,min}}{1.5\ h_{ef}}$$ — Eq. (D-11)

$$\psi_{ed,N} = 0.7 + 0.3 \frac{12.0}{1.5\ (10.0)} = 0.94$$

Determine $\psi_{c,N}$: — D.5.2.6

$\psi_{c,N} = 1.0$ for locations where concrete cracking is likely to occur (i.e., the edge of the foundation)

Determine $\psi_{cp,N}$ — D.5.2.7

For cast-in-place anchors, $\psi_{cp,N} = 1.0$

Determine N_b: — D.5.2.2

$$N_b = 24\sqrt{f_c'}\ h_{ef}^{1.5} = 24\ \sqrt{4000}\ (10.0)^{1.5} = 48{,}000\ \text{lb}$$ — Eq. (D-7)

Substituting into Eq. (D-5):

$$\phi\ N_{cbg} = 0.70\left[\frac{972}{900}\right](1.0)\ (0.94)\ (1.0)\ (1.0)\ (48{,}000) = 34{,}111\ \text{lb}$$

c. Pullout strength (ϕN_{pn}) — D.5.3

$\phi N_{pn} = \phi\ \psi_{c,P}\ N_p$ — Eq. (D-14)

where:

$\phi = 0.70$, Condition B always applies for pullout strength — D.4.4(c)ii

$\psi_{c,P} = 1.0$, cracking may occur at the edges of the foundation — *D.5.3.6*

N_p for the hex head bolts:

$N_p = A_{brg}\, 8\, f'_c$ — *Eq. (D-15)*

$A_{brg} = 0.654$ in.^{2}, for 3/4 in. hex head bolt (see Table 34-2)

Substituting into Eq. (D-14) and Eq. (D-15) with 2 bolts (ϕN_{pn})

$\phi N_{pn} = 2\,(0.70)\,(1.0)\,(0.654)\,(8)\,(4000) = 29{,}299$ lb

d. Concrete side-face blowout strength (ϕN_{sb}) — *D.5.4*

The side-face blowout failure mode must be investigated when the edge distance (c) is less than $0.4\,h_{ef}$ — *D.5.4.1*

$0.4\,h_{ef} = 0.4\,(10) = 4.0$ in. < 12.0 in.

Therefore, the side-face blowout strength is not applicable (N/A)

Summary of design strengths based on steel strength, concrete breakout strength, pullout strength, and side-face blowout strength for tension:

Steel strength, (ϕN_{sa}):	29,058 lb ← controls	*D.5.1*
Embedment strength - concrete breakout, (ϕN_{cbg}):	34,111 lb	*D.5.2*
Embedment strength - pullout, (ϕN_{pn}):	29,299 lb	*D.5.3*
Embedment strength - side-face blowout, (ϕN_{sb}):	N/A	*D.5.4*

Therefore:

$\phi N_n = 29{,}058$ lb and is controlled by a ductile steel element as required in D.3.3.4

For seismic load in a region of moderate or high seismic risk, the design tensile strength is $0.75\phi N_n$: — *D.3.3.3*

$0.75\,\phi N_n = 0.75\,(29{,}058) = 21{,}794$ lb and is controlled by a ductile steel element

Check if $N_{ua} \le 0.75\phi N_n$

21,600 lb < 21,794 lb   O.K. for tension

6. Determine the design shear strength (ϕV_n) — *D.6*

a. Steel strength, (ϕV_{sa}): — *D.6.1*

$\phi V_{sa} = \phi\, n\, 0.6\, A_{se}\, f_{uta}$ — *Eq. (D-20)*

where:

$\phi = 0.65$ — D.4.4(a)ii

Per Table 34-1, the ASTM F 1554 Grade 36 meets the Ductile Steel Element definition of Section D.1.

$A_{se} = 0.334 \text{ in.}^2$ (see Table 34-2)

$f_{uta} = 58{,}000$ psi (see Table 34-1)

Substituting:

$\phi V_{sa} = 0.65\ (2)\ (0.6)\ (0.334)\ (58{,}000) = 15{,}110$ lb

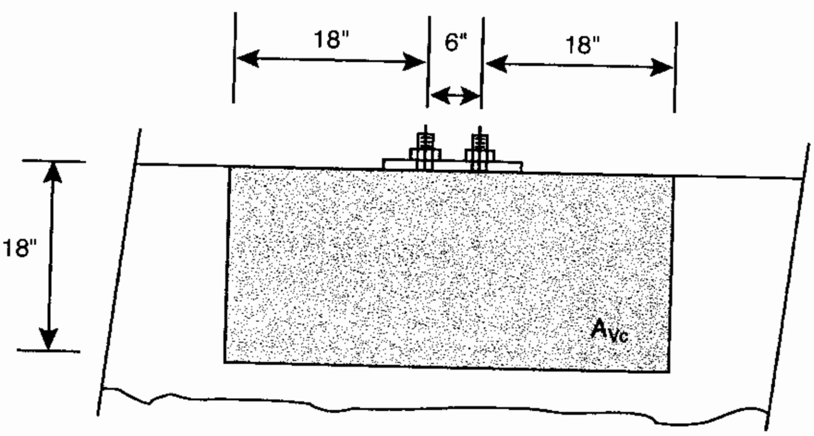
b. Concrete breakout strength (ϕV_{cbg}): — D.6.2

$$\phi V_{cbg} = \phi \frac{A_{Vc}}{A_{Vco}} \psi_{ec,V}\ \psi_{ed,V}\ \psi_{c,V}\ V_b$$ — Eq. (D-22)

Since no supplementary reinforcement has been provided, $\phi = 0.70$ — D.4.4(c)i

Determine A_{Vc} and A_{Vco}: — D.6.2.1

A_{Vc} is the projected area of the shear failure surface on the free edge toward which shear is directed. The projected area is determined by a rectangle with edges bounded by 1.5 c_{a1} (1.5×12.0 = 18.0 in.) and free edges of the concrete from the centerlines of the anchors and surface of the concrete.



$A_{Vc} = (18 + 6 + 18)(18) = 756\ \text{in.}^2$	
$A_{Vco} = 4.5\ c_{a1}^2 = 4.5\ (12)^2 = 648\ \text{in.}^2$	*Eq. (D-23)*
Check: $A_{Vc} \le nA_{Vco}$ $756 < 2(648)$ O.K.	
Determine $\psi_{ec,V}$:	*D.6.2.5*
$\psi_{ec,V} = 1.0$ (no eccentricity in the connection)	
Determine $\psi_{ed,V}$:	*D.6.2.6*
$\psi_{ed,V} = 1.0$ (no orthogonal free edge)	*Eq. (D-27)*
Determine $\psi_{c,V}$:	
$\psi_{c,V} = 1.0$ for locations where concrete cracking is likely to occur (i.e., the edge of the foundation)	
Determine V_b for an anchor:	
$V_b = 7\left(\frac{\ell_e}{d_o}\right)^{0.2}\sqrt{d_o}\ \sqrt{f'_c}\ c_{a1}^{1.5}$	*Eq. (D-24)*
where:	
ℓ_e = load bearing length of the anchor for shear, not to exceed $8d_o$	*2.1*
For this problem $8d_o$ will control:	
$\ell_e = 8d_o = 8\ (0.75) = 6.0$ in. < 10 in. therefore, use $8d_o$	
Substituting into Eq. (D-24):	
$V_b = (7)\left(\frac{8\ (0.75)}{0.75}\right)^{0.2}\sqrt{0.75}\ \sqrt{4000}\ \ (12.0)^{1.5} = 24{,}157\ \text{lb}$	
Substituting into Eq. (D-22):	
$\phi V_{cbg} = 0.70\left(\frac{756}{648}\right)(1.0)\ (1.0)\ (1.0)\ (24{,}157) = 19{,}728\ \text{lb}$	

c. Pryout strength (ϕV_{cpg})	D.6.3
Note: The pryout failure mode is normally only a concern for shallow, stiff anchors. Since this example problem addresses both shear directed toward the free edge and shear directed inward from the free edge, the pryout strength will be evaluated.	
$\phi V_{cpg} = \phi\, k_{cp}\, N_{cbg}$	Eq. (D-30)
where:	
$\phi = 0.70$, Condition B always applies for pryout strength	D.4.4(c)i
$k_{cp} = 2.0$ for $h_{ef} > 2.5$ in.	
From Step 5(b) above	
$\phi\, N_{cbg} = \phi\, \frac{A_{Nc}}{A_{Nco}}\, \psi_{ec,N}\, \psi_{ed,N}\, \psi_{c,N}\, \psi_{cp,N}\, N_b$	Eq. (D-5)
$N_{cbg} = \left[\frac{972}{900}\right](1.0)\,(0.94)\,(1.0)\,(1.0)\,(48{,}000) = 48{,}730$ lb	
Substituting into Eq. (D-30):	
$\phi V_{cpg} = 0.70\,(2.0)\,(48{,}730) = 68{,}222$ lb	

Summary of design strengths based on steel strength, concrete breakout strength, and pryout strength for shear:

Steel strength, (ϕV_{sa}):	15,110 lb ← controls	D.6.1
Embedment strength - concrete breakout, (ϕV_{cbg}):	19,728 lb	D.6.2
Embedment strength - pryout, (ϕV_{cp}):	68,222 lb	D.6.3

Therefore:

$\phi V_n = 15{,}110$ lb and is controlled by a ductile steel element as required in D.3.3.4

For seismic load in a region of moderate or high seismic risk, the design shear strength is $0.75\phi V_n$:	D.3.3.3

$0.75\,\phi V_n = 0.75\,(15{,}110) = 11{,}333$ lb and is controlled by a ductile steel element

Check if $V_{ua} \le 0.75\phi V_n$

5000 lb < 11,333 lb O.K. for shear

7. Required edge distances, spacings, and thicknesses to preclude splitting failure *D.8*

   Since cast-in-place anchors are not likely to be highly torqued, the minimum cover requirements of 7.7 apply.

   Per 7.7 the minimum clear cover for a 3/4 in. bar is 1-1/2 in. when exposed to earth or weather. The clear cover provided for the bolt exceeds this requirement with the 12 in. edge distance to the bolt centerline O.K.

8. Summary

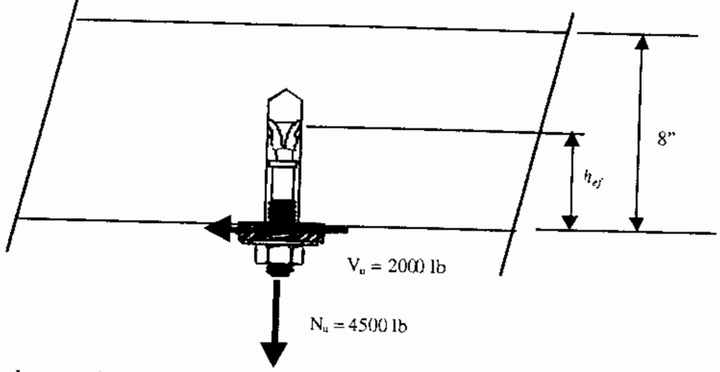
   Use 3/4 in. diameter ASTM F 1554 Grade 36 hex head anchors with $h_{ef} = 10$ in.

Note: OSHA Standard 29 CFR Part 1926.755 requires that column anchorages use at least four anchors and be able to sustain a minimum eccentric gravity load of 300 pounds located 18 in. from the face of the extreme outer face of the column in each direction. The load is to be applied at the top of the column. The intent is that the column be able to sustain an iron worker hanging off the side of the top of the column. This connection will satisfy the OSHA requirement but calculations are not included in the example.

Example 34.8—Single Post-Installed Anchor in Tension and ShearAway from Edges

Design a single post-installed mechanical anchor installed in the bottom of an 8 in. slab to support a factored 4500 lb tension load and a factored 2000 lb shear load (seismic loads from regions of moderate to high seismic risk are not included).

$f'_c = 4000$ psi



Note: This example for a single post-installed mechanical anchor is provided at the end of the design examples of Part 34 since additional calculations to account for group effects, edge conditions, eccentricity, and tension/shear interaction covered in the previous examples for cast-in-place anchors are essentially the same as for post-installed mechanical anchors.

Similarities between post-installed mechanical anchors and cast-in-place anchors:

- For group and edge conditions, A_{Nc}, A_{Nco}, A_{Vc}, and A_{Vco} are determined in the same manner.
- For eccentric loads, $\psi_{ec,N}$ and $\psi_{ec,V}$ are determined in the same manner.
- For edge effects, $\psi_{ed,N}$ and $\psi_{ed,V}$ are determined in the same manner.
- For anchors used in areas where concrete cracking may occur, $\psi_{c,N}$ and $\psi_{c,V} = 1.0$.

The unique properties of post-installed mechanical anchors are provided by the ACI 355.2 product evaluation report (refer to the sample in Table 34-3 for anchor data for a fictitious post-installed torque-controlled mechanical expansion anchor). The unique properties associated with each post-installed mechanical anchor product are:

- effective embedment length h_{ef}
- effective cross sectional area A_{se} in tension and shear
- specified yield strength f_{ya} and specified ultimate strength f_{uta}
- minimum edge distance $c_{a,min}$ for the anchor
- minimum member thickness h_{min} for the anchor
- minimum spacing s for the anchor
- critical edge distance c_{ac} for $\psi_{ep,N}$ with uncracked concrete design (D.5.2.7)
- category of the anchor for determination of the appropriate ϕ factor for embedment strength
- coefficient for basic concrete breakout strength k_c for use in Eq. (D-7)
- factor $\psi_{c,N}$ for uncracked concrete design
- pullout strength N_p of the anchor

Calculations and Discussion	Code Reference

1. The solution to this example is found by assuming the size of the anchor, then checking compliance with the design provisions. Try the fictitious 5/8 in. post-installed torque-controlled mechanical expansion anchor with a 4.5 in. effective embedment depth, shown in Table 34-3.

Example 34.8 (cont'd)	Calculations and Discussion	Code Reference

2. This is a tension/shear interaction problem where values for both the design tensile strength (ϕN_n) and design shear strength (ϕV_n) will need to be determined. ϕN_n is the smallest of the design tensile strengths as controlled by steel (ϕN_{sa}), concrete breakout (ϕN_{cb}), pullout (ϕN_{pn}), and side-face blowout (ϕN_{sb}). ϕV_n is the smallest of the design shear strengths as controlled by steel (ϕV_{sa}), concrete breakout (ϕV_{cb}), and pryout (ϕV_{cp}). *D.7* *D.4.1.2*

3. Determine the design tensile strength (ϕN_n) *D.5*

   a. Steel strength, (ϕN_{sa}): *D.5.1*

   $\phi N_{sa} = \phi\ n\ A_{se}\ f_{uta}$ *Eq. (D-3)*

   where:

   $\phi = 0.75$ *D.4.4(a)i*

   As shown in Table 34-3, this anchor meets ductile steel requirements.

   $A_{se} = 0.226$ in.^{2} (see Table 34-3)

   $f_{uta} = 75{,}000$ psi (see Table 34-3)

   Note: Per D.5.1.2, f_{uta} shall not be taken greater than $1.9f_{ya}$ or 125,000psi. From Table 34-3, $f_{ya} = 55{,}000$ psi and $1.9f_{ya} = 1.9(55{,}000) = 104{,}500$ psi, therefore use the specified minimum f_{uta} of 75,000 psi. *D.5.1.2*

   Substituting:

   $\phi N_{sa} = 0.75\ (1)\ (0.226)\ (75{,}000) = 12{,}712$ lb

   b. Concrete breakout strength (ϕN_{cb}): *D.5.2*

   $$\phi N_{cb} = \phi \frac{A_{Nc}}{A_{Nco}} \psi_{ed,N}\ \psi_{c,N}\ \psi_{cp,N}\ N_b$$ *Eq. (D-4)*

   where:

   $\phi = 0.65$ *D.4.4*

   From Table 34-3, this post-installed anchor is Category 1 and no supplementary reinforcement has been provided.

$\frac{A_{Nc}}{A_{Nco}}$ and $\psi_{ed,N}$ terms are 1.0 for single anchors away from edges

$\psi_{c,N} = 1.0$ and $\psi_{cp,N} = 1.0$ for locations where concrete cracking is likely to occur (i.e., the bottom of the slab)

$N_b = k_c \sqrt{f_c'}\ h_{ef}^{1.5}$ — Eq. (D-7)

where:

$k_c = 17$

Note: $k_c = 17$ for post-installed anchors unless the ACI 355.2 product evaluation report indicates a higher value may be used. For the case of this torque-controlled mechanical expansion anchor, $k_c = 17$ per Table 34-3. — RD.5.2.2

$h_{ef} = 4.5$ in. (Table 34-3)

Therefore,

$N_b = 17\sqrt{4000}\ 4.5^{1.5} = 10{,}264$ lb

Substituting:

$\phi N_{cb} = 0.65\ (1.0)\ (1.0)\ (1.0)\ (10{,}264) = 6672$ lb

c. Pullout strength (ϕN_{pn}) — D.5.3

$\phi N_{pn} = \phi\ \psi_{c,P}\ N_p$ — Eq. (D-14)

where:

$\phi = 0.65$, Category 1 and no supplementary reinforcement has been provided — D.4.4

$\psi_{c,P} = 1.0$, cracking may occur at the edges of the foundation — D.5.3.6

$N_p = 8211$ lb (see Table 34-3)

Substituting:

$\phi N_{pn} = 0.65\ (1.0)\ (8211) = 5337$ lb

d. Concrete side-face blowout strength (ϕN_{sb}) — D.5.4

This anchor is not located near any free edges therefore the side-face blowout strength is not applicable. — D.5.4.1

Summary of steel strength, concrete breakout strength, pullout strength, and side-face blowout strength for tension:

Steel strength, (ϕN_{sa}):	12,712 lb	*D.5.1*
Embedment strength - concrete breakout, (ϕN_{cb}):	6672 lb	*D.5.2*
Embedment strength - pullout, (ϕN_{pn}):	5337 lb ← controls	*D.5.3*
Embedment strength - side-face blowout, (ϕN_{sb}):	N/A	*D.5.4*

Therefore:

$\phi N_n = 5337$ lb

4. Determine the design shear strength (ϕV_n) *D.6*

a. Steel strength, (ϕV_{sa}): *D.6.1*

$\phi V_{sa} = \phi\, n\, (0.6 A_{se}\, f_{uta})$ *Eq. (D-20)*

where:

$\phi = 0.65$

As shown in Table 34-3, this anchor meets ductile steel requirements.

$A_{se} = 0.226$ in.^{2} (see Table 34-3)

$f_{uta} = 75{,}000$ psi (see Table 34-3)

Note: Per D.5.1.2, f_{uta} shall not be taken greater than $1.9 f_{ya}$ or 125,000 psi. From Table 3, $f_{ya} = 55{,}000$ psi and $1.9 f_{ya} = 1.9(55{,}000) = 104{,}500$ psi. Therefore, use the specified minimum f_{uta} of 75,000 psi.

Substituting:

$\phi V_{sa} = 0.65\ (1)\ (0.6)\ (0.226)\ (75{,}000) = 6610$ lb

b. Concrete breakout strength (ϕV_{cb}): *D.6.2*

This anchor is not located near any free edges therefore the concrete breakout for shear is not applicable.

c. Pryout strength (ϕV_{cp}) *D.6.3*

$\phi V_{cp} = \phi\, k_{cp}\, N_{cb}$ *Eq. (D-29)*

where:

$\phi = 0.65$, Category 1 and no supplementary reinforcement has been provided *D.4.4*

$k_{cp} = 2.0$ for $h_{ef} > 2.5$ in.

$$N_{cb} = \frac{A_{Nc}}{A_{Nco}} \psi_{ed,N}\, \psi_{c,N}\, \psi_{cp,N}\, N_b$$ *Eq. (D-4)*

From Step 3 above:

$N_{cb} = (1.0)\,(1.0)\,(1.0)\,(10{,}264) = 10{,}264$ lb

Substituting into Eq. (D-29):

$\phi V_{cp} = 0.65\,(2.0)\,(10{,}264) = 13{,}343$ lb

Summary of steel strength, concrete breakout strength, and pryout strength for shear:

Steel strength, (ϕV_{sa}):	6610 lb ← controls	*D.6.1*
Embedment strength - concrete breakout, (ϕV_{cb}):	N/A	*D.6.2*
Embedment strength - pryout, (ϕV_{cp}):	13,343 lb	*D.6.3*

Therefore:

$\phi V_n = 6610$ lb

5. Check tension and shear interaction *D.7*

If $V_{ua} \leq 0.2\phi V_n$ then the full tension design strength is permitted *D.7.1*

$V_{ua} = 2000$ lb

$0.2\phi V_n = 0.2\,(6610) = 1322$ lb

V_{ua} exceeds $0.2\phi V_n$, the full tension design strength is not permitted

If $N_{ua} \leq 0.2\phi N_n$ then the full shear design strength is permitted *D.7.2*

$N_{ua} = 4500$ lb

$0.2\phi N_n = 0.2\,(5337) = 1067$ lb

N_{ua} exceeds $0.2\phi N_n$, the full shear design strength is not permitted

The interaction equation must be used *D.7.3*

$$\frac{N_{ua}}{\phi N_n} + \frac{V_{ua}}{\phi V_n} \leq 1.2$$ *Eq. (D-29)*

$$\frac{4500}{5337}+\frac{2000}{6610}=0.84+0.30=1.14<1.2 \quad \text{O.K.}$$

6. Required edge distances, spacings, and thickness to preclude splitting failure *D.8*

Since this anchor is located away from edges, only the limits on embedment length h_{ef} related to member thickness are applicable. Per D.8.5, h_{ef} shall not exceed 2/3 of the member thickness or the member thickness less 4 in. D.8 does permit the use of larger values of h_{ef} provided product-specific tests have been performed in accordance with ACI 355.2.

As shown in Table 34-3, the ACI 355.2 product evaluation report for this anchor provides the minimum thickness as 1.5 h_{ef} = 1.5(4.5) = 6.75 in. which is less than the 8 in. provided O.K.

7. Summary

The fictitious 5/8 in. diameter post-installed torque-controlled mechanical expansion anchor with 4.5 in. effective embedment depth shown in Table 34-3 is O.K. for the factored tension and shear loads.

Also from PCA

The following publications may be of interest to readers of this report:

2003 Analysis of Revisions to the IBC — Structural Provisions (LT289)
ACI 318-05 Building Code Requirements for Structural Concrete and Commentary (LT287)
Circular Concrete Tanks without Prestressing (IS072)
Column Shortening in Tall Buildings—Prediction and Compensation (EB108)
Concrete Floor Systems—Guide to Estimating and Economizing (SP041)
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