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NOTES ON

ACI 318-05

BUILDING CODE REQUIREMENTS FOR STRUCTURAL CONCRETE

with Design Applications

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PCA

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About the building on the cover

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Preface

The first edition of this reference manual was developed to aid users in applying the provisions of the 1971 edition of "Building Code Requirements for Reinforced Concrete (ACI 318-71)." The second through fifth editions updated the material in conformity with provisions of the 1977 code edition, the 1980 code supplement, and the 1983 and 1989 code editions, respectively. The sixth, seventh and eighth editions addressed the 1995, 1999, and 2002 editions of "Building Code Requirements for Structural Concrete (ACI 318-95), (ACI 318-99), and (ACI 318-02)." Through eight editions, much of the initial material has been revised to better emphasize the subject matter, and new chapters added to assist the designer in proper application of the ACI 318 design provisions.

This ninth edition reflects the contents of "Building Code Requirements for Structural Concrete (ACI 318-05)." The text and design examples have been revised to reflect, where possible, comments received from users of the""Notes" who suggested improvements in wording, identified errors, and recommended items for inclusion or deletion.

The primary purpose for publishing this manual is to assist the engineer and architect in the proper application of the ACI 318-05 design standard. The emphasis is placed on "how-to-use" the code. For complete background information on the development of the code provisions, the reader is referred to the""Commentary on Building Code Requirements for Structural Concrete (ACI 318R-05)" which, starting with the 1989 edition, has been published together with the code itself under the same cover.

This manual is also a valuable aid to educators, contractors, materials and products manufacturers, building code authorities, inspectors, and others involved in the design, construction, and regulation of concrete structures.

Although every attempt has been made to impart editorial consistency to the thirty-four chapters, some inconsistencies probably still remain. A few typographical and other errors are probably also to be found. PCA would be grateful to any reader who would bring such errors and inconsistencies to our attention. Other suggestions for improvement are also most sincerely welcome.

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David A. Fanella of Graef, Anhalt, Schloemer Associates, Inc. developed Example 29.8.

Dale McFarlane of PCA staff played a crucial role in the production of this publication. He was responsible for the word processing, layout and formatting of this large and complex manuscript. His assistance is very much appreciated.

Finally, sincere gratitude must be expressed to the authors and contributors of various parts of the first through eighth editions of the "Notes." Their initial work is carried over into this edition, although their names are no longer separately identified with the various parts. Robert F. Mast, BERGER/ABAM, Federal Way, WA, updated Parts 5, 6, 7, 8, 24, and 25, of the 2002 edition, all pertaining to application of the Unified Design Provisions. Last, but not least, James Doyle, Consultant, Chaplin, KY, performed a thorough review of Part 10, Deflections, for the 2002 edition.

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General Requirements

A significant renaming of the ACI 318 standard took place with the 1995 edition; in the document title, "Reinforced Concrete," was changed to "Structural Concrete" in recognition of the then new Chapter 22 - Structural Plain Concrete. Prior to the '95 code, design and construction requirements for structural members of plain concrete were contained in a separate companion document to ACI 318, designated ACI 318.1. The requirements for structural plain concrete of the former ACI 318.1 code are now incorporated in Chapter 22.

1.1* SCOPE

As the name implies, *Building Code Requirements for Structural Concrete (ACI 318-05)* is meant to be adopted by reference in a general building code, to regulate the design and construction of buildings and structures of concrete. Section 1.1.1 emphasizes the intent and format of the ACI 318 document and its status as part of a legally adopted general building code. The ACI 318 code has no legal status unless adopted by a state or local jurisdiction having power to regulate building design and construction through a legally appointed building official. It is also recognized that when the ACI code is made part of a legally adopted general building code, that general building code may modify some provisions of ACI 318 to reflect local conditions and requirements. For areas where there is no general building code, there is no law to make ACI 318 the "code." In such cases, the ACI code defines minimum acceptable standards of design and construction practice, even though it has no legal status.

A provision in 1.1.1, new to ACI 318-02 and unchanged in ACI 318-05 requires that the minimum specified compressive strength of concrete be not less than 2500 psi. This provision is also included in 5.1.1. While the commentary does not explain why this provision was added, it was most likely included because an identical requirement was in *The BOCA National Building Code* (NBC), and *Standard Building Code* (SBC) for several editions, and it was also adapted into the 2000 *International Building Code* (IBC) and remains in the 2003 IBC.

Also new to 1.1.1 of ACI 318-02 and unchanged in ACI 318-05 is a statement that "No maximum specified compressive strength (of concrete) shall apply unless restricted by a specific code provision." The impetus for adding this was the fact that some local jurisdictions, most notably in southern California, were in effect, if not formally, imposing maximum limits on strength of concrete used in structures in regions of high seismic risk (UBC Seismic Zone 3 or 4). Committee 318 felt that it was advisable to add the statement to make it known to regulators that possible need for limitations on concrete strength are considered when new code provisions are introduced, and unless concrete strength is specifically limited by other provisions of ACI 318, no maximum upper limit on strength is deemed necessary. The Committee has been making adjustments in the standard on an ongoing basis to account for sometimes differing properties of high-strength concrete.

In the past, most jurisdictions in the United States adopted one of the three following model building codes, now referred to as *legacy codes*, to regulate building design and construction. *The BOCA National Building Code* (NBC), published by the Building Officials and Code Administrators International[1.1], was used primarily in the

*Section numbers correspond to those of ACI 318-05.

northeastern states; the *Standard Building Code* (SBC), published by the Southern Building Code Congress International[1.2], was used primarily in the southeastern states; and the *Uniform Building Code* (UBC), published by the International Conference of Building Officials[1.3], was used mainly in the central and western United States. All three of these model codes used the ACI 318 standard to regulate design and construction of structural elements of concrete in buildings or other structures. *The BOCA National Building Code* and the *Standard Building Code* adopted ACI 318 primarily by reference, incorporating only the construction requirements (Chapter 4 through 7) of ACI 318 directly within Chapter 19 of their documents. The *Uniform Building Code* reprinted ACI 318 in its entirety in Chapter 19. It is essential that designers of concrete buildings in jurisdictions still regulated by the UBC refer to Chapter 19, as some ACI 318 provisions were modified and some provisions were added to reflect, in most cases, more stringent seismic design requirements. To clearly distinguish where the UBC differed from ACI 318, the differing portions of UBC Chapter 19 were printed in italics.

Many states and local jurisdictions that formerly adopted one of the three legacy codes, have adopted the *International Building Code* (IBC), developed by the International Code Council[1.A]. The 2000 edition (first edition) of the IBC adopted ACI 318-99 by reference, and the 2003 edition of the IBC[1.A] adopts ACI 318-02 by reference." Portions of Chapters 3 – 7 of ACI 318 have been included in IBC Sections 1903 – 1907. A few modifications have been made to the reproduced ACI 318 provisions and these are indicated by the text printed in italics. Additional modifications to provisions in other Chapters of ACI 318 are contained in IBC Section 1908. Many of these were necessary to coordinate ACI 318 provisions for seismic design (Chapter 21) with the IBC's seismic design provisions.

As this book goes to press, it is anticipated that the 2006 edition of the IBC will adopt ACI 318-05 by reference. In addition, most of the text from ACI 318 had been transcribed into IBC Sections 1903 – 1907 will be removed and replaced with references to the ACI code. IBC Section 1908 will continue to contain modifications to the provisions of ACI 318, most of which are related to seismic design issues.

In the fall of 2002, the National Fire Protection Association (NFPA) issued the first edition (2003) of its *Building Construction and Safety Code NFPA 5000*[1.B] which adopted ACI 318-02 by reference. While there were no modifications to ACI 318 within the first edition of NFPA 5000, it adopted the modifications to ACI 318 contained in Section A.9.9 of ASCE 7-02[1.C]. As this book goes to press, the 2006 edition of NFPA 5000 is nearing completion and it is anticipated that it will adopt ACI 318-05 by reference and the modifications to ACI 318-05 contained in Section 14.2 of ASCE 7-05, including its Supplement Number 1. Only a few jurisdictions scattered throughout the country have adopted NFPA 5000 and it appears that it will not be able to supplant the IBC as the model of choice.

Whichever building code governs the design, be it a model code or locally developed code, the prudent designer should always refer to the governing code to determine the edition of ACI 318 that is adopted and if there are any modifications to it.

Seismic Design Practice — Earthquake design requirements in two of the three legacy codes were based on the 1991 edition of the *NEHRP (National Earthquake Hazards Reduction Program) Recommended Provisions for the Development of Seismic Regulations for New Buildings.*[1.5] *The BOCA National Building Code* (NBC) and the *Standard Building Code* (SBC) incorporated the NEHRP recommended provisions into the codes, with relatively few modifications. The *Uniform Building Code* (UBC), published by the International Conference of Building Officials which traditionally followed the lead of the Structural Engineers Association of California (SEAOC), had its seismic provisions based on the *Recommended Lateral Force Requirements and Commentary*[1.6] (the SEAOC "Blue Book") published by the Seismology Committee of SEAOC. The *SEAOC Blue Book* in its 1996 and 1999 editions, adopted many of the features of the 1994 NEHRP provisions.[1.E]

The designer should be aware that there were important differences in design methodologies between the UBC and the NBC and SBC for earthquake design. Even with the different design methodologies, it is important to note that a building designed under the NBC or SBC earthquake design criteria and the UBC criteria provided a similar level of safety and that the two sets of provisions (NBC and SBC versus UBC) were substantially equivalent.[1.7]

The seismic design provisions of the 2000 edition of the *International Building Code*, were based on the 1997 edition of the *NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures.*[1.8] Major differences between the 2000 IBC and the NBC and SBC seismic provisions, that were based on the '91 NEHRP Provisions[1.5], included:

1. Seismic ground motion maps of the 1991 edition were replaced with spectral response acceleration maps at periods of 0.2 second and 1.0 second.

2. The 1991 maps gave ground motion parameters that had a 10% probability of exceedance in 50 years (i.e., approximately a 475-year return period). The 1997 maps were based on a maximum considered earthquake (MCE), and for most regions the MCE ground motion was defined with a uniform likelihood of exceedance of 2% in 50 years (return period of about 2500 years).

3. Seismic detailing requirements, were triggered by building use and estimated ground motion on rock in the '91 edition; the trigger was revised to include the amplifying effects of soft soils overlying rock. This might require buildings on soft soils in areas that were traditionally considered to be subject to low or moderate seismic hazard to be detailed for moderate and high seismic risk, respectively.

4. In the '91 edition, the amplifying effects of soft soils were ignored in calculating the design base shear for short period buildings. These effects were now taken into consideration, and resulted in significant increases in base shear for short period buildings on soft soils in areas subject to low seismic hazard.

5. A reliability/redundancy factor was introduced for buildings subject to high seismic risk. This was done to force designers to either add redundancies to the seismic force-resisting system or to pay a penalty in the form of designing for a higher base shear.

6. It became a requirement to design every building for a lateral force at each floor equal to 1% of the effective seismic weight at that level. Seismic design of buildings subject to negligible or very low seismic risk (e.g., located in Seismic Zone 0, or assigned to SPC A) has traditionally not been required by building codes. This new requirement meant that in areas where seismic design had traditionally been ignored (e.g., south Florida, and much of Texas), designers now needed to make sure that these so-called index forces did not control the design of the lateral force-resisting system. These index forces instead of wind are liable to control design of the lateral force-resisting system of larger concrete buildings, such as parking structures, or long narrow buildings, such as hotels/motels.

For a comprehensive comparison of the major differences between the seismic design requirements of the 2000 IBC, and the last editions of the NBC, SBC and UBC, see *Impact of the Seismic Design Provisions of the International Building Code*[1.9].

"The seismic design requirements of the 2003 IBC are based on ASCE 7-02, which in turn is based on the 2000 edition of the *NEHRP (National Earthquake Hazards Reduction Program) Recommended Provisions for Seismic Regulations for New Buildings and Other Strcutures*[1.D]. A comprehensive discussion of changes in the structural provisions from the 2000 to the 2003 IBC has been provided in Ref. 1.F. The 2003 IBC saw the beginning of a philosophical shift from the code containing almost all the seismic design provisions, as was the case with the 2000 IBC, to one in which the code only has the simplified design provisions. For design of buildings requiring other than simplified analysis procedured, the 2003 IBC references ASCE-7-02. It is anticipated that the 2006 IBC will carry this shift to its conclusion and remove virtually all the seismic design provions from the code and reference the provisions of ASCE 7-05, including its Supplement Number 1. It should be pointed out that ASCE 7-05 will be based on the 2003 edition of the *NEHRP (National Earthquake Hazards Reduction Program) Recommended Provisions for Seismic Regulations for New Buildings and Other Strcutures*[1.E]. Supplement Number 1 to ASCE 7-05 updates the seismic design provisions by referencing the latest editions of material design standards, such as ACI 318-05.

For seismic design, the 2003 edition of NFPA 5000 adopts by reference ASCE 7-02. It is anticipated that the 2006 edition of NFPA 5000 will reference ASCE 7-05, including Supplement Number 1."

Differences in Design Methodology — The UBC earthquake design force level was based on the seismic zone, the structural system, and the building use (occupancy). These design considerations were used to determine a design base shear. As the anticipated level of ground shaking increased, the design base shear increased. Similarly, as the need for post-disaster functionality increased, the design base shear was increased.

As with the UBC, the NBC and the SBC provisions increased the design base shear as the level of ground shaking increased. In the NBC and SBC, this was done not through a seismic zone factor Z, but through a coefficient A_v representing effective peak velocity-related acceleration or a coefficient A_a representing effective peak acceleration (for definitions of these terms, see the Commentary to the NEHRP Provisions[1.5]). These two quantities were given on separate contour maps that took the place of the seismic zoning map of the UBC. The NBC and the SBC utilized a "seismic performance category" (SPC) that took into account the level of seismicity and the building occupancy. Based on the SPC of the building, different design criteria such as drift limits and detailing requirements were specified. The IBC provision also increase the design base shear as the level of ground shaking increases. However, in the IBC, the A_a and A_v maps are replaced with spectral response acceleration maps at periods of 0.2 second and 1.0 second, respectively, the IBC replaces the "seismic performance category" of the NBC and the SBC with a "seismic design category" (SDC). This is more than a change of terminology, because in addition to considering the occupancy of the structure and the estimated ground motion on rock, also considered is the modification of ground motion due to the amplifying effects of soft soils overlying rock. Based on the SDC of the building, different design criteria such as drift limits and detailing requirements are specified. As in the UBC, the NBC and the SBC, the IBC earthquake provisions factor into design the effects of site geology and soil characteristics and the type and configuration of the structural framing system.

Another major difference between the provisions of the 1994 and earlier editions of the UBC and those of the IBC, NBC and SBC is in the magnitude of the design base shear. The designer should note that the earthquake design forces of the IBC, NBC and SBC, and the 1994 and earlier editions of the UBC cannot be compared by simply looking at the numbers, since one set of numbers is based on strength design and the other set is based on working or allowable stress design. NBC and SBC design earthquake forces were strength level while pre-1997 UBC forces were service load level. IBC also provides strength level design earthquake forces. The difference shows up in the magnitude of the response modification coefficient, commonly called the "R" factor. In the NBC and SBC provisions, the term was R; in the IBC, the term is R; in the pre-1997 UBC it was R_w, with the "w" subscript signifying "working" load level design forces. The difference also becomes apparent in the load factors to be applied to the earthquake force effects (E). In the NBC and SBC, the load factor for earthquake force effects was 1.0, as it is in the IBC. In the pre-1997 UBC, for reinforced concrete design, a load factor of 1.4 was applied to the earthquake force effects. Thus, for reinforced concrete, when comparing the base shear calculated by the pre-1997 UBC with that calculated by the 2000 or 2003 IBC, or 1993, 1996 or 1999 NBC, or the 1994, 1997 or 1999 SBC, the designer must multiply the UBC base shear by 1.4.

The seismic design force of the 1997 UBC was at strength level, rather than service level. The change was accomplished by changing the former response modification factors, R_w, to strength-based R-factors, similar to those found in the IBC, NBC and SBC. Since the load combinations of Section 9.2 of ACI 318-95, reproduced in Section 1909.2 of the 1997 UBC, were intended to be used with service level loads, the UBC had to adopt strength-based load combinations that were intended to be used with strength level seismic forces. Therefore, the 1997 UBC required that when concrete elements were to be designed for seismic forces or the effects thereof, the strength-based load combinations of UBC Section 1612.2.1 must be used. These load combinations were based on the load combinations of ASCE 7-95[1.10]. The 1997 UBC also required that when concrete elements were being designed for seismic forces or the effects thereof using the UBC load combination, a multiplier of 1.1 must be applied to amplify the required strengths. This was felt to be necessary at the time because of a presumed incompatibility between the strength reduction factors of Section 9.3 of ACI 318 and the strength design load combination of ASCE 7-95 that were incorporated into the 1997 UBC. After actual seismic designs were performed using the 1997 UBC provisions, it was apparent that use of the 1.1 multiplier resulted in overly conservative designs when compared to the 1994 UBC. Based on a study of the appropriateness of using the multiplier, the SEAOC Seismology Committee has gone on record recommending that it not be used. For additional information on this subject, see Ref. 1.11. The multiplier has now been removed from the 2001 California Building Code[1.G], which is based on the 1997 UBC.

The vertical distribution of base shear along the height of a building also differs between the UBC and the IBC, NBC and SBC. For shorter buildings (with a fundamental period less than or equal to 0.7 second), the UBC required that the design base shear be distributed to the different floor levels along the height in proportion to the product of the weights assigned to floor levels and the heights of the floors above the building base (in accordance with the first mode of vibration of the building). For taller buildings (with fundamental period greater than 0.7 second), the design base shear was divided into two parts. The first part was applied as a concentrated force at the top of the building (to account for higher modes of vibration), with the magnitude being in proportion to the fundamental period of the building, this concentrated force was limited to 25% of the design base shear. The remainder of the design base shear was required to be distributed as specified for shorter buildings. In the NBC and SBC, a fraction of the base shear was applied at a floor level in proportion to the product of weight applied to the floor and height (above the base) raised to the power k, where k is a coefficient based on building period. The IBC and SBC specify a k of 1 (linear distribution of V) for $T \leq 0.5$ sec. These loads specified a k of 2 (parabolic distribution of V) for $T \geq 2.5$ sec. For 0.5 sec. $< T <$ 2.5 sec., two choices were available. One might interpolate between a linear and a parabolic distribution by finding a k-value between 1 and 2, depending upon the period; or one might use a parabolic distribution ($k = 2$), which is always more conservative. The IBC uses the same distribution as the NBC and the SBC.

Lastly, the detailing requirements, also termed ductility or toughness requirements, which are applicable to structures in regions of moderate to high seismic risk, or assigned to intermediate or high seismic performance or design categories, were similar in the three legacy codes. These requirements are essential to impart to buildings the ability to deform beyond the elastic limit and to undergo many cycles of extreme stress reversals. Fortunately, for reinforced concrete structures, all three legacy codes adopted and the IBC now adopts the ACI 318 standard including Chapter 21 — Special Provisions for Seismic Design. However, the designer will need to refer to the governing model code for any modifications to the ACI 318 seismic requirements. Portions of UBC Chapter 19 that differ substantially from the ACI were printed in italics. The NBC and SBC also included some modifications to the ACI document, most notably for prestressed concrete structures assigned to SPC D or E. Likewise, the 2000 IBC included modifications to ACI 318 in Section 1908, most of which recognize precast concrete systems not in Chapter 21 of ACI 318-99 for use in structures assigned to SDC D, E or F. Section 1908 of the 2003 IBC contains fewer modifications partly because design provisions for precast concrete structures in SDC, D, E or F included in ACI 318-02.

Metric in Concrete Construction — Metric is back. In 1988, federal law mandated the metric system as the preferred system of measurement in the United States. In July 1990, by executive order, all federal agencies were required to develop specific timetables for transition to metric. Some federal agencies involved in construction generally agreed to institute the use of metric units in the design of federal construction by January 1994.

The last editions of the three legacy codes featured and the IBC features both inch-pound (U.S. Customary) and SI-metric (Systeme International) units. The "soft" metric equivalents were or are given in the three legacy codes, generally in parentheses after the English units.

It is noteworthy that when metric conversion was first proposed in the 1970s, some of the standards-writing organizations began preparing metric editions of some of their key documents. The American Concrete Institute first published a "hard" metric companion edition to the ACI 318 standard, ACI 318M-83, in 1983. The current ACI 318 standard is available as ACI 318-05 (U.S. Customary units) and ACI 318M-05 (SI-metric units). ACI 318-H05, for the first time, is a soft metric, rather than a hard metric, version of ACI 318-05. Within the same time period, the American Society for Testing and Materials (ASTM) published metric companions to many of its ASTM standards. For example, Standard Specifications A 615M and A 706M for steel bars for concrete reinforcement were developed as metric companions to A 615 and A 706. The older editions of these metric standards were in rounded metric (hard metric) numbers and included ASTM standard metric reinforcing bars. Due to the expense of maintaining two inventories, one for bars in inch-pound units and another for bars in hard metric units, reinforcing bar manufacturers convinced the standards writers to do away with the hard metric standards and develop metric standards based upon soft conversion of ASTM standard inch-pound bars. The latest editions of the ASTM metric reinforcing bar standards reflect this philosophy. Since all federally financed projects have to be designed and constructed in metric, bar manufacturers decided in 1997 that rather than produce the same bars with two different systems of designating size and strength (i.e., inch-pound and metric), they would produce bars with only one system of marking and that would be the system prescribed for the soft metric converted bars. Thus, it is now commonplace to see reinforcing bars with metric size and strength designations on a job that was designed in inch-pound units. It is important to remember that if this occurs on your job, the bars are identical to the inch-pound bars that were specified, except for the markings designating size and strength.

This Ninth edition of the "Notes" is presented in the traditional U.S. Customary units. Largely because of the large volume of this text, unlike in most other PCA publications, no soft metric conversion has been included.

1.1.6 Soil-Supported Slabs

Prior to the 1995 edition of the code, it did not explicitly state whether soil-supported slabs, commonly referred to as slabs-on-grade or slabs-on-ground, were regulated by the code. They were explicitly excluded from the 1995 edition of ACI 318 "...unless the slab transmits vertical loads from other portions of the structure to the soil." The 1999 edition expanded the scope by regulating slabs-on-grade that "... transmit vertical loads or *lateral forces* from other portions of the structure to the soil." Mat foundation slabs and other slabs on ground which help support the structure vertically and/or transfer lateral forces from the supported structure to the soil should be designed according to the applicable provisions of the code, especially Chapter 15 - Footings. The design methodology for typical slabs-on-grade differs from that for building elements, and is addressed in References 1.12 and 1.13. Reference 1.12 describes the design and construction of concrete floors on ground for residential, light industrial, commercial, warehouse, and heavy industrial buildings. Reference 1.13 gives guidelines for slab thickness design for concrete floors on grade subject to loadings suitable for factories and warehouses.

In addition to the modification to 1.1.6, a new Section 21.8, Foundations, was added in Chapter 21 — Special Provisions for Seismic Design in the 1999 edition of ACI 318. Due to sections being added to Chapter 21 in the 2002 edition, these provisions are now in 21.10. Section 21.10.3.4 indicates that "slabs on grade that resist seismic forces from walls or columns that are part of the lateral-force-resisting system shall be designed as structural diaphragms in accordance with 21.9." In this location of Chapter 21, the provisions only apply in regions of high seismic risk, or to structures assigned to high seismic performance or design categories. In regions of low or moderate seismic risk, or for structures assigned to low or intermediate seismic performance or design categories, the provisions of Chapters 1 through 18, or Chapter 22 apply to such slabs, by virtue of the new provision in 1.1.6 (see Table 1-3).

1.1.8 Special Provisions for Earthquake Resistance

Since publication of the 1989 code, the special provisions for seismic design have been located in the main body of the code to ensure adoption of the special seismic design provisions when a jurisdiction adopts the ACI code as part of its general building code. With the continuing high interest nationally in the proper design of buildings for earthquake performance, the code's emphasis on seismic design of concrete buildings continues with this edition. Chapter 21 represents the latest in special seismic detailing of reinforced concrete buildings for earthquake performance.

The landmark volume, *Design of Multistory Concrete Buildings for Earthquake Motions* by Blume, Newmark, and Corning[1.H], published by the Portland Cement Association (PCA) in 1961, gave major impetus to the design and construction of concrete buyildings in regions of high seismicity. In the decades since, significant strides have been made in the earthquake resistant design and construction of reinforced concrete buildings. Significant developments have occurred in the building codes arena as well. However, a comprehensive guide to aid the designer in the detailed seismic design of concrete buildings was not available until PCA published *Design of Concrete Buildings for Earthquake and Wind Forces* by S.K. Ghosh and August W. Domel, Jr. in 1992[1J].

That design manual illustrated the detailed design of reinforced concrete buildings utilizing the various structural systems recognized in U.S. seismic codes. All designs were according to the provisions of the 1991 edition of the *Uniform Building Code* (UBC), which had adopted, with modifications, the seismic detailing requirements of the 1989 edition of *Building Code Requirements for Reinforced Concrete* (ACI 318-89, Revised 1992). Design of the same building was carriedd out for regions of high, moderate, and low seismicity, and for wind, so that it would be apparent how design and detailing changed with increased seismic risk at the site of the structure.

The above publication was updated to the 1994 edition of the UBC, in which ACI 318-89, Revised 1992, remained the reference standard for concrete design and construction, although a new procedure for the design of reinforced concrete shear walls in combined bending and axial compression was introduced in the UBC itself. The updated publication by S.K. Ghosh, August W. Domel, Jr., and David A. Fanella was issued by PCA in 1995.

Since major changes occurred between the 1994 and 1997 editions of the UBC as discussed above, a new book titled *Design of Concrete Buildings for Earthquake and Wind Forces According to the 1997 Uniform Building Code*[1.15] was developed. It discussed the major differences in the design requirements between the 1994 and the 1997 editions of the UBC. Three different types of concrete structural framing systems were designed and detailed for earthquake forces representing regions of high seismicity (Seismic Zones 3 and 4). Although the design examples focused on regions of high seismicity, one chapter discussed the detailing requirements for structures located in regions of low, moderate, and high seismicity. Design of the basic structural systems for wind was also illustrated. As in this "Notes" publication, the emphasis has placed on "how-to-use" the various seismic design and detailing provisions of the latest and possibly the last UBC.

PCA publication, *Design of Low-Rise Concrete Buildings for Earthquake Forces*[1.16], was a companion document to that described above; however, its focus was on designing concrete buildings under the 1996 and 1997 editions of *The BOCA National Building Code* (NBC) and the *Standard Building Code* (SBC), respectively. As indicated previously, the seismic provisions of the last editions of the NBC and SBC were almost identical, and were based on the 1991 edition of the *NEHRP Recommended Provisions for the Development of Seismic Regulations for New Buildings*.[1.5] With the two exceptions noted below, the book was also applicable to the 1993 and 1999 editions of the NBC, and the 1994 and 1999 editions of the SBC. The only difference between the loading requirements of the 1993 and the 1996 and 1999 NBC was that the load combinations to be used for seismic design under the 1993 edition of the NBC were identical to those that had to be used under all three editions of the SBC. Whereas, the 1996 and 1999 NBC adopted by reference the strength design load combinations of ASCE 7-95[1.9]. The second exception was that different editions of ACI 318 were adopted by the various editions of the codes as illustrated in the table below.

Model Code	Edition	Edition of ACI 318 adopted by Model Code
NBC	1993	1989, Revised 1992
	1996	1995
	1999	1995
SBC	1994	1989
	1997	1995
	1999	1995
IBC	2000	1999
IBC	2003	2002
NFPA 5000	2003	2002

Since designing for seismic forces in areas that had traditionally adopted the NBC or SBC was relatively new, the book provided excellent background information for the structural engineer. Since the overwhelming majority of all buildings constructed in this country are low-rise, that was the focus of this book. For its purpose, low-rise was defined as less than 65 feet in height or having a fundamental period of vibration of less than 0.7 second.

To assist the designer in understanding and using the special detailing requirements of Chapter 21 of the Code, PCA developed a publication titled *Seismic Detailing of Concrete Buildings*[1.17]. Numerous tables and figures illustrated the provisions for buildings located in regions of moderate and high seismic risk – IBC Seismic Design Categories C, D, E and F. While the book was based on the '99 edition of the Code, which was referenced by the 2000 IBC, most of the provisions are applicable to ACI 318-02 and ACI 318-05.

In recent years, the building code situation in this country has changed drastically. The seismic design provisions of the IBC represent revolutionary changes from those of model codes it was developed to replace. This created a need for a new publication similar to the volume first issued by PCA in 1992. To fill that need, PCA and the International Code Council (ICC) published *Seismic and Wind Design of Concrete Buildings*: 2000 IBC, ASCE 7-98, ACI 318-99 by S.K. Ghosh and David A. Fanella in 2003[1.K].

An update of the above publication to the 2003 IBC, *Seismic and Wind Design of Concrete Buildings*: 2003 IBC, ASCE 7-02, ACI 318-02 by S.K. Ghosh, David A. Fanella, and Xuemei Liang[1.L] has recently been published by PCA and ICC. In Chapter 1, an introduction to earthquake-resistant design is provided, along with summaries of the seismic and wind design provisions of the 2003 IBC. Chapter 2 is devoted to an office building utilizing a dual shear wall-frame interactive system in one direction and a moment-resisting frame in the orthogonal direction. Designs for Seismic Design Categories (SDC) A, C, D, and E are illustrated in both directions. Chapter 3 features a residential building, which utilizes a shear-wall frame interactive system in SDC A and B and a building frame system for lateral resistance in SDC C, D, and E. Chapter 4 presents the design of a school building with a moment-resisting frame system in SDC B, C, and D. A residential building utilizing a bearing wall system is treated in Chapter 5. Design is illustrated for SDC A, B, C, D, and E. The final (sixth) chapter is devoted to design of a precast parking structure utilizing the building frame system in SDC B, C, and D. While design is always for the combination of gravity, wind, and seismic forces, wind forces typically govern the design in the low seismic design categories (particularly A), and earthquake forces typically govern in the high seismic design categories (particularly D and above). Detailing requirements depend on the seismic design category, regardless of whether wind or seismic forces govern the design. This publication is designed to provide an appreciation on how design and detailing change with changes in the seismic design category.

1.1.8.1 Structures at Low Seismic Risk — For concrete structures located in regions of low seismic hazard or assigned to low seismic performance or design categories (no or minor risk of damage—Sesimic Design Category A or B), no special design or detailing is required; thus, the general requirements of the code, excluding Chapter 21, apply. Concrete structures proportioned by the general requirements of the code are considered to have a level of toughness adequate for low earthquake intensities.

The designer should be aware that the general requirements of the code include several provisions specifically intended to improve toughness, in order to increase resistance of concrete structures to earthquake and other catastrophic or abnormal loads. For example, when a beam is part of the lateral force-resisting system of a structure, a portion of the positive moment reinforcement must be anchored at supports to develop its yield strength (see 12.11.2). Similarly, hoop reinforcement must be provided in certain types of beam-column connections (see 11.11.2). Other design provisions introduced since publication of the 1971 code, such as those requiring minimum shear reinforcement (see 11.5.5) and improvements in bar anchorage and splicing details (Chapter 12), also increase toughness and the ability of concrete structures to withstand reversing loads due to earthquakes. With publication of the 1989 code, provisions addressing special reinforcement for structural integrity (see 7.13) were added, to enhance the overall integrity of concrete structures in the event of damage to a major supporting element or abnormal loading.

1.1.8.2 Structures at Moderate/Intermediate or High Seismic Risk — For concrete structures located in regions of moderate seismic hazard, or assigned to intermediate seismic performance or design categories (moderate risk of damage—Seismic Design Category C), 21.12 includes certain reinforcing details, in addition to those contained in Chapters 1 through 18, that are applicable to reinforced concrete moment frames (beam-column or slab-column framing systems) required to resist earthquake effects. Reflecting terminology that has been in use in the model codes over at least the past decade, frames detailed in accordance with 21.12 are now referred to as *Intermediate Moment Frames*. These so-called "intermediate" reinforcement details will serve to accommodate an appropriate level of inelastic behavior if the frame is subjected to an earthquake of such magnitude as to require it to perform inelastically. There are no design or detailing requirements in addition to those of Chapters 1 through 18 for other structural components of structures at moderate seismic risk (including structural walls (shearwalls)) regardless of whether they are assumed in design to be part of the seismic-force-resisting system or not. Structural walls proportioned by the general requirements of the code are considered to have sufficient toughness at drift levels anticipated in regions of moderate seismicity.

The type of framing system provided for earthquake resistance in a structure at moderate seismic risk will dictate whether any special reinforcement details need to be incorporated in the structure.

If the lateral force-resisting system consists of moment frames, the details of 21.12 for *Intermediate Moment Frames* must be provided, and 21.2.2.3 shall also apply. Note that even if a load combination including wind load effects (see 9.2.1) governs design versus a load combination including earthquake force effects, the intermediate reinforcement details must still be provided to ensure a limited level of toughness in the moment resisting frames. Whether or not the specified earthquake forces govern design, the frames are the only defense against the effects of an earthquake.

For a combination frame-shearwall structural system, inclusion of the intermediate details will depend on how the earthquake loads are "assigned" to the shearwalls and the frames. If the total earthquake forces are assigned to the shearwalls, the intermediate detailing of 21.12 is not required for the frames. If frame-shearwall interaction is considered in the analysis, with some of the earthquake forces to be resisted by the frames, then the intermediate details of 21.12 are required to toughen up the frame portion of the dual framing system. Model codes have traditionally considered a dual system to be one in which at least 25% of the design lateral forces are capable of being resisted by the moment frames. If structural walls resist total gravity and lateral load effects, no intermediate details are required for the frames; the general requirements of the code apply.

For concrete structures located in regions of high seismic hazard, or assigned to high seismic performance or design categories (major risk of damage—Seismic Design Category D, E, or F), all structural components must satisfy the applicable special proportioning and detailing requirements of Chapter 21 (excluding 21.12 and 21.13). If, for purposes of design, some of the frame members are not considered as part of the lateral force resisting system, special consideration is still required in the proportioning and detailing of these frame members (see 21.11). The special provisions for seismic design of Chapter 21 are intended to provide a monolithic reinforced concrete structure with adequate toughness to respond inelastically under severe earthquake motions.

Unlike previous editions of the Code, the 2002 and 2005 edition specifically addresses precast concrete systems for use in structures in regions of moderate or high seismic hazard, or in structures assigned to intermediate or high seismic performance or design categories. A special moment frame can either be cast-in-place or erected with precast elements. A precast concrete special moment frame must comply with all the requirements for cast-in-place frames (21.2 through 21.5), plus 21.6. In addition, the requirements for an ordinary moment frame must be satisfied (Chapters 1 through 18). Since there is no provision for an intermediate moment frame made of precast elements, by implication such frames erected in structures in regions of moderate seismic hazard, or assigned to intermediate seismic performance or design categories must either be special moment frames, or be qualified under the performance criteria of 21.2.1.5. In the 2002 code, the definition of "ordinary moment frame" was revised to clarify that such a frame can either be cast-in-place or constructed with precast elements, both of which must comply with Chapters 1 through 18.

Two new precast structural walls were added to the 2002 code; an intermediate precast structural wall, and a special precast structural wall. The intermediate precast structural wall must comply with Chapters 1 through 18, plus 21.13. Section 21.13 does not address the wall itself, but covers the connection between individual wall panels, and the connection of wall panels to the foundation. Wherever precast wall panels are used to resist seismic lateral forces in structures in regions of moderate seismic hazard or assigned to intermediate seismic performance or design categories, they must comply with the requirements for an intermediate precast structural wall, or special precast structural wall. By implication, a wall composed of precast elements designed in accordance with Chapters 1 through 18, but not complying with either of these requirements can only be used in structures in regions of low seismic hazard, or in structures assigned to low seismic performance or design categories.

The special precast structural wall must comply with Chapters 1 through 18, plus 21.2, 21.7, 21.13.2 and 21.13.3. Wherever precast wall panels are used to resist seismic lateral forces in structures in regions of high seismic hazard, or assigned to high seismic performance or design categories, they must comply with the requirements for a special precast structural wall.

The ACI 318 proportioning and detailing requirements for lateral force-resisting structural systems of reinforced concrete are summarized in Table 1-1.

Table 1-1 Sections of Code to be Satisfied

Component resisting earthquake effect unless otherwise noted		Level of Seismic Hazard or Assigned Seismic Performance of Design Categories as Defined in Code Section Indicated		
		Low/A,B/A,B 21.2.1.2	Intermediate/C/C 21.2.1.3	High/D,E/D,E,F 21.2.1.4
Frame members	Cast-in-Place	Ch. 1–18	Ch. 1–18, 21.2.2.3, 21.12	Ch. 1–18, 21.2–21.5
	Precast	Ch. 1–18	Note 1	Ch. 1–18, 21.2–21.6
Structural walls and	Cast-in-Place	Ch. 1-18, or Ch. 22	Ch. 1–18	Ch. 1–18. 21.2, 21.7
	Precast	Ch. 1–18, or Ch. 22	Ch. 1–18, 21.13	Ch. 1–18, 21.2, 21.7, 21.8
Structural diaphragms and trusses		Ch. 1–18	Ch. 1–18	Ch. 1–18, 21.2, 21.9
Foundations		Ch. 1–18, or Ch. 22	Ch. 1–18	Ch. 1–18, 21.2, 21.10, 22.10
Frame members assumed not to resist earthquake forces		Ch. 1–18	Ch. 1–18	Ch. 1–18, 21.11

Note 1: There are no provisions for constructing an intermediate moment frame with precast elements. See 21.2.1.5.

1.1.8.3 Seismic Hazard Level Specified in General Building Code — This code has traditionally addressed levels of seismic hazard as "low," "moderate," or "high." Precise definitions of seismic hazard levels are under the jurisdiction of the general building code, and have traditionally been designated by zones (related to intensity of ground shaking). The model codes specify which sections of Chapter 21 must be satisfied, based on the seismic hazard level. As a guide, in the absence of specific requirements in the general building code, seismic hazard levels and seismic zones generally correlate as follows:

Seismic Hazard Level	Seismic Zone
Low	0 and 1
Moderate	2
High	3 and 4

The above correlation of seismic hazard levels and seismic zones refers to the *Uniform Building Code*[1.3].

However, with the adoption of the 1991 NEHRP Provisions into *The BOCA National Building Code* and the *Standard Building Code*, the designer needed to refer to the governing model code to determine appropriate seismic hazard level and corresponding special provisions for earthquake resistance. The NBC, SBC, and '91 NEHRP Provisions, on which the seismic design requirements of the two legacy model codes were based, assigned a building to a Seismic Performance Category (SPC). The SPC expressed hazard in terms of the nature and use of the building and the expected ground shaking on rock at the building site. To determine the SPC of a structure, one had to first determine its Seismic Hazard Exposure Group. Essential facilities were assigned to Seismic Hazard Exposure Group III, assembly buildings and other structures with a large number of occupants were assigned to Group II. Buildings and other structures not assigned to Group II or III, were considered to belong to Group I (see the governing code for more precise definitions of these Seismic Hazard Exposure Groups). The next step was to determine the effective peak velocity-related acceleration coefficient, A_v, given on a contour map that formed part of the NBC and the SBC. With these two items, the structure's SPC could be determined from a table in the governing code that was similar to Table 1-2, which is reproduced from the 1991 NEHRP Provisions.

Table 1-2 Seismic Performance Category[1.5]

Value of A_v	Seismic Hazard Exposure Group		
	I	II	III
$A_v < 0.05$	A	A	A
$0.05 \leq A_v < 0.10$	B	B	C
$0.10 \leq A_v < 0.15$	C	C	C
$0.15 \leq A_v < 0.20$	C	D	D
$0.20 \leq A_v$	D	D	E

"In the 2000 and 2003 editions of the *International Building Code,* the seismic design requirements are based on the 1997 and 2000 edition of the NEHRP Provisions[1.8], respectively. In the IBC the seismic hazard is expressed in a manner that is similar to that of the NBC and the SBC, but with one important difference." The IBC also considers the amplifying effects of softer soils on ground shaking in assigning seismic hazard. The terminology used in the IBC for assigning hazard and prescribing detailing and other requirements is the Seismic Design Category (SDC). The SDC of a building is determined in a manner similar to the SPC in the NBC and the SBC. First the building is assigned to a Seismic Use Group (SUG), which is the same as the Seismic Hazard Exposure Group of the NBC and the SBC. At this point the IBC process becomes more involved. Instead of determining one mapped value of expected ground shaking, two spectral response acceleration values are determined from two different maps; one for a short (0.2 second) period and the other for a period of 1 second. These values are then adjusted for site soil effects and multiplied by two-thirds to arrive at design spectral acceleration values. Knowing the SUG and the design spectral response acceleration values (S_{DS} and S_{D1}), Knowing the SUG and the design spectral response acceleration values,one enters two different tables to determine the SDC based on the two design values. The governing SDC is the higher of the two, if they differ.

"As a guide, for purposes of determining the applicability of special proportioning and detailing requirements of Chapter 21 of the ACI code, Table 1-3 shows the correlation between UBC seismic zones; the Seismic Performance Categories of the NBC, SBC, 1994 (and earlier) NEHRP and ASCE 7-95 (and earlier); and the Seismic Design Categories of the IBC, NFPA 5000, 1997 (and later) NEHRP, and ASCE 7-98 (and later)."

1.2 DRAWINGS AND SPECIFICATIONS

If the design envisioned by the engineer is to be properly implemented in the field, adequate information needs to be included on the drawings or in the specifications, collectively known as the construction documents. The code has for many editions included a list of items that need to be shown on the construction documents.

1.2.1 Items Required to be Shown

The information required to be included as a part of the construction documents remains essentially unchanged from the 1999 code; however, in the 2002 code item "e" was expanded to require that anchors be shown on the drawings. Enough information needs be shown so anchors can be installed with the embedment depth and edge distances the engineer assumed in the design. In addition, where "supplemental reinforcement" (see definition in D.1) was assumed in the design, the location of the reinforcement with respect to the anchors needs to be indicated.

Table 1-3 — Correlation Between Seismic Hazard Levels of ACI 318 and Other Codes and Standards

Code, Standard or Resource Document And Edition	Assigned Seismic Performance or Design Categories and Level of Seismic Risk as Defined in Code Section		
	Low (21.2.1.2)	Moderate/Intermediate (21.2.1.3)	High (21.2.1.4)
BOCA National Building Code 1993, 1996, 1999	SPC[1] A, B	SPC C	SPC D, E
Standard Building Code 1994, 1997, 1999	SPC A, B	SPC C	SPC D, E
Uniform Building Code 1991, 1994, 1997	Seismic Zone 0, 1	Seismic Zone 2	Seismic Zone 3, 4
International Building Code 2000, 2003	SDC[2] A, B	SDC C	SDC D, E, F
NFPA 5000-2003	SDC[2] A, B	SDC C	SDC D, E, F
ASCE[3] 7-93, 7-95	SPC[1] A, B	SPC C	SPC D, E
NEHRP[4] 1991, 1994	SPC[1] A, B	SPC C	SPC D, E
ASCE[3] 7-98, 7-02	SDC[2] A, B	SDC C	SDC D, E, F
NEHRP[5] 1997	SDC[2] A, B	SDC C	SDC D, E, F

1. *SPC = Seismic Performance Category as defined in building code, standard or resource document*
2. *SDC = Seismic Design Category as defined in building code, standard or resource document*
3. *Minimum Design Loads for Buildings and Other Structures*
4. *NEHRP (National Earthquake Hazards Reduction Program) Recommended Provisions for Seismic Regulations for New Buildings*
5. *NEHRP (National Earthquake Hazards Reduction Program) Recommended Provisions for Seismic Regulations for New Buildings and Other Structures*

1.3 INSPECTION

The ACI code requires that concrete construction be inspected as required by the legally adopted general building code. In the absence of inspection requirements in the general building code or in an area where a building code has not been adopted, the provisions of 1.3 may serve as a guide to providing an acceptable level of inspection for concrete construction. In cases where the building code is silent on this issue or a code has not been adopted, concrete construction, at a minimum; should be inspected by a registered design professional, someone under the supervision of a registered design professional, or a qualified inspector. Individuals professing to be qualified to perform these inspections should be required to demonstrate their competence by becoming certified. Voluntary certification programs for inspectors of concrete construction have been established by the American Concrete Institute (ACI), and International Code Council (ICC). Other similar certification programs may also exist.

The IBC, adopted extensively in the U.S. to regulate building design and construction, and NFPA 5000 require varying degrees of inspection of concrete construction. However, administrative provisions such as these are

*Commentary section numbers are preceded by an "R" (e.g., R1.3.5 refers to Comentary Section R1.3.5).

frequently amended when the model code is adopted locally. The engineer should refer to the specific inspection requirements contained in the general building code having jurisdiction over the construction.

In addition to periodic inspections performed by the building official or his representative, inspections of concrete structures by special inspectors may be required; see discussion below on 1.3.5. The engineer should check the local building code or with the local building official to ascertain if special inspection requirements exist within the jurisdiction where the construction will be occurring. Degree of inspection and inspection responsibility should be set forth in the contract documents. However, it should be pointed out that most codes with provisions for special inspections do not permit the contractor to retain the special inspector. Normally they require that the owner enter into a contract with the special inspector. Therefore, if the frequency and type of inspections are shown in the project's construction documents, it should be made clear that the costs for providing these services are not to be included in the bid of the general contractor.

1.3.4 Records of Inspection

Inspectors and inspection agencies will need to be aware of the wording of 1.3.4. Records of inspection must be preserved for two years after completion of a project, or longer if required by the legally adopted general building code. Preservation of inspection records for a minimum two-year period after completion of a project is to ensure that records are available, should disputes or discrepancies arise subsequent to owner acceptance or issuance of a certificate of occupancy, concerning workmanship or any violations of the approved construction documents, or the general building code requirements.

1.3.5 Special Inspections

Continuous inspection is required for placement of all reinforcement and concrete for special moment frames (beam and column framing systems) resisting earthquake-induced forces in structures located in regions of high seismic hazard, or in structures assigned to high seismic performance or design categories. Special moment frames of cast-in-place concrete must comply with the 21.2 - 21.5. Special moment frames constructed with precast concrete elements must comply with the additional requirements of 21.6. For information on how the model building codes in use in the U.S. assign seismic hazard, see Table 1-3. The code stipulates that the inspections must be made by a qualified inspector under the supervision of the engineer responsible for the structural design or under the supervision of an engineer with demonstrated capability for supervising inspection of special moment frames resisting seismic forces in regions of high seismic hazard, or in structures assigned to high seismic performance or design categories. R1.3.5* indicates that qualification of inspectors should be acceptable to the jurisdiction enforcing the general building code.

This inspection requirement is patterned after similar provisions contained in the *The BOCA National Building Code* (NBC), *International Building Code* (IBC), *Standard Building Code* (SBC), and the *Uniform Building Code* (UBC), referred to in those codes as "special inspections." The specially qualified inspector must "demonstrate competence for inspection of the particular type of construction requiring special inspection." See Section 1.3 above for information on voluntary certification programs for concrete special inspectors. Duties and responsibilities of the special inspector are further outlined as follows:

1. Observe the work for conformance with the approved construction documents.
2. Furnish inspection reports to the building official, the engineer or architect of record, and other designated persons.
3. Submit a final inspection report indicating whether the work was in conformance with the approved construction documents and acceptable workmanship.

The requirement for special inspections by a specially qualified special inspector was long a part of the *Uniform Building Code*; however, it was adopted much later in the NBC and the SBC. With the adoption of the NEHRP recommended earthquake provisions by the IBC, NFPA 5000, NBC and the SBC, the need for special inspections came to the forefront. An integral part of the NEHRP provisions is the requirement for special inspections of the seismic-force resisting systems of buildings in intermediate and high seismic performance or design categories.

By definition, special inspection by a special inspector implies continuous inspection of construction. For concrete construction, special inspection is required during placement of all reinforcing steel, during the taking of samples of concrete used for fabricating strength test cylinders, and during concrete placing operations. The special inspector need not be present during the entire time reinforcing steel is being placed, provided final inspection of the in-place reinforcement is performed prior to concrete placement. Generally, special inspections are not required for certain concrete work when the building official determines that the construction is of a minor nature or that no special hazard to public safety exists. Special inspections are also not required for precast concrete elements manufactured under plant control where the plant has been prequalified by the building official to perform such work without special inspections.

Another "inspection" requirement in the IBC, NFPA 5000, and UBC that was not part of the NBC or the SBC is the concept of "structural observation". Under the UBC, structural observation was required for buildings located in high seismic risk areas (Seismic Zone 3 or 4). Under the IBC, it was required for more important structures assigned to seismic design category D, E or F, or sited in an area where the basic wind speed exceeds 110 miles per hour (3-second gust speed). NFPA 5000 has requirements that are similar to those of the IBC. Under the UBC, the owner is required to retain the engineer or architect in responsible charge of the structural design work or another engineer or architect designated by the engineer or architect responsible for the structural design to perform visual observation of the structural framing system at significant stages of construction and upon completion, for general conformance to the approved plans and specifications. Under the IBC and NFPA 5000, any registered design professional qualified to perform the work can be retained for the purpose of making structural observations. At the completion of the project, and prior to issuance of the certificate of occupancy, the engineer or architect is required to submit a statement in writing to the building official indicating that the site visits have been made and noting any déficiencies that have not been corrected.

With ever-increasing interest in inspection of new building construction in the U.S., especially in high seismic risk areasand high wind areas, the designer will need to review the inspection requirements of the governing general building code, and ascertain the role of the engineer in the inspection of the construction phase.

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Materials, Concrete Quality

CHAPTER 3—MATERIALS

UPDATE FOR THE '05 CODE

New to the '05 Edition of the Code is a change of the term "welded wire fabric" to "welded wire reinforcement" to correct a common misinterpretation that welded wire fabric is not an appropriate alternate to conventional reinforcing bars.

3.1 TESTS OF MATERIALS

Provisions in 3.1.3 (and in 1.3.4) require the inspecting engineer and architect responsible for maintaining availability of complete test records during construction. The provisions of 3.1.3 also require that records of tests of materials and of concrete must be retained by the inspector for two years after completion of a project, or longer if required by the locally adopted building code. Retention of test records for a minimum two-year period after completion of a project is to ensure that records are available should questions arise (subsequent to owner acceptance or issuance of the certificate of occupancy) concerning quality of materials and of concrete, or concerning any violations of the approved plans and specifications or of the building code.

This is required because engineers and architect do not normally inspect concrete, whereas inspectors are typically hired for this purpose. The term "inspector" is defined in 1.3.1. For many portions of the United States, the term "inspector" may be assumed to be the "special inspector", as defined in the legally adopted building codes. When a special inspector is not employed, other arrangements with the code official will be necessary to insure the availability and retention of the test records.

3.2 CEMENTS

Cement used in the work must correspond to that on which the selection of concrete proportions for strength and other properties was based. This may simply mean the same type of cement or it may mean cement from the same source. In the case of a plant that has determined the standard deviation from tests involving cements from several sources, the former would apply. The latter would be the case if the standard deviation of strength tests used in establishing the required target strength was based on one particular type of cement from one particular source.

In ACI 318-02, ASTM C 1157 (Performance Specification for Blended hydraulic Cement) was recognized for the first time. The ASTM C 1157 standard differs from ASTM C 150 and ASTM C 595 in that it does not establish the chemical composition of the different types of cements. However, individual constituents used to manufacture ASTM C 1157 cements must comply with the requirements specified in the standard. The standard also provides for several optional requirements, including one for cement with low reactivity to alkali-reactive aggregates.

Shrinkage-compensating concrete, made using expansive cement conforming to ASTM C 845 (specification for expansive hydraulic cement), minimizes the potential for drying shrinkage cracks. Expansive cement expands slightly during the early hardening period after initial setting. When expansion is restrained by reinforcement, expansive cement concrete can also be used to (1) compensate for volume decrease due to drying shrinkage, (2) induce tensile stress in the reinforcement (post-tensioning), and (3) stabilize the long term dimensions of post-tensioned structures with respect to original design. The major advantage of using expansive cement in concrete is in the control and reduction of drying shrinkage cracks.

The proportions of the concrete mix assume additional importance when expansive cement is used in conjunction with some admixtures. The beneficial effects of using expansive cement may be less or may have the opposite effect when some admixtures are used in concrete containing expansive cement. Section 3.6.8 flags this concern. Trial mixtures should be made with the selected admixtures and other ingredients of expansive cement concrete to observe the effects of the admixtures on the properties of the fresh and the hardened concrete.

Also, when expansive cement concrete is specified, the design professional must consider certain aspects of the design that may be affected. Code sections related to such design considerations include:

- Section 8.2.4 - Effects of forces due to expansion of shrinkage-compensating concrete must be given consideration in addition to all the other effects listed.

- Section 9.2.7 - Structural effects due to expansion of shrinkage-compensating concrete must be included in T which is included in the load combinations of Eq. (9-2).

3.3 AGGREGATES

The nominal maximum aggregate size is limited to (i) one-fifth the narrowest dimension between sides of forms, (ii) one-third the depth of the slab, and (iii) three-quarters the minimum clear spacing between reinforcing bars or prestressing tendons or ducts. The limitations on nominal maximum aggregate size may be waived if the workability and methods of consolidation of the concrete are such that the concrete can be placed without honeycomb or voids. The engineer must decide whether the limitations on maximum size of aggregate may be waived.

3.4 WATER

Over the past numbers of years environmental regulations associated with the disposal of water from concrete production operations have caused larger amounts of non-potable water (i.e., sources not fit for human consumption) to be used as mixing water in hydraulic cement concrete. Use of this water source needs to be limited by the solids content in the water. A new ASTM standard [2.2], which has not as yet been incorporated into ACI 318, provides a test method for this measurement by means of measuring water density.

In addition to limiting the amount of solids in mixing water, maximum concentrations of other materials that impact the quality of concrete must be limited. These include levels of chloride ion, sulfates, and alkalies. Another ASTM standard [2.3], which has also not as yet been incorporated into ACI 318, provides upper limits for these materials, as well as the total solids content in mixing water.

The chief concern over high chloride content is the possible effect of chloride ions on the corrosion of embedded reinforcing steel or prestressing tendons, as well as concrete containing aluminum embedments or which are cast against stay-in-place galvanized metal forms. Limitations placed on the maximum concentration of chloride ion that are contributed by the ingredients including water, aggregates, cement, and admixtures are given in Chapter 4, Table 4.4.1. These limitations that specifically apply to corrosion protection of reinforcement are measured in water soluble chloride ion in concrete, percent by weight of cement. The previously cited ASTM standard limits the chloride ions in ppm (parts per million) and only applies to that contributed by the mixing water.

3.5 STEEL REINFORCEMENT

3.5.2 Welding of Reinforcement

ACI 318-05 references the latest edition of the Structural Welding Code for Reinforcing Steel - ANSI/AWS D1.4-98. All welding of reinforcing bars must be performed in strict compliance with the D1.4 requirements. Recent revisions to D1.4 deserve notice. Most notably, the preheat requirements for A 615 steel bars require consideration if the chemical composition of the bars is not known. See discussion on 12.14.3 in Part 4.

The engineer should especially note the welding restrictions of 21.2.7 for reinforcement in earthquake force resisting structural members in buildings in regions of high seismic risk or in structures assigned to high seismic performance or design categories. Because these structural elements may perform beyond the elastic range of response, under potentially extreme effects of major earthquakes, welding of reinforcing steel, especially welded splices, must be performed in strict adherence with the welding procedures outlined in ANSI/AWS D1.4. These procedures include adequate inspection.

Section R3.5.2 provides guidance on welding to existing reinforcing bars (which lack mill test reports) and on field welding of cold drawn wire and welded wire. Cold drawn wire is used as spiral reinforcement, and wires or welded wire reinforcement may occasionally be field welded. Special attention is necessary when welding cold drawn wire to address possible loss of its yield strength and ductility. Electric resistance welding, as covered by ASTM A 185 and A 497, is an acceptable welding procedure used in the manufacture of welded wire reinforcement. When welded splices are used in lieu of required laps, pull tests of representative samples or other methods should be specified to determine that an acceptable level of specified strength of steel is provided. "Tack" welding (welding of cross bars) of deformed bars or wire reinforcement is not permitted unless authorized by the engineer (see 7.5.4).

The last paragraph of R3.5.2 states that welding of wire is not covered in ANSI/AWS D1.4. Actually, ANSI/AWS D1.4 addresses the welding of all forms of steel reinforcement, but lacks certain critical information for wire or welded wire reinforcemnt (e.g., preheats and electrode selection are not discussed). However, it is recommended that field welding of wire and welded wire reinforcement follow the applicable provisions of ANSI/AWS D1.4, such as certification of welders, inspection procedures, and other applicable welding procedures.

3.5.3 Deformed Reinforcement

Only deformed reinforcement as defined in Chapter 2 may be used for nonprestressed reinforcement, except that plain bars and plain wire may be used for spiral reinforcement. Welded plain wire reinforcement is included under the code definition of deformed reinforcement. Reinforcing bars rolled to ASTM A 615 specifications are the most commonly specified for construction. Rail and axle steels (ASTM A 616 and ASTM A 617, respectively) were deleted from ACI 318-02 and replaced by ASTM A 996 (Specification for Rail-Steel and Axle-Steel Deformed Bars for Concrete Reinforcement). Deformed reinforcement meeting ASTM A 996 is marked with the letter *R* and must meet more restrictive provisions for bend tests than was required by the previous two specification standards that ASTM A996 replaced. Rail steel (ASTM A 996) is not generally available, except in a few areas of the country.

ASTM A 706 covers low-alloy steel deformed bars (Grade 60 only) intended for special applications where welding or bending or both are of importance. Reinforcing bars conforming to A 706 should be specified wherever critical or extensive welding of reinforcement is required, and for use in reinforced concrete structures located in regions of high seismic risk or in structures assigned to high seismic performance or design categories where more bendability and controlled ductility are required. The special provisions of Chapter 21 for seismic design require that reinforcement resisting earthquake-induced flexural and axial forces in frame members and in wall boundary elements forming parts of structures located in regions of high seismic risk or in structures assigned to high seismic performance or design categories comply with ASTM A 706 (see 21.2.5). Grades 40 and 60 ASTM A 615 bars are also permitted in these members if (a) the ratio of actual ultimate tensile strength to the actual tensile yield strength is not less than 1.25, and (b) the actual yield strength based on mill tests does not exceed the specified yield strength by more than 18,000 psi (retests are not permitted to exceed the specified yield strength by more than an additional 3000 psi).

Before specifying A 706 reinforcement, local availability should be investigated. Most rebar producers can make A 706 bars, but generally not in quantities less than one heat of steel for each bar size ordered. A heat of steel varies from 50 to 200 tons, depending on the mill. A 706 in lesser quantities of single bar sizes may not be immediately available from any single producer. Notably, A 706 is being specified more and more for reinforced concrete structures in high seismic risk areas (see Table 1-1 in Part 1). Not only are structural engineers specifying it for use in earthquake-resisting elements of buildings, but also for reinforced concrete bridge structures. Also, A 706 has long been the choice of precast concrete producers because it is easier and more cost effective for welding, especially in the various intricate bearing details for precast elements. This increased usage should impact favorably on the availability of this low-alloy bar.

Section 9.4 permits designs based on a yield strength of reinforcement up to a maximum of 80,000 psi. Currently there is no ASTM specification for a Grade 80 reinforcement. However, deformed reinforcing bars No. 6 through No. 18 with a yield strength of 75,000 psi (Grade 75) are included in the ASTM A 615 specification. "Section 3.5.3.2 requires that the yield strength of deformed bars with a specified yield strength greater than 60,000 psi be taken as the stress corresponding to a strain of 0.35 percent. The 0.35 percent strain limit is to ensure that the elasto-plastic stress-strain curve assumed in 10.2.4 will not result in unconservative values of member strength. Therefore, the designer should be aware that if ASTM A 615, Grade 75 bars are specified, the project specifications need to include a requirement that the yield strength of the bars shall be determined in accordance with Section 9.2.2 of the ASTM A 615 specification." Certified mill test reports should be obtained from the supplier when Grade 75 bars are used. Before specifying Grade 75, local availability should be investigated. The higher yield strength No. 6 through No. 18 bars are intended primarily as column reinforcement. They are used in conjunction with higher strength concrete to reduce the size of columns in high-rise buildings and other applications where high capacity columns are required. Wire used to manufacture both plain and deformed welded wire reinforcement can have a specified yield strength in excess of 60,000 psi. It is permissible to take advantage of the higher yield strength provided the specified yield strength, f_y, used in the design corresponds to the stress at a strain of 0.35 percent.

In recent years manufacturers of reinforcing bars have switched their production entirely to soft metric bars. The physical dimensions (i.e., diameter, and height and spacing of deformations) of the soft-metric bars are no different than the inch-pound bars that were manufactured for many years. The only difference is that the bar size mark that is rolled onto the bar is based on SI metric units. Metric bar sizes and bar marks are based on converting the bar's inch-pound diameter to millimeters and rounding to the nearest millimeter. For example, a No. 4, or 1/2-in. diameter bar, becomes a No. 13 bar since its diameter is 12.7 mm. See Table 2-1 for a complete listing of all 11 ASTM standard reinforcing bar sizes.

ASTM standard specifications A 615, A 706 and A 996 have requirements for bars in both inch-pound and SI metric units; therefore, they have dual designations (e.g., ASTM A 706/A 706M). Each specification provides criteria for one or more grades of steel, which are summarized in Table 2-2.

The minimum required yield strength of the steel used to produce the bars has been changed slightly within ASTM A 615M. The latest edition of the ASTM A 615M bar specifications have a Grade 280, or 280 megapascals (MPa) minimum yield strength, which was previously designated as Grade 300. Soft converting Grade 40 or 40,000 psi yield strength steel will result in a metric yield strength of 275.8 MPa (1,000 psi = 6.895 MPa), which is more closely designated as Grade 280, than the previous Grade 300 designation.

Table 2-1 Inch-Pound and Soft Metric Bar Sizes

Inch-Pound		Metric	
Size No.	Dia. (in.)	Size No.	Dia. (mm)
3	0.375	10	9.5
4	0.500	13	12.7
5	0.625	16	15.9
6	0.750	19	19.1
7	0.875	22	22.2
8	1.000	25	25.4
9	1.128	29	28.7
10	1.270	32	32.3
11	1.410	36	35.8
14	1.693	43	43.0
18	2.257	57	57.3

Table 2-2 ASTM Specifications - Grade and Min. Yield Strength

ASTM Specification	Grade/Minimum Yield Strength	
	Inch-Pound (psi)	Metric (MPa)
A 615 and A 615M	40/40,000 60/60,000 75/75,000	280/280 420/420 520/520
A 996 and A 996M	40/40,000 50/50,000 60/60,000	280/280 350/350 420/420
A 706/A 706M	60/60,000	420/420

When design and construction proceed in accordance with the ACI 318 Code, using customary inch-pound units, the use of soft metric bars will have only a very small effect on the design strength or allowable load-carrying capability of members. For example, where the design strength of a member is a function of the steel's specified yield strength, f_y, the use of soft metric bars increases the strength approximately 1.5% for grade 420 [(420 - 413.7)/413.7].

3.5.3.5 - 3.5.3.6 Welded Plain and Deformed Wire Reinforcement—On occasion, building department plan reviewers have questioned the use of welded wire reinforcement as an alternative to conventional reinforcing bars for structurally reinforced concrete applications. This usually occurs during the construction phase when reinforcing bars shown on the structural drawings are replaced with welded wire reinforcement through a change order. The code officials' concern probably stems from the commonly accepted industry terminology for welded wire reinforcement used as "nonstructural" reinforcement for the control of crack widths for slabs-on-ground.

Wire sizes for welded wire reinforcement range from W1.4 (10 gauge) to W4 (4 gauge). Plain wire is denoted by the letter "W" followed by a number indicating cross-sectional area in hundredths of a square inch. Styles of welded wire reinforcement used to control crack widths in residential and light industrial slabs-on-grade are 6 x 6 W1.4 x W1.4, 6 x 6 W2 x W2, 6 x 6 W2.9 x W2.9 and 6 x 6 W4 x W4. These styles of welded wire reiforcement weigh 0.21 lb, 0.30, 0.42 lb and 0.55 lb per square foot respectively, and are manufactured in rolls, although they are also available in sheets. Smaller wire sizes are not typically used as an alternative to conventional reinforcing bars. Welded wire reinforcement used for structural reinforcement is typically made with a wire size larger than W4. The term "welded wire reinforcement" has replaced the term "welded wire fabric" in this edition of the code to help correct this misinterpretation.

Substitution of welded wire reinforcement for reinforcing bars may be requested for construction or economic considerations. Whatever the reason, both types of reinforcement, either made with welded wire or reinforcing bars are equally recognized and permitted by the code for structural reinforcement. Both welded deformed wire reinforcement and welded plain wire reinforcement are included under the code definition for deformed reinforcement. Welded deformed wire reinforcement utilizes wire deformations plus welded intersections for bond and anchorage. (Deformed wire is denoted by the letter "D" followed by a number indicating cross-sectional area in hundredths of a square inch.) Welded plain wire reiforcement bonds to concrete by positive mechanical anchorage at each wire intersection. This difference in bond and anchorage for plain versus deformed reinforcement is reflected in the development of lap splices provisions of Chapter 12.

3.5.3.7 Coated Reinforcement—Appropriate references to the ASTM specifications for coated reinforcement, A 767 (galvanized) and A 775 (epoxy-coated), are included in the code to reflect increased usage of coated bars. Coated welded wire reinforcement is available with an epoxy coating (ASTM A 884), with wire galvanized before welding (ASTM A 641), and with welded wire galvanized after welding (ASTM A 123). The most common coated bars and welded wire are epoxy-coated reinforcement for corrosion protection. Epoxy-coated reinforcement provides a viable corrosion protection system for reinforced concrete structures. Usage of epoxy-coated reinforcement has become commonplace for many types of reinforced concrete construction such as parking garages (exposed to deicing salts), wastewater treatment plants, marine structures, and other facilities located near coastal areas where the risk of corrosion of reinforcement is higher because of exposure to seawater—particularly if the climate is warm and humid.

Designers specifying epoxy-coated reinforcing bars should clearly outline in the project specifications special hardware and handling methods to minimize damage to the epoxy coating during handling, transporting, and placing coated bars, and placing of concrete.[2.5, 2.6] Special hardware and handling methods include:

1. Using nylon lifting slings, or padded wire rope slings.
2. Using spreader bars for lifting bar bundles, or lifting bundles at the third points with nylon or padded slings. Bundling bands should be made of nylon, or be padded.
3. Storing coated bars on padded or wooden cribbing.
4. Not dragging coated bars over the ground, or over other bars.
5. Minimizing walking on coated bars and dropping tools or other construction materials during or after placing the bars.
6. Using bar supports of an organic material or wire bar supports coated with an organic material such as epoxy or vinyl compatible with concrete.
7. Using epoxy- or plastic-coated tie wire, or nylon-coated tie wire to minimize damage or cutting into the bar coating.
8. Setting up, supporting and moving concrete conveying and placing equipment carefully to minimize damage to the bar coating.

Project specifications should also address field touch-up of the epoxy coating after bar placement. Permissible coating damage and repair are included in the ASTM A 775 and in Ref. 2.5. Reference 2.6 contains suggested project specification provisions for epoxy-coated reinforcing bars.

The designer should be aware that epoxy-coated reinforcement requires increased development and splice lengths for bars in tension (see 12.2.4.3).

3.6 ADMIXTURES

3.6.9 Silica Fume

Silica fume (ASTM C 1240) gets its name because it extracted from the fumes of electric furnaces that produce ferrosilicon or silicon metal. By the time it is collected and prepared as an admixture for concrete it has become a very finely divided solid-microsilica. Silica fume is generally used in concrete for one or more of the following reasons. When used in conjunction with high-range water reducing admixtures, it makes it possible to produce concrete with compressive strengths of 20,000 psi (138 MPa) or higher. It is also used to achieve a very dense cement paste matrix to reduce the permeability of concrete. This provides better corrosion protection to reinforcing steel, particularly when the concrete will be subject to direct or indirect applications of deicing chemicals, such as in bridge decks or in parking garages, respectively.

Mix proportioning, production methods (mixing and handling), and the placing and curing procedures for silica fume concrete require a more concentrated quality control effort than for conventional concretes. It is imperative that the engineer, concrete supplier, and the contractor work as a team to ensure consistently high quality when silica fume concrete is specified.

Note, concrete containing silica fume can be almost black, dark gray, or practically unchanged from the color of cement, depending on the dosage of silica fume. The greatest differences in color will occur in concretes made with cements that are light in color. Mix proportions may also affect variations in color. If color difference is a problem (architectural concrete), the darkest brand of cement available should be used, and different trial mixtures should be tried during the mix design process.

CHAPTER 4—DURABILITY REQUIREMENTS

GENERAL CONSIDERATIONS

The special exposure conditions addressed by the code are located exclusively in Chapter 4 - Durability Requirements, to emphasize the importance of special exposures on concrete durability. The code provisions for concrete proportioning and strength evaluation are located in Chapter 5 - Concrete Quality, Mixing, and Placing. As stated in 5.1.1, selection of concrete proportions must be established to provide for both (a) resistance to special exposures as required by Chapter 4, and (b) conformance with the strength requirements of Chapter 5.

Resistance to special exposures is addressed in 4.2 - Freeze-thaw exposure, 4.3 - Sulfate exposure, and 4.4 - Corrosion protection of reinforcement. Conformance with strength test requirements is addressed in 5.6 - Evaluation and acceptance of concrete. Depending on design and exposure requirements, the lower of the water-cementitious materials ratios required for the structural design requirements and for the concrete exposure conditions must be specified (see Example 2.1)

Unacceptable deterioration of concrete structures in many areas due to severe exposure to freezing and thawing, to deicing salts used for snow and ice removal, to sulfate in soil and water, and to chloride exposure have warranted a stronger code emphasis on the special exposure requirements. Chapter 4 directs special attention to the need for considering concrete durability, in addition to concrete strength.

In the context of the code, durability refers to the ability of concrete to resist deterioration from the environment or the service in which it is placed. Properly designed and constructed concrete should serve its intended function without significant distress throughout its service life. The code, however, does not include provisions for especially severe exposures such as to acids or high temperatures, nor is it concerned with aesthetic considerations such as surface finishes. Items like these, which are beyond the scope of the code, must be covered specifically in the project specifications. Concrete ingredients and proportions must be selected to meet the minimum requirements stated in the code and the additional requirements of the construction documents.

In addition to the proper selection of cement, adequate air entrainment, maximum water-cementitious materials ratio, and limiting chloride ion content of the materials, other requirements essential for durable concrete exposed to adverse environments include: low slump, adequate consolidation, uniformity, adequate cover of reinforcement, and sufficient moist curing to develop the potential properties of the concrete.

4.1 WATER-CEMENTITIOUS MATERIALS RATIO

The traditional "water-cement" ratio was renamed "water-cementitious materials ratio" starting with the 1989 edition of code in recognition of the other cementitious materials permitted by the code to satisfy the code limitations on w/c for concrete durability. The notation, "w/c", is commonly used for "water-cementitious materials" in the term w/c ratio. The definition for "cementitious materials" (see Chapter 2) permits the use of cementitious materials other than portland cement and blended hydraulic cements to satisfy the code's w/c limitation. For the calculation of w/c ratios, as limited by the code, the "cementitious materials" may include:

- portland cement (ASTM C 150)
- blended hydraulic cement (ASTM C 595 and ASTM C 1157)
- expansive hydraulic cement (ASTM C 845)

either by themselves, or in combination with:

- fly ash (ASTM C 618)
- raw or calcinated natural pozzolans (ASTM C 618)
- ground granulated blast-furnace slag (ASTM C 989)
- silica fume (ASTM C 1240)

4.2 FREEZING AND THAWING EXPOSURES

For concrete that will be exposed to freezing and thawing while moist or to deicer salts, air-entrained concrete must be specified with minimum air contents for severe and moderate exposure, as set forth in Table 4.2.1. Project specifications should allow the air content of the delivered concrete to be within (-1.5) and (+1.5) percentage points of Table 4.2.1 target values. Severe exposure is a cold climate where the concrete may be exposed almost continuously to wet freeze-thaw conditions or where deicing salts are used. Examples include pavements, bridge decks, sidewalks, and parking garages. For severe exposure conditions, the code also imposes a maximum limit on the w/c ratio and minimum f'_c (See 4.4.2). A moderate exposure is a cold climate where concrete may be exposed to freezing, but will only occasionally be exposed to moisture prior to freezing, and where no deicing salts are used. Examples are certain exterior walls, beams, girders and slabs not in direct contact with soil.

Intentionally entraining air in concrete significantly improves the resistance of hardened concrete to freezing when exposed to water and deicing salts. Concrete that is dry or contains only a small amount of moisture is essentially not affected by even a large number of cycles of freezing and thawing. Sulfate resistance is also improved by air entrainment.

The entrainment of air in concrete can be accomplished by adding an air-entraining admixture at the mixer, by using an air-entraining cement, or by a combination of both. Air-entraining admixtures, added at the mixer, must conform to ASTM C 260 (3.6.4); air-entraining cements must comply with the specifications in ASTM C 150 and C 595 (3.2.1). Air-entraining cements are sometimes difficult to obtain; and their use has been decreasing as the popularity of air-entraining admixtures has increased. ASTM C 94, Standard Specifications for Ready Mixed Concrete, which is adopted by reference in the ACI code (5.8.2), requires that air content tests be conducted. The frequency of these tests is the same as required for strength evaluation. Samples of concrete must be obtained in accordance with ASTM C 172 and tested in accordance with ASTM C 173 or C 231.

Normal weight concrete that will be exposed to freeze-thaw conditions while wet and exposed to deicing salts must be proportioned so that both a maximum w/c ratio and a minimum compressive strength are provided (Table 4.2.2). Requiring both criteria helps to ensure that the desired durability will actually be obtained in the field. Generally, the required average concrete strength, f'_{cr}, used to develop the mix design will be 500 to 700 psi higher than the specified compressive strength, f'_c. It is also more difficult to accurately determine the w/c ratio of concrete during production then controlling compressive strength. Thus, when selecting an f'_c, it should be reasonable consistent with the w/c ratio required for durability. Using this approach, minimum strengths required for durability provide an effective backup quality control check to the w/c ratio limitation which is more essential to durability.

A minimum strength only is specified for lightweight aggregate concrete, due to the variable absorption characteristics of lightweight aggregates, which makes the calculated w/c ratios meaningless.

Concrete used in water-retaining structures or exposed to severe exposure conditions as described above must be virtually impermeable or watertight. Low permeability not only improves freezing and thawing resistance, especially in the presence of deicing salts, but also improves the resistance of concrete to chloride ion penetration. Concrete that is intended to have low permeability to water must be proportioned so that the specified w/c ratio does not exceed 0.50. If concrete is to be exposed to freezing and thawing in a moist condition, the specified w/c ratio must be no more than 0.45. Also, for corrosion protection of reinforcement in concrete exposed to deicing salts (wet freeze-thaw conditions), and in concrete exposed to seawater (including seawater spray), the concrete must be proportioned so that the specified w/c ratio does not exceed 0.40.

For the above exposure conditions, the corresponding minimum concrete strengths indicated in Table 4.2.2 must also be satisfied for normal- and lightweight, aggregate concretes. Design Example 2.1 illustrates mix proportioning to satisfy both a w/c ratio and a strength requirement for concrete durability.

4.2.3 Concrete Exposed to Deicing Chemicals

Table 4.2.3 limits the type and amount of portland cement replacement permitted in concrete exposed to deicing salts. The amount of fly ash or other pozzolan, or both, is limited to 25 percent of the total weight of cementitious materials. Slag and silica fume are similarly limited to 50 percent and 10 percent, respectively, of the total weight. If fly ash (or other pozzolan) plus slag and silica fume are used as partial cement replacement, the total weight of the combined replacement materials cannot exceed 50 percent of the total weight of cementitious materials, with the maximum percentage of each type of replacement not to exceed the individual percentage limitations. If slag is excluded from the cement replacement combination, the total weight of the combined replacement cannot exceed 35 percent, with the individual percentages of each also not to be exceeded.

As an example: If a reinforced concrete element is to be exposed to deicing salts, Table 4.2.2 limits the w/c ratio to 0.40. If the mix design requires 280 lb of water to produce an air-entrained concrete mix of a given slump, the total weight of cementitious materials cannot be less than 280/0.40 = 700 lbs. The 700 lbs of "cementitious materials" may be all portland cement or a combination of portland cement and fly ash, pozzolan, slag, or silica fume.

If fly ash is used as portland cement replacement, the maximum amount of fly ash is limited to 0.25 (700) = 175 lbs, maintaining the same w/c = 280/(525+175) = 0.40.

If slag is the total replacement, the maximum is limited to 0.50 (700) = 350 lbs, with w/c = 280/(350+350) = 0.40.

If the cement replacement is a combination of fly ash and slag, the maximum amount of the combination is limited to 0.50 (700) = 350 lbs, with the fly ash portion limited to 0.25 (700) = 175 lbs of the total combination, with w/c = 280/(350+175+175) = 0.40.

If the cement replacement is a combination of fly ash and silica fume (a common practice in high performance concrete), the maximum amount of the combination is limited to 0.35 (700) = 245 lbs, and the silica fume portion limited to 0.10 (700) = 70 lbs, with w/c = 280/(385+245+70) = 0.40.
Obviously, other percentages of cement replacement can be used so long as the combined and individual percentages of Table 4.2.3 are not exceeded.

It should be noted that the portland cement replacement limitations apply only to concrete exposed to the potential damaging effects of deicing chemicals. Research has indicated that fly ash, slag, and silica fume can reduce concrete permeability and chloride ingress by providing a more dense and impermeable cement paste. As to the use of fly ash and other pozzolans, and especially silica fume, it is noteworthy that these cement replacement admixtures are commonly used in high performance concrete (HPC) to decrease permeability and increase strength.

4.3 SULFATE EXPOSURES

Sulfate attack of concrete can occur when it is exposed to soil, seawater, or groundwater having a high sulfate content. Measures to reduce sulfate attack include the use of sulfate-resistant cement. The susceptibility to sulfate attack is greater for concrete exposed to moisture, such as in foundations and slabs on ground, and in structures directly exposed to seawater. For concrete that will be exposed to sulfate attack from soil or water, sulfate-resisting cement must be specified. Table 4.3.1 lists the appropriate types of sulfate-resisting cements and maximum water-cementitious materials ratios and corresponding minimum concrete strengths for various exposure conditions. Degree of exposure is based on the amount of water-soluble sulfate concentration in soil or on the amount of sulfate concentration in water. Note that Table 4.3.1 lists seawater under "moderate exposure," even though it generally contains more than 1500 ppm of sulfate concentration. The reason is that the presence of chlorides in seawater inhibits the expansive reaction that is characteristic of sulfate attack.[2.1]

In selecting a cement type for sulfate resistance, the principal consideration is the tricalcium aluminate (C_3A) content. Cements with low percentages of C_3A are especially resistant to soils and waters containing sulfates. Where precaution against moderate sulfate attack is important, as in drainage structures where sulfate concentrations in groundwater are higher than normal, but not necessarily severe (0.10 - 0.20 percent), Type II portland cement (maximum C_3A content of eight percent per ASTM C 150) must be specified.

Type V portland cement must be specified for concrete exposed to severe sulfate attack—principally where soils or groundwaters have a high sulfate content. The high sulfate resistance of Type V cement is attributed to its low tricalcium aluminate content (maximum C_3A content of five percent).

Certain blended cements (C 595) also provide sulfate resistance. Other types of cement produced with low C_3A contents are usable in cases of moderate to severe sulfate exposure. Sulfate resistance also increases with air-entrainment and increasing cement contents (decreasing water-cementitious materials ratios).

Before specifying a sulfate resisting cement, its availability should be checked. Type II cement is usually available, especially in areas where resistance to moderate sulfate attack is needed. Type V cement is available only in particular areas where it is needed to resist severe and very severe sulfate environments. Blended cements may not be available in many areas.

4.4 CORROSION PROTECTION OF REINFORCEMENT

Chlorides can be introduced into concrete through its ingredients: mixing water, aggregates, cement, and admixtures, or through exposure to deicing salts, seawater, or salt-laden air in coastal environments. The chloride ion content limitations of Table 4.4.1 are to be applied to the chlorides contributed by the concrete ingredients, not to chlorides from the environment surrounding the concrete (chloride ion ingress). Chloride ion limits are the responsibility of the concrete production facility which must ensure that the ingredients used in the production of concrete (cement, water, aggregate, and admixtures) result in concrete with chloride ion contents within the limits given for different exposure conditions. When testing is performed to determine chloride ion content of the individual ingredients, or samples of the hardened concrete, test procedures must conform to ASTM C 1218, as indicated in 4.4.1. In addition to a high chloride content, oxygen and moisture must be present to induce the corrosion process. The availability of oxygen and moisture adjacent to embedded steel will vary with the in-service exposure condition, which varies among structures, and between different parts of the same structure.

If significant amounts of chlorides may be introduced into the hardened concrete from the concrete materials to be used, the individual concrete ingredients, including water, aggregates, cement, and any admixtures, must be tested to ensure that the total chloride ion concentration contributed from the ingredients does not exceed the limits of Table 4.4.1. These limits have been established to provide a threshold level to avoid corrosion of the embedded reinforcement prior to service exposure. Chloride limits for corrosion protection also depend upon the type of construction and the environment to which the concrete is exposed during its service life, as indicated in Table 4.4.1.

Chlorides are present in variable amounts in all of the ingredients of concrete. Both water soluble and insoluble chlorides exist; however, only water soluble chlorides induce corrosion. Tests are available for determining either the water soluble chloride content or the total (soluble plus insoluble) chloride content. The test for soluble chloride is more time-consuming and difficult to control, and is therefore more expensive than the test for total chloride. An initial evaluation of chloride content may be obtained by testing the individual concrete ingredients for total (soluble plus insoluble) chloride content. If the total chloride ion content is less than that permitted by Table 4.4.1, water-soluble chloride need not be determined. If the total chloride content exceeds the permitted value, testing of samples of the hardened concrete for water-soluble chloride content will need to be performed for direct comparison with Table 4.4.1 values. Some of the soluble chlorides in the ingredients will react with the cement during hydration and become insoluble, further reducing the soluble chloride ion content, the corrosion-inducing culprit. Of the total chloride ion content in hardened concrete, only about 50 to 85 percent is water soluble; the rest is insoluble. Note that hardened concrete should be at least 28 days of age before sampling.

Chlorides are among the more abundant materials on earth, and are present in variable amounts in all of the ingredients of concrete. Potentially high chloride-inducing materials and conditions include: use of seawater as mixing water or as washwater for aggregates, since seawater contains significant amounts of sulfates and chlorides; use of marine-dredged aggregates, since such aggregates often contain salt from the seawater; use of aggregates that have been contaminated by salt-laden air in coastal areas; use of admixtures containing chloride, such as calcium chloride; and use of deicing salts where salts may be tracked onto parking structures by vehicles. The engineer needs to be cognizant of the potential hazard of chlorides to concrete in marine environments or other exposures to soluble salts. Research has shown that the threshold value for a water soluble chloride content of concrete necessary for corrosion of embedded steel can be as low as 0.15 percent by weight of cement. When chloride content is above this threshold value, corrosion is likely if moisture and oxygen are readily available. If chloride content is below the threshold value, the risk of corrosion is low.

Depending on the type of construction and the environment to which it is exposed during its service life, and the amount and extent of protection provided to limit chloride ion ingress, the chloride level in concrete may increase with age and exposure. Protection against chloride ion ingress from the environment is addressed in 4.4.2, with reference to Table 4.2.2. A maximum water-cementitious materials ratio of 0.40 and a minimum strength of 5000 psi must be provided for corrosion protection of "reinforcement in concrete exposed to chlorides from deicing chemicals, brackish water, seawater or spray from these sources." Resistance to corrosion of embedded steel is also improved with an increase in the thickness of concrete cover. Section R7.7.5 recommends a minimum concrete cover of 2 in. for cast-in-place walls and slabs, and 2-1/2 in. for other members, where concrete will be exposed to external sources of chlorides in service. For plant-produced precast members, the corresponding recommended minimum concrete covers are 1-1/2 in. and 2 in., respectively.

Other methods of reducing environmentally caused corrosion include the use of epoxy-coated reinforcing steel[2.4, 2.5, 2.6], corrosion-inhibiting admixtures, surface treatments, and cathodic protection. Epoxy coating of reinforcement prevents chloride ions from reaching the steel. Corrosion-inhibiting admixtures attempt to chemically arrest the corrosive reaction. Surface treatments attempt to stop or reduce chloride ion penetration at the exposed concrete surface. Cathodic protection methods reverse the corrosion current flow through the concrete and reinforcing steel. It should be noted that, depending on the potential severity of the chloride exposure, and the type and importance of the construction, more than one of the above methods may be combined to provide "added" protection. For example, in prestressed parking deck slabs in cold climates where deicing salts are used for snow and ice removal, all conventional reinforcement and the post-tensioning tendons may be epoxy-coated, with the entire tendon system including the anchorages encapsulated in a watertight protective system especially manufactured for aggressive environments. In addition, special high performance (impermeable) concrete may be used, with the entire deck surface covered with a multi-layer membrane surface treatment. Such extreme protective measures may be cost-effective, considering the alternative. Performance tests for chloride permeability of concrete mixtures may also be used to assure corrosion resistance. ASTM C 1202, which was introduced starting with the 2002 edition of the code, provides a test method for an electrical indication of concrete's ability to resist chloride ion penetration. It is based on AASHTO T 277-83, which was previously referenced in the code.

CHAPTER 5—CONCRETE QUALITY, MIXING, AND PLACING

UPDATE FOR THE '05 CODE

New to the '05 Edition of the Code is the change of the term '"standard deviation" to "sample standard deviation" because from a scientific point of view, the Code provides a check of a sample standard deviation, and not a true (absolute) standard deviation for each concrete mixture. The new notation of sample standard deviation is "s_s".

5.1.1 Concrete Proportions for Strength

Concrete mix designs are proportioned for strength based on probabilistic concepts that are intended to ensure that adequate strength will be developed in the concrete. It is emphasized in 5.1.1 that the required average compressive strength, f'_{cr}, of concrete produced must exceed the larger of the value of f'_c specified for the structural design requirements and the minimum strength required for the special exposure conditions set forth in Chapter 4. Concrete proportioned by the code's probabilistic approach may produce strength tests which fall below the specified compressive strength, f'_c. Section 5.1.1 introduces this concept by noting that it is the code's intent to "minimize frequency of strength below f'_c." If a concrete strength test falls below f'_c, the acceptability of this lower strength concrete is provided for in Section 5.6.3.3.

A minimum 2500 psi specified compressive strength, f'_c›, is required by Section 5.1.1 of the code. This makes the code consistent with minimum provisions that are contained in several legacy model building codes, and the *International Building Code* (IBC).

5.1.3 Test Age for Strength of Concrete

Section 5.1.3 permits f'_c to be based on tests at ages other than the customary 28 days. If other than 28 days, the test age for f'_c must be indicated on the design drawings or in the specifications. Higher strength concretes, exceeding 6000 psi compression strength, are often used in tall buildings can justifiably have test ages longer than the customary 28 days. For example, in high-rise structures requiring high-strength concrete, the process of construction is such that the columns of the lower floors are not fully loaded until a year or more after commencement of construction. For this reason, specified compressive strengths, f'_c, based on 56- or 90-day test results are commonly specified.

5.2 SELECTION OF CONCRETE PROPORTIONS

Recommendations for proportioning concrete mixtures are given in detail in *Design and Control of Concrete Mixtures.*[2.1] Recommendations for selecting proportions for concrete are also given in detail in "Standard

Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete" (ACI 211.1)[2.7] and "Standard Practice for Selecting Proportions for Structural Lightweight Concrete" (ACI 211.2).[2.8]

The use of field experience or laboratory trial batches (see 5.3) is the preferred method for selecting concrete mixture proportions. When no prior experience or trial batch data are available, permission may be granted by the registered design professional to base concrete proportions on "other experience or information" as prescribed in 5.4.

5.3 PROPORTIONING ON THE BASIS OF FIELD EXPERIENCE AND/OR TRIAL MIXTURES

5.3.1 Sample Standard Deviation

For establishing concrete mixture proportions, emphasis is placed on the use of laboratory trial batches or field experience as the basis for selecting the required water-cementitious materials ratio. The code emphasizes a statistical approach to establishing the required average compressive strength of concrete, f'_{cr}, or "target strength" required to ensure attainment of the specified compressive strength, f'_c. If an applicable sample standard deviation, s_s, from strength tests of the concrete is known, the target strength level for which the concrete must be proportioned is established. Otherwise, the proportions must be selected to produce a conservative target strength sufficient to allow for a high degree of variability in strength test results. For background information on statistics as it relates to concrete, see "Recommended Practice for Evaluation of Compression Test Results of Concrete"[2.9] and "Statistical Product Control."[2.10]

Concrete used in background tests to determine sample standard deviation is considered to be "similar" to that specified, if it was made with the same general types of ingredients, under no more restrictive conditions of control over material quality and production methods than are specified to exist on the proposed work, and if its specified strength does not deviate by more than 1000 psi from that f'_c specified. A change in the type of concrete or a significant increase in the strength level may increase the sample standard deviation. Such a situation might occur with a change in the type of aggregate; i.e., from natural aggregate to lightweight aggregate or vice versa, or with a change from non-air-entrained concrete to air-entrained concrete. Also, there may be an increase in sample standard deviation when the average strength level is raised by a significant amount, although the increment in sample standard deviation should be somewhat less than directly proportional to the strength increase. When there is reasonable doubt as to its reliability, any estimated sample standard deviation used to calculate the required average strength should always be on the conservative (high) side.

Sample standard deviations are normally established by at least 30 consecutive tests on representative materials. If less than 30, but at least 15 tests are available, Section 5.3.1.2 provides for a proportional increase in the calculated sample standard deviation as the number of consecutive tests decrease from 29 to 15.

Statistical methods provide valuable tools for assessing the results of strength tests. It is important that concrete technicians understand the basic language of statistics and be capable of effectively utilizing the tool to evaluate strength test results.

Figure 2-1 illustrates several fundamental statistical concepts. Data points represent six (6) strength test results* from consecutive tests on a given class of concrete. The horizontal line represents the average of tests that is designated $\bar{X}$. The average is computed by adding all test values and dividing by the number of values summed; i.e., in Fig. 2-1:

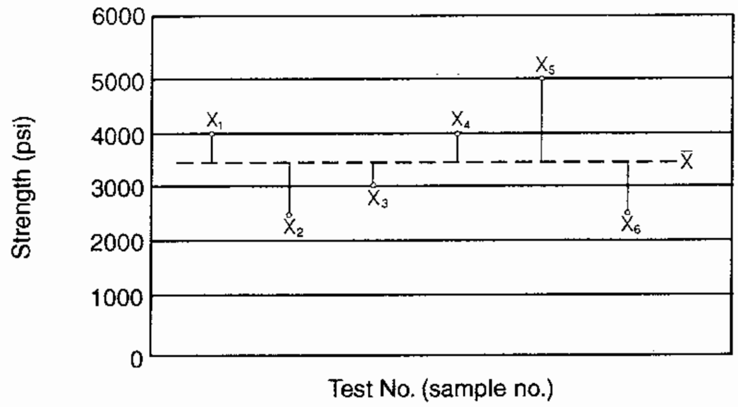

Figure 2-1 Illustration of Statistical Terms

$$\overline{X} = (4000 + 2500 + 3000 + 4000 + 5000 + 2500)/6 = 3500 \text{ psi}$$

The average $\overline{X}$ gives an indication of the overall strength level of the concrete tested.

It would also be informative to have a single number which would represent the variability of the data about the average. The up and down deviations from the average (3500 psi) are given as vertical lines in Fig. 2-1. If one were to accumulate the total length of the vertical lines without regard to whether they are up or down, and divide that total length by the number of tests, the result would be the average length, or the average distance from the average strength:

$$(500 + 1000 + 500 + 500 + 1500 + 1000)/6 = 833 \text{ psi}$$

This is one measure of variability. If concrete test results were quite variable, the vertical lines would be long. On the other hand, if the test results were close, the lines would be short.

In order to emphasize the impact of a few very high or very low test values, statisticians recommend the use of the square of the vertical line lengths. The square root of the sum of the squared lengths divided by one less than the number of tests (some texts use the number of tests) is known as the standard deviation. This measure of variability is commonly designated by the letter s_S. Mathematically, s_S is expressed as:

$$s_S = \sqrt{\frac{\Sigma\left(X - \overline{X}\right)^2}{n - 1}}$$

where

s_S = standard deviation, psi
Σ indicates summation
X = an individual strength test result, psi
$\overline{X}$ = average strength, psi
n = number of tests

For example, for the data in Fig. 2-1, the sample standard deviation would be:

$$s_S = \sqrt{\frac{\left(X_1 - \overline{X}\right)^2 + \left(X_2 - \overline{X}\right)^2 + \left(X_3 - \overline{X}\right)^2 + \left(X_4 - \overline{X}\right)^2 + \left(X_5 - \overline{X}\right)^2 + \left(X_6 - \overline{X}\right)^2}{6 - 1}}$$

* A strength test result is the average of the strengths of two cylinders made from the same batch of concrete and tested at the same time.

which is calculated below.

				Deviation $(X - \overline{X})$ (length of vertical lines)	$(X - \overline{X})^2$ (length squared)
4000	-	3500	=	+ 500	+ 250,000
2500	-	3500	=	- 1000	+ 1,000,000
3000	-	3500	=	- 500	+ 250,000
4000	-	3500	=	+ 500	+ 250,000
5000	-	3500	=	+ 1500	+ 2,250,000
2500	-	3500	=	- 1000	+ 1,000,000
				Total	+ 5,000,000

$$s_S = \sqrt{\frac{5,000,000}{5}} = 1,000 \text{ psi (a very large value)}$$

For concrete strengths in the range of 3000 to 4000 psi, the expected sample standard deviation, representing different levels of quality control, will range as follows:

Sample Standard Deviation	Representing
300 to 400 psi	Excellent Quality Control
400 to 500 psi	Good
500 to 600 psi	Fair
> 600 psi	Poor Quality Control

For the very-high-strength, so called high-performance concrete (HPC), with strengths in excess of 10,000 psi, the expected sample standard deviation will range as follows:

300 to 500 psi	Excellent Quality Control
500 to 700 psi	Good
> 700 psi	Poor Quality Control

Obviously, it would be time consuming to actually calculate s_S in the manner described above. Most hand-held scientific calculators are programmed to calculate sample standard deviation directly. The appropriate mathematical equations are programmed into the calculator with the user simply entering the statistical data (test values), then pressing the appropriate function key to obtain sample standard deviation directly. Example 2.2 illustrates a typical statistical evaluation of strength test results.

The coefficient of variation, V, is simply the standard deviation expressed as a percentage of the average value. The mathematical formula is:

$$V = \frac{s_S}{\overline{X}} \times 100\%$$

For the test results of Fig. 2-1:

$$V = \frac{1000}{3500} \times 100 = 29\%$$

Standard deviation may be computed either from a single group of successive tests of a given class of concrete or from two groups of such tests. In the latter case, a <u>statistical average</u> value of standard deviation is to be used, calculated by usual statistical methods as follows:

$$s_{s3} = \sqrt{\frac{(n_1 - 1)(s_{s1})^2 + (n_2 - 1)(s_{s2})^2}{n_{total} - 2}}$$

where

n_1 = number of samples in group 1

n_2 = number of samples in group 2

$n_{total} = n_1 + n_2$

s_{s1} or s_{s2} is calculated as follows:

$$s_s = \sqrt{\frac{(X_1 - \overline{X})^2 + (X_2 - \overline{X})^2 + \ldots + (X_n - \overline{X})^2}{n - 1}}$$

For ease of computation,

$$s_s = \sqrt{\frac{X_1^2 + X_2^2 + X_3^2 + \ldots + X_n^2 - n\overline{X}^2}{n - 1}}$$

or

$$s_s = \sqrt{\frac{\left(X_1^2 + X_2^2 + X_3^2 + \ldots + X_n^2\right) - \frac{(X_1 + X_2 + X_3 + \ldots + X_n)^2}{n}}{n - 1}}$$

where $X_1, X_2, X_3, \ldots X_n$ are the individual strength test results and n is the total number of strength tests.

5.3.2 Required Average Strength

Where the concrete production facility has a record based on at least 30 consecutive strength tests representing materials and conditions similar to those expected (or a record based on 15 to 29 consecutive tests with the calculated sample standard deviation modified by the applicable factor from Table 5.3.1.2), the strength used as the basis for selecting concrete proportions for specified compressive strengths, f'_c, equal to or greater than 5000 psi
must be the larger of:

$f'_{cr} = f'_c + 1.34s_s$ *Table 5.3.2.1, (5-1)*

and $f'_{cr} = f'_c + 2.33s_s - 500$ *Table 5.3.2.1, (5-2)*

For specified compressive strengths, f'_c over 5000, the strength used as the basis for selecting concrete proportions must be the larger of:

$f'_{cr} = f'_c + 1.34s_s$ *Table 5.3.2.1, (5-1)*

and $f'_{cr} = 0.90f'_c + 2.33s_s$ *Table 5.3.2.1, (5-3)*

If the sample standard deviation is unknown, the required average strength f'_{cr} used as the basis for selecting concrete proportions must be determined from Table 5.3.2.2:

For f'_c	less than 3000 psi	$f'_{cr} = f'_c + 1000$ psi
	between 3000 and 5000 psi	$f'_{cr} = f'_c + 1200$ psi
	greater than 5000 psi	$f'_{cr} = f'_c + 1.10f'_c + 700$ psi

Formulas for calculating the required target strengths are based on the following criteria:

1. A probability of 1 in 100 that the average of 3 consecutive strength tests will be below the specified strength, f'_c: $f'_{cr} = f'_c + 1.34s_s$, and

2. A probability of 1 in 100 that an individual strength test will be more than 500 psi below the specified strength f'_c, : $f'_{cr} = f'_c + 2.33s_s - 500$ (for concrete strengths not over 5000 psi), and

3. A probability of 1 in 100 that an individual strength test will be more than $0.90f'_c$ below the specified strength f'_c (for concrete strengths in excess of 5000 psi): $f'_{cr} = 0.90f'_c + 2.33s_s$.

Criterion (1) will produce a higher required target strength than Criterion (2) for low to moderate standard deviations, up to 500 psi. For higher standard deviations, Criterion (2) will govern.

The average strength provisions of Section 5.3.2 are intended to provide an acceptable level of assurance that concrete strengths are satisfactory when viewed on the following basis: (1) the average of strength tests over an appreciable time period (three consecutive tests) is equal to or greater than the specified compressive strength, f'_c; or (2) an individual strength test is not more than 500 psi below (for concrete strengths not over 5000 psi); or (3) an individual strength test is not more than $0.90f'_c$ below (for concrete strengths in excess of 5000 psi).

5.3.3 Documentation of Average Compressive Strength

Mix approval procedures are necessary to ensure that the concrete furnished will actually meet the strength requirements. The steps in a mix approval procedure can be outlined as follows:

1. Determine the expected sample standard deviation from past experience.

   a. This is done by examining a record of 30 consecutive tests made on a similar mix.

   b. If it is difficult to find a similar mix on which 30 consecutive tests have been conducted, the sample standard deviation can be computed from two mixes, if the total number of tests equals or exceeds 30. The sample standard deviations are computed separately and then averaged by the statistical averaging method already described.

2. Use the sample standard deviation to select the appropriate target strength from the larger of Table 5.3.2.1 (5-1), (5-2) and (5-3).

   a. For example, if the sample standard deviation is 450 psi, then overdesign must be by the larger of:

      $1.34\,(450) = 603$ psi
      $2.33\,(450) - 500 = 549$ psi

      Thus, for a 3000 psi specified strength, the average strength used as a basis for selecting concrete mixture proportions must be 3600 psi.

b. Note that if no acceptable test record is available, the average strength must be 1200 psi greater than f'_c (i.e., 4200 psi average for a specified 3000 psi concrete), see Table 5.3.2.2.

3. Furnish data to document that the mix proposed for use will give the average strength needed. This may consist of:

   a. A record of 30 tests of field concrete. This would generally be the same test record that was used to document the sample standard deviation, but it could be a different set of 30 results; or

   b. Laboratory strength data obtained from a series of trial batches.

Where the average strength documentation for strengths over 5000 psi are based on laboratory trail mixtures, it is permitted to increase f'_{cr} calculated in Table 5.3.2.2 to allow for a reduction in strength from laboratory trials to actual concrete production.

Section 5.3.3.2(c) permits tolerances on slump and air content when proportioning by laboratory trial batches. The tolerance limits are stated at maximum permitted values, because most specifications, regardless of form, will permit establishing a maximum value for slump or air content. The wording also makes it clear that these tolerances on slump and air content are to be applied only to laboratory trial batches. Selection of concrete proportions by trial mixtures is illustrated in Example 2.3.

5.4 PROPORTIONING WITHOUT FIELD EXPERIENCE OR TRIAL MIXTURES

When no field or trail mixture data are available, "other experience or information" may be used to select a water-cementitious materials ratio. This mixture proportioning option, however, is permitted only when approved by the project engineer/architect. Note that this option must, of necessity, be conservative, requiring a rather high target overstrength (overdesign) of 1200 psi. If, for example, the specified strength is 3000 psi, the strength used as the basis for selecting concrete mixture proportions (water-cementitious materials ratio) must be based on 4200 psi. In the interest of economy of materials, the use of this option for mix proportioning should be limited to relatively small projects where the added cost of obtaining trial mixture data is not warranted. Note also that this alternative applies only for specified compressive strengths of concrete up to 5000 psi; for higher concrete strengths, proportioning by field experience or trial mixture data is required. The '99 Edition of the code limited the maximum strength proportioned without field experience or trial mixtures to 4000 psi.

5.6 EVALUATION AND ACCEPTANCE OF CONCRETE

5.6.1 Laboratory and Field Technicians

The concrete test procedures prescribed in the code require personnel with specific knowledge and skills. Experience has shown that only properly trained field technicians and laboratory personnel who have been certified under nationally recognized programs can consistently meet the standard of control that is necessary to provide meaningful test results. Section 5.6.1 of the code requires that tests performed on fresh concrete at the job site and procedures required to prepare concrete specimens for strength tests must be performed by a "qualified field testing technician". Commonly performed field tests which will require qualified field testing technicians include; unit weight, slump, air content and temperature; and making and curing test specimens. Field technicians in charge of these duties may be qualified through certification in the ACI Concrete Field Testing Technician – Grade I Certification Program.

Section 5.6.1 also requires that "qualified laboratory technicians" must perform all required laboratory tests. Laboratory technicians performing concrete testing may be qualified by receiving certification in accordance with requirements of ACI Concrete Laboratory Testing Technician, Concrete Strength Testing Technician, or the requirements of ASTM C 1007.

The following discussion on Chapter 5 of code addresses the selection of concrete mixture proportions for strength, based on probabilistic concepts.

5.6.2 Frequency of Testing

Proportioning concrete by the probabilistic basis of the code requires that a statistically acceptable number of concrete strength tests be provided. Requiring that strength tests be performed according to a prescribed minimum frequency provides a statistical basis.

The code minimum frequency criterion for taking samples for strength tests** , based on a per day and a per project criterion (the more stringent governs***) for each class of concrete, is summarized below.

5.6.2.1 Minimum Number of Strength Tests Per Day—This number shall be no less than:

- Once per day, nor less than,
- Once for each 150 cu yds of concrete placed, nor less than,
- Once for each 5000 sq ft of surface area of slabs or walls placed.

5.6.2.2 Minimum Number of Strength Tests Per Project —This number shall not be less than:

- Five strength tests from five (5) randomly selected batches or from each batch if fewer than five batches.

If the total quantity of concrete placed on a project is less than 50 cu yds, 5.6.2.3 permits strength tests to be waived by the building official.

According to the ASTM Standard for making concrete test specimens in the field (ASTM C 31), test cylinders should be 6×12 in., unless required otherwise by the project specifications.

With the increased use of very-high-strength concretes (in excess of 10,000 psi), the standard 6×12 in. cylinder requires very high capacity testing equipment which is not readily available in many testing laboratories. Consequently, most projects that specify very-high-strength concrete specifically permit the use of the smaller 4×8 in. cylinders for strength specimens. The 4×8 in. cylinder requires about one-half the testing capacity of the 6×12 in. specimen. Also, most precast concrete producers use the 4×8 in. cylinders for in-house concrete quality control.

It should be noted that the total number of cylinders cast for a project will normally exceed the code minimum number needed to determine acceptance of concrete strength (two cylinders per strength test). A prudent total number for a project may include additional cylinders for information (7-day tests) or to be field cured to check early strength development for form stripping, plus one or two in reserve, should a low cylinder break occur at the 28-day acceptance test age.

Example 2.4 illustrates the above frequency criteria for a large project (5.6.2.1 controls). Example 2.5 illustrates a smaller project (5.6.2.2 controls).

5.6.3.3 Acceptance of Concrete—The strength level of an individual class of concrete is considered satisfactory if both of the following criteria are met:

1. No single test strength (the average of the strengths of at least two cylinders from a batch) shall be more than 500 psi below the specified compressive strength when f'_c is 5000 psi or less (i.e., less than 2500 psi for a

** Strength test = average of two cylinder strengths (see 5.6.1.4).
*** On a given project, if total volume of concrete is such that frequency of testing required by 5.6.1.1 would provide less than five tests for a given class of concrete, the per project criterion will govern.

specified 3000 psi concrete strength); or is more than 10 percent below f'_c if over 5000 psi.

2. The average of any three consecutive test strengths must equal or exceed the specified compressive strength f'_c.

Examples 2.6 and 2.7 illustrate "acceptable" and "low strength" strength test results, respectively, based on the above code acceptance criteria.

5.6.5 Investigation of Low-Strength Test Results

If the average of three strength test results in a row is below the specified strength, steps must be taken to increase the strength level of the concrete (see 5.6.3.4). If a single strength test result falls more than 500 psi below the specified strength when f'_c is 5000 or less, or is more than 10 percent below f'_c if over 5000 psi, there may be more serious problems, and an investigation is required according to the procedures outlined in 5.6.5 to ensure structural adequacy.

Note that for acceptance of concrete, a single strength test result (one "test") is always the average strength of 2 cylinders broken at the designated test age, usually 28 days. Due to the many potential variables in the production and handling of concrete, concrete acceptance is <u>never</u> based on a single cylinder break. Two major reasons for low strength test results[3.1] are: (1) improper handling and testing of the cylinders – found to contribute to the majority of low strength investigations; and (2) reduced concrete strength due to an error in production, or the addition of too much water to the concrete at the job site. The latter usually occurs because of delays in placement or requests for a higher slump concrete. High air content due to an over-dosage of air entraining admixture at the batch plant has also contributed to low strength.

If low strength is reported, it is imperative that the investigation follows a logical sequence of possible cause and effect. All test reports should be reviewed and results analyzed before any action is taken. The pattern of strength test results should be studied for any clue to the cause. Is there any indication of actual violation of the specifications? Look at the slump, air content, concrete and ambient temperatures, number of days cylinders were left in the field and under what curing conditions, and any reported cylinder defects.

If the deficiency justifies investigation, testing accuracy should be verified first, and then the structural requirements compared with the measured strength. Of special interest in the early investigation should be the handling and testing of the test cylinders. Minor discrepancies in curing cylinders in mild weather will probably not affect strength much, but if major violations occur, large reductions in strength may result. Almost all deficiencies involving handling and testing of cylinders will lower strength test results. A number of simultaneous violations may contribute to significant reductions. Examples include: extra days in the field; curing over 80°F; frozen cylinders; impact during transportation; delay in moist curing at the lab; improper caps; and insufficient care in breaking cylinders.

For in-place concrete investigation, it is essential to know where in the structure the "tested concrete" is located and which batch (truck) the concrete is from. This information should be part of the data recorded at the time the test cylinders were molded. If test results are found deficient, in-place strength testing may be necessary to ascertain compliance with the code and construction documents. If strength is greater than that actually needed, there is little point in investigating the in-place strength. However, if testing procedures conform to the standards and the test results indicate that concrete strength is lower than required for the member in question, further investigation of the in-place concrete may be required (see 5.6.5).

The laboratory should be held responsible for deficiencies in its procedures. Use of qualified lab personnel is essential. Personnel sampling concrete, making test cylinders and operating lab equipment must be qualified by the ACI certification program or equivalent (see 5.6.1).

If core testing should be required, core drilling from the area in question should be performed according to the procedures outlined in ASTM C 42. The testing of cores requires great care in the operation itself and in the interpretation of the results. Detailed procedures are given in ASTM C 42. The following highlights proper core drilling and testing procedures:

1. Wait 14 days (minimum) before core drilling.
2. Drill 3 cores from the questionable area.
3. Drill cores with a diamond bit.
4. Drill core with a diameter of 2-1/2 in. (minimum) or 2 × maximum aggregate size.
5. Avoid any reinforcing steel in the drilled cores.
6. Drill a minimum core length of 1 × core diameter, but preferably 2 × core diameter.
7. If possible, drill completely through member.
8. Allow 2 in. extra length at the core end to be broken out.
9. Use wooden wedges to remove end portions to be broken out.
10. Saw broken ends to plane surfaces.
11. If concrete is dry under service conditions, air dry the cores for 7 days (60 to 70°F, 60% relative humidity). Test the cores dry.
12. If concrete is wet under service conditions, soak the cores in water (73.4 ± 3°F) for 40 hours. Test the cores wet.
13. Cap the core ends with 1/8 in. thick (or less) capping material.
14. Accurately center the core in the testing machine.
15. Correct the strength for length-to-diameter ratio less than 2, as shown below (interpolate between listed values):

Length-to-Diameter Ratio	Strength Correction Factor
1.94 - 2.10	1.00
1.75	0.98
1.50	0.96
1.25	0.93
1.00	0.87

In addition to the procedures contained in ASTM C 42, the Commentary to 5.6.5 cautions that where a water-cooled bit is used to obtain cores, the coring process causes a moisture gradient between the exterior and interior of the core, which will adversely affect the core's compressive strength. Thus, a restriction on the commencement of core testing is imposed to provide a minimum time for the moisture gradient to dissipate.

There were several significant changes to the '02 Edition of the code that affect the storage and testing of drilled cores. The provisions in 5.6.5.3 have been completely revised to require that immediately after drilling, cores must have any surface water removed by wiping and be placed in watertight bags or containers prior to transportation and storage. The cores must be tested no earlier than 48 hours, nor more than 7 days after coring unless approved by the registered design professional. In prior editions, storage conditions and restrictions on when testing could be performed were different for concrete in structures that would be "dry" or "superficially wet" under service conditions.

In evaluating core test results, the fact that core strengths may not equal the strength specified for molded cylinders should not be a cause for concern. Specified compressive strengths, f'_c, allow a large margin for the unknowns of placement and curing conditions in the field as well as for normal variability. For cores actually taken from the structure, the unknowns have already exerted their effect, and the margin of measured strength above required strength can logically be reduced.

Section 5.6.5.4 states that the concrete will be considered structurally adequate if the average strength of three cores is at least 85 percent of f'_c, with no single core strength less than 75 percent of the specified compressive strength. The concrete can be considered acceptable from the standpoint of strength if the core test results for a given location meet these requirements. The structural engineer should examine cases where core strength values fail to meet the above criteria, to determine if there is cause for concern over structural adequacy. If the results of properly made core tests are so low as to leave structural integrity in doubt, further action may be required.

As a last resort, load tests may be required to check the adequacy of structural members which are seriously in doubt. Generally such tests are suited only for flexural members—floors, beams, and the like—but they may sometimes be applied to other members. In any event, load testing is a highly specialized endeavor that should be performed and interpreted only by an engineer fully qualified in the proper techniques. Load testing procedures and criteria for their interpretation are given in code Chapter 20.

In those rare cases where a structural element fails the load test or where structural analysis of unstable members indicates an inadequacy, appropriate corrective measures must be taken. The alternatives, depending on individual circumstances, are:

- Reducing the load rating to a level consistent with the concrete strength actually obtained.
- Augmenting the construction to bring its load-carrying capacity up to original expectations. This might involve adding new structural members or increasing the size of existing members.
- Replacing the unacceptable concrete.

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2.2 ASTM C 1603, "Standard Test Method for Measurement of Solids in Water", American Society for Testing and Materials, West Conshohocken, PA, 2004.

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2.4 ASTM D 3963-99, "Standard Specification for Fabrication and Jobsite Handling of Epoxy-Coated Reinforcing Steel Bars", American Society for Testing and Materials, West Conshohocken, PA, 1999.

2.5 ASTM A 775-04, "Standard Specification for Epoxy-Coated Steel Reinforcing Bars", American Society for Testing and Materials, West Conshohocken, PA, 2004.

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2.7 ACI Committee 211.1, "Standard Practice for Selecting Proportions for Normal, Heavyweight and Mass Concrete (ACI 211.1-91 (Reapproved 2002)," American Concrete Institute, Farmington Hills, MI, 2002.

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2.9 ACI Committee 214, "Recommended Practice for Evaluation of Compression Test Results of Concrete (ACI 214-77, Reapproved 1997)," American Concrete Institute, Farmington Hills, MI, 1997.

2.10 "Guideline Manual to Quality Assurance / Quality Control" NRMCA Publication 190, National Ready Mixed Concrete Association, Silver Springs, MD, 1999.

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Example 2.1—Selection of Water-Cementitious Materials Ratio for Strength and Durability

Concrete is required for a loading dock slab that will be exposed to moisture in a severe freeze-thaw climate, but not subject to deicers. A specified compressive strength f'_c of 3000 psi is used for structural design. Type I cement with 3/4-in. maximum size normal weight aggregate is specified.

Calculations and Discussion	Code Reference
1. Determine the required minimum strength and maximum w/c ratio for the proposed concrete work to satisfy both design strength and exposure requirements.	*5.2.1*
For concrete exposed to freezing and thawing in a moist condition, Table 4.2.2 requires a maximum water-cementitious materials ratio of 0.45, and a minimum strength f'_c of 4500 psi.	*4.2.2*
Since the required strength for the exposure conditions is greater than the required strength for structural design (f'_c = 3000 psi), the strength for the exposure requirements (f'_c = 4500 psi) governs.	
2. Select a w/c ratio to satisfy the governing required strength, f'_c = 4500 psi.	
Concrete exposed to freezing and thawing must be air-entrained, with air content indicated in Table 4.2.1. For concrete in a cold climate and exposed to wet freeze-thaw conditions, a target air content of 6% is required for a 3/4-in. maximum size aggregate.	*4.2.1*
Selection of water-cementitious materials ratio for required strength should be based on trial mixtures or field data made with actual job materials, to determine the relationship between w/c ratio and strength.	*5.3*
Assume that the strength test data of Example 2.2, with an established sample standard deviation of 353 psi, represent materials and conditions similar to those expected for the proposed concrete work: a. normal weight, air-entrained concrete b. specified strength (4000 psi) within 1000 psi of that required for the proposed work (4500 psi) c. 30 strength test results.	*5.3.1.1*
For a sample standard deviation of 353 psi, the required average compressive strength f'_{cr} to be used as the basis for selection of concrete proportions must be the larger of	*5.3.2*
$f'_{cr} = f'_c + 1.34s_s = 4500 + 1.34\,(353) = 5000$ psi, or	*Eq. 5-1*
$f'_{cr} = f'_c + 2.33s_s - 500 = 4500 + 2.33\,(353) - 500 = 4800$ psi	*Eq. 5-2*
Therefore, f'_{cr} = 5000 psi.	

Note: The average strength required for the mix design should equal the specified strength plus an allowance to account for variations in materials; variations in methods of mixing, transporting, and placing the concrete; and variations in making, curing, and testing concrete cylinder specimens. For this example, with a sample standard deviation of 353 psi, an allowance of 500 psi for all those variations is made.

Typical trial mixture or field data strength curves are given in Ref. 2.1. Using the field data strength curve, Fig. 9-2 of Ref. 2.1, reproduced in Fig. 2-2, the required water-cementitious materials ratio (w/c) approximately equals 0.38 for an f'_{cr} of 5000 psi. (Use of the typical data curve of Fig. 2-2 is for illustration purposes only; a w/c versus required strength curve that is reflective of local materials and conditions should be used in an actual design situation.)

Since the required w/c ratio of 0.38 for the 4500 psi specified strength is less than the 0.45 required by Table 4.2.2, the 0.38 value must be used to establish the mixture proportions. Note that the specified strength, f'_c = 4500 psi, is the strength that is expected to be equaled or exceeded by the average of any set of three consecutive strength tests, with no individual test more than 500 psi below the specified 4500 psi strength. *5.6.3.3*

As a follow up to this example, the test records of Example 2.2 could probably be used (by the concrete producer) to demonstrate that the concrete mix for which the records were generated will produce the required average strength f'_{cr} of the concrete work for this project. For the purpose of documenting the average strength potential of the concrete mix, the concrete producer need only select 10 consecutive tests from the total of 30 tests that represent a higher average than the required average of 5000 psi. Realistically, the average of the total 30 test results (4835 psi) is close enough to qualify the same concrete mix for the proposed work. *5.3.3*

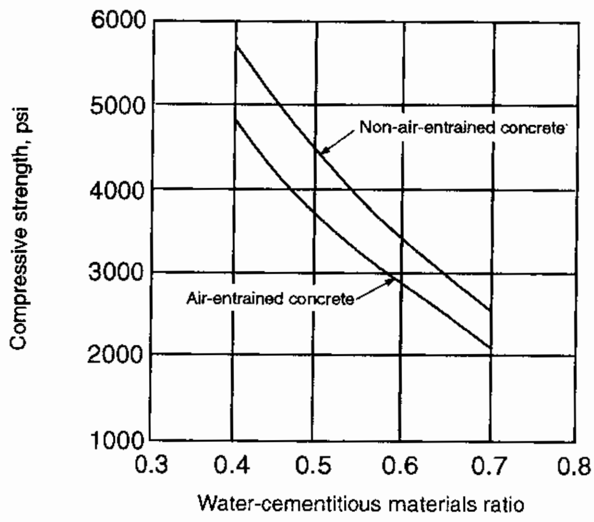


Figure 2-2 Typical Trial Mixture or Field Data Strength Curves

Example 2.2—Strength Test Data Report

Calculate the mean and sample standard deviation for the 30 strength tests results given below, using the formula for sample standard deviation given in R5.3.1. The project specifications call for column concrete to be normal weight, air-entrained, with a specified strength of 4000 psi.

Test No.	Date of Test	28-Day #1	28-Day #2	28-Day Average	28-Day Average (3-Consecutive)
1	05-March-04	4640	4770	4705	
2	06-March-04	4910	5100	5005	
3	10-March-04	4570	4760	4665	4792
4	12-March-04	4800	5000	4900	4857
5	13-March-04	5000	4900	4950	4838
6	17-March-04	4380	4570	4475	4775
7	19-March-04	4630	4820	4725	4717
8	21-March-04	4800	4670	4735	4645
9	25-March-04	5020	4940	4980	4813
10	28-March-04	4740	4900	4820	4845
11	30-March-04	4300	4110	4205	4668
12	02-April-04	4280	3620	3950	4325
13	05-April-04	4740	4880	4810	4322
14	08-April-04	4870	5040	4955	4592
15	09-April-04	4590	4670	4630	4798
16	15-April-04	4420	4690	4555	4713
17	16-April-04	4980	5070	5025	4737
18	19-April-04	4900	4860	4880	4820
19	20-April-04	5690	5570	5630	5178
20	22-April-04	5310	5310	5310	5273
21	24-April-04	5080	4970	5025	5322
22	28-April-04	4640	4440	4540	4958
23	01-May-04	5090	5080	5085	4883
24	03-May-04	5430	5510	5470	5032
25	07-May-04	5290	5360	5325	5293
26	10-May-04	4700	4770	4735	5177
27	11-May-04	4880	5040	4960	5007
28	15-May-04	5000	4890	4945	4880
29	16-May-04	4810	4670	4740	4882
30	18-May-04	4250	4400	4325	4670

Calculations and Discussion	Code Reference

Computation of the mean strength and sample standard deviation is shown in the following table. The sample standard deviation of 353 psi represents excellent quality control for the specified 4000 psi concrete.

Note that the concrete supplied for this concrete work satisfies the acceptance criteria of 5.6.3.3; no single strength test (28-day average of two cylinders) falls below the specified strength (4000 psi) by more than 500 psi (3500 psi), and the average of each set of 3 consecutive strength tests exceeds the specified strength (4000 psi).

Test No.	28-day Strength, X, psi	$X - \bar{X}$, psi	$(X - \bar{X})^2$
1	4705	-130	16,900
2	5005	170	28,900
3	4665	-170	28,900
4	4900	65	4,225
5	4950	115	13,225
6	4475	-360	129,600
7	4725	-110	12,100
8	4735	-100	10,000
9	4980	145	21,025
10	4820	-15	225
11	4205	-630	396,900
12	3950	-885	783,225
13	4810	-25	625
14	4955	100	10,000
15	4630	-205	42,025
16	4555	-280	78,400
17	5025	190	36,100
18	4880	45	2,025
19	5630	795	632,025
20	5310	475	225,625
21	5025	190	36,100
22	4540	-295	87,025
23	5085	250	62,500
24	5470	635	403,225
25	5325	490	240,100
26	4735	-100	10,000
27	4960	125	15,625
28	4945	110	12,100
29	4740	-95	9,025
30	4325	-510	260,100
Σ	145,060		3,607,850

Number of Tests = *30*
Maximum Strength = *5630 psi*
Minimum Strength = *3950 psi*

Mean Strength $= \dfrac{145{,}060}{30} = 4835$ *psi*

Sample Standard Deviation $= \sqrt{\dfrac{3{,}607{,}850}{29}} = 353$ *psi*

The single low strength test (3950 psi) results from the very low break for cylinder #2 (3620 psi) of test No. 12. The large disparity between cylinder #2 and cylinder #1 (4280 psi), both from the same batch, would seem to indicate a possible problem with the handling and testing procedures for cylinder #2.

Interestingly, the statistical data from the 30 strength test results can be filed for use on subsequent projects to establish a mix design, where the concrete work calls for normal weight, air-entrained concrete with a specified strength within 1000 psi of the specified 4000 psi value (3000 to 5000 psi). The target strength for mix proportioning would be calculated using the 353 psi sample standard deviation in code Eqs. (5-1) and (5-2). The low sample standard deviation should enable the "ready-mix company" to produce an economical mix for similar concrete work. The strength test data of this example are used to demonstrate that the concrete mix used for this project qualifies for the proposed concrete work of Example 2.1.

Example 2.3—Selection of Concrete Proportions by Trial Mixtures

Establish a water-cementitious materials ratio for a concrete mixture on the basis of the specified compressive strength of the concrete to satisfy the structural design requirements.

Project Specifications:

f'_c = 3000 psi (normal weight) at 28 days

3/4-in. max. size aggregate

5% total air content

4 in. max slump

Kona sand and gravel

Type I Portland Cement

Assume no strength test records are available to establish a target strength for selection of concrete mixture proportions. The water-cementitious materials ratio is to be determined by trial mixtures. See 5.3.3.2.

Calculations and Discussion	Code Reference

1. Without strength test results, use Table 5.3.2.2 to establish a target strength, f'_{cr}. *5.3.2.2*

   For $f'_c = 3000$ psi,
   $f'_{cr} = f'_c + 1200 = 3000 + 1200 = 4200$ psi

2. Trial Mixture Procedure *5.3.3.2*

   Trial mixtures should be based on the same materials as proposed for the concrete work. Three (3) concrete mixtures with three (3) different water-cementitious materials ratios (w/c) should be made to produce a range of strengths that encompass the target strength f'_{cr}. The trial mixtures should have a slump within ± 0.75 in. of the maximum specified (3.25 to 4.75 in.), and a total air content within ± 0.5% of the volume required by the project specifications (4.5 to 5.5%). Three (3) test cylinders per trial mixture should be made and tested at 28 days. The test results are then plotted to produce a strength versus w/c ratio curve to be used to establish an appropriate w/c ratio for the target strength f'_{cr}.

   To illustrate the trial mixture procedure, assume trial mixtures and test data as shown in Table 2-3. Based on the test results plotted in Fig. 2-3 for the three trial mixtures, the maximum w/c ratio to be used as the basis for proportioning the concrete mixture with a target strength, f'_{cr}, of 4200 psi by interpolation, is 0.49.

   Using a water-cementitious materials ratio of 0.49 to produce a concrete with a specified strength of 3000 psi results in a significant overdesign. Referring to Fig. 2-2, Example 2.1, for a w/c ratio of 0.49, a strength level approximating 3800 psi can be expected for air-entrained concrete. The required extent of mix overdesign, when sufficient strength data are not available to establish a sample standard deviation, should be apparent.

Table 2-3 Trial Mixture Data

Trial Mixtures	Batch No. 1	Batch No. 2	Batch No. 3
Selected w/c ratio	0.45	0.55	0.65
Measured slump, in.	3.75	4.25	4.50
Measured air content, %	4.4	5.3	4.8
Test results, psi:			
Cylinder #1	4650	3900	2750
Cylinder #2	4350	3750	2900
Cylinder #3	4520	3650	2850
Average	4510	3770	2830

As strength test data become available during construction, the amount by which the value of f'_{cr} must exceed the specified value of f'_c (1200 psi) may be reduced using a sample standard deviation calculated from the actual job test data, producing a more economical concrete mix. 5.5

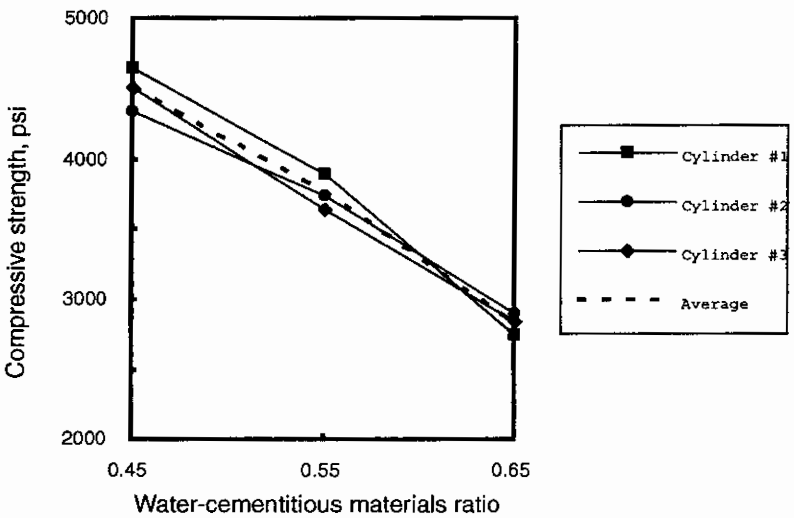


Figure 2-3 Trial Mixture Strength Curve

Example 2.4—Frequency of Testing

Determine the minimum number of test cylinders that must be cast to satisfy the code minimum sampling frequency for strength tests. Concrete placement = 200 cu yd per day for 7 days, transported by 10 cu yd truck mixers. This is a larger project where the minimum number of test cylinders per day of concrete placement (see 5.6.2.1) is greater than the minimum number per project (see 5.6.2.2).

Calculations and Discussion	Code Reference
1. Total concrete placed on project = 200 (7) = 1400 cu yd	
2. Total truck loads (batches) required ≈ 1400/10 ≈ 140	
3. Truck loads required to be sampled per day = 200/150 = 1.3	*5.6.2.1*
4. 2 truck loads must be sampled per day	
5. Total truck loads required to be sampled for project = 2 (7) = 14	
6. Total number of cylinders required to be cast for project = 14 (2 cylinders per test) = 28 (minimum)	*5.6.2.4*

It should be noted that the total number of cylinders required to be cast for this project represents a code required minimum number only that is needed for determination of acceptable concrete strength. Addition cylinders should be cast to provide for 7-day breaks, to provide field cured specimens to check early strength development for form removal or for determining when to post-tension prestressing tendons, and to keep one or two in reserve, should a low cylinder break occur at 28-day.

Example 2.5—Frequency of Testing

Determine the minimum number of test cylinders that must be cast to satisfy the code minimum sampling frequency for strength tests. Concrete is to be placed in a 100 ft × 75 ft × 7-1/2 in. slab, and transported by 10 cu yd truck mixers. This is a smaller project where the minimum required number of test cylinders is based on the frequency criteria of 5.6.2.2.

Calculations and Discussion	Code Reference
1. Total surface area placed $= 100 \times 75 = 7500$ sq ft	
2. Total concrete placed on project $= 7500 \times 7.5 \times \frac{1 \text{ ft}}{12 \text{ in.}}/27 = 174$ cu yd	
3. Total truck loads (batches) required $\approx 174/10 \approx 18$	
4. Required truck loads sampled per day $= 174/150 = 1.2$ $= 7500/5000 = 1.5$	*5.6.2.1*
5. But not less than 5 truck loads (batches) per project	*5.6.2.2*
6. Total number of cylinders cast for project = 5 (2 cylinders per test) = 10 (minimum)	*5.6.2.4*

It should again be noted that the total number of cylinders cast represents a code required minimum number only for acceptance of concrete strength. A more prudent total number for a project may include additional cylinders.

Example 2.6—Acceptance of Concrete

The following table lists strength test data from 5 truck loads (batches) of concrete delivered to the job site. For each batch, two cylinders were cast and tested at 28 days. The specified strength of the concrete f'_c is 4000 psi. Determine the acceptability of the concrete based on the strength criteria of 5.6.3.3.

Test No.	Cylinder #1	Cylinder #2	Test Average	Average of 3 Consecutive Tests
1	4110	4260	4185	—
2	3840	4080	3960	—
3	4420	4450	4435	4193
4	3670	3820	3745	4047
5	4620	4570	4595	4258

Calculations and Discussion	Code Reference

The average of the two cylinder breaks for each batch represents a single strength test result. Even though the lowest of the five strength test results (3745 psi) is below the specified strength of 4000 psi, the concrete is considered acceptable because it is not below the specified value by more than 500 psi for concrete with an f'_c not over 5000 psi; i.e., not below 3500 psi. The second acceptance criterion, based on the average of three (3) consecutive tests, is also satisfied by the three consecutive strength test averages shown. The procedure to evaluate acceptance based on 3 strength test results in a row is shown in the right column. The 4193 psi value is the average of the first 3 consecutive test results: (4185 + 3960 + 4435)/3 = 4193 psi. The average of the next 3 consecutive tests is calculated as (3960 + 4435 + 3745)/3 = 4047 psi, after the 4185 psi value is dropped from consideration. The average of the next 3 consecutive values is calculated by dropping the 3960 psi value. For any number of strength test results, the consecutive averaging is simply a continuation of the above procedure. Thus, based on the code acceptance criteria for concrete strength, the five strength tests results are acceptable, both on the basis of individual test results and the average of three consecutive test results.

Example 2.7—Acceptance of Concrete

The following table lists strength test data from 5 truck loads (batches) of concrete delivered to the job site. For each batch, two cylinders were cast and tested at 28 days. The specified strength of the concrete f'_c is 4000 psi. Determine the acceptability of the concrete based on the strength criteria of 5.6.3.3.

Test No.	Cylinder #1	Cylinder #2	Test Average	Average of 3 Consecutive Tests
1	3620	3550	3585	—
2	3970	4060	4015	—
3	4080	4000	4040	3880*
4	4860	4700	4780	4278
5	3390	3110	3250**	4023

**Average of 3 consecutive tests below f'_c (4000 psi).*
***One test more than 500 psi below specified value.*

Calculations and Discussion	Code Reference

Investigation of low-strength test results is addressed in 5.6.5. If average of three "tests" in a row dips below the specified strength, steps must be taken to increase the strength of the concrete. If a "test" falls more than 500 psi below the specified strength for concrete with an f'_c not over 5000 psi, there may be more serious problems, requiring an investigation to ensure structural adequacy; and again, steps taken to increase the strength level. For investigations of low strength, it is imperative that the location of the questionable concrete in the structure be known, so that the engineer can make an evaluation of the low strength on the structural adequacy of the member or element. *5.6.5*

Based on experience,[2.11] the major reasons for low strength test results are (1) improper sampling and testing, and (2) reduced concrete quality due to an error in production, or the addition of too much water to the concrete at the job site, caused by delays in placement or requests for wet or high slump concrete. High air content can also be a cause of low strength.

The test results for the concrete from Truck 5 are below the specified value, especially the value for cylinder #2, with the average strength being only 3250 psi. (Note that no acceptance decisions are based on the single low cylinder break of 3110 psi. Due to the many variables in the production, sampling and testing of concrete, acceptance or rejection is always based on the average of at least 2 cylinder breaks.)

3

Details of Reinforcement

GENERAL CONSIDERATIONS

Good reinforcement details are vital to satisfactory performance of reinforced concrete structures. Standard practice for reinforcing steel details has evolved gradually. The Building Code Committee (ACI 318) continually collects reports of research and practice related to structural concrete, suggests new research needed, and translates the results into specific code provisions for details of reinforcement.

The ACI Detailing Manual[3.1] provides recommended methods and standards for preparing design drawings, typical details, and drawings for fabrication and placing of reinforcing steel in reinforced concrete structures. Separate sections of the manual define responsibilities of both the engineer and the reinforcing bar detailer. The CRSI Manual of Standard Practice[3.2] provides recommended industry practices for reinforcing steel. As an aid to designers, Recommended Industry Practices for Estimating, Detailing, Fabrication, and Field Erection of Reinforcing Materials are included in Ref. 3.2, for direct reference in project drawings and specifications. The WRI Structural Welded Wire Fabric Detailing Manual[3.3] provides information on detailing welded wire reinforcement systems.

7.1 STANDARD HOOKS

Requirements for standard hooks and minimum finished inside bend diameters for reinforcing bars are illustrated in Tables 3-1 and 3-2. The standard hook details for stirrups and ties apply to No. 8 and smaller bar sizes only.

Table 3-1 Standard Hooks for Primary Reinforcement

Bar size, No.	Min. finished bend dia.[a]
3 through 8	$6d_b$
9, 10, 11	$8d_b$
14 and 18	$10d_b$

[a] Measured on inside of bar.

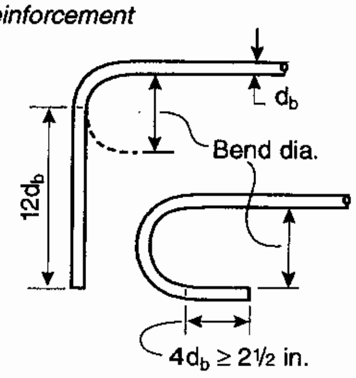
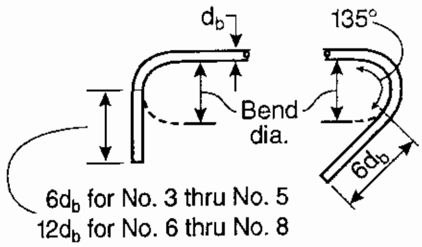


Table 3-2 Standard Hooks for Stirrups and Tie Reinforcement

Bar size, No.	Min. finished bend dia.[b]
3 through 5	$4d_b$
6 through 8	$6d_b$

[b] Measured on inside of bar.



Moment resisting frames used to resist seismic lateral forces in regions of high seismic risk or in structures assigned to high seismic performance or design categories (see Table 1-1), must be designed as special moment frames as defined in 21.1. In special moment frames, detailing of transverse reinforcement in beams and columns must comply with 21.3.3 and 21.4.4, respectively. Except for circular hoops which are required to have seismic hooks with a 90-degree bend on the free ends, the ends of hoops and crossed ties must terminate in seismic hooks with 135-degree bends. These hooks are necessary to effectively anchor the free ends within the confined core so satisfactory performance is achieved in areas of members where inelastic behavior may occur. See Part 29 of this publication for discussion and illustrations of this special detailing requirement.

7.2 MINIMUM BEND DIAMETERS

Minimum bend diameter for a reinforcing bar is defined as "the diameter of bend measured on the *inside* of the bar." Minimum bend diameters, expressed as multiples of bar diameters, are dependent on bar size; for No. 3 to No. 8 bars, the minimum bend diameter is 6 bar diameters; for No. 9 to No. 11 bars, the minimum bend diameter is 8 bar diameters; and for No. 14 and No. 18 bars, the minimum bend diameter is 10 bar diameters. Exceptions to these provisions are:

1. For stirrups and ties in sizes No. 5 and smaller, the minimum bend diameter is 4 bar diameters. For No. 6 through No. 8 stirrups and ties, the minimum bend diameter is 6 bar diameters.
2. For welded wire reinforcement used for stirrups and ties, the inside diameter of the bend must not be less than four wire diameters for deformed wire larger than D6 and two wire diameters for all other wire. Welded intersections must be at least four wire diameters away from bends with inside diameters of less than eight wire diameters.

7.3 BENDING

All reinforcement must be bent cold unless otherwise permitted by the engineer. For unusual bends, special fabrication including heating may be required and the engineer must give approval to the techniques used.

7.3.2 Field Bending of Reinforcing Bars

Reinforcing bars partially embedded in concrete are frequently subjected to bending and straightening in the field. Protruding bars often must be bent to provide clearance for construction operations. Field bending and straightening may also be required because of incorrect fabrication or accidental bending. According to 7.3.2, bars partially embedded in concrete must not be field bent without authorization of the engineer unless shown on the plans. Test results[3.4] provide guidelines for field bending and straightening, and heating if necessary, of bars partially embedded in concrete. As an aid to the engineer on proper procedure, the recommendations of Ref. 3.4 are stated below. ASTM A 615 Grade 60 deformed bars were used in the experimental work on which the recommendations are based.

1. Field bending/straightening should be limited to bar sizes No. 11 and smaller. Heat should be applied for bending/straightening bar sizes No. 6 through No. 11, or for bending/straightening bar sizes No. 5 and smaller when those bars have been previously bent. Previously unbent bars of sizes No. 5 and smaller may be bent/straightened without heating.

2. A bending tool with bending diameter as shown in Table 3-3(a) should be used. Any bend should be limited to 90 degrees.

3. In applying heat for field bending/straightening, the steel temperature should be at or above the minimum temperature shown in Table 3-3(b) at the end of the heating operation, and should not exceed the maximum temperature shown during the heating operation.

4. In applying heat for field bending/straightening, the entire length of the portion of the bar to be bent (or the entire length of the bend to be straightened) should be heated plus an additional 2 in. at each end. For bars larger than No. 9, two heat tips should be used simultaneously at opposite sides of the bar to assure a uniform temperature throughout the thickness of the bar.

5. Before field bending/straightening, the significance of possible reductions in the mechanical properties of bent/straightened bars, as indicated in Table 3-3(c), should be evaluated.

7.5 PLACING REINFORCEMENT

7.5.1 Support for Reinforcement

Support for reinforcement, including tendons and post-tensioning ducts, is required to adequately secure the reinforcement against displacement during concrete placement. The CRSI Manual of Standard Practice[3.2] gives an in-depth treatise on types and typical sizes of supports for reinforcement. Types and typical sizes of wire bar supports are illustrated in Table 3-4. In addition to wire bar supports, bar supports are also available in precast concrete, cementitious fiber-reinforced and plastic materials. If the concrete surface will be exposed during service, consideration must be given to the importance of the appearance of the concrete surface and the environment to which it will be exposed. For example, if the concrete surface will be exposed directly to the weather or to a humid environment, it is likely that rust spots or stains will eventually show if unprotected bright steel barsupports are used. As outlined in the CRSI manual, bar supports are available in four classes of protection, depending on their expected exposure and the amount of corrosion protection required. Based on current industry practice, the available classes of protection are:

Class 1 — Maximum Protection
Plastic protected bar supports intended for use in situations of moderate to severe exposure and/or situations requiring light grinding (1/16 in. maximum) or sandblasting of the concrete surface.

Class 1A — Maximum Protection (For Use With Epoxy-Coated Reinforcement Bars) Epoxy-, vinyl-, or plastic coated bright basic wire bar supports intended for use in situations of moderate to maximum exposure where no grinding or sandblasting of the concrete surface is required. Generally, they are used when epoxy-coated reinforcing bars are required.

Class 2 — Moderate Protection
Stainless steel protected steel wire bar supports intended for use in situations of moderate exposure and/or situations requiring light grinding (1/16 in. maximum) or sandblasting of the concrete surface. The bottom of each leg is protected with a stainless steel tip.

Class 3 — No Protection
Bright basic wire bar supports with no protection against rusting. Unprotected wire bar supports are intended for use in situations where surface blemishes can be tolerated, or where supports do not come into contact with a concrete surface which is exposed.

Table 3-3 Field Bending and Straightening of Reinforcing Bars[3.4]

(a) Ratio of Bend Diameter to Bar Diameter

Bar Size, No.	Bend inside diameter/bar diameter	
	Not Heated	Heated
3, 4, 5	8	8
6, 7, 8, 9	Not permitted	8
10, 11	Not permitted	10

(b) Temperature Limits for Heating Bars

Bar Size, No.	Minimum Temperature (°F)	Maximum Temperature (°F)
3, 4	1200	1250
5, 6	1350	1400
7, 8, 9	1400	1450
10, 11	1450	1500

(c) Percent Reduction in Mechanical Properties of Bent and Straightened Bars

Bending Condition	Bar Size, No.	% Yield Strength Reduction	% Ultimate Tensile Strength Reduction	% Elongation Reduction
Cold	3, 4	—	—	20
Cold	5	5	—	30
Hot	All sizes	10	10	20

The engineer will need to specify the proper class of protection in the project specifications. It should be noted that the support system for reinforcement is usually detailed on the reinforcement placing drawings prepared by the "rebar" fabricator. The support system, including the proper class of protection, should be reviewed by the engineer, noting that the bar support size also dictates the cover provided for the reinforcement.

Use of epoxy-coated reinforcing bars will require bar supports made of a dielectrical material, or wire supports coated with a dielectrical material such as epoxy or vinyl, which is compatible with concrete. See discussion on 3.5.3.7, in Part 2 of this document, concerning special hardware and handling to minimize damage to the epoxy coating during handling, transporting, and placing epoxy-coated bars.

Commentary R7.5.1 emphasizes the importance of rigidly supporting the beam stirrups, in addition to the main flexural reinforcement, directly on the formwork. If not supported directly, foot traffic during concrete placement can push the web reinforcement down onto the forms, resulting in loss of cover and potential corrosion problems. It should be noted that the CRSI Manual of Standard Practice[3.2], often referenced in the design documents for placing reinforcing bars, does not specifically address this need for direct web reinforcement support. The placing drawings, usually prepared by the bar fabricator, should show a typical section or detail, so that this support requirement is clear and not overlooked by the ironworkers.

A word of caution on reinforcement displacement during concrete placing operations. If concrete placement is by pumping, it is imperative that the pipelines and the pipeline support system be supported above and independently of the chaired reinforcement by "chain-chairs" or other means. There must be no contact, direct or indirect, with the chaired reinforcement; otherwise, the surging action of the pipeline during pumping operations can, and most assuredly will, completely dislodge the reinforcement. This potential problem is especially acute in relatively thin slab members, especially those containing tendons, where the vertical placement of the reinforcement is most critical. The project specifications should specifically address this potential concrete placement problem.

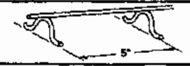
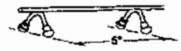
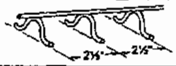
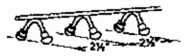
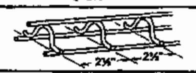
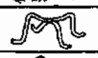
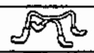
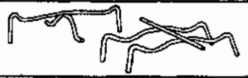
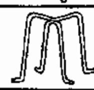
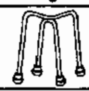
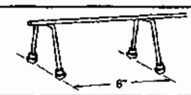
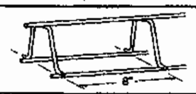
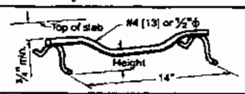
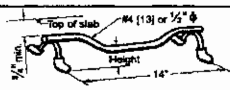
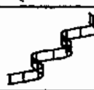
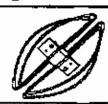
7.5.2 Tolerances in Placing Reinforcement

The code provides tolerances applied simultaneously to concrete cover and member effective depth, d. With dimension "d" being the most structurally important dimension, any deviation in this dimension, especially for members of lesser depth, can have an adverse effect on the strength provided in the completed construction. The permitted variation from the effective depth d takes this strength reduction into account, with a smaller permitted variation for shallower members. The permitted tolerances are also established to reflect common construction techniques and practices. The critical dimensional tolerances for locating the longitudinal reinforcement are illustrated in Table 3-5, with two exceptions:

1. Tolerance for clear distance to formed soffits must not exceed minus 1/4 in.

2. Tolerance for cover must not exceed minus one-third the minimum concrete cover required in the design drawings and specifications. See Example 3.1

Table 3-4 Types and Sizes of Wire Bar Supports[3.2]

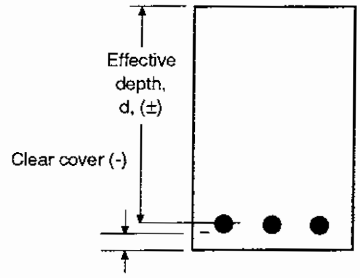
SYMBOL	BAR SUPPORT ILLUSTRATION	BAR SUPPORT ILLUSTRATION PLASTIC CAPPED OR DIPPED	TYPE OF SUPPORT	TYPICAL SIZES
SB		CAPPED 	Slab Bolster	¾, 1, 1½, and 2 in. heights in 5 ft and 10 ft lengths
SBU*			Slab Bolster Upper	Same as SB
BB		CAPPED 	Beam Bolster	1, 1½, 2 to 5 in. heights in increments of ¼ in. in lengths of 5 ft
BBU*			Beam Bolster Upper	Same as BB
BC		DIPPED 	Individual Bar Chair	¾, 1, 1½, and 1¾ in. heights
JC		DIPPED  DIPPED	Joist Chair	4, 5, and 6 in. widths and ¾, 1 and 1½ in. heights
HC		CAPPED 	Individual High Chair	2 to 15 in. heights in increments of ¼ in.
HCM*			High Chair for Metal Deck	2 to 15 in. heights in increments of ¼ in.
CHC		CAPPED 	Continuous High Chair	Same as HC in 5 ft and 10 ft lengths
CHCU*			Continuous High Chair Upper	Same as CHC
CHCM*			Continuous High Chair for Metal Deck	Up to 5 in. heights in increments of ¼ in.
JCU**		DIPPED 	Joist Chair Upper	14 in. span; heights −1 in. thru +3½ in. vary in ¼ in. increments
CS			Continuous Support	1½ to 12 in. in increments of ¼ in. in lengths of 6'-8"
SBC			Single Bar Centralizer (Friction)	6 in. to 24 in. diameter

*Usually available in Class 3 only, except on special order.
**Usually available in Class 3 only, with upturned or end-bearing legs.

1 in. = 25.4 mm
1 ft = 304.8 mm

Table 3-5 Critical Dimensional Tolerances for Placing Reinforcement

Effective Depth d	Tolerance on d	Tolerance on Min. Cover
d ≤ 8 in.	± 3/8 in.	- 3/8 in.
d > 8 in.	± 1/2 in.	- 1/2 in.



For ends of bars and longitudinal location of bends, the tolerance is ± 2 in., except at discontinuous ends of corbels and brackets where the tolerance is ± 1/2 in. At the discontinuous ends of other members the tolerance is permitted to be ± 1 in. The tolerance for minimum cover in 7.5.2.1 shall also apply. These tolerances are illustrated in Fig. 3-1.

Note that a plus (+) tolerance increases the dimension and a minus (-) tolerance decreases the dimension. Where only a minus tolerance is indicated on minimum cover, there is no limit in the other direction. Quality control during construction should be based on the more restrictive of related tolerances.

In addition to the code prescribed rebar placing tolerances, the engineer should be familiar with ACI Standard 117, *Standard Tolerances for Concrete Construction and Materials.*[3.5] ACI 117 includes tolerances for all measured dimensions, quantities and concrete properties used in concrete construction. The ACI 117 document is intended to be used by direct reference in the project specifications; therefore it is written in a specification format.

The designer must specify and clearly identify cover tolerances as the needs of the project dictate. For example, if concrete is to be exposed to a very aggressive environment, such as deicing chemicals, where the amount of concrete cover to the reinforcement may be a critical durability consideration, the engineer may want to indicate closer tolerances on concrete cover than those permitted by the code, or alternatively, specify a larger cover in recognition of expected variation in the placing of the reinforcement.

7.5.4 "Tack" Welding

Note that welding of crossing bars (tack welding) for assembly of reinforcement is prohibited except as specifically authorized by the engineer. By definition, a tack weld is a small spotweld to facilitate fabrication or field installation of reinforcement, and is not intended as a structural weld. Tack welding can lead to local embrittlement of the steel, and should never be done on reinforcement required by design. As noted in 3.5.2, all welding of reinforcement must conform to controlled welding procedures specified in AWS D1.4, including proper preheat (if required), and welding with electrodes meeting requirements of final welds.

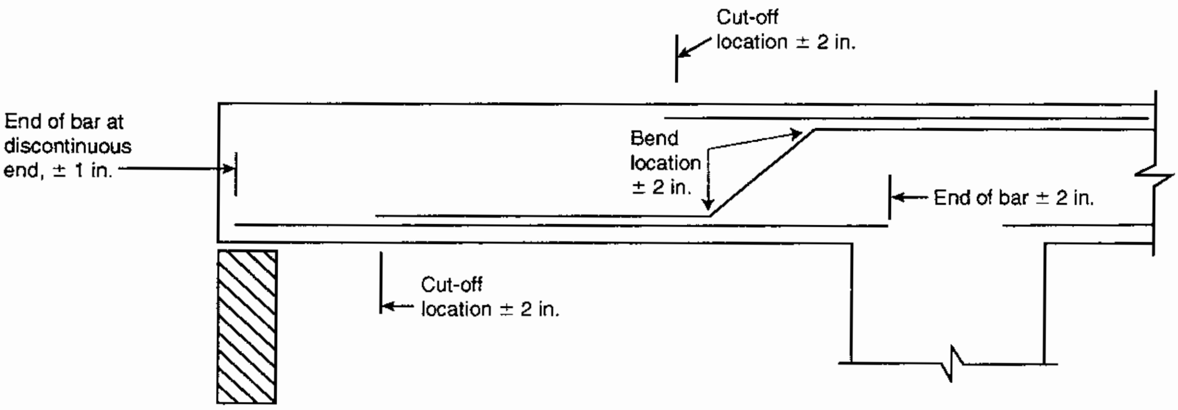


Figure 3-1 Tolerances for Bar Bend and Cutoff Locations

7.6 SPACING LIMITS FOR REINFORCEMENT

Spacing (clear distance) between bars must be as follows:

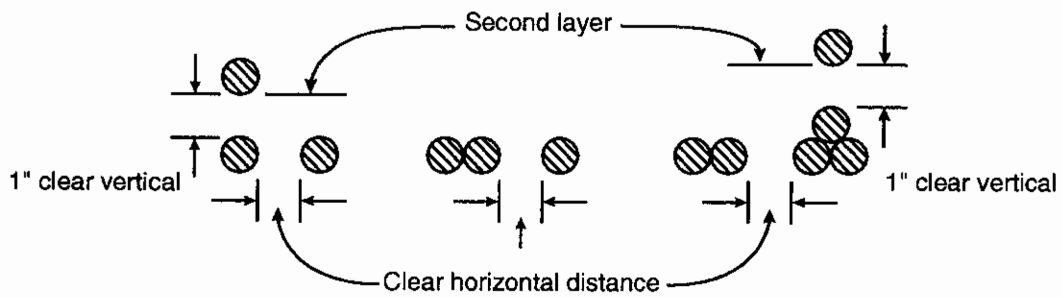
Minimum Spacing

For members with parallel bars in a layer, the clear spacing between bars must be at least one bar diameter but not less than 1 in.; and for reinforcement in two or more layers, bars in the upper layers must be directly above bars in the bottom layer, with at least 1 in. clear vertically between layers. For spirally reinforced and tied reinforced compression members, the clear distance between longitudinal bars must be at least 1-1/2 bar diameters, but not less than 1-1/2 in. These spacing requirements also apply to clear distance between contact-lap-spliced single or bundled bars and adjacent splices or bars. Section 3.3.2, which contains spacing requirements based on maximum nominal aggregate size, may also be applicable. Clear distances between bars are illustrated in Table 3-6.

Maximum Spacing

In walls and slabs other than concrete joists, primary flexural reinforcement must not be spaced greater than 3 times the wall or slab thickness nor 18 in.

Table 3-6 Minimum Clear Distances Between Bars, Bundles, or Tendons



Reinforcement Type	Type Member	Clear Distance
Deformed bars	Flexural members	$d_b \geq 1$ in.
	Compression members, tied or spirally reinforced	$1.5\ d_b \geq 1.5$ in.
Pretensioning tendons	Wires	$4d_b$
	Strands*	$3d_b$

*When $f'_{ci} \geq 4000$ psi, center to center = 1-3/4 in. for 1/2 in. strands
= 2 in. for 0.6 in. strands

7.6.6 Bundled Bars

For isolated situations requiring heavy concentration of reinforcement, bundles of standard bar sizes can save space and reduce congestion for placement and consolidation of concrete. In those situations, bundling of bars in columns is a means to better locating and orienting the reinforcement for increased column capacity; also, fewer ties are required if column bars are bundled.

Bundling of bars (parallel reinforcing bars in contact, assumed to act as a unit) is permitted, but only if such bundles are enclosed by ties or stirrups. Some limitations are placed on the use of bundled bars as follows:

1. No. 14 and No. 18 bars cannot be bundled in beams.

2. If individual bars in a bundle are cut off within the span of beams, such cutoff points must be staggered at least 40 bar diameters.

3. A maximum of two bundled bars in any one plane is implied (three or four adjacent bars in one plane are not considered as bundled bars).

4. For spacing and concrete cover based on bar diameter, d_b, a unit of bundled bars must be treated as a single bar with diameter derived from the total area of all bars in the bundle. Equivalent diameters of bundled bars are given in Table 3-7.

5. A maximum of four bars may be bundled (See Fig. 3-2).

6. Bundled bars must be enclosed within stirrups or ties.

Table 3-7 Equivalent Diameters of Bundled Bars, in.

Bar Size, No.	Bar Diameter	2-Bar Bundle	3-Bar Bundle	4-Bar Bundle
6	0.750	1.06	1.30	1.50
7	0.875	1.24	1.51	1.75
8	1.000	1.42	1.74	2.01
9	1.128	1.60	1.95	2.26
10	1.270	1.80	2.20	2.54
11	1.410	1.99	2.44	2.82
14	1.693	2.39	2.93	3.39

Figure 3-2 Possible Reinforcing Bar Bundling Schemes

7.6.7 Prestressing Steel and Ducts

Prior to the '99 code, distances between prestressed steel were specified in terms of minimum clear distances. The '99 and subsequent codes specifies distances between prestressed steel in terms of minimum center-to-center spacing and requires $4d_b$ for strands and $5d_b$ for wire. When the compressive strength of the concrete at the time of prestress transfer, f'_{ci} is 4000 psi or greater, the minimum center-to-center spacing can be reduced to 1-3/4 in. for strands 1/2-in. nominal diameter or smaller and 2 in. for strands 0.6-in. nominal diameter. These changes were made as a result of research sponsored by the Federal Highway Administration. Center-to-center spacing is now specified because that is the way it was measured in the research. In addition, converting to clear spacing is awkward and unnecessary, and templates used by precast manufacturers have always been fabricated based on center-to-center dimensions. Closer vertical spacing and bundling of prestressed steel is permitted in the middle portion of the span if special care in design and fabrication is employed. Post-tensioning ducts may be bundled if concrete can be satisfactorily placed and provision is made to prevent the tendons from breaking through the duct when tensioned.

7.7 CONCRETE PROTECTION FOR REINFORCEMENT

Concrete cover or protection requirements are specified for members cast against earth, in contact with earth or weather, and for interior members not exposed to weather. Starting with the '02 code, the location of the cover requirements for cast-in-place concrete (prestressed) was reorganized. Cast-in-place concrete (prestressed) immediately follows cast-in-place (nonprestressed). They are then followed by the cover requirements for precast concrete manufactured under plant control conditions. In some cases slightly reduced cover or protection is permitted under the conditions for cast-in-place (prestressed) and precast concrete manufactured under plant control control conditions than permitted for cast-in-place concrete (non-prestressed). The term "manufactured under plant controlled conditions" does not necessarily mean that precast members must be manufactured in a plant. Structural elements precast at the job site (e.g., tilt-up concrete walls) will also qualify for the lesser cover if the control of form dimensions, placing of reinforcement, quality of concrete, and curing procedure are equivalent to those normally expected in a plant operation. Larger diameter bars, bundled bars, and prestressed tendons require greater cover. Corrosive environments or fire protection may also warrant increased cover. Section 18.3.3, which was introduced in the '02 code, requires that prestressed flexural members be classified as Class U (uncracked), Class C (cracked), or Class T (transition between uncracked and cracked). Section 7.7.5.1, also new to the '02 code, requires the cover of 7.7.2 be increased 50% for prestressed members classified as Class C or T where the members are exposed to corrosive environments or other severe exposure conditions. The requirement to increase the cover by 50% may be waived if the precompressed zone is not in tension under sustained load. The designer should take special note of the commentary recommendations (R7.7.5) for increased cover where concrete will be exposed to external sources of chlorides in service, such as deicing salts and seawater. As noted in R7.7, alternative methods of protecting the reinforcement from weather may be used if they provide protection equivalent to the additional concrete cover required in 7.7.1(b), 7.7.2(b), and 7.7.3(a), as compared to 7.7.1(c), 7.7.2(c), and 7.7.3(b), respectively.

7.8 SPECIAL REINFORCEMENT DETAILS FOR COLUMNS

Section 7.8 covers the special detailing requirements for offset bent longitudinal bars and steel cores of composite columns.

Where column offsets of less than 3 in. are necessary, longitudinal bars may be bent, subject to the following limitations:

1. Slope of the inclined portion of an offset bar with respect to the axis of column must not exceed 1 in 6 (see Fig. 3-3).

2. Portions of bar above and below an offset must be parallel to axis of column.

3. Horizontal support at offset bends must be provided by lateral ties, spirals, or parts of the floor construction. Ties or spirals, if used, shall be placed not more than 6 in. from points of bend (see Fig. 3-3). Horizontal support provided must be designed to resist 1-1/2 times the horizontal component of the computed force in the inclined portion of an offset bar.

4. Offset bars must be bent before placement in the forms.

When a column face is offset 3 in. or more, longitudinal column bars parallel to and near that face must not be offset bent. Separate dowels, lap spliced with the longitudinal bars adjacent to the offset column faces, must be provided (see Fig. 3-3). In some cases, a column might be offset 3 in. or more on some faces, and less than 3 in. on the remaining faces, which could possibly result in some offset bent longitudinal column bars and some separate dowels being used in the same column.

Steel cores in composite columns may be detailed to allow transfer of up to 50 percent of the compressive load in the core by direct bearing. The remainder of the load must be transferred by welds, dowels, splice plates, etc. This should ensure a minimum tensile capacity similar to that of a more common reinforced concrete column.

7.9 CONNECTIONS

Enclosures must be provided for splices of continuing reinforcement, and for end anchorage of reinforcement terminating at beam and column connections. This confinement may be provided by the surrounding concrete or internal closed ties, spirals, or stirrups.

7.10 LATERAL REINFORCEMENT FOR COMPRESSION MEMBERS

7.10.4 Spirals

Minimum diameter of spiral reinforcement in cast-in-place construction is 3/8 in. and the clear spacing must be between the limits of 1 in. and 3 in. This requirement does not preclude the use of a smaller minimum diameter

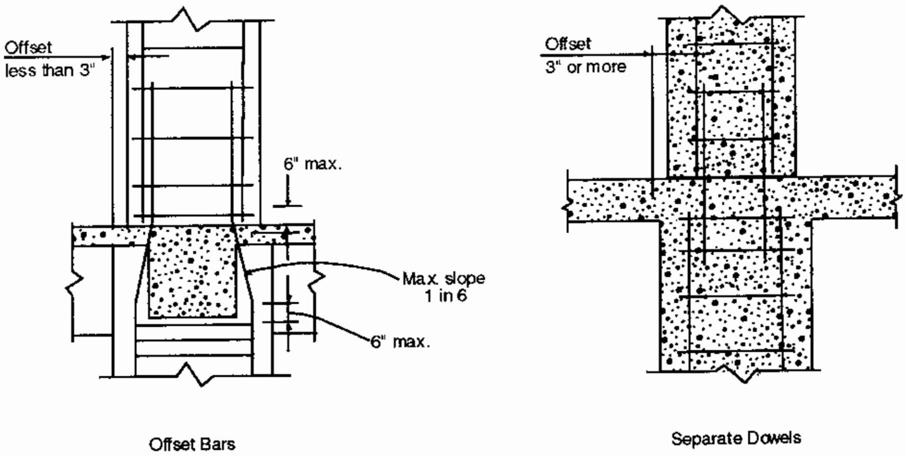


Figure 3-3 Special Column Details

for precast units. Beginning with the '99 code, full mechanical splices complying with 12.14.3 are allowed. Previously, only lap splices and full welded splices were permitted. Editions of the code prior to the '99 required lap splices to be 48 bar or wire diameters, regardless of whether the bar or wire was plain or deformed, or uncoated or epoxy-coated. The '99 code was revised to require that lap splices of plain uncoated and epoxy-coated deformed bar or wire be 72 bar or wire diameters. The required lap splice length for plain uncoated and epoxy-coated deformed bar or wire is permitted to be reduced to 48 bar or wire diameters provided the ends of the lapped bars or wires terminate in a standard 90 degree hook as required for stirrups and ties (7.1.3). The lap splice length for deformed uncoated bar or wire remains unchanged at 48 bar or wire diameters, as does the requirement that the minimum lap splice length be not less than 12 in. Anchorage of spiral reinforcement must be provided by 1-1/2 extra turns at each end of a spiral unit.

Spiral reinforcement must extend from the top of footing or slab in any story to the level of the lowest horizontal reinforcement in slabs, drop panels, or beams above. If beams or brackets do not frame into all sides of the column, ties must extend above the top of the spiral to the bottom of the slab or drop panel (see Fig. 3-4). In columns with capitals, spirals must extend to a level where the diameter or width of capital is twice that of the column.

Spirals must be held firmly in place, at proper pitch and alignment, to prevent displacement during concrete placement. Prior to ACI 318-89, the code specifically required spacers for installation of column spirals. Section 7.10.4.9 now simply states that "spirals shall be held firmly in place and true to line." This performance provision permits alternative methods, such as tying, to hold the fabricated cage in place during construction, which is current practice in most areas where spirals are used. The original spacer requirements were moved to the commentary to provide guidance where spacers are used for spiral installation. Note that the project specifications should cover the spacer requirements (if used) or the tying of the spiral reinforcement.

7.10.5 Ties

In tied reinforced concrete columns, ties must be located at no more than half a tie spacing above the floor slab or footing and at no more than half a tie spacing below the lowest horizontal reinforcement in the slab or drop panel above. If beams or brackets frame from four directions into a column, ties may be terminated not more than 3 in. below the lowest horizontal reinforcement in the shallowest of such beams or brackets (see Fig. 3-5). Minimum size of lateral ties in tied reinforced columns is related to the size of the longitudinal bars. Minimum tie sizes are No. 3 for non-prestressed longitudinal bars No. 10 and smaller, and No. 4 for No. 11 longitudinal bars and larger and for bundled bars. The following restrictions also apply: spacing must not exceed 16 longitudinal bar diameters, 48 tie bar diameters, or the least dimension of the column; every corner bar and alternate bar must have lateral support provided by the corner of a tie or crosstie with an included angle of not more than 135 degree. No unsupported bar shall be farther than 6 in. from a supported bar (see Fig. 3-6). Note that the 6-in. clear distance is measured along the tie.

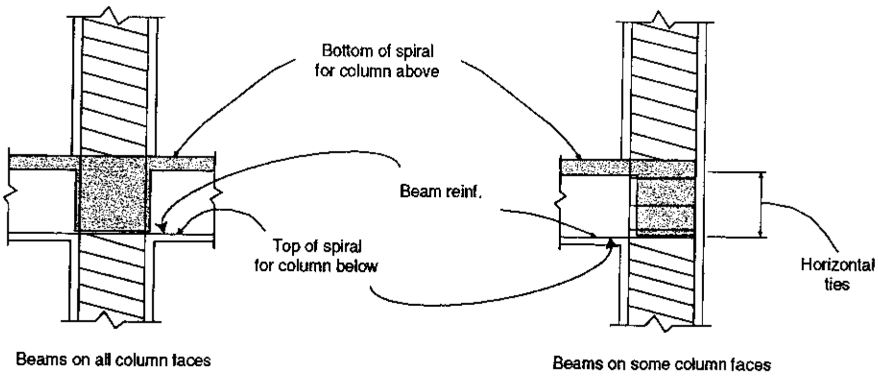


Figure 3-4 Termination of Spirals

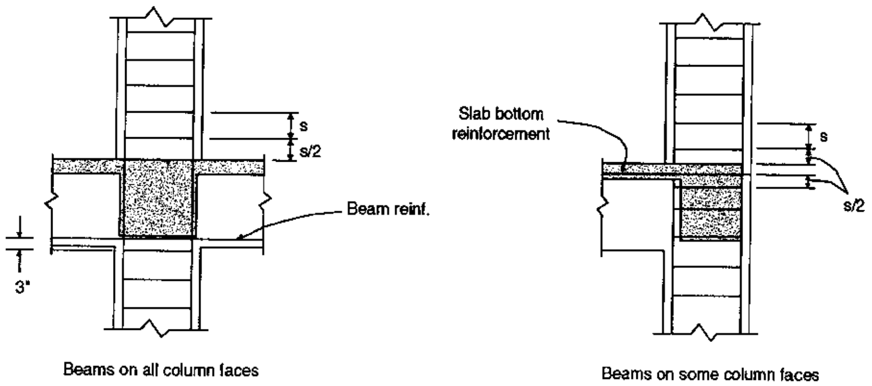


Figure 3-5 Termination of Column Ties

Welded wire reinforcement and continuously wound bars or deformed wire reinforcement of equivalent area may be used for ties. Where main reinforcement is arranged in a circular pattern, it is permissible to use complete circular ties at the specified spacing. This provision allows the use of circular ties at a spacing greater than that specified for spirals in spirally reinforced columns. Anchorage at the end of a continuously wound bar or wire reinforcement should be by a standard hook or by one additional turn of the tie pattern.

Where anchor bolts are provided in the tops of columns or pedestals to attach other structural members, the code requires that these bolts be confined by lateral reinforcement that is also surrounding at least four of the vertical bars in the column or pedestal for continuity of the load transfer at the connection. The lateral ties are required to be a minimum of two No. 4 or three No. 3 bars and must be distributed within the top 5 in. of the column or pedestal.

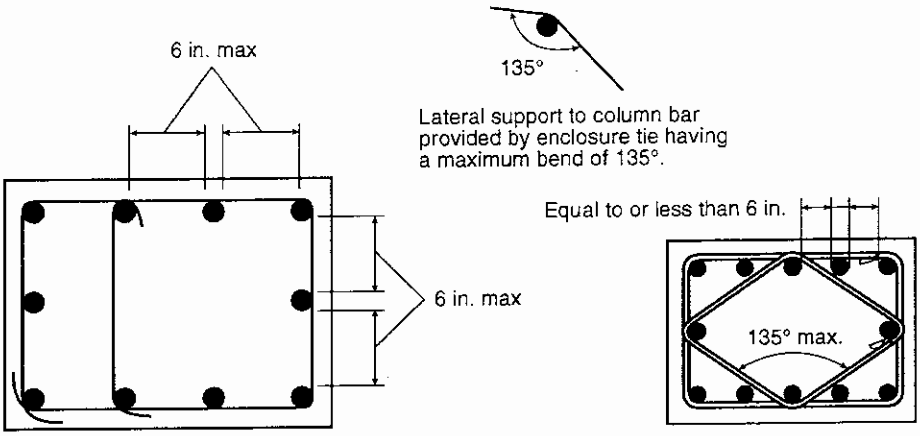


Figure 3-6 Lateral Support of Column Bars by Ties

7.11 LATERAL REINFORCEMENT FOR FLEXURAL MEMBERS

Where compression reinforcement is used to increase the flexural strength of a member (10.3.5.1), or to control long-term deflection [Eq. (9-11)], 7.11.1 requires that such reinforcement be enclosed by ties or stirrups. The purpose of the ties or stirrups is to prevent buckling of the compression reinforcement. Requirements for size and spacing of the ties or stirrups are the same as for ties in tied columns. Welded wire reinforcement of equivalent area may be used. The ties or stirrups must extend throughout the distance where the compression reinforcement is required for flexural strength or deflection control. Section 7.11.1 is interpreted not to apply to reinforcement located in a compression zone to help assemble the reinforcing cage or hold the web reinforcement in place during concrete placement.

Enclosing reinforcement required by 7.11.1 is illustrated by the U-shaped stirrup in Fig. 3-7, for a continuous beam, in the negative moment region; the continuous bottom portion of the stirrup satisfies the enclosure intent of 7.11.1 for the two bottom bars shown. A completely closed stirrup is ordinarily not necessary, except in cases of high moment reversal, where reversal conditions require that both top and bottom longitudinal reinforcement be designed as compression reinforcement.

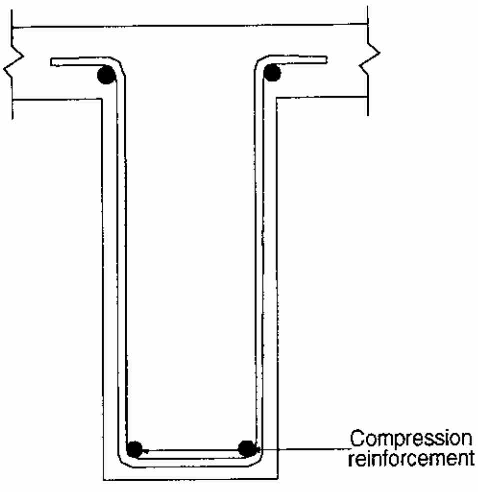


Figure 3-7 Enclosed Compression Reinforcement in Negatve Moment Region

Torsion reinforcement, where required, must consist of completely closed stirrups, closed ties, spirals, or closed cages of welded wire reinforcement as required by 11.6.4.

7.11.3 Closed Ties or Stirrups

According to 7.11.3, a closed tie or stirrup is formed either in one piece with overlapping 90-degree or 135-degree end hooks around a longitudinal bar, or in one or two pieces with a Class B lap splice, as illustrated in Fig. 3-8. The one-piece closed stirrup with overlapping end hooks is not practical for placement. Neither of the closed stirrups shown in Fig. 3-8 is considered effective for members subject to high torsion. Tests have shown that, with high torsion, loss of concrete cover and subsequent loss of anchorage result if the 90-degree hook and lap splice details are used where confinement by external concrete is limited. See Fig. 3-9. The ACI Detailing Manual[3.1] recommends the details illustrated in Fig. 3-10 for closed stirrups used as torsional reinforcement.

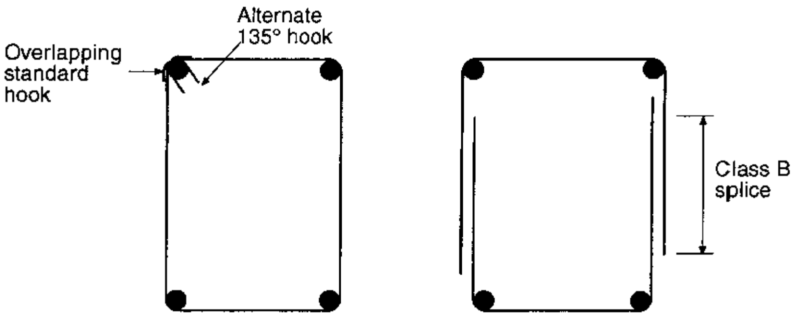


Figure 3-8 Code Definition of Closed Tie or Stirrup

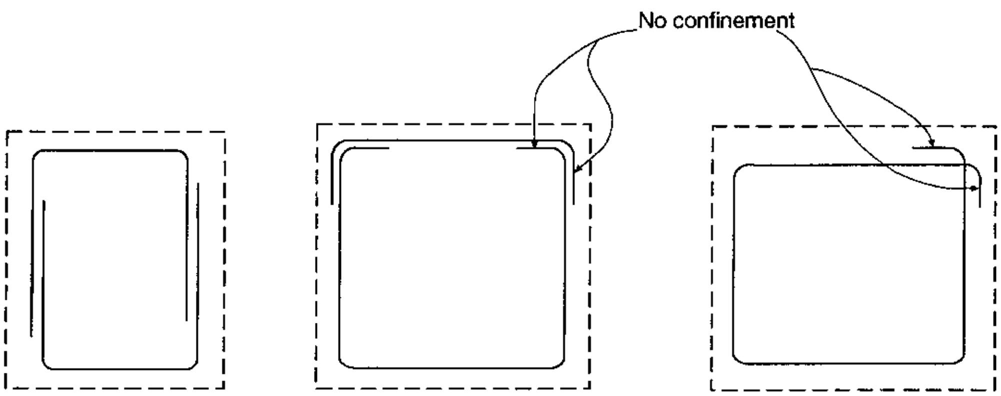


Figure 3-9 Closed Stirrup Details Not Recommended for Members Subject to High Torsion

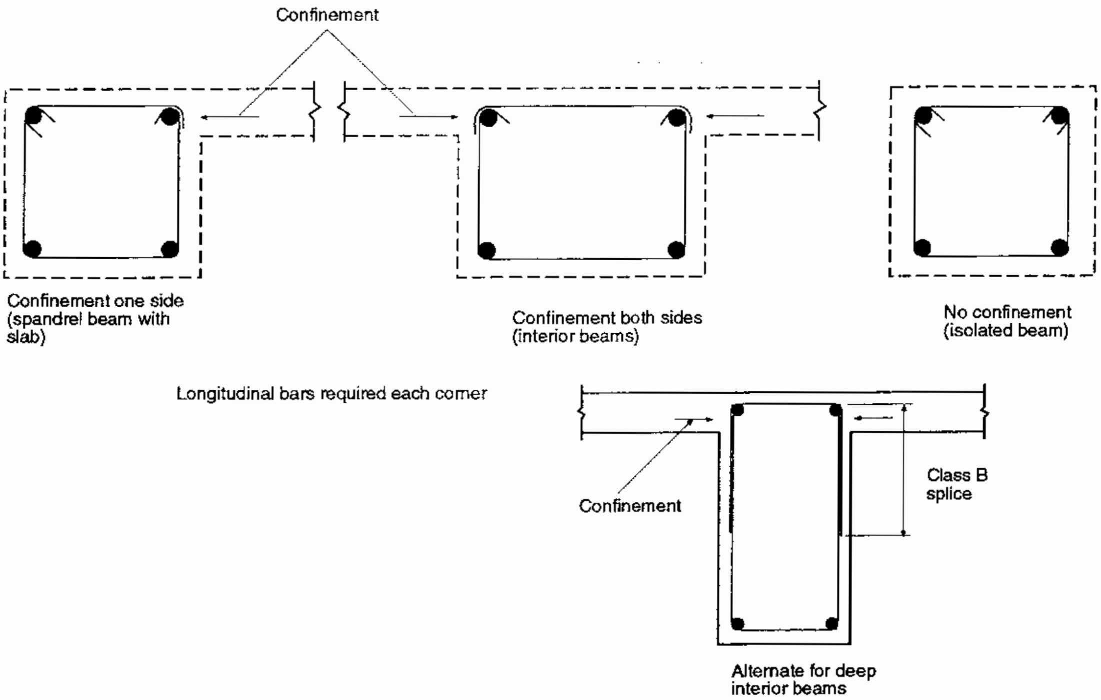


Figure 3-10 Two-Piece Closed Stirrup Details [3.1]
Recommended for Members Subject to High Torsion

7.12 SHRINKAGE AND TEMPERATURE REINFORCEMENT

Minimum shrinkage and temperature reinforcement normal to primary flexural reinforcement is required for structural floor and roof slabs (not slabs on ground) where the flexural reinforcement extends in one direction only. Minimum steel ratios, based on the gross concrete area, are:

1. 0.0020 for Grades 40 and 50 deformed bars;

2. 0.0018 for Grade 60 deformed bars or welded wire reinforcement;

3. $0.0018 \times 60{,}000/f_y$ for reinforcement with a yield strength greater than 60,000 psi; but not less than 0.0014.

Spacing of shrinkage and temperature reinforcement must not exceed 5 times the slab thickness nor 18 in. Splices and end anchorages of such reinforcement must be designed for the full specified yield strength. The minimum steel ratios cited above do not apply where prestressed steel is used.

Bonded or unbonded prestressing tendons may be used for shrinkage and temperature reinforcement in structural slabs (7.12.3). The tendons must provide a minimum average compressive stress of 100 psi on the gross concrete area, based on effective prestress after losses. Spacing of tendons must not exceed 6 ft. Where the spacing is greater than 54 in., additional bonded reinforcement must be provided at slab edges.

7.13 REQUIREMENTS FOR STRUCTURAL INTEGRITY

Structures capable of safely supporting all conventional design loads may suffer local damage from severe local abnormal loads, such as explosions due to gas or industrial liquids; vehicle impact; impact of falling objects; and local effects of very high winds such as tornadoes. Generally, such abnormal loads or events are not design considerations. The overall integrity of a reinforced concrete structure to withstand such abnormal loads can be substantially enhanced by providing relatively minor changes in the detailing of the reinforcement. The intent of 7.13 is to improve the redundancy and ductility of structures. This is achieved by providing, as a minimum, some continuity reinforcement or tie between horizontal framing members. In the event of damage to a major supporting element or an abnormal loading event, the integrity reinforcement is intended to confine any resulting damage to a relatively small area, thus improving overall stability.

It is not the intent of 7.13 that a structure be designed to resist general collapse caused by gross misuse or to resist severe abnormal loads acting directly on a large portion of the structure. General collapse of a structure as the result of abnormal events such as wartime or terrorist bombing, and landslides, are beyond the scope of any practical design.

7.13.1 General Structural Integrity

Since accidents and misuse are normally unforseeable events, they cannot be defined precisely; likewise, providing general structural integrity to a structure is a requirement that cannot be stated in simple terms. The performance provision..."members of a structure shall be effectively tied together to improve integrity of the overall structure," will require a level of judgment on the part of the design engineer, and will generate differing opinions among engineers as to how to effectively provide a general structural integrity solution for a particular framing system. It is obvious that all conditions that might be encountered in design cannot be specified in the code. The code, however, does set forth specific examples of certain reinforcing details for cast-in-place joists, beams, and two-way slab construction.

With damage to a support, top reinforcement which is continuous over the support, but not confined by stirrups, will tend to tear out of the concrete and will not provide the catenary action needed to bridge the damaged support. By making a portion of the bottom reinforcement in beams continuous over supports, some catenary action can be provided. By providing some continuous top and bottom reinforcement in edge or perimeter beams, an entire structure can be tied together; also, the continuous tie provided to perimeter beams of a structure will toughen the exterior portion of a structure, should an exterior column be severely damaged. Other examples of ways to detail for required integrity of a framing system to carry loads around a severely damaged member can be cited. The design engineer will need to evaluate his particular design for specific ways of handling the problem. The concept of providing general structural integrity is discussed in the Commentary of ASCE 7, *Minimum Design Loads for Buildings and Other Structures.*[3.6] The reader is referred to that document for further discussion of design concepts and details for providing general structural integrity.

7.13.2 Cast-in-Place Joists and Beams

Since 1989, the code requires continuous reinforcement in beams around the perimeter of the structure for structural integrity. The required amount is a minimum of one-sixth the tension reinforcement for negative moment at the support and one-fourth of the tension reinforcement for positive moment at the midspan. In either case the code requires a minimum of two bars and, mechanical and welded splices are explicitly permitted for splicing of continuous reinforcement in cast-in-place joists and beams. Figures 3-11 through 3-13 illustrate the required reinforcing details for the general case of cast-in-place joists and beams.

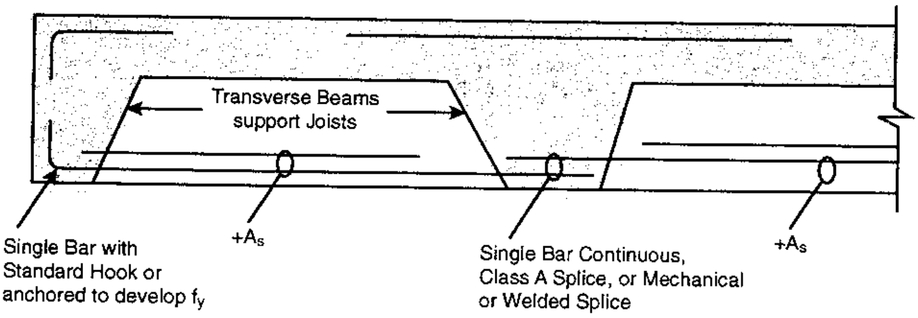


Figure 3-11 Continuity Reinforcement for Joist Construction

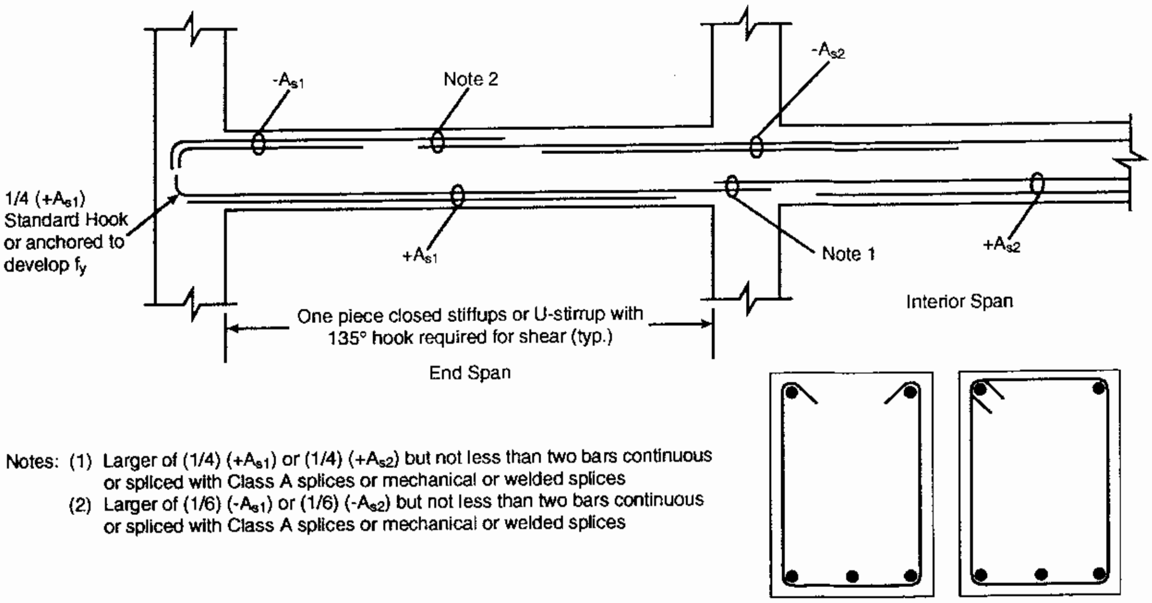
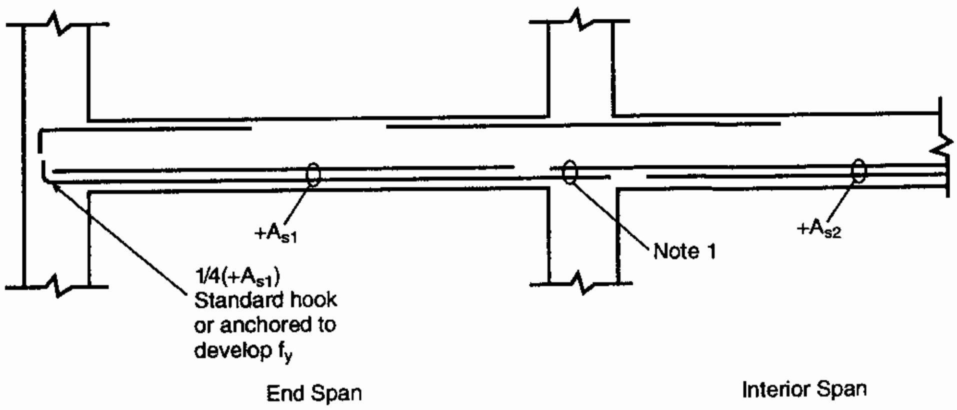


Figure 3-12 Continuity Reinforcement for Perimeter Beams



Note: (1) Larger of (1/4)(+A_{s1}) or (1/4)(+A_{s2}) but not less than two bars continuous or spliced with Class A splices or mechanical or welded splices

Figure 3-13 Continuity Reinforcement for Other Beams without Closed Stirrups

7.13.3 Precast Concrete Construction

While the requirements for structural integrity introduced in ACI 318-89 were prescriptive for cast-in-place construction, the '89 code provided only performance requirements for precast construction. This approach was made necessary because precast structures can be built in a lot of different ways. The code requires tension ties for precast concrete buildings of all heights. Connections that rely solely on friction due to gravity forces are not permitted.

The general requirement for structural integrity (7.13.1) states that "...members of a structure shall be effectively tied together...". The '89 commentary cautioned that for precast concrete construction, connection details should be arranged so as to minimize the potential for cracking due to restrained creep, shrinkage, and temperature movements. Ref. 3.7 contains information on industry practice for connections and detailing requirements. Prescriptive requirements recommended by the PCI for precast concrete bearing wall buildings are given in Ref. 3.8. Prescriptive structural integrity requirements for precast concrete structures were introduced for the first time in Chapter 16 of ACI 318-95 (see discussion in Part 23 of this publication).

7.13.4 Lift-Slab Construction

Section 7.13.4 refers the code user to 13.3.8.6 and 18.12.6 for lift-slab construction.

REFERENCES

3.1 *ACI Detailing Manual* — 2004, Publication SP-66(04), American Concrete Institute, Farmington Hills, MI, 2004.

3.2 *Manual of Standard Practice*, 27th edition, Concrete Reinforcing Steel Institute, Schaumburg, IL, 2001.

3.3 *Structural Welded Wire Fabric Detailing Manual*, WWR-600, Wire Reinforcement Institute, McLean, VA, 1994.

3.4 Babaei, K., and Hawkins, N.M., "Field Bending and Straightening of Reinforcing Steel," *Concrete International: Design and Construction*, V. 14, No. 1, January 1992.

3.5 *Standard Specification for Tolerances for Concrete Construction and Materials*, ACI 117-90, (reapproved 2002), American Concrete Institute, Farmington Hills, MI, 2002.

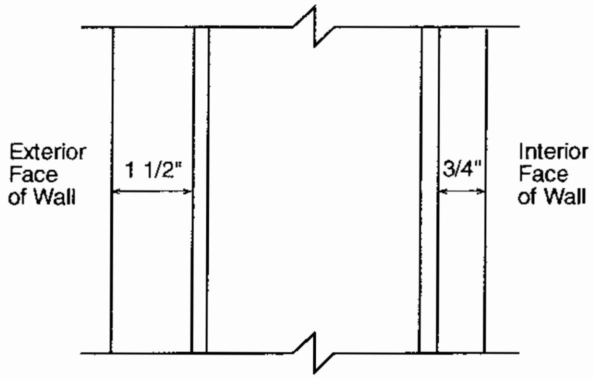
3.6 *Minimum Design Loads for Buildings and Other Structures*, (ASCE 7-02), American Society of Civil Engineers, Reston, VA, 2002.

3.7 *Design and Typical Details of Connections for Precast and Prestressed Concrete*, Publication MNL-123-88, Precast/Prestressed Concrete Institute, Chicago, IL, 1988.

3.8 PCI Building Code Committee, "Proposed Design Requirements for Precast Concrete," *PCI Journal*, V. 31, No. 6, Nov.-Dec. 1986, pp. 32-47.

Example 3.1—Placing Tolerance for Rebars

For the wall section shown below, with specified clear concrete cover indicated, determine the minimum cover permitted in construction, including the code tolerances on concrete cover.



Calculations and Discussion	Code Reference

Tolerance on concrete cover is minus 1/2 in., but in no case may the tolerance be more than 1/3 the specified concrete cover. *7.5.2.1*

1. For the exterior face, a measured 1 in. cover (1-1/2 - 1/2) is permitted. Actual bar placement may be within 1 in. of the side forms.

2. For the interior face, a measured 1/2 in. cover (3/4 - 1/4) is permitted. For the 3/4 in. specified cover, the tolerance limit is (1/3) (3/4) = 1/4 in. < 1/2 in.

As noted in the ACI 117 Standard[3.5], tolerances are a means to establish permissible variation in dimension and location, giving both the designer and the contractor parameters within which the work is to be performed. They are the means by which the designer conveys to the contractor the performance expectations.

4

Development and Splices of Reinforcement

GENERAL CONSIDERATIONS

The development length concept for anchorage of deformed bars and deformed wire in tension, is based on the attainable average bond stress over the length of embedment of the reinforcement. This concept requires the specified minimum lengths or extensions of reinforcement beyond all locations of peak stress in the reinforcement. Such peak stresses generally occur in flexural members at the locations of maximum stress and where adjacent reinforcement terminates or is bent.

The strength reduction factor ϕ is not used in Chapter 12 of the code since the specified development lengths already include an allowance for understrength.

12.1 DEVELOPMENT OF REINFORCEMENT—GENERAL

Development length or anchorage of reinforcement is required on both sides of a location of peak stress at each section of a reinforced concrete member. In continuous members, for example, reinforcement typically continues for a considerable distance on one side of a critical stress location so that detailed calculations are usually required only for the side where the reinforcement is terminated.

Until further research is completed and to ensure ductility and safety of structures built with high strength concrete, starting with the 1989 code, the term $\sqrt{f'_c}$ has been limited to 100 psi. Existing design equations for development of straight bars in tension and compression, and standard hooks in tension, are all a function of $\sqrt{f'_c}$. These equations were developed from results of tests on reinforcing steel embedded in concrete with compressive strengths of 3000 to 6000 psi. ACI Committee 318 was prudent in limiting $\sqrt{f'_c}$ at 100 psi pending completion of tests to verify applicability of current design equations to bars in high strength concrete.

12.2 DEVELOPMENT OF DEFORMED BARS AND DEFORMED WIRE IN TENSION

The provisions of 12.2 are based on the work of Orangun, Jirsa, and Breen[4.1], and Sozen and Moehle.[4.2] Development length of straight deformed bars and wires in tension, expressed in terms of bar or wire diameter, is given in 12.2.3 by the general equation:

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \lambda}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b \qquad \textit{Eq. (12-1)}$$

where

ℓ_d = development length, in.

d_b = nominal diameter of bar or wire, in.

f_y = specified yield strength of nonprestressed bar or wire, psi

f'_c = specified compressive strength of concrete, psi

ψ_t = **reinforcement location factor**
= 1.3 for horizontal reinforcement placed such that more than 12 in. of fresh concrete is cast below the development length or splice
= 1.0 for other reinforcement

ψ_e = **coating factor**
= 1.5 for epoxy-coated bars or wires with cover less than $3d_b$ or clear spacing less than $6d_b$
= 1.2 for all other epoxy-coated bars or wires
= 1.0 for uncoated reinforcement

The product of ψ_t and ψ_e need not be taken greater than 1.7.

ψ_s = **reinforcement size factor**
= 0.8 for No. 6 and smaller bars and deformed wires
= 1.0 for No. 7 and larger bars

λ = **lightweight aggregate concrete factor**
= 1.3 when lightweight aggregate concrete is used, or
= $6.7\sqrt{f'_c}/f_{ct}$, but not less than 1.0, when f_{ct} is specified
= 1.0 for normal weight concrete

c_b = **spacing or cover dimension**, in.
= the smaller of (1) distance from center of bar or wire being developed to the nearest concrete surface, and (2) one-half the center-to-center spacing of bars or wires being developed

K_{tr} = **transverse reinforcement index**

$$= \frac{A_{tr} f_{yt}}{1500sn}$$

where

A_{tr} = total cross-sectional area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed, in.^{2}

f_{yt} = specified yield strength of transverse reinforcement, psi

s = maximum spacing of transverse reinforcement within ℓ_d, center-to-center, in.

n = number of bars or wires being developed along the plane of splitting

Note that the term $\left(\frac{c_b + K_{tr}}{d_b}\right)$ cannot be taken greater than 2.5 (12.2.3) to safeguard against pullout type failures. In the 1989 and earlier editions of the code, the expression $0.03 d_b f_y / \sqrt{f'_c}$ was specified to prevent pullout type failures.

As a design simplification, it is conservative to assume $K_{tr} = 0$, even if transverse reinforcement is present. If a clear cover of $2d_b$ and a clear spacing between bars being developed of $4d_b$ is provided, variable "c" would equal $2.5d_b$. For the preceding conditions, even if $K_{tr} = 0$, the term $\left(\frac{c_b + K_{tr}}{d_b}\right)$ would equal 2.5.

The term $\left(\frac{c_b + K_{tr}}{d_b}\right)$ in the denominator of Eq. (12-1) accounts for the effects of small cover, close bar spacing, and confinement provided by transverse reinforcement. To further simplify computation of ℓ_d, preselected values for term $\left(\frac{c_b + K_{tr}}{d_b}\right)$ were chosen starting with the 1995 code. As a result, Equation (12-1) can take the simplified forms specified in 12.2.2, and shown below in Table 4-1. For discussion purposes only, the four equations are identified in this table as Equations A through D. Note that these identifiers do not appear in the code. In Eqs. A and B, the term $\left(\frac{c_b + K_{tr}}{d_b}\right) = 1.5$, while in Eqs. C and D, $\left(\frac{c_b + K_{tr}}{d_b}\right) = 1.0$. Equations A and C include a reinforcement size factor $\psi_s = 0.8$. The 20 percent reduction is based on comparisons with past provisions and numerous test results.

Equations A and B can only be applied if one of the following two different sets of conditions is satisfied:

Table 4-1 Development Lengths ℓ_d Specified in 12.2.2

	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	(Eq. A) $\left(\frac{f_y \psi_t \psi_e \lambda}{25\sqrt{f'_c}}\right) d_b$	(Eq. B) $\left(\frac{f_y \psi_t \psi_e \lambda}{20\sqrt{f'_c}}\right) d_b$
Other cases	(Eq. C) $\left(\frac{3 f_y \psi_t \psi_e \lambda}{50\sqrt{f'_c}}\right) d_b$	(Eq. D) $\left(\frac{3 f_y \psi_t \psi_e \lambda}{40\sqrt{f'_c}}\right) d_b$

Set #1

The following three conditions must simultaneously be satisfied:

1. The clear spacing of reinforcement being developed or spliced should not be less than the diameter of reinforcement being developed, d_b,
2. The clear cover for reinforcement being developed should not be less than d_b, and
3. Minimum amount of stirrups or ties throughout ℓ_d should not be less than the minimum values specified in 11.5.5.3 for beams or 7.10.5 for columns.

Set #2

The following two conditions must simultaneously be satisfied:

1. The clear spacing of reinforcement being developed or spliced should not be less than $2d_b$, and
2. The clear cover should not be less than d_b.

If all the conditions of Set #1 or of Set #2 cannot be satisfied, then Eqs. C or D must be used. Note that Eq. D is identical to Eq. (12-1) with $\left(\frac{c_b + K_{tr}}{d_b}\right) = 1.0$ and reinforcement size factor $\gamma = 1.0$.

Although Eqs. A through D are easier to use than Eq. (12-1), the term $\left(\frac{c_b + K_{tr}}{d_b}\right)$ can only assume the value of 1.0 (Eqs. C and D) or 1.5 (Eqs. A and B). On the other hand, Eq. (12-1) may require a little extra effort, but the value of expression $\left(\frac{c_b + K_{tr}}{d_b}\right)$ can be as high as 2.5. Therefore, the development lengths ℓ_d computed by Eq. (12-1) could be substantially shorter than development lengths computed from the simplified equations of 12.2.2.

The development lengths of Table 4-1 can be further simplified for specific conditions. For example, for Grade 60 reinforcement (f_y = 60,000 psi) and different concrete compressive strengths, assuming normal weight concrete ($\lambda = 1.0$) and uncoated ($\psi_e = 1.0$) bottom bars or wires ($\psi_t = 1.0$), values of ℓ_d as a function of d_b can be determined as shown in Table 4-2.

Table 4-2 Development Length ℓ_d for Grade 60, Uncoated, Bottom Reinforcement in Normal Weight Concrete

	f'_c psi	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars being developed or spliced not less than d_b, clear cover not less than d_b, and beam stirrups or column ties throughout ℓ_d not less than the code minimum or Clear spacing of bars being developed or spliced not less than $2d_b$ and clear cover not less than d_b	3000	$44d_b$	$55d_b$
	4000	$38d_b$	$47d_b$
	5000	$34d_b$	$42d_b$
	6000	$31d_b$	$39d_b$
	8000	$27d_b$	$34d_b$
	10,000	$24d_b$	$30d_b$
Other cases	3000	$66d_b$	$82d_b$
	4000	$57d_b$	$71d_b$
	5000	$51d_b$	$64d_b$
	6000	$46d_b$	$58d_b$
	8000	$40d_b$	$50d_b$
	10,000	$36d_b$	$45d_b$

As in previous editions of the code, development length of straight deformed bars or wires, including all modification factors must not be less than 12 in.

12.2.5 Excess Reinforcement

Reduction in ℓ_d may be permitted by the ratio [(A_s required)/(A_s provided)] when excess reinforcement is provided in a flexural member. Note that this reduction does not apply when the full f_y development is required, as for tension lap splices in 7.13, 12.15.1, and 13.3.8.5, development of positive moment reinforcement at supports in 12.11.2, and for development of shrinkage and temperature reinforcement according to 7.12.2.3. Note also that this reduction in development length is not permitted for reinforcement in structures located in regions of high seismic risk or for structures assigned to high seismic performance or design categories (see 21.2.1.4).

Reduced ℓ_d computed after applying the excess reinforcement according to 12.2.5 must not be less than 12 in.

12.3 DEVELOPMENT OF DEFORMED BARS AND DEFORMED WIRE IN COMPRESSION

Shorter development lengths are required for bars in compression than in tension since the weakening effect of flexural tension cracks in the concrete is not present. The development length for deformed bars or deformed wire in compression is $\ell_{dc} = 0.02 d_b f_y / \sqrt{f'_c}$, but not less than $0.0003 d_b f_y$ or 8 in. Note that ℓ_{dc} may be reduced where excess reinforcement is provided (12.3.3(a)) and where "confining" ties or spirals are provided around the reinforcement (12.3.3(b)). Note that the tie and spiral requirements to permit the 25 percent reduction in development length are somewhat more restrictive than those required for "regular" column ties in 7.10.5 and less restrictive than those required for spirals in 7.10.4. For reference, compression development lengths for Grade 60 bars are given in Table 4-3.

Table 4-3 Compression Development Length ℓ_{dc} (inches) for Grade 60 Bars

Bar Size	f'_c (Normal Weight Concrete), psi		
No.	3000	4000	≥ 4444 *
3	8.2	7.1**	6.8**
4	11.0	9.5	9.0
5	13.7	11.9	11.3
6	16.4	14.2	13.5
7	19.2	16.6	15.8
8	21.9	19.0	18.0
9	24.7	21.4	20.3
10	27.8	24.1	22.9
11	30.9	26.8	25.4
14	37.1	32.1	30.5
18	49.4	42.8	40.6

* *For $f'_c \geq 4444$ psi, minimum basic development length $0.0003d_bf_y$ governs; for Grade 60 bars, $\ell_{dc} = 18d_b$.*

** *Development length ℓ_{dc} (including applicable modification factors) must not be less than 8 in.*

12.4 DEVELOPMENT OF BUNDLED BARS

Increased development length for individual bars within a bundle, whether in tension or compression, is required when 3 or 4 bars are bundled together. The additional length is needed because the grouping makes it more difficult to mobilize resistance to slippage from the "core" between the bars. The modification factor is 1.2 for a 3-bar bundle, and 1.33 for a 4-bar bundle. Other pertinent requirements include 7.6.6.4 concerning cut-off points of individual bars within a bundle, and 12.14.2.2 relating to lap splices of bundled bars.

Where the factors of 12.2 are based on bar diameter d_b, a unit of bundled bars must be treated as a single bar of a diameter derived from the total equivalent area. See Table 3-7 in Part 3 of this document.

12.5 DEVELOPMENT OF STANDARD HOOKS IN TENSION

The current provisions for hooked bar development were first introduced in the 1983 code. They represented a major departure from the hooked-bar anchorage provisions of earlier codes in that they uncoupled hooked-bar anchorages from straight bar development and gave total hooked-bar embedment length directly. The current provisions not only simplify calculations for hook anchorage lengths but also result in a required embedment length considerably less, especially for the larger bar sizes, than that required by earlier codes. Provisions are given in 12.5 for determining the development length of deformed bars with standard end hooks. End hooks can only be considered effective in developing bars in tension, and not in compression (see 12.1.1 and 12.5.5). Only "standard" end hooks (see 7.1) are considered; anchorage capacity of end hooks with larger end diameters cannot be determined by the provisions of 12.5.

In applying the hook development provisions, the first step is to calculate the development length of the hooked bar, ℓ_{dh} from 12.5.2. This length is then multiplied by the applicable modification factor or factors of 12.5.3. Development length ℓ_{dh} is measured from the critical section to the outside end of the standard hook, i.e., the straight embedment length between the critical section and the start of the hook, plus the radius of bend of the hook, plus one-bar diameter. For reference, Fig. 4-1 shows ℓ_{dh} and the standard hook details (see 7.1) for all standard bar sizes. For 180 degree hooks normal to exposed surfaces, the embedment length should provide for a minimum distance of 2 in. beyond the tail of the hook.

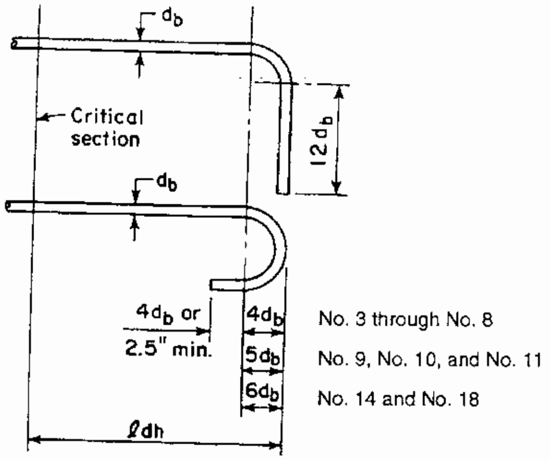


Figure 4-1 Development Length ℓ_{dh} of Standard Hooks

12.5.2 Development Length ℓ_{dh}

The development length, ℓ_{dh}, for standard hooks in tension is given in 12.5.2 as:

$$\ell_{dh} = \left(\frac{0.02\psi_e \lambda f_y}{\sqrt{f'_c}}\right) d_b$$

where $\psi_e = 1.2$ for epoxy-coated reinforcement[4.3] and $\lambda = 1.3$ for lightweight aggregate concrete. For other cases, ψ_e and λ are equal to 1.0.

Table 4-4 lists the development length of hooked bars embedded in normal weight concrete with different specified compressive strengths and uncoated Grade 60 reinforcing bars.

*Table 4-4 Development Length ℓ_{dh} (inches) of Standard Hooks for Uncoated Grade 60 Bars**

Bar Size	f'_c (Normal Weight Concrete), psi					
No.	3000	4000	5000	6000	8000	10,000
3	8.2	7.1	6.4	5.8	5.0	4.5
4	11.0	9.5	8.5	7.7	6.7	6.0
5	13.7	11.9	10.6	9.7	8.4	7.5
6	16.4	14.2	12.7	11.6	10.1	9.0
7	19.2	16.6	14.8	13.6	11.7	10.5
8	21.9	19.0	17.0	15.5	13.4	12.0
9	24.7	21.4	19.1	17.5	15.1	13.5
10	27.8	24.1	21.6	19.7	17.0	15.2
11	30.9	26.8	23.9	21.8	18.9	16.9
14	37.1	32.1	28.7	26.2	22.7	20.3
18	49.5	42.8	38.3	35.0	30.3	27.1

* Development length ℓ_{dh} (including modification factors) must not be less than the larger of $8d_b$ or 6 in.

12.5.3 Modification Factors

The ℓ_{dh} modification factors listed in 12.5.3 account for:

- Favorable confinement conditions provided by increased cover (12.5.3(a))
- Favorable confinement provided by transverse ties or stirrups to resist splitting of the concrete (12.5.3(b) and (c))

- More reinforcement provided than required by analysis (12.5.3(d))

The side cover (normal to plane of hook), and the cover on bar extension beyond 90 degree hook referred to in 12.5.3(a) are illustrated in Fig. 4-2.

Note that requirements for 90-degree and 180-degree hooks are clarified in 12.5.3 of the 2002 code. Figures R12.5.3 (a) and R12.5.3 (b) illustrate the cases where the modification factor of 12.5.3 (b) may be used.

After multiplying the development length ℓ_{dh} by the applicable modification factor or factors, the resulting development length ℓ_{dh} must not be less than the larger of $8d_b$ or 6 in.

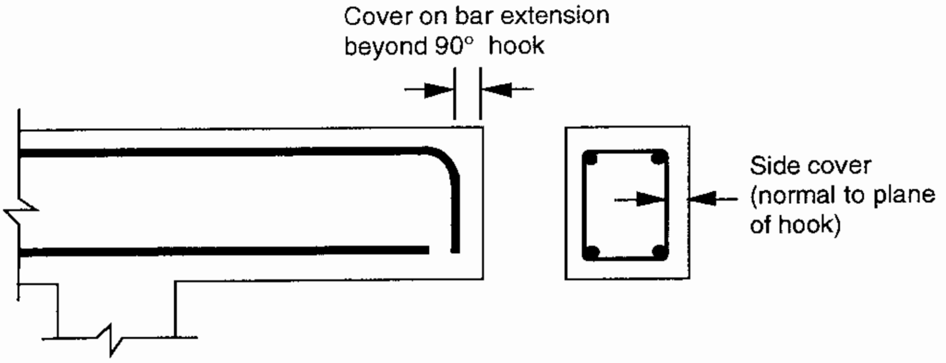


Figure 4-2 Concrete Covers Referenced in 12.5.3(a)

12.5.4 Standard Hook at Discontinuous Ends

Section 12.5.4 is a special provision for hooked bars terminating at discontinuous ends of members, such as at the ends of simply-supported beams, at free ends of cantilevers, and at ends of members framing into a joint where the member does not extend beyond the joint. If the full strength of a hooked bar must be developed, and both side cover and top (or bottom) cover over the hook are less than 2.5 in., 12.5.4 requires the hook to be enclosed within ties or stirrup for the full development length, ℓ_{dh}. Spacing of the ties or stirrup must not exceed $3d_b$, where d_b is the diameter of the hooked bar. In addition, the modification factor of 0.8 for confinement provided by ties or stirrups (12.5.3(b) and (c)) does not apply to the special condition covered by 12.5.4. At discontinuous ends of slabs with concrete confinement provided by the slab continuous on both sides normal to the plane of the hook, the provisions of 12.5.4 do not apply.

12.6 MECHANICAL ANCHORAGE

Section 12.6 permits the use of mechanical devices for development of reinforcement, provided their adequacy without damaging the concrete has been confirmed by tests. Section 12.6.3 reflects the concept that development of reinforcement may consist of a combination of mechanical anchorage plus additional embedment length of the reinforcement. For example, when a mechanical device cannot develop the design strength of a bar, additional embedment length must be provided between the mechanical device and the critical section.

12.7 DEVELOPMENT OF WELDED DEFORMED WIRE REINFORCEMENT IN TENSION

For welded deformed wire reinforcement, development length is measured from the critical section to the end of the wire. As specified in 12.7.1, development of welded deformed wire is computed as the product of ℓ_d from 12.2.2 or 12.2.3 times a wire reinforcementfactor from 12.7.2 or 12.7.3. Where provided reinforcement is more than required, development length can be reduced by 12.2.5. In applying 12.2.2 or 12.2.3 to epoxy-coated deformed wire reinforcement, a coating factor $\psi_e = 1.0$ can be used. The resulting development length ℓ_d

cannot be less than 8 in., except in computation of lap splice lengths (see 12.18) and development of web reinforcement (see 12.13). Figure 4-3 shows the development length requirements for welded deformed wire reinforcement.

To apply the wire reinforcement factor of 12.7.2 to the development length of deformed wire reinforcement requires at least one cross wire located within the development length at a distance no less than 2 in. from the critical section. The wire reinforcement factor given in 12.7.2 is the greater of $(f_y - 35{,}000)/f_y$ or $5d_b/s$, but need not be taken greater than 1.0. s is the spacing between the wires to be developed.

If there is no cross wire within the development length, or the cross wire is less than 2 in. from the critical section, the development length of welded deformed wire reinforcement must be computed from 12.2.2 or 12.2.3. For this condition, the wire reinforcement factor must be taken equal to 1.0 (see 12.7.3).

According to ASTM A497, welded deformed steel wire reinforcement may consist solely of deformed steel wire (ASTM A496), or welded deformed steel wire reinforcement (ASTM A496) in one direction in combination with plain steel wire (ASTM A82) in the orthogonal direction. In the latter case, the reinforcement must be developed according to 12.8 for plain wire reinforcement.

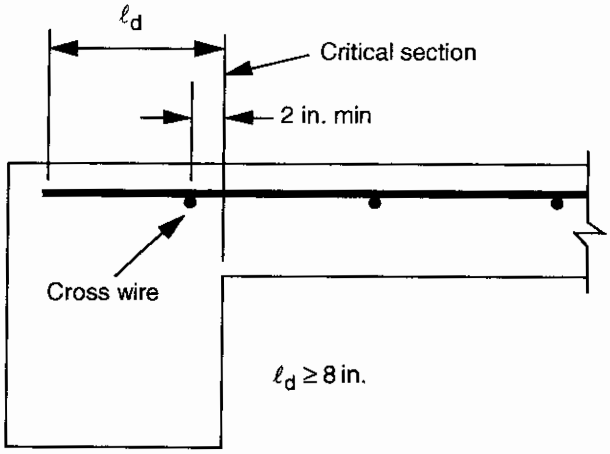


Figure 4-3 Development of Welded Deformed Wire Reinforcement

12.8 DEVELOPMENT OF WELDED PLAIN WIRE REINFORCEMENT IN TENSION

For welded plain wire reinforcement, the development length is measured from the point of critical section to the outermost cross wire. Full development of plain reinforcement ($A_w f_y$) is achieved by embedment of at least two cross wires beyond the critical section, with the closer cross wire located not less than 2 in. from the critical section. Section 12.8 further requires that the length of embedment from critical section to outermost cross wire not be less than $\ell_d = 0.27(A_b/s)(f_y/\sqrt{f'_c})\lambda$, nor less than 6 in. If more reinforcement is provided than that required by analysis, the development length ℓ_d may be reduced by the ratio of (A_s required)/(A_s provided). The 6 in. minimum development length does not apply to computation of lap splice lengths (see 12.19). Figure 4-4 shows the development length requirements for welded plain wire reinforcement.

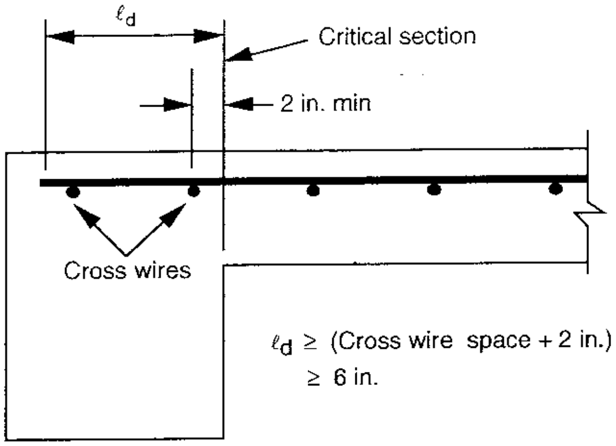


Figure 4-4 Development of Welded Plain Wire Reinforcement

For fabrics made with smaller wires, embedment of two cross wires, with the closer cross wire not less than 2 in. from the critical section, is usually adequate to develop the full yield strength of the anchored wires. Fabrics made with larger (closely spaced) wires will require a longer embedment ℓ_d.

For example, check fabric 6 × 6-W4 × W4 with f'_c = 3000 psi, f_y = 60,000 psi, and normal weight concrete (λ = 1.0).

$$\ell_d = 0.27 \times (A_b/s_s) \times \left(f_y/\sqrt{f'_c}\right) \times \lambda$$

$$= 0.27 \times (0.04/6) \times (60{,}000/\sqrt{3000}) \times 1.0 = 1.97 \text{ in.}$$

$$< 6 \text{ in.}$$

$$< (1 \text{ space} + 2 \text{ in.}) \quad \text{governs}$$

Two cross wire embedment plus 2 in. is satisfactory (see Fig. 4-5).

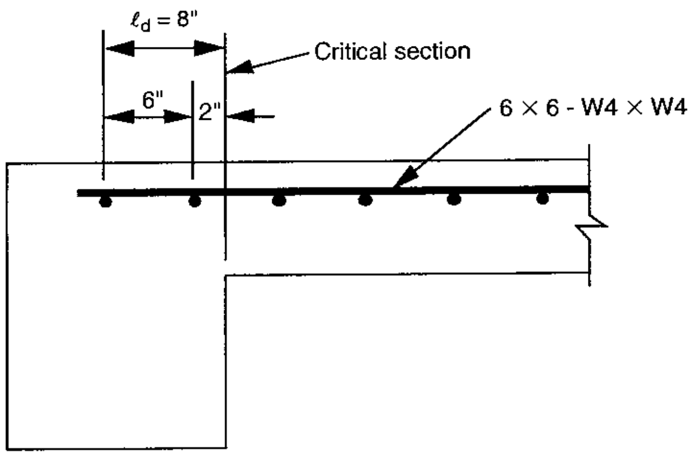


Figure 4-5 Development of 6 × 6-W4 × W4 Welded Wire Reinforcement

Check fabric 6 × 6-W20 × W20:

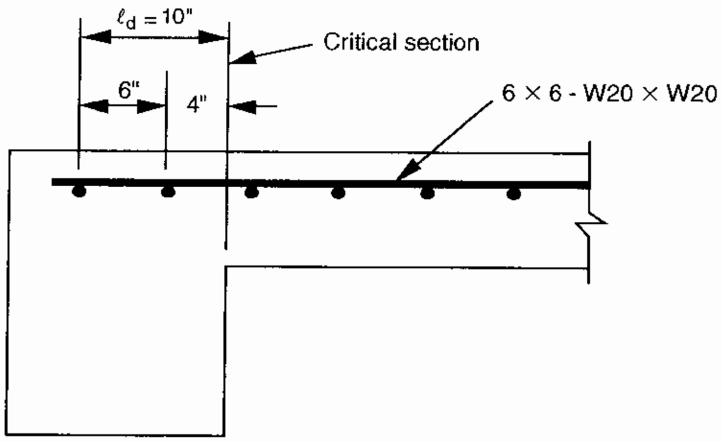
$$\ell_d = 0.27 \times (0.20/6) \times \left(60{,}000/\sqrt{3000}\right) \times 1.0 = 9.9 \text{ in.}$$

$$> 6 \text{ in.}$$

$$> (1 \text{ space} + 2 \text{ in.})$$

As shown in Fig. 4-6, an additional 2 in. beyond the two cross wires plus 2 in. embedment is required to fully develop the W20 fabric. If the longitudinal spacing is reduced to 4 in. (4 × 6-W20 × W20), a minimum ℓ_d of 15 in. is required for full development, i.e. 3 cross wires plus 3 in. embedment.

References 4.4 and 4.5 provide design aids for welded wire reinforcement, including development length tables for both deformed and plain welded wire reinforcement.



Note: If end support is not wide enough for straight embedment, the development length ℓ_d may be bent down (hooked) into support.

Figure 4-6 Development of 6 × 6-W20 × W20 Fabric

12.9 DEVELOPMENT OF PRESTRESSING STRAND

Prestressed concrete members may be either pretensioned or post-tensioned. In post-tensioned applications, development of tendons is accomplished through mechanical anchorage. Tendons may include strands, wires or high-strength bars.

In pretensioned members, tendons typically consist of seven-wire strands. Development length ℓ_d (in inches) of strands is specified in 12.9.1 and is computed from Eq. (12-2), which was formerly in R12.9:

$$\ell_d = \left(\frac{f_{se}}{3000}\right) d_b + \left(\frac{f_{ps} - f_{se}}{1000}\right) d_b \qquad \textit{Eq. (12-2)}$$

where f_{ps} = stress in prestressed reinforcement at nominal strength, psi
f_{se} = effective stress in prestressed reinforcement after all prestress losses, psi
d_b = nominal diameter of strand, in.

The expressions in parentheses are dimensionless.

The term $\left(\frac{f_{se}}{3000}\right) d_b$ represents the transfer length of the strand (ℓ_t), i.e., the distance over which the strand should be bonded to the concrete to develop f_{se} in the strand. The second term, $\left[\left(f_{ps}-f_{se}\right)/1000\right] d_b$, represents the flexural bond length, i.e., the additional length over which the strand should be bonded so that a stress f_{ps} may develop in the strand at nominal strength of the member.

Where bonding of one or more strands does not extend to the end of the member, critical sections may be at locations other than those where full design strength is required (see 12.9.2). In such cases, a more detailed analysis may be required. Similarly, where heavy concentrated loads occur within the strand development length, critical sections may occur away from the section that is required to develop full design strength.

Note that two times the development length specified in 12.9.1 is required for "debonded" strands (12.9.3) when the member is designed allowing tension in the precompressed tensile zone under service load conditions.

In some pretensioned applications, total member length may be shorter than two times the development length. This condition may be encountered in very short precast, prestressed concrete members. In such cases, the strands will not be able to develop f_{ps}. Maximum usable stress in underdeveloped strands can be derived as illustrated in Fig. 4-7. The maximum strand stress, f_{max}, at distance ℓ_x from girder end can be determined for the condition of $\ell_t < \ell_x < \ell_d$ as follows:

$$
\begin{aligned}
f_{max} &= f_{se} + \Delta f \\
&= f_{se} + \frac{(f_{ps} - f_{se})}{(f_{ps} - f_{se})\, d_b}\left(\ell_x - \frac{f_{se}}{3000}\, d_b\right) \\
&= f_{se} + \frac{\ell_x}{d_b} - \frac{f_{se}}{3000}
\end{aligned}
$$

Therefore,

$$f_{max} = \frac{\ell_x}{d_b} + \frac{2}{3000}\, f_{se}$$

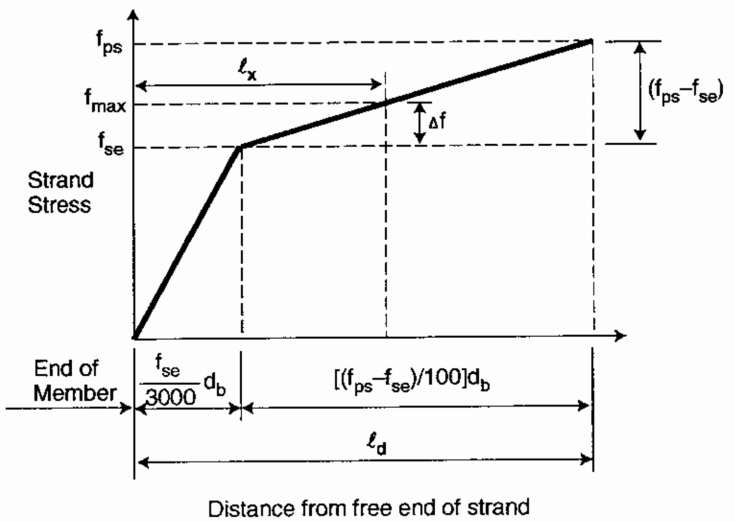
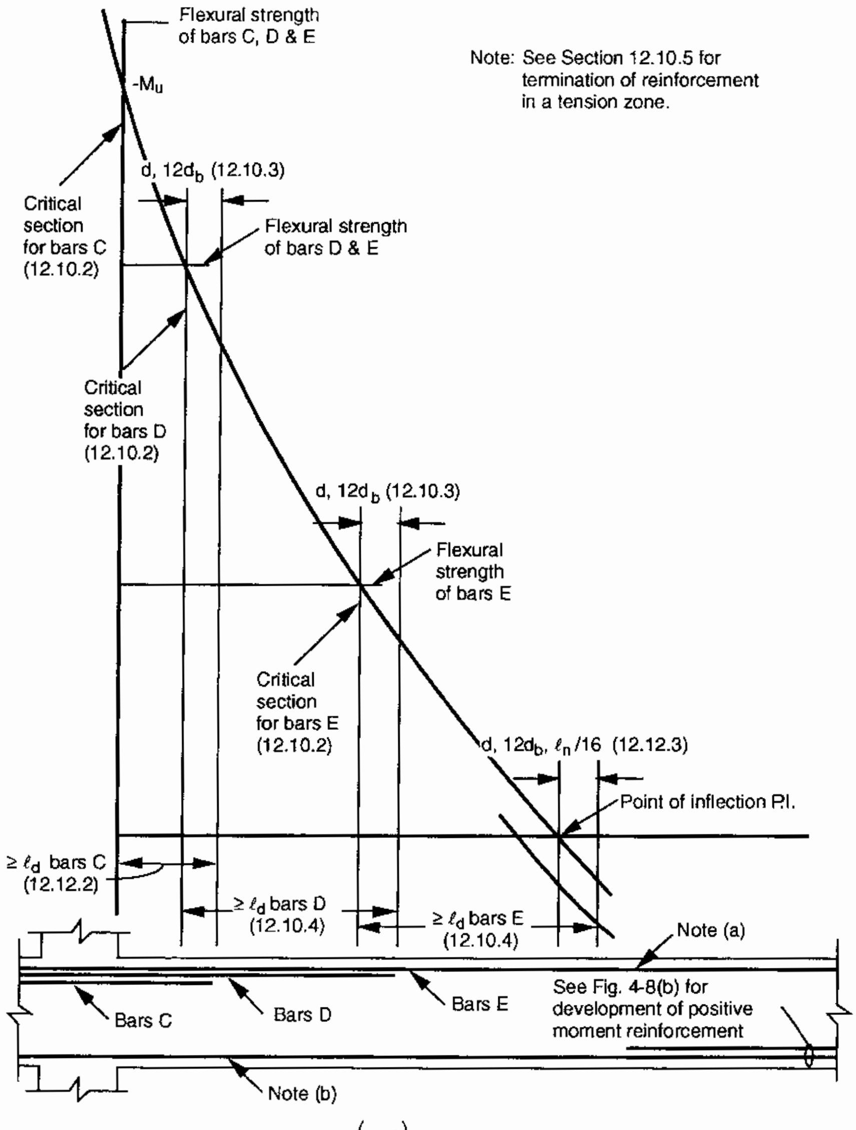


Figure 4-7 Strand Transfer and Development Lengths

12.10 DEVELOPMENT OF FLEXURAL REINFORCEMENT—GENERAL

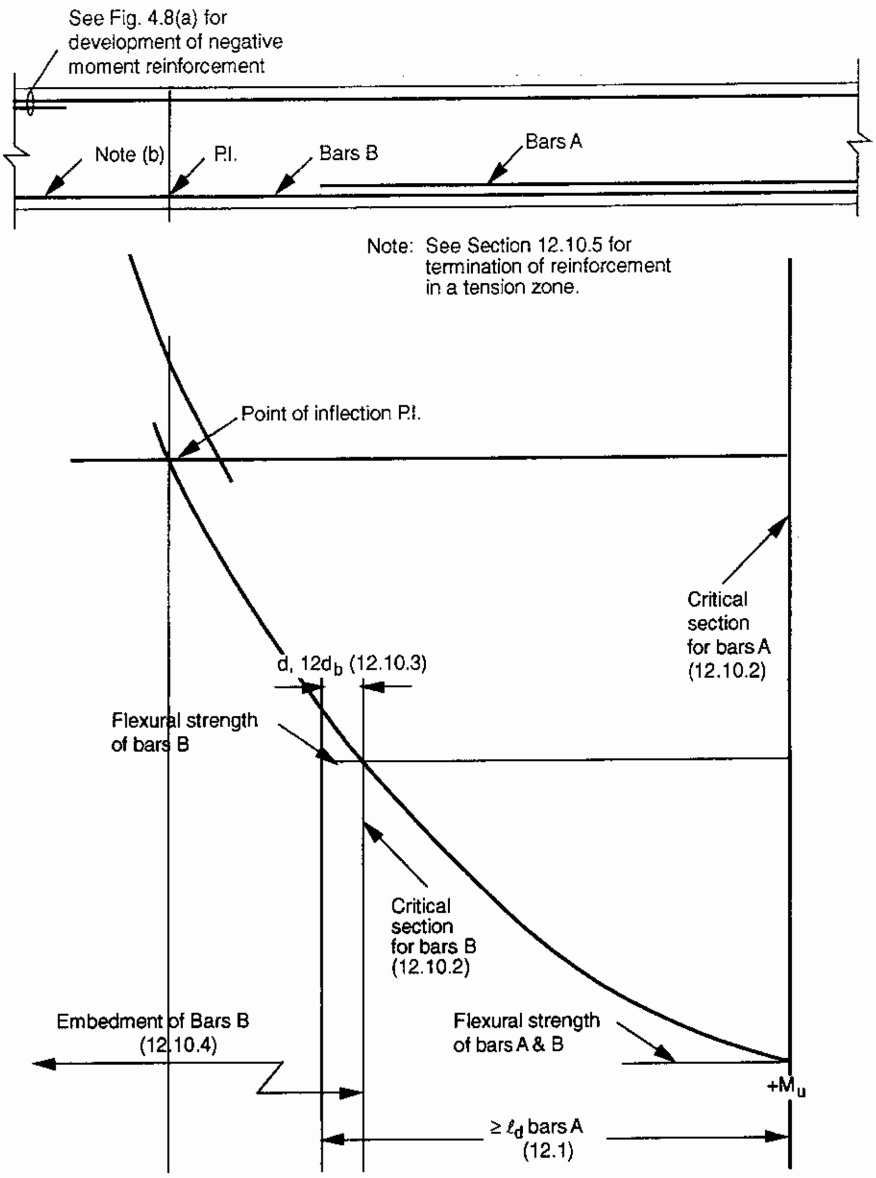
Section 12.10 gives the basic requirements for providing development length of reinforcement from the points of maximum or critical stress. Figures 4-8(a) and (b) illustrate typical critical sections and code requirements for development and termination of flexural reinforcement in a continuous beam. Points of maximum positive and negative moments $\left(M_u^+ \text{ and } M_u^-\right)$ are critical sections, from which adequate anchorage ℓ_d must be provided. Critical sections are also at points within the span where adjacent reinforcement is terminated; continuing bars must have adequate anchorage ℓ_d from the theoretical cut-off points of terminated bars (see 12.10.4). Note also that terminated bars must be extended beyond the theoretical cut-off points in accordance with 12.10.3. This extension requirement is to guard against possible shifting of the moment diagram due to load variation, settlement of supports, and other unforeseen changes in the moment conditions. Development lengths ℓ_d are determined from 12.2.



Note (a): Portion of total negative reinforcement $\left(A_s^-\right)$ must be continuous (or spliced with a Class A splice or a mechanical or welded splice satisfying 12.14.3) along full length of perimeter beams (7.13.2.2).

(a) Negative Moment Reinforcement

Figure 4-8 Development of Positive and Negative Moment Reinforcement



Note (b): Portion of total positive reinforcement $\left(A_s^+\right)$ must be continuous (or spliced with a Class A splice or a mechanical or welded splice satisfying 12.14.3) along full length of perimeter beams and of beams without closed stirrups (7.13.2.2). See also 7.13.2.4.

(b) Positive Moment Reinforcement

Figure 4-8 Development of Positive and Negative Moment Reinforcement — continued —

Sections 12.10.1 and 12.10.5 address the option of anchoring tension reinforcement in a compression zone. Research has confirmed the need for restrictions on terminating bars in a tension zone. When flexural bars are cut off in a tension zone, flexural cracks tend to open early. If the shear stress in the area of bar cut-off and tensile stress in the remaining bars at the cut-off location are near the permissible limits, diagonal tension cracking tends to develop from the flexural cracks. One of the three alternatives of 12.10.5 must be satisfied to reduce the possible occurrence of diagonal tension cracking near bar cut-offs in a tension zone. Section 12.10.5.2 requires excess stirrup area over that required for shear and torsion. Requirements of 12.10.5 are not intended to apply to tension splices.

Section 12.10.6 is for end anchorage of tension bars in special flexural members such as brackets, members of variable depth, and others where bar stress, f_s, does not decrease linearly in proportion to a decreasing moment. In Fig. 4-9, the development length ℓ_d into the support is probably less critical than the required development length. In such a case, safety depends primarily on the outer end anchorage provided. A welded cross bar of equal diameter should provide an effective end anchorage. A standard end hook in the vertical plane may not be effective because an essentially plain concrete corner might exist near the load and could cause localized failure. Where brackets are wide and loads are not applied too close to the corners, U-shaped bars in a horizontal plane provide effective end hooks.

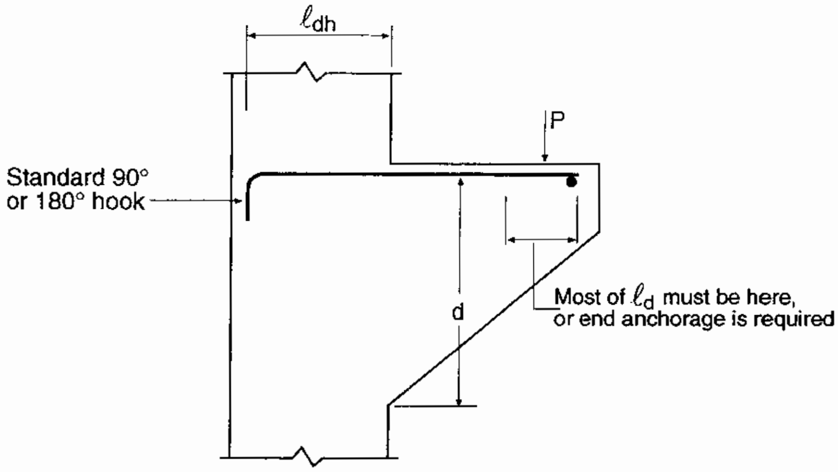


Figure 4-9 Special Member Largely Dependent on End Anchorage

12.11 DEVELOPMENT OF POSITIVE MOMENT REINFORCEMENT

To further guard against possible shifting of moments due to various causes, 12.11.1 requires specific amounts of positive moment reinforcement to be extended along the same face of the member into the support, and for beams, to be embedded into the support at least 6 in. The specified amounts are one-third for simple members and one-fourth for continuous members. In Fig. 4-8(b), for example, the area of Bars "B" would have to be at least one-fourth of the area of reinforcement required at the point of maximum positive moment M_u^+.

Section 12.11.2 is intended to assure ductility in the structure under severe overload, as might be experienced in a strong wind or earthquake. In a lateral load resisting system, full anchorage of the reinforcement extended into the support provides for possible stress reversal under such overload. Anchorage must be provided to develop the full yield strength in tension at the face of the support. The provision will require such members to have bottom bars lapped at interior supports or hooked at exterior supports. The full anchorage requirement does not apply to any excess reinforcement provided at the support.

Section 12.11.3 limits bar sizes for the positive moment reinforcement at simple supports and at points of inflection. In effect, this places a design restraint on flexural bond stress in areas of small moment and large shear. Such a condition could exist in a heavily loaded beam of short span, thus requiring large size bars to be developed within a short distance. Bars should be limited to a diameter such that the development length ℓ_d computed for f_y according to 12.2 does not exceed $(M_n/V_u) + \ell_a$ (12.11.3). The limit on bar size at simple supports is waived if the bars have standard end hooks or mechanical anchorages terminating beyond the centerline of the support. Mechanical anchorages must be equivalent to standard hooks.

The length (M_n/V_u) corresponds to the development length of the maximum size bar permitted by the previously used flexural bond equation. The length (M_n/V_u) may be increased 30% when the ends of the bars are confined by a compressive reaction, such as provided by a column below, but not when a beam frames into a girder.

For the simply-supported beam shown in Fig. 4-10, the maximum permissible ℓ_d for Bars "a" is 1.3 $M_n/V_u + \ell_a$. This has the effect of limiting the size of bar to satisfy flexural bond. Even though the total embedment length from the critical section for Bars "a" is greater than 1.3 $M_n/V_u + \ell_a$, the size of Bars "a" must be limited so that $\ell_d \leq 1.3\ M_n/V_u + \ell_a$. Note that M_n is the nominal flexural strength of the cross-section (without the ϕ factor). As noted previously, larger bar sizes can be accommodated by providing a standard hook or mechanical anchorage at the end of the bar within the support. At a point of inflection (see Fig. 4-11), the positive moment reinforcement must have a development length ℓ_d, as computed by 12.2, not to exceed the value of $(M_n/V_u) + \ell_a$, with ℓ_a not greater than d or $12d_b$, whichever is greater.

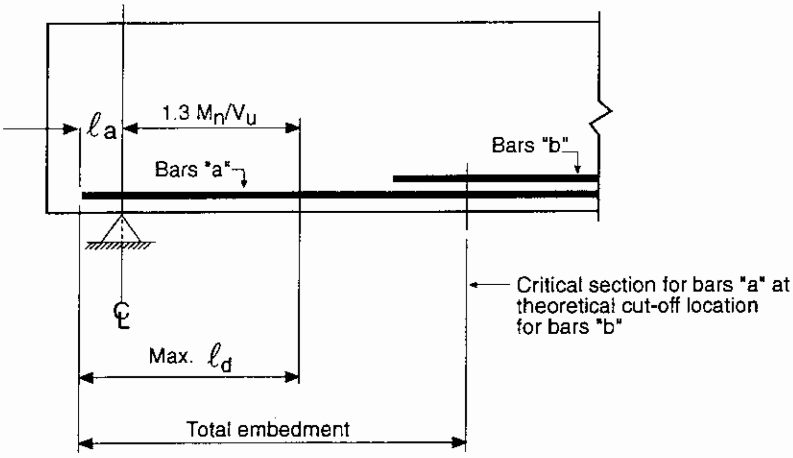


Figure 4-10 Development Length Requirements at Simple Support (straight bars)

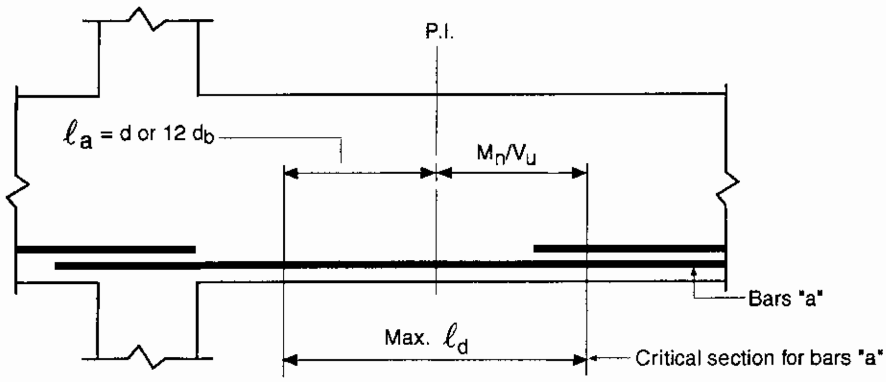
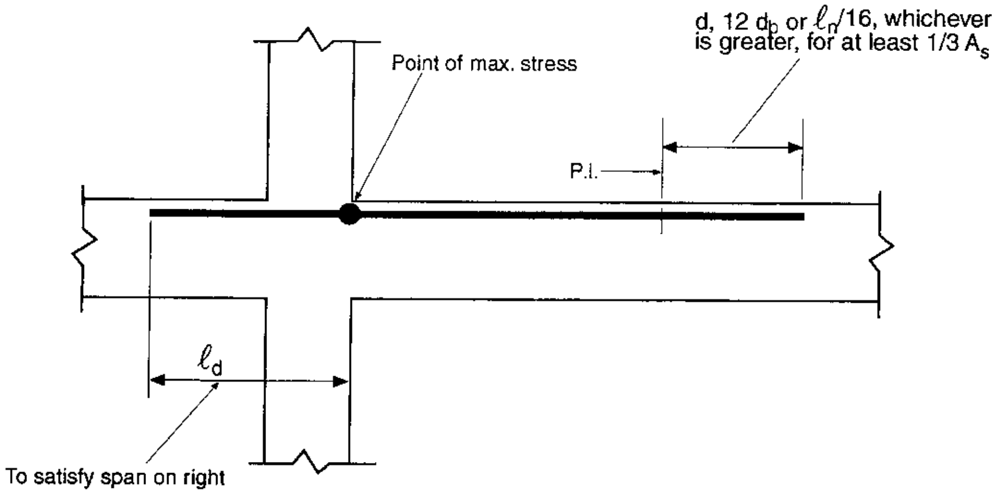


Figure 4-11 Concept for Determining Maximum Size of Bars "a" at Point of Inflection (12.11.3)

Sections 12.11.4 and 12.12.4 address development of positive and negative moment reinforcement in deep flexural members. The provisions specify that at simple supports of deep beams, positive moment tension reinforcement should be anchored to develop its specified yield strength f_y in tension at the face of the support. However, if the design is carried out using the strut-and-tie method of Appendix A, this reinforcement shall be anchored in accordance with A.4.3. At interior supports of deep beams, both positive and negative moment tension reinforcement shall be continuous or be spliced with that of the adjacent spans.

12.12 DEVELOPMENT OF NEGATIVE MOMENT REINFORCEMENT

The requirements in 12.12.3 guard against possible shifting of the moment diagram at points of inflection. At least one-third of the negative moment reinforcement provided at a support must be extended a specified embedment length beyond a point of inflection. The embedment length must be the effective depth of the member d, $12d_b$, or 1/16 the clear span, whichever is greater, as shown in Figs. 4-8 and 4-12. The area of Bars "E" in Fig. 4-8(a) must be at least one-third the area of reinforcement provided for $-M_u$ at the face of the support. Anchorage of top reinforcement in tension beyond interior support of continuous members usually becomes part of the adjacent span top reinforcement, as shown in Fig. 4-12.



(Usually such anchorage becomes part of adjacent beam reinforcement)

Figure 4-12 Anchorage into Adjacent Beam

Standard end hooks are an effective means of developing top bars in tension at exterior supports as shown in Fig. 4-13. Code requirements for development of standard hooks are discussed above in 12.5.

12.13 DEVELOPMENT OF WEB REINFORCEMENT

Stirrups must be properly anchored so that the full tensile force in the stirrup can be developed at or near mid-depth of the member. To function properly, stirrups must be extended as close to the compression and tension surfaces of the member as cover requirements and proximity of other reinforcement permit (12.13.1). It is equally important for stirrups to be anchored as close to the compression face of the member as possible because flexural tension cracks initiate at the tension face and extend towards the compression zone as member strength is approached.

The ACI code anchorage details for stirrups have evolved over many editions of the code and are based primarily on past experience and performance in laboratory tests. For No. 5 bar and smaller, stirrup anchorage is provided by a standard stirrup hook (90 degree bend plus $6d_b$ extension at free end of bar)* around a longitudinal bar (12.13.2.1). The same anchorage detail is permitted for the larger stirrup bar sizes, No. 6, No. 7, and No. 8, in Grade 40. Note that for the larger bar sizes, the 90 degree hook detail requires a $12d_b$ extension at the free end of the bar (7.1.3(b)). Fig. 4-14 illustrates the anchorage requirement for U-stirrups fabricated from deformed bars and deformed wire.

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* For structures located in regions of high seismic risk, stirrups required to be hoops must be anchored with a 135-degree bend plus $6d_b$ (but not less than 3 in.) extension. See definition of seismic hook in 21.1.

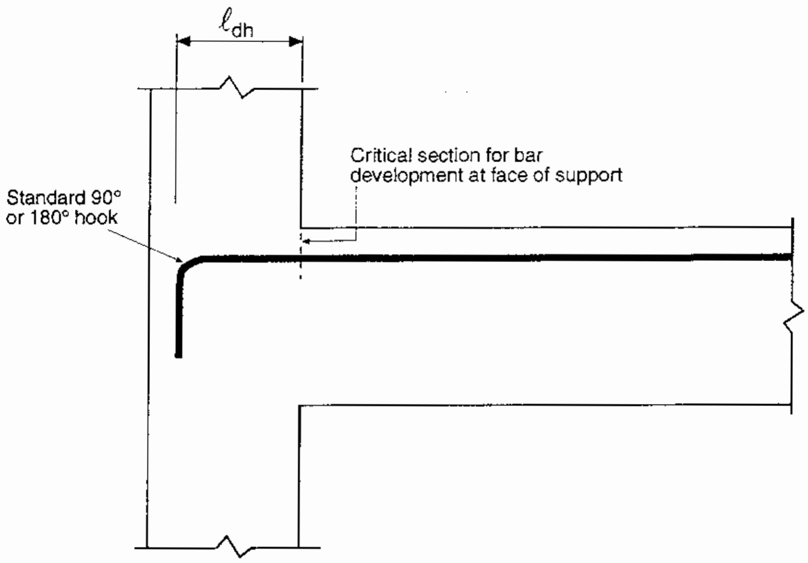


Figure 4-13 Anchorage into Exterior Support with Standard Hook

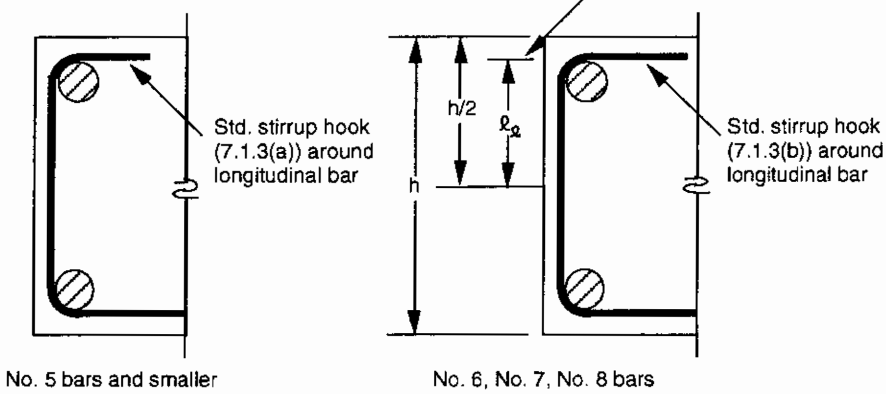


Figure 4-14 Anchorage Details for U-Stirrups (Deformed Bars and Deformed Wires)

For the larger stirrup bar sizes (No. 6, No. 7, or No. 8) in Grade 60, in addition to a standard stirrup hook, an embedment of $0.014d_b f_{yt} / \sqrt{f'_c}$ between midheight of member and outside end of hook is required. The available embedment length, denoted ℓ_ℓ, must be checked to ensure adequate anchorage at the higher bar force (see 12.13.2.2). The embedment length required is illustrated in Fig. 4-14 and listed in Table 4-5. Minimum depth of member required to accommodate No. 6, No. 7, or No. 8 stirrups fabricated in Grade 60 is also shown in Table 4-6. For practical size of beams where the loads are of such magnitude to require No. 6, No. 7, or No. 8 bar sizes for shear reinforcement, the embedment length required should be easily satisfied, and the designer need only be concerned with providing a standard stirrup hook around a longitudinal bar for proper stirrup end anchorage.

Provisions of 12.13.2.3 covering the use of welded plain wire reinforcement as simple U-stirrups are shown in Fig. 4-15. Requirements for stirrup anchorage (12.13.2.4) detail for straight single leg stirrups formed with welded plain or deformed wire reinforcement is shown in Fig. 4-16. Anchorage of the single leg is provided primarily by the longitudinal wires. Use of welded wire reinforcement for shear reinforcement has become commonplace in the precast, prestressed concrete industry.

Table 4-5 Embedment Length ℓ_{ℓ} (in.) for Grade 60 Stirrups

Bar Size No.	Concrete Compressive Strength f'_c, psi					
	3000	4000	5000	6000	8000	10,000
6	11.5	10.0	8.9	8.1	7.0	6.3
7	13.4	11.6	10.4	9.5	8.2	7.4
8	15.3	13.3	11.9	10.8	9.4	8.4

Table 4-6 Minimum Depth of Member (in.) to Accommodate Grade 60 No. 6, No. 7, and No. 8 Stirrups

Clear cover to stirrup (in.)	Bar Size No.	Concrete Compressive Strength f'_c, psi					
		3000	4000	5000	6000	8000	10,000
1-1/2	6	26	23	21	20	17	16
	7	30	27	24	22	20	18
	8	34	30	27	25	22	20
2	6	27	24	22	21	18	17
	7	31	28	25	23	21	19
	8	35	31	28	26	23	21

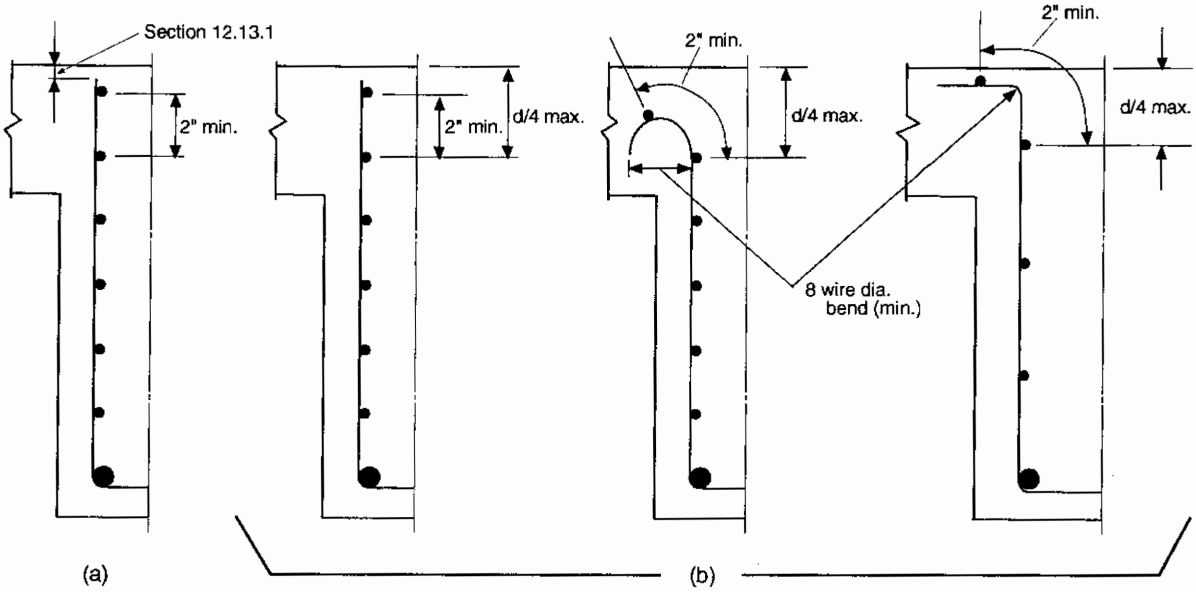


Figure 4-15 Anchorage Details for Welded Plain Wire Reinforcement U-Stirrups (12.13.2.3)

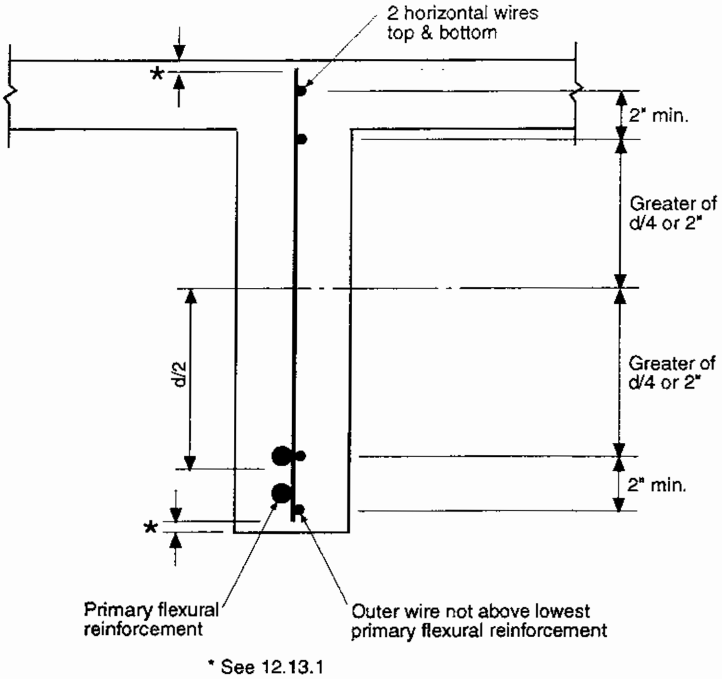


Figure 4-16 Anchorage Details for Welded Wire Reinforcement Single Leg Stirrups (12.13.2.4)

Note that 12.13.3 requires that each bend in the continuous portion of U-stirrups must enclose a longitudinal bar. This requirement is usually satisfied for simple U-stirrups, but requires special attention in bar detailing when multiple U-stirrups are used.

Clarifications of anchorage of web reinforcement made in the 1989 code eliminated the possibility of anchoring web reinforcement without hooking the stirrup around a longitudinal bar. Inquiries have shown that some designers routinely use small bars in joists without hooking them around a longitudinal bar, particularly a continuously bent single leg stirrup called a W-stirrup, accordion stirrup, or snake. To recognize this practice, 12.13.2.5 was introduced starting with the 1995 code.

12.13.4 Anchorage for Bent-Up Bars

Section 12.13.4 gives anchorage requirements for longitudinal (flexural) bars bent up to resist shear. If the bent-up bars are extended into a tension region, the bent-up bars must be continuous with the longitudinal reinforcement. If the bent-up bars are extended into a compression region, the required anchorage length beyond mid-depth of the member (d/2) must be based on that part of f_{yt} required to satisfy Eq. (11-17). For example, if f_{yt} = 60,000 psi and calculations indicate that 30,000 psi is required to satisfy Eq. (11-17), the required anchorage length $\ell'_d = (30{,}000/60{,}000)\ell_d$, where ℓ_d is the tension development length for full f_y per 12.2. Fig. 4-17 shows the required anchorage length ℓ'_d.

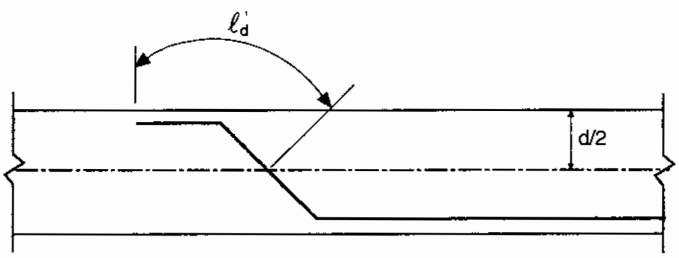


Figure 4-17 Anchorage for Bent-Up Bars

12.13.5 Closed Stirrups or Ties

Section 12.13.5 gives requirements for lap splicing double U-stirrups or ties (without hooks) to form a closed stirrup. Legs are considered properly spliced when the laps are $1.3\ell_d$ as shown in Fig. 4-18, where ℓ_d is determined from 12.2.

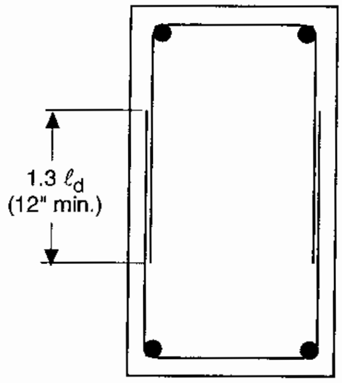
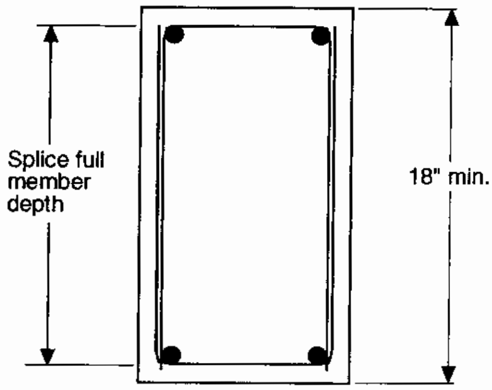


Figure 4-18 Overlapping U-Stirrups to Form Closed Unit

Alternatively, if a lap splice of $1.3\ell_d$ cannot fit within the depth of shallow members, provided that depth of members is at least 18 in., double U-stirrups may be used if each leg extends the full available depth of the member and the force in each leg does not exceed 9000 lb ($A_b f_{yt} \leq 9000$ lb.; see Fig. 4-19).

If stirrups are designed for the full yield strength f_y, No. 3 and 4 stirrups of Grade 40 and only No. 3 of Grade 60 satisfy the 9000 lb limitation.



No. 3: 0.11(40,000) = 4400 lb.
No. 4: 0.20(40,000) = 8000 lb.
No. 5: 0.31(40,000) = 12,400 lb.

No. 3: 0.11(60,000) = 6600 lb.
No. 4: 0.20(60,000) = 12,000 lb.

Figure 4-19 Lap Splice Alternative for U-Stirrups

12.14 SPLICES OF REINFORCEMENT—GENERAL

The splice provisions require the engineer to show clear and complete splice details in the contract documents. The structural drawings, notes and specifications should clearly show or describe all splice locations, types permitted or required, and for lap splices, length of lap required. The engineer cannot simply state that all splices shall be in accordance with the ACI 318 code. This is because many factors affect splices of reinforcement, such as the following for tension lap splices of deformed bars:

- bar size
- bar yield strength
- concrete compressive strength
- bar location (top bars or other bars)
- normal weight or lightweight aggregate concrete
- spacing and cover of bars being developed
- enclosing transverse reinforcement
- epoxy coating
- number of bars spliced at one location

It is virtually impossible for a reinforcing bar detailer to know what splices are required at a given location in a structure, unless the engineer explicitly illustrates or defines the splice requirements. Section 12.14.1 states: "Splices of reinforcement shall be made only as required or permitted on the design drawings, or in specifications, or as authorized by the engineer."

Two industry publications are suggested as design reference material for proper splicing of reinforcement. Reference 4.4 provides design aid data in the use of welded wire reinforcement, including development length and splice length tables for both deformed and plain wire reinforcement. Reference 4.5 provides accepted practices in splicing reinforcement; use of lap, mechanical, and welded splices are described, including simplified design data for lap splice lengths.

12.14.2 Lap Splices

Lap splices are not permitted for bars larger than No. 11, either in tension or compression, except:

- No. 14 and No. 18 bars in compression only may be lap spliced to No. 11 and smaller bars (12.16.2), and
- No. 14 and No. 18 bars in compression only may be lap spliced to smaller size footing dowels (15.8.2.3).

Section 12.14.2.2 gives the provisions for lap splicing of bars in a bundle (tension or compression). The lap lengths required for individual bars within a bundle must be increased by 20 percent and 33 percent for 3- and 4-bar bundles, respectively. Overlapping of individual bar splices within a bundle is not permitted. Two bundles must not be lap-spliced as individual bars.

Bars in flexural members may be spliced by noncontact lap splices. To prevent a possible unreinforced section in a spaced (noncontact) lap splice, 12.14.2.3 limits the maximum distance between bars in a splice to one-fifth the lap length, or 6 in. whichever is less. Contact lap splices are preferred for the practical reason that when the bars are wired together, they are more easily secured against displacement during concrete placement.

12.14.3 Mechanical and Welded Splices

Section 12.14.3 permits the use of mechanical or welded splices. A full mechanical splice must develop, in tension or compression, at least 125 percent of the specified yield strength of the bar (12.14.3.2). In a full welded splice, the bars must develop in tension at least 125 percent of the specified yield strength of the bar (12.4.3.4). ANSI/AWS D1.4 allows indirect welds where the bars are not butted. Note that ANSI/AWS D1.4 indicates that wherever practical, direct butt splices are preferable for No. 7 and larger bars. Use of mechanical or welded splices having less than 125 percent of the specified yield strength of the bar is limited to No. 5 and smaller bars (12.14.3.5) in regions of low computed stress. Mechanical and welded splices not meeting 12.14.3.2 and 12.14.3.4 are limited to No. 5 and smaller bars due to the potentially brittle nature of failure at these welds.

Section 12.14.3.3 requires all welding of reinforcement to conform to *Structural Welding Code-Reinforcing Steel* (ANSI/AWS D1.4). Section 3.5.2 requires that the reinforcement to be welded must be indicated on the drawings, and the welding procedure to be used must be specified. To carry out these code requirements properly, the engineer should be familiar with provisions in ANSI/AWS D1.4 and the ASTM specifications for reinforcing bars.

The standard rebar specifications ASTM A615, A616 and A617 do not address weldability of the steel. No limits are given in these specifications on the chemical elements that affect weldability of the steels. A key item in ANSI/AWS D1.4 is carbon equivalent (C.E.). The minimum preheat and interpass temperatures specified in ANSI/AWS D1.4 are based on C.E. and bar size. Thus, as indicated in 3.5.2 and R3.5.2, when welding is required, the ASTM A615, A616 and A617 rebar specifications must be supplemented to require a report of the chemical composition to assure that the welding procedure specified is compatible with the chemistry of the bars.

ASTM A706 reinforcing bars are intended for welding. The A706 specification contains restrictions on chemical composition, including carbon, and C.E. is limited to 0.55 percent. The chemical composition and C.E. must be reported. By limiting C.E. to 0.55 percent, little or no preheat is required by ANSI/AWS D1.4. Thus, the engineer does not need to supplement the A706 specification when the bars are to be welded. However, before specifying ASTM A706 reinforcing bars, local availability should be investigated.

Reference 4.5 contains a detailed discussion of welded splices. Included in the discussion are requirements for other important items such as field inspection, supervision, and quality control.

The ANSI/AWS D1.4 document covers the welding of reinforcing bars only. For welding of wire to wire, and of wire or welded wire reinforcement to reinforcing bars or structural steels, such welding should conform to applicable provisions of ANSI/AWS D1.4 and to supplementary requirements specified by the engineer. Also, the engineer should be aware that there is a potential loss of yield strength and ductility of low carbon cold-drawn wire if wire is welded by a process other than controlled resistance welding used in the manufacture of welded wire reinforcement.

In the discussion of 7.5 in Part 3 of this document, it was noted that welding of crossing bars (tack welding) is not permitted for assembly of reinforcement unless authorized by the engineer. An example of tack welding would be a column cage where the ties are secured to the longitudinal bars by small arc welds. Such welding can cause a metallurgical notch in the longitudinal bars, which may affect the strength of the bars. Tack welding seems to be particularly detrimental to ductility (impact resistance) and fatigue resistance. Reference 4.5 recommends: "Never permit field welding of crossing bars ('tack' welding, 'spot' welding, etc.). Tie wire will do the job without harm to the bars."

12.15 SPLICES OF DEFORMED BARS AND DEFORMED WIRE IN TENSION

Tension lap splices of deformed bars and deformed wire are designated as Class A and B with the length of lap being a multiple of the tensile development length ℓ_d. The two-level splice classification (Class A & B) is intended to encourage designers to splice bars at points of minimum stress and to stagger lap splices along the length of the bars to improve behavior of critical details.

The development length ℓ_d (12.2) used in the calculation of lap length must be that for the full f_y because the splice classifications already reflect any excess reinforcement at the splice location (factor of 12.2.5 for excess A_s must not be used). The minimum length of lap is 12 in.

For lap splices of slab and wall reinforcement, effective clear spacing of bars being spliced at the same location is taken as the clear spacing between the spliced bars (R12.15.1). This clear spacing criterion is illustrated in Fig. 4-20(a). Spacing for noncontact lap splices (spacing between lapped bars not greater than (1/5) lap length nor 6 in.) should be considered the same as for contact lap splices. For lap splices of column and beam bars, effective clear spacing between bars being spliced will depend on the orientation of the lapped bars; see Fig. 4-20(b) and (c), respectively.

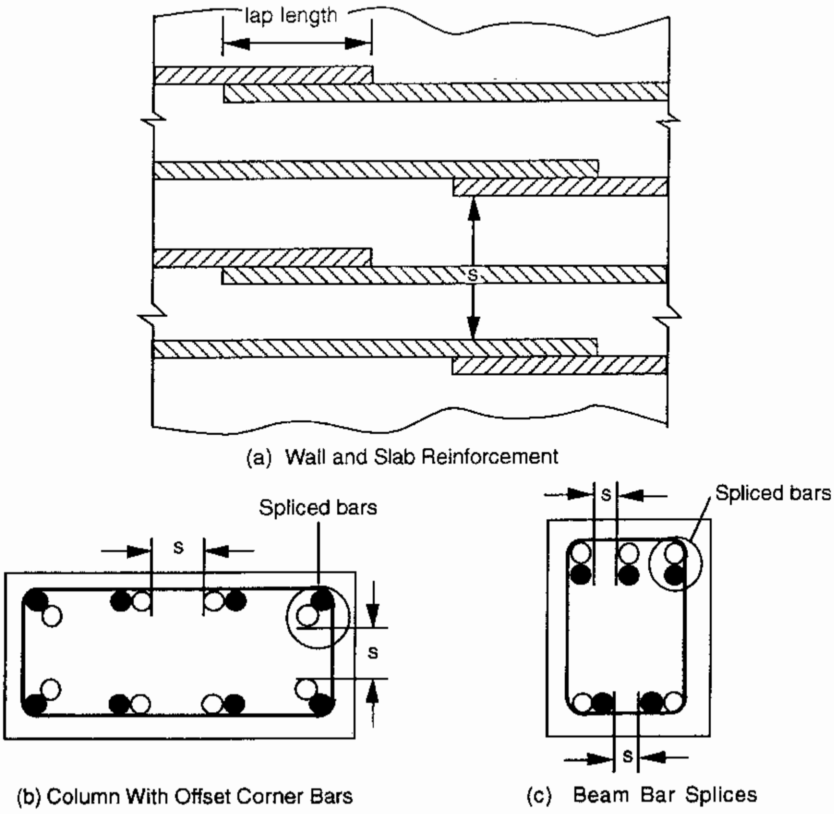


Figure 4-20 Effective Clear Spacing of Spliced Bars

The designer must specify the class of tension lap splice to be used. The class of splice depends on the magnitude of tensile stress in the reinforcement and the percentage of total reinforcement to be lap spliced within any given splice length as shown in Table 4-7. If the area of tensile reinforcement provided at the splice location is more than twice that required for strength (low tensile stress) and 1/2 or less of the total steel area is lap spliced within the required splice length, a Class A splice may be used. Both splice conditions must be satisfied, otherwise, a Class B splice must be used. In other words, if the area of reinforcement provided at the splice location is less than twice that required for strength (high tensile stress) and/or more than 1/2 of the total area is to be spliced within the lap length, a Class B splice must be used.

Table 4-7 Tension Lap Splice Conditions (at splice location)

CLASS A...1.0ℓ_d	CLASS B...1.3ℓ_d
(A_s provided) $\geq$ 2 (A_s required) and percent A_s spliced $\leq$ 50	All other conditions

Mechanical or welded splices conforming to 12.14.3 may be used in lieu of tension lap splices. Section R12.15.3 clarifies that such splices need not be staggered although such staggering is encouraged where the area of reinforcement provided is less than twice that required by analysis.

Section 12.15.4 emphasizes that mechanical and welded splices not meeting the requirements of 12.14.3.2 and 12.14.3.4, respectively, are only allowed for No. 5 bars and smaller, and only if certain conditions are met (see 12.15.4.1 and 12.15.4.2).

Splices in tension tie members are required to be made with a full mechanical or welded splice with a 30 in. stagger between adjacent bar splices. See definition of "tension tie member" in R12.15.5.

12.16 SPLICES OF DEFORMED BARS IN COMPRESSION

Since bond behavior of reinforcing bars in compression is not complicated by the potential problem of transverse tension cracking in the concrete, compression lap splices do not require such strict provisions as those specified for tension lap splices. Tests have shown that the strength of compression lap splices depends primarily on end bearing of the bars on the concrete, without a proportional increase in strength even when the lap length is doubled. Thus, the code requires significant longer lap length for bars with a yield strength greater than 60,000 psi.

12.16.1 Compression Lap Splices

Calculation of compression lap splices was simplified starting with the '89 code by removing the redundant calculation for development length in compression. For compression lap splices, 12.16.1 requires the minimum lap length to be simply $0.0005d_bf_y$ for $f_y = 60,000$ psi or less, but not less than 12 in. For reinforcing bars with a yield strength greater than 60,000 psi, a minimum lap length of $(0.0009f_y - 24)\ d_b$ but not less than 12 in. is specified. Lap splice lengths must be increased by one-third for concrete with a specified compressive strength less than 3000 psi.

As noted in the discussion of 12.14.2, No. 14 and No. 18 bars may be lap spliced, in compression only, to No. 11 and smaller bars or to smaller size footing dowels. Section 12.16.2 requires that when bars of a different size are lap spliced in compression, the length of lap must be the compression development length of the larger bar, or the compression lap splice length of the smaller bar, whichever is the longer length.

12.16.4 End-Bearing Splices

Section 12.16.4 specifies the requirements for end-bearing compression splices. End-bearing splices are only permitted in members containing closed ties, closed stirrups or spirals (12.16.4.3). Section R12.16.4.1 cautions the engineer in the use of end-bearing splices for bars inclined from the vertical. End-bearing splices for compression bars have been used almost exclusively in columns and the intent is to limit use to essentially vertical bars because of the field difficulty of getting adequate end bearing on horizontal bars or bars significantly inclined from the vertical. Mechanical or welded splices are also permitted for compression splices and must meet the requirements of 12.14.3.2 or 12.14.3.4, respectively.

12.17 SPECIAL SPLICE REQUIREMENTS FOR COLUMNS

The special splice requirements for columns were significantly simplified in the '89 code. The column splice requirements simplify the amount of calculations that are required compared to previous provisions by assuming that a compression lap splice (12.17.2.1) has a tensile capacity of at least one-fourth f_y (R12.17).

The column splice provisions are based on the concept of providing some tensile resistance at all column splice locations even if analysis indicates compression only at a splice location. In essence, 12.17 establishes the required tensile strength of spliced longitudinal bars in columns. Lap splices, butt-welded splices, mechanical or end-bearing splices may be used.

12.17.2 Lap Splices in Columns

Lap splices are permitted in column bars subject to compression or tension. Type of lap splice to be used will depend on the bar stress at the splice location, compression or tension, and magnitude if tension, due to all factored load combinations considered in the design of the column. Type of lap splice to be used will be governed by the load combination producing the greatest amount of tension in the bars being spliced. The design requirements for lap splices in column bars can be illustrated by a typical column load-moment strength interaction as shown in Fig. 4-21.

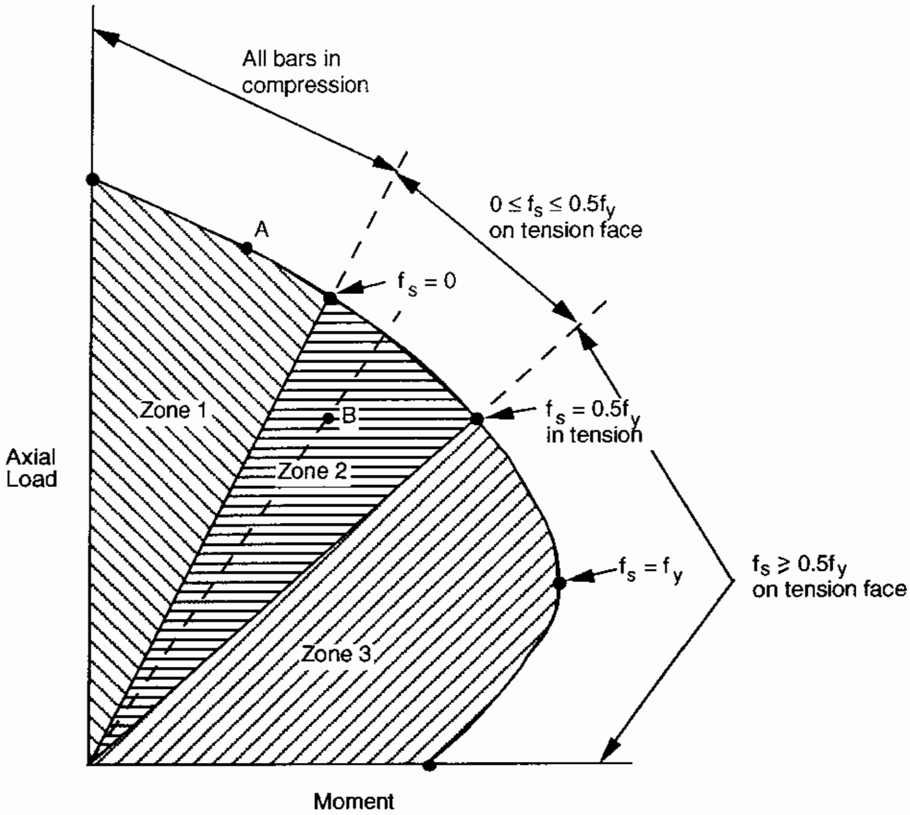


Figure 4-21 Special Splice Requirements for Columns

Bar stress at various locations along the strength interaction curve define segments of the strength curve where the different types of lap splices may be used. For factored load combinations along the strength curve, bar stress can be readily calculated to determine type of lap splice required. However, a design dilemma exists for load combinations that do not fall exactly on the strength curve (below the strength curve) as there is no simple exact method to calculate bar stress for this condition.

A seemingly rational approach is to consider factored load combinations below the strength curve as producing bar stress of the same type, compression or tension, and of the same approximate magnitude as that produced along the segment of the strength curve intersected by radial lines (lines of equal eccentricity) through the load combination point. This assumption becomes more exact as the factored load combinations being investigated fall nearer to the actual strength interaction curve of the column. Using this approach, zones of "bar stress" can be established as shown in Fig. 4-21.

For factored load combinations in Zone 1 of Fig. 4-21, all column bars are considered to be in compression. For load combinations in Zone 2 of the figure, bar stress on the tension face of the column is considered to vary from zero to 0.5f_y in tension. For load combinations in Zone 3, bar stress on the tension face is considered to be greater than 0.5f_y in tension. Type of lap splice to be used will then depend on which zone, or zones, all factored load combinations considered in the design of the column are located. The designer need only locate the factored load combinations on the load-moment strength diagram for the column and bars selected in the design to determine type of lap splice required. Use of load-moment design charts in this manner will greatly facilitate the design of column bar splices. For example, if factored gravity load combination governed design of the column, say Point A in Fig. 4-21, where all bars are in compression, but a load combination including wind, say Point B in Fig. 4-21, produces some tension in the bars, the lap splice must be designed for a Zone 2 condition (bar stress is tensile but does not exceed $0.5f_y$ in tension).

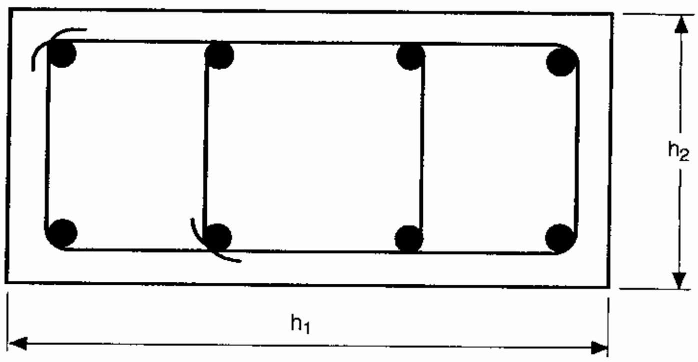
The design requirements for lap splices in columns are summarized in Table 4-8. Note that the compression lap splice permitted when all bars are in compression (see 12.17.2.1) considers a compression lap length adequate as a minimum tensile strength requirement. See Example 4.8 for design application of the lap splice requirements for columns.

Table 4-8 Lap Splices in Columns

12.17.2.1—Bar stress in compression (Zone 1)*	Use compression lap splice (12.16) modified by factor of 0.83 for ties (12.17.2.4) or 0.75 for spirals (12.17.2.5).
12.17.2.2—Bar stress ≤ $0.5f_y$ in tension (Zone 2)*	Use Class B tension lap splice (12.15) if more than 1/2 of total column bars spliced at same location. or Use Class A tension lap splice (12.15) if not more than 1/2 of total column bars spliced at same location. Stagger alternate splices by ℓ_d.
12.17.2.3—Bar stress > $0.5f_y$ in tension (Zone 3)*	Use Class B tension lap splice (12.15).

** For Zones 1, 2, and 3, see Fig. 4-21.*

Sections 12.17.2.4 and 12.17.2.5 provide reduction factors for the compression lap splice when the splice is enclosed throughout its length by ties (0.83 reduction factor) or by a spiral (0.75 reduction factor). Spirals must meet the requirements of 7.10.4 and 10.9.3. When ties are used to reduce the lap splice length, the ties must have a minimum effective area of 0.0015hs. The tie legs in both directions must provide the minimum effective area to permit the 0.83 modification factor. See Fig. 4-22. The 12 in. minimum lap length also applies to these permitted reductions.



(perpendicular to h_1 dimension) 4 tie bar areas ≥ $0.0015h_1s$
(perpendicular to h_2 dimension) 2 tie bar areas ≥ $0.0015h_2s$

Figure 4-22 Application of 12.17.2.4

With the "basic" lap length for compression lap splices a function of bar diameter d_b and bar yield strength f_y, and three modification factors for ties and spirals and for lower concrete strength, it is convenient to establish compression lap splices simply as a multiple of bar diameter.

For Grade 60 bars .... $30d_b$
enclosed within ties .... $25d_b$
enclosed within spirals .... $22.5d_b$
For Grade 75 bars .... $43.5d_b$
enclosed within ties .... $36d_b$
enclosed within spirals .... $33d_b$

but not less than 12 in. For f'_c less than 3000 psi, multiply by a factor of 1.33. Compression lap splice tables for the standard bar sizes can be readily developed using the above values.

12.17.3 Mechanical or Welded Splices in Columns

Mechanical or welded splices are permitted in column bars where bar stress is either compressive or tensile for all factored load combinations (Zones 1, 2, and 3 in Fig. 4-21). "Full" mechanical or "full" welded splices must be used; that is, the mechanical or welded splice must develop at least 125 percent of the bar yield strength, $1.25A_bf_y$. Use of mechanical or welded splices of lesser strength is permitted for splicing bars No. 5 and smaller in tension, in accordance with 12.15.4.

12.17.4 End Bearing Splices in Columns

End bearing splices are permitted for column bars stressed in compression for all factored load combinations (Zone 1 in Fig. 4-21). Even though there is no calculated tension, a minimum tensile strength of the continuing (unspliced) bars must be maintained when end bearing splices are used. Continuing bars on each face of the column must provide a tensile strength of $A_sf_y/4$, where A_s is the total area of bars on that face of the column. Thus, not more than 3/4 of the bars can be spliced on each face of the column at any one location. End bearing splices must be staggered or additional bars must be added at the splice location if more than 3/4 of the bars are to be spliced.

12.18 SPLICES OF WELDED DEFORMED WIRE REINFORCEMENT IN TENSION

For tension lap splices of deformed wire reinforcement, the code requires a minimum lap length of $1.3\ell_d$, but not less than 8 in. Lap length is measured between the ends of each reinforcement sheet. The development length ℓ_d is the value calculated by the provisions in 12.7. The code also requires that the overlap measured between the outermost cross wires be at least 2 in. Figure 4-23 shows the lap length requirements.

If there are no cross wires within the splice length, the provisions in 12.15 for deformed wire must be used to determine the length of the lap.

Section 12.18.3 provides requirements for splicing welded wire reinforcement, including deformed wires in one direction and plain wires in the orthogonal direction.

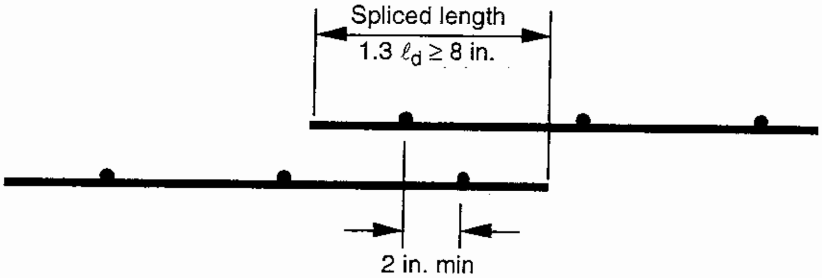
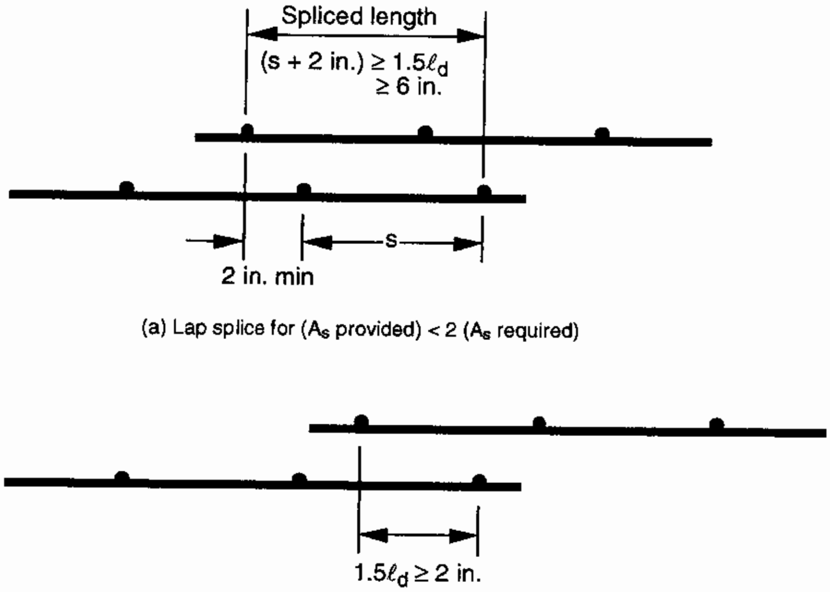


Figure 4-23 Lap Splice Length for Deformed Wire Fabric

12.19 SPLICES OF WELDED PLAIN WIRE REINFORCEMENT IN TENSION

The minimum length of lap for tension lap splices of plain wire reinforcement is dependent upon the ratio of the area of reinforcement provided to that required by analysis. Lap length is measured between the outermost cross wires of each reinforcement sheet. The required lap lengths are shown in Fig. 4-24.



(a) Lap splice for (A_s provided) < 2 (A_s required)

(b) Lap Splice for (A_s provided) ≥ 2 (A_s required)

Figure 4-24 Lap Splice Length for Plain Wire Reinforcement

CLOSING REMARKS

One additional comment concerning splicing of temperature and shrinkage reinforcement at the exposed surfaces of walls or slabs: one must assume all temperature and shrinkage reinforcement to be stressed to the full specified yield strength f_y. The purpose of this reinforcement is to prevent excess cracking. At some point in the member, it is likely that cracking will occur, thus fully stressing the temperature and shrinkage reinforcement. Therefore, all splices in temperature and shrinkage reinforcement must be assumed to be those required for development of yield tensile strength. A Class B tension lap splice must be provided for this steel.

REFERENCES

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4.2 Sozen, M.A., and Moehle, J.P., "Selection of Development and Lap-Splice Lengths of Deformed Reinforcing Bars in Concrete Subjected to Static Loads," Report to PCA and CRSI, PCA R&D Serial No. 1868, March 1990.

4.3 Hamad, B. S., Jirsa, J. O., and D'Abreu, N. I., "Effect of Epoxy Coating on Bond and Anchorage of Reinforcement in Concrete Structures," Research Report 1181-1F, Center for Transportation Research, University of Texas at Austin, Dec. 1990, 242 pp.

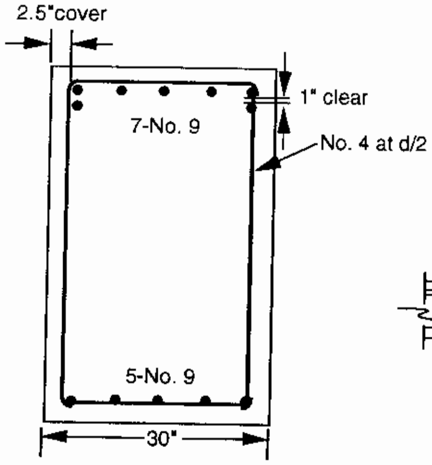
4.4 *Manual of Standard Practice, Stuctural Welded Wire Reinforcement*, WWR-500, 5th Edition, Wire Reinforcement Institute, Findlay, OH, 1999, 27 pp.

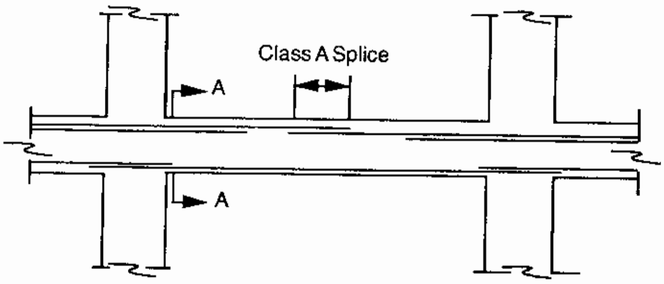
4.5 *Reinforcement Anchorages and Splices*, 4th Edition, Concrete Reinforcing Steel Institute, Schaumburg, IL, 1997.

Example 4.1—Development of Bars in Tension

A beam at the perimeter of the structure has 7-No. 9 top bars over the support. Structural integrity provisions require that at least one-sixth of the tension reinforcement be made continuous, but not less than 2 bars (7.13.2.2). Bars are to be spliced with a Class A splice at midspan. Determine required length of Class A lap splice for the following two cases:

Case A - Development computed from 12.2.2
Case B - Development computed from 12.2.3





Assume:

Lightweight concrete
2.5 in. clear cover to stirrups
Epoxy-coated bars
f'_c = 4000 psi
f_y = 60,000 psi
b = 30 in. (with bar arrangement as shown)

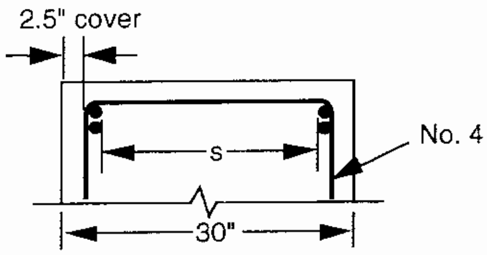
Calculations and Discussion	Code Reference
It is assumed that development of negative moment reinforcement has been satisfied and, therefore, top bars are stopped away from midspan.	*12.12.3*
Minimum number of top bars to be made continuous for structural integrity is 1/6 of 7 bars provided, i.e., 7/6 bars or, a minimum of 2 bars. Two corner bars will be spliced at midspan.	*7.13.2.2*
Class A lap splice requires a $1.0\ell_d$ length of bar lap	*12.15.1*

Nominal diameter of No. 9 bar = 1.128 in.

<u>CASE A - Section 12.2.2</u>

Refer to Table 4-1. For bars No. 7 and larger, either Eq. B or Eq. D apply. To determine if Eq. B or Eq. D governs, determine clear cover and clear spacing for bars being developed.

Clear spacing between spliced bars (corner bars)
= [30 - 2 (cover) - 2 (No. 4 stirrup) - 2 (No. 9 bar)]
= [30 - 2 (2.5) - 2 (0.5) - 2(1.128)]
= 21.7 in.
= $19.3d_b$



Clear cover to spliced bar = 2.5 + 0.5 = 3.0 in. = $2.7d_b$

As clear spacing > $2d_b$ and clear cover > d_b, Eq. B applies.

$$\ell_d = \left(\frac{f_y \psi_t \psi_e \lambda}{20\sqrt{f'_c}}\right) d_b$$ 12.2.2

ψ_t = 1.3 for top bar 12.2.4

ψ_e = 1.5 for epoxy-coated bar with cover less than $3d_b$ 12.2.4

$\psi_t\psi_e = 1.3 \times 1.5 = 1.95$; however, product of α and β need not be taken greater than 1.7. 12.2.4

λ = 1.3 for lightweight aggregate concrete 12.2.4

$$\ell_d = \frac{60{,}000\ (1.7)\ (1.3)}{20\sqrt{4000}}\ (1.128)$$

= 118.3 in.

Class A splice = $1.0\ell_d$ = 118.3 in.

<u>**CASE B - Section 12.2.3**</u>

Application of Eq. (12-1) requires a little more computations, but can result in smaller development lengths.

$$\ell_d = \left(\frac{3}{40}\ \frac{f_y}{\sqrt{f'_c}}\ \frac{\psi_t\psi_e\psi_s\lambda}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right) d_b$$ Eq. (12-1)

Parameter "c_b" is the smaller of (1) distance from center of bar being developed to the nearest concrete surface, and (2) one-half the center-to-center spacing of bars being developed. Also, note that the term $\left(\frac{c_b + K_{tr}}{d_b}\right)$ cannot exceed 2.5.

Distance from center of bar or wire being developed to the nearest concrete surface
= clear cover to spliced bar + 1/2 bar diameter

$= 2.7d_b + 0.5d_b = 3.2d_b$

Center-to-center spacing $=$ clear spacing $+ 1.0d_b = 19.3d_b + 1.0d_b = 20.3d_b$

Therefore, c is the smaller of $3.2d_b$ and $0.5\ (20.3d_b)$, i.e. $3.2d_b$

No need to compute K_{tr} as c/d_b is greater than 2.5

$\gamma = 1.0$ for No. 7 bar and larger

$$\ell_d = \frac{3\ (60{,}000)\ (1.7)\ (1.0)\ (1.3)}{40\sqrt{4000}\ (2.5)}\ (1.128)$$

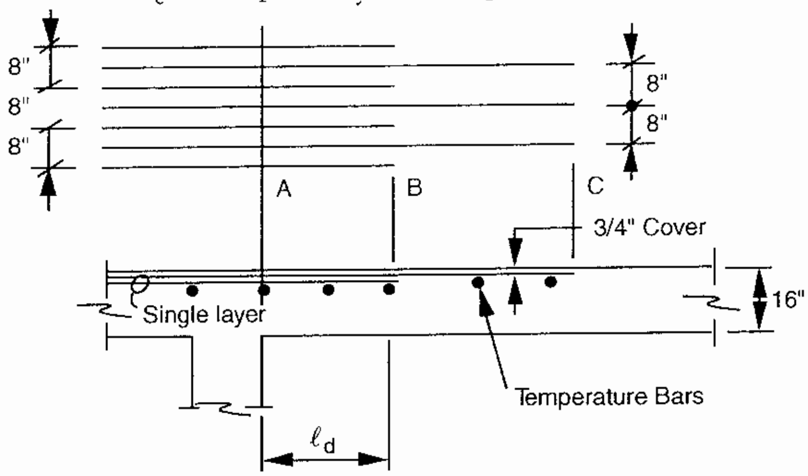
$= 71.0$ in.

Class A splice $= 1.0\ell_d = 71.0$ in.

The extra computations required to satisfy the general Eq. (12-1) of 12.2.3 can lead to substantial reductions in tension development or splice lengths compared to values computed from the simplified procedure of 12.2.2.

Example 4.2—Development of Bars in Tension

Calculate required tension development length for the No. 8 bars (alternate short bars) in the "sand-lightweight" one-way slab shown below. Use $f'_c = 4000$ psi and $f_y = 60{,}000$ psi, and uncoated bars.



Calculations and Discussion	Code Reference

Calculations for this example will be performed using 12.2.2 and 12.2.3.

Assume short bars are developed within distance AB while long bars are developed within BC.

Nominal diameter of No. 8 bar is 1.00 in.

A. Development length by 12.2.2

Center-to-center spacing of bars being developed = 8 in. = $8d_b$

Clear cover = 0.75 in. = $0.75d_b$

As clear cover is less than d_b, and bar size is larger than No. 7, Eq. D of Table 4-1 applies.

$$\ell_d = \left(\frac{3f_y\psi_t\psi_e\lambda}{40\sqrt{f'_c}}\right)d_b$$ *12.2.2*

ψ_t = 1.3 for top bar

ψ_e = 1.0 for uncoated bars

λ = 1.3 for lightweight concrete

$$\ell_d = \frac{3\ (60{,}000)\ (1.3)\ (1.0)\ (1.3)}{40\sqrt{4000}}\ (1.0) = 120.3 \text{ in.}$$

B. Development length by 12.2.3

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s\lambda}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b$$ *Eq. (12-1)*

ψ_t = 1.3 for top bar

ψ_e = 1.0 for uncoated bars

ψ_s = 1.0 for No. 7 and larger bars

λ = 1.3 for lightweight concrete

Center-to-center spacing of bars being developed = 8 in. = $8d_b$
Clear spacing between bars being developed = 8 - 1 = 7 in. = $7d_b$

Clear cover = 0.75 in. = $0.75d_b$
Distance "c" from center of bar to concrete surface = 0.75 + 0.5 = 1.25 in. = $1.25d_b$ (governs)

= $8d_b/2 = 4d_b$ (center -to-center spacing/2)

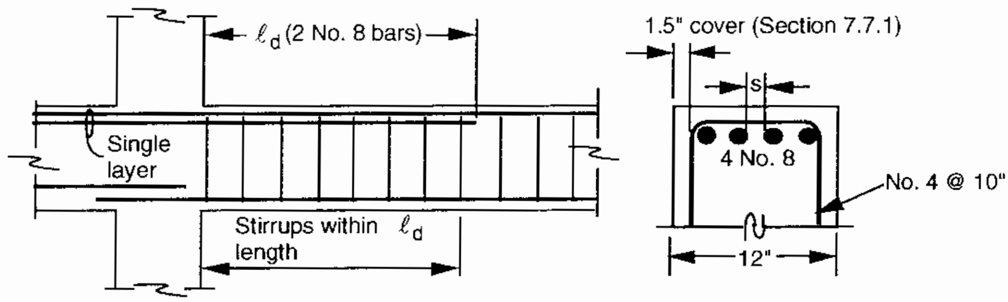
c_b = $1.25d_b$ (computed above)

$$K_{tr} = \frac{A_{tr}f_{yt}}{1500sn} = 0 \text{ (no transverse reinforcement)}$$

$$\ell_d = \frac{3\,(60{,}000)\,(1.3)\,(1.0)\,(1.0)\,(1.3)}{40\sqrt{4000}\,(1.25)}(1.0) = 96.2 \text{ in.}$$

Example 4.3—Development of Bars in Tension

Calculate required development length for the inner 2 No. 8 bars in the beam shown below. The 2 No. 8 outer bars are to be made continuous along full length of beam. Use $f_c' = 4000$ psi (normal weight concrete) and $f_y =$ 60,000 psi, and uncoated bars. Stirrups provided satisfy the minimum code requirements for beam shear reinforcement.



Calculations and Discussion	Code Reference

Calculations for this example will be performed using 12.2.2 and 12.2.3.

Nominal diameter of No. 8 bar = 1.00 in.

A. Development length by 12.2.2

Clear spacing = [12 - 2 (cover) - 2 (No. 4 stirrups) - 4 (No. 8 bars)]/3 spaces
= [12 - 2 (1.5) - 2 (0.50) - 4 (1.00)]/3
= 1.33 in.
= $1.33d_b$

Clear cover = 1.5 + 0.5 = 2.0 in. = $2d_b$

Refer to Table 4-1. Clear spacing between bars being developed more than d_b, clear cover more than d_b, and minimum stirrups provided. Eq. B of Table 4-1 applies.

$$\ell_d = \left(\frac{f_y \psi_t \psi_e \lambda}{20\sqrt{f_c'}}\right) d_b$$ *12.2.2*

ψ_t = 1.3 for top bar

ψ_e = 1.0 for uncoated bars

λ = 1.0 for normal weight concrete

$$\ell_d = \frac{(60{,}000)\ (1.3)\ (1.0)\ (1.0)}{20\sqrt{4000}}\ (1.0) = 61.7 \text{ in.}$$

B. Development length by 12.2.3

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s\lambda}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b$$ Eq. (12-1)

ψ_t = 1.3 for top bar

ψ_e = 1.0 for uncoated bars

ψ_s = 1.0 for No. 7 and larger bars

λ = 1.0 for normal weight concrete

Clear spacing = $1.33d_b$
Center-to-center spacing of bars being developed = 1.33 + 1.0 in. = 2.33 in. = $2.33d_b$

Clear cover = 1.50 + 0.5 = 2.0 in. = $2d_b$
Distance from center of bar to concrete surface = 1.5 + 0.5 + 0.5 = 2.5 in. = $2.5d_b$

c_b = the smaller of (1) distance from center of bar being developed to the nearest concrete surface ($2.5d_b$), and of (2) one-half the center-to-center spacing of bars being developed ($2.33d_b$/ 2 = $1.17d_b$)

$c_b = 1.17d_b$

$$K_{tr} = \frac{A_{tr}f_{yt}}{1500sn}$$

A_{tr} (2-No. 4) = 2 × 0.2 = 0.4 in.^{2}

s = 10 in. spacing of stirrups

n = 2 bars being developed

$$K_{tr} = \frac{0.4\,(60{,}000)}{1500\,(10)\,(2)} = 0.80 \text{ in.} = 0.80d_b$$

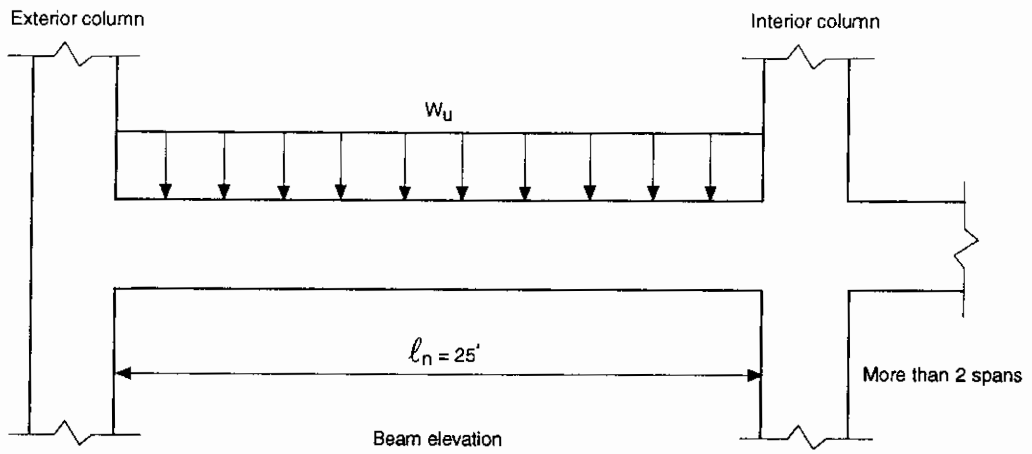
$$\left(\frac{c_b + K_{tr}}{d_b}\right) = \frac{1.17 + 0.80}{1.0} = 1.97 < 2.5 \quad \text{O.K.}$$ 12.2.3

$$\ell_d = \frac{3\,(60{,}000)\,(1.3)\,(1.0)\,(1.0)\,(1.0)}{40\,\sqrt{4000}\,(1.97)}\,(1.0) = 47.0 \text{ in.}$$

Example 4.4—Development of Flexural Reinforcement

Determine lengths of top and bottom bars for the exterior span of the continuous beam shown below. Concrete is normal weight and bars are Grade 60. Total uniformly distributed factored gravity load on beam is w_u = 6.0 kips/ft (including weight of beam).

f'_c = 4000 psi
f_y = 60,000 psi
b = 16 in.
h = 22 in.
Concrete cover = 1 1/2 in.



Calculations and Discussion	Code Reference

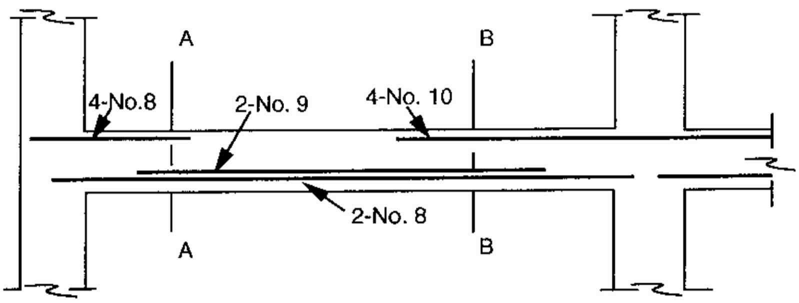
1. Preliminary design for moment and shear reinforcement

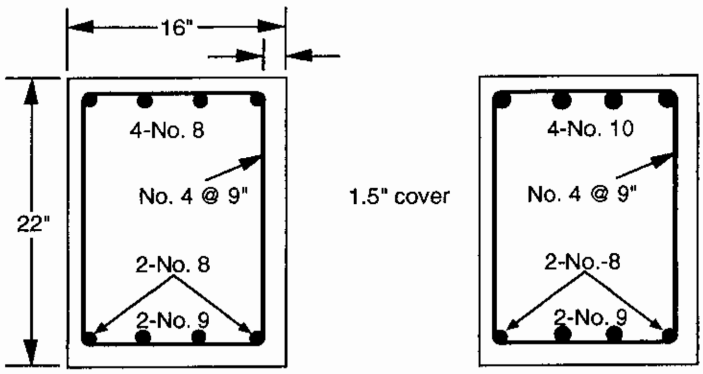
a. Use approximate analysis for moment and shear — *8.3.3*

Location	Factored moments & shears
Interior face of exterior support	$-M_u = w_u\ell_n^2/16 = 6\ (25^2)/16 = -234.4$ ft - kips
End span positive	$+M_u = w_u\ell_n^2/14 = 6\ (25^2)/14 = 267.9$ ft - kips
Exterior face of first interior support	$-M_u = w_u\ell_n^2/10 = 6\ (25^2)/10 = -375.0$ ft - kips
Exterior face of first interior support	$V_u = 1.15w_u\ell_n/2 = 1.15\ (6)\ (25)/2 = 86.3$ kips

b. Determine required flexural reinforcement using procedures of Part 7 of this publication. With 1.5 in. cover, No. 4 bar stirrups, and No. 9 or No. 10 flexural bars, d ≈ 19.4 in.

M_u	A_s required	Bars	A_s provided
-234.4 ft-kips	2.93 in.^{2}	4 No. 8	3.16 in.^{2}
+267.9 ft-kips	3.40 in.^{2}	2 No. 8 2 No. 9	3.58 in.^{2}
-375.0 ft-kips	5.01 in.^{2}	4 No. 10	5.08 in.^{2}





Section A-A Section B-B

c. Determine required shear reinforcement

V_u at "d" distance from face of support: *11.1.3.1*

$V_u = 86.3 - 6\,(19.4/12) = 76.6$ kips

$\phi V_c = \phi\left(2\sqrt{f'_c}b_w d\right) = 0.75 \times 2\sqrt{4{,}000} \times 16 \times 19.4/1{,}000 = 29.5$ kips *11.1.3.1*

Try No. 4 U-stirrups @ 7 in. spacing $< s_{max} = \dfrac{d}{2} = 9.7$ in. *11.5.4.1*

$\phi V_s = \dfrac{\phi A_v f_y d}{s} = 0.75\,(0.40)\,(60)\,(19.4)/7 = 49.9$ kips *11.5.6.2*

$\phi V_n = \phi V_c + \phi V_s = 29.5 + 49.9 = 79.4$ kips > 76.6 kips O.K.

Distance from support where stirrups not required:

$$V_u < \frac{\phi V_c}{2} = \frac{29.5}{2} = 14.8 \text{ kips}$$ 11.5.5.1

$V_u = 86.3 - 6x = 14.8$ kips

$x = 11.9$ ft $\approx$ 1/2 span

Use No. 4 U-stirrups @ 7 in. (entire span)

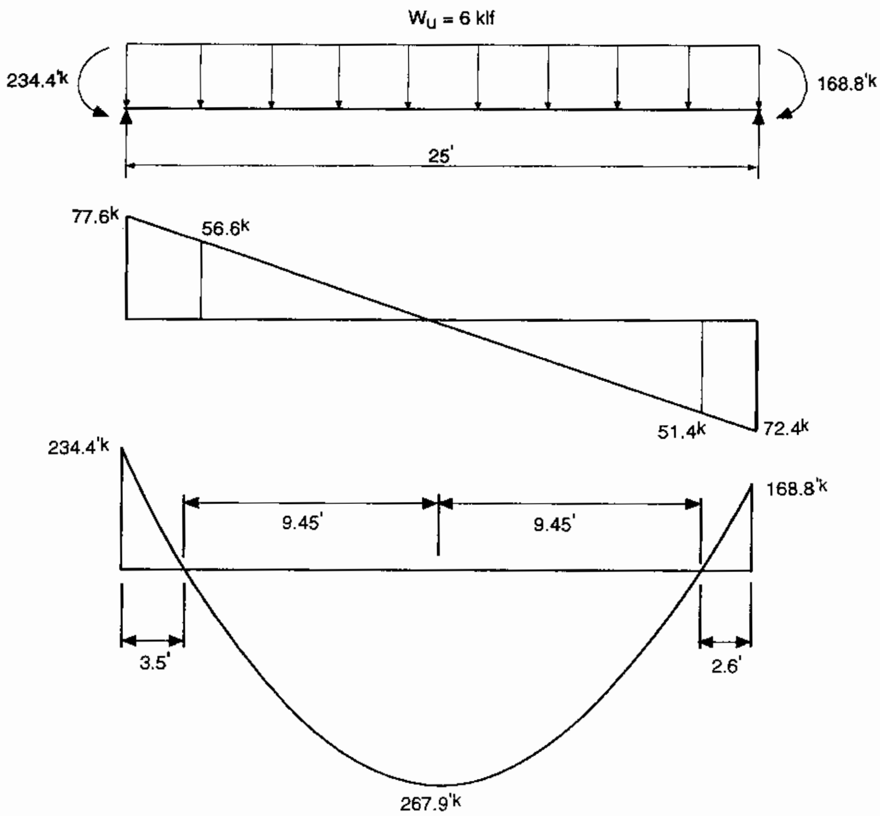
2. Bar lengths for bottom reinforcement

   a. Required number of bars to be extended into supports. 12.11.1

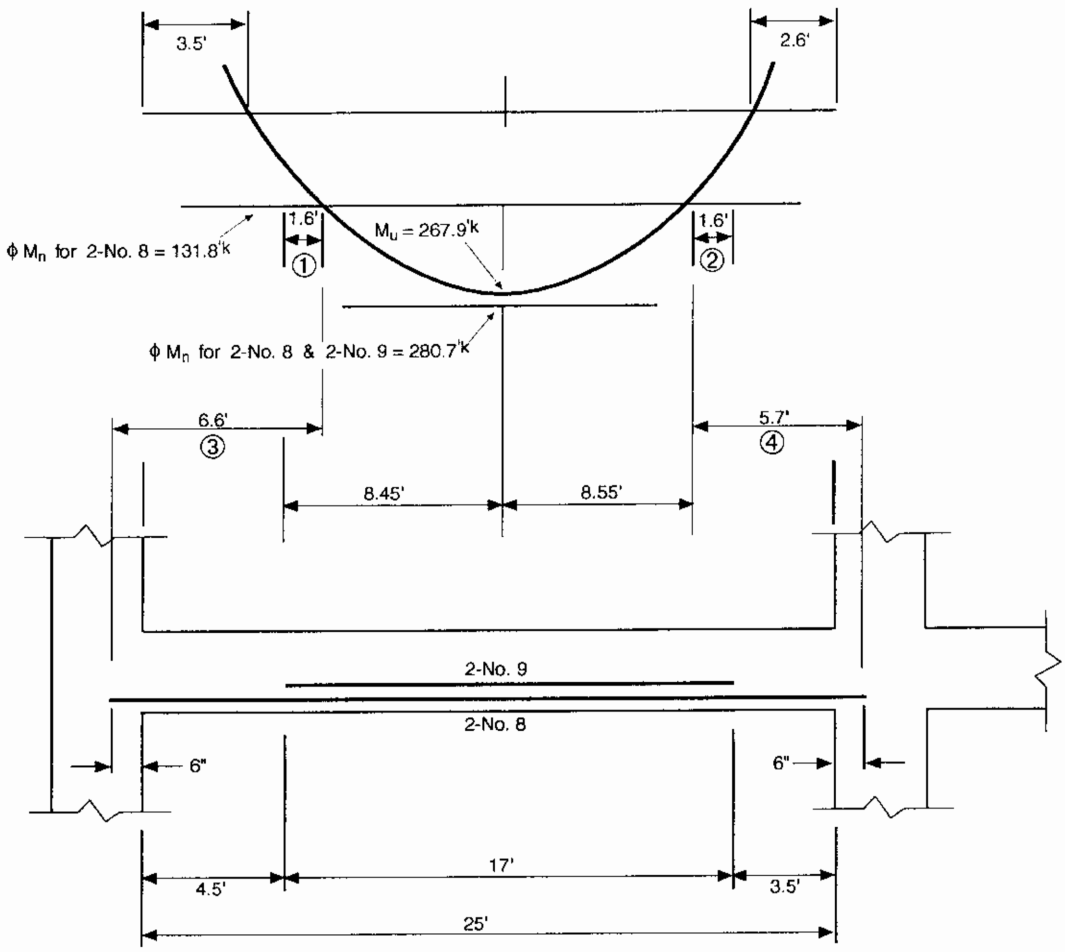
   One-fourth of ($+A_s$) must be extended at least 6 in. into the supports. With a longitudinal bar required at each corner of the stirrups (12.13.3), at least 2 bars should be extended full length. Extend the 2-No. 8 bars full span length (plus 6 in. into the supports) and cut off the 2-No. 9 bars within the span.

   b. Determine cut-off locations for the 2 No. 9 bars and check other development requirements.

   Shear and moment diagrams for loading condition causing maximum factored positive moment are shown below.



The positive moment portion of the M_u diagram is shown below at a larger scale, including the design moment strengths ϕM_n for the total positive A_s (2-No. 8 and 2-No. 9) and for 2-No. 8 bars separately. For 2-No. 8 and 2-No. 9, ϕM_n = 280.7 ft-kips. For 2-No. 8, ϕM_n = 131.8 ft-kips.



As shown, the 2-No. 8 bars extend full span length plus 6 in. into the supports. The 2-No. 9 bars are cut off tentatively at 4.5 ft and 3.5 ft from the exterior and interior supports, respectively. These tentative cutoff locations are determined as follows:

Dimensions (1) and (2) must be the larger of d or $12d_b$: *12.10.3*

d = 19.4 in. = 1.6 ft (governs)

$12d_b$ = 12 (1.128) = 13.5 in.

Within the development length ℓ_d, only 2-No. 8 bars are being developed (2-No. 9 bars are already developed in length 8.45 ft)

Development for No. 8 corner bars, see Table 4-2.

$\ell_d = 47d_b = 47\,(1.0) = 47 \text{ in.} = 3.9 \text{ ft}$

Dimension (3): 6.6 ft > 3.9 ft O.K.
Dimension (4): 5.7 ft > 3.9 ft O.K.

Check required development length ℓ_d for 2-No. 9 bars. Note that 2-No. 8 bars are already developed in length 4 ft from bar end.

Clear spacing between 2-No. 9 bars

$[16 - 2(1.5) - 2(0.5) - 2(1.0) - 2(1.128)]/3 = 2.58 \text{ in.} = 2.29d_b > 2d_b$

For No. 9 bar, $\ell_d = 47d_b$ *Table 4-2*

$= 47\,(1.128) = 53 \text{ in.} = 4.4 \text{ ft} < 8.45 \text{ ft}$ O.K.

For No. 8 bars, check development requirements at points of inflection (PI): *12.11.3*

$$\ell_d \le \frac{M_n}{V_u} + \ell_a$$ *Eq. (12-3)*

For 2-No. 8 bars, $M_n = 131.8/0.9 = 146.4$ ft-kips

At left PI, $V_u = 77.6 - 6(3.5) = 56.6$ kips

ℓ_a = larger of $12d_b = 12\,(1.0) = 12$ in. or d = 19.4 in. (governs)

$$\ell_d \le \frac{146.4 \times 12}{56.6} + 19.4 = 50.5 \text{ in.}$$

For No. 8 bars, $\ell_d = 47 \text{ in.} < 50.5 \text{ in.}$ O.K.

At right PI, $V_u = 56.8$ kips; by inspection, the development requirements for the No. 8 bars are O.K.

With both tentative cutoff points located in a zone of flexural tension, one of the three conditions of 12.10.5 must be satisfied.

At left cutoff point (4.5 ft from support):

$V_u = 77.6 - (4.5 \times 6) = 50.6$ kips

$\phi V_n = 79.4$ kips (No. 4 U-stirrups @ 7 in.)

$2/3\,(79.4) = 52.9 \text{ kips} > 50.6 \text{ kips}$ O.K. *12.10.5.1*

For illustrative purposes, determine if the condition of 12.10.5.3 is also satisfied:

M_u = 54.1 ft-kips at 4.5 ft from support

A_s required = 0.63 in.^{2}

For 2-No. 8 bars, A_s provided = 1.58 in.^{2}

1.58 in.^{2} > 2 (0.63) = 1.26 in.^{2} O.K. *12.10.5.3*

3/4 (79.4) = 59.6 kips > 50.6 kips O.K. *12.10.5.3*

Therefore, 12.10.5.3 is also satisfied at cutoff location.

At right cutoff point (3.5 ft from support):

$V_u = 72.4 - (3.5 \times 6) = 51.4$ kips

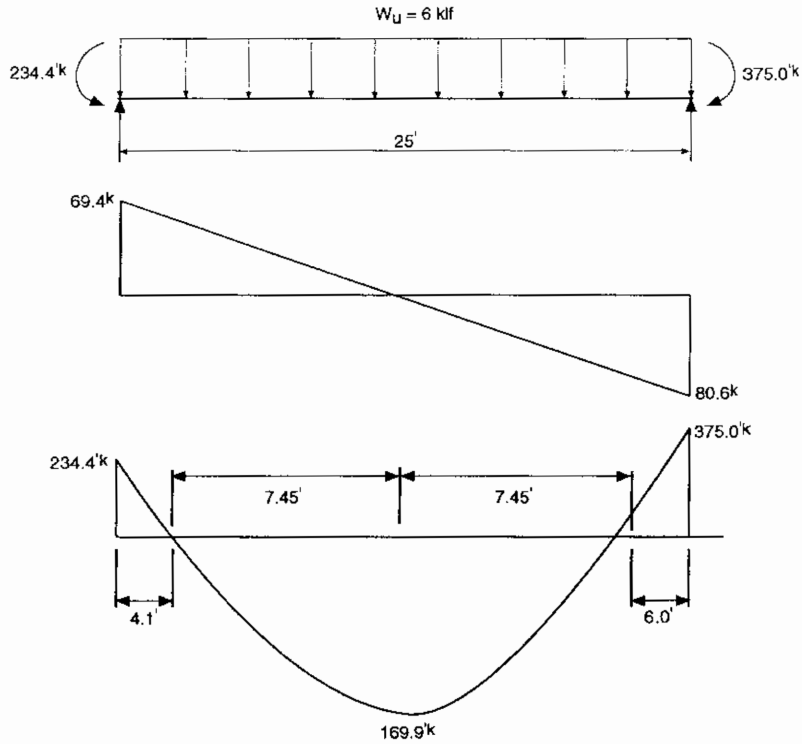
2/3 (ϕV_n) = 52.9 kips > 51.4 kips O.K. *12.10.5.1*

Summary: The tentative cutoff locations for the bottom reinforcement meet all code development requirements. The 2-No. 9 bars × 17 ft would have to be placed unsymmetrically within the span. To assure proper placing of the No. 9 bars, it would be prudent to specify a 18 ft length for symmetrical bar placement within the span, i.e., 3.5 ft from each support. The ends of the cut off bars would then be at or close to the points of inflection, thus, eliminating the need to satisfy the conditions of 12.10.5 when bars are terminated in a tension zone. The recommended bar arrangement is shown at the end of the example.

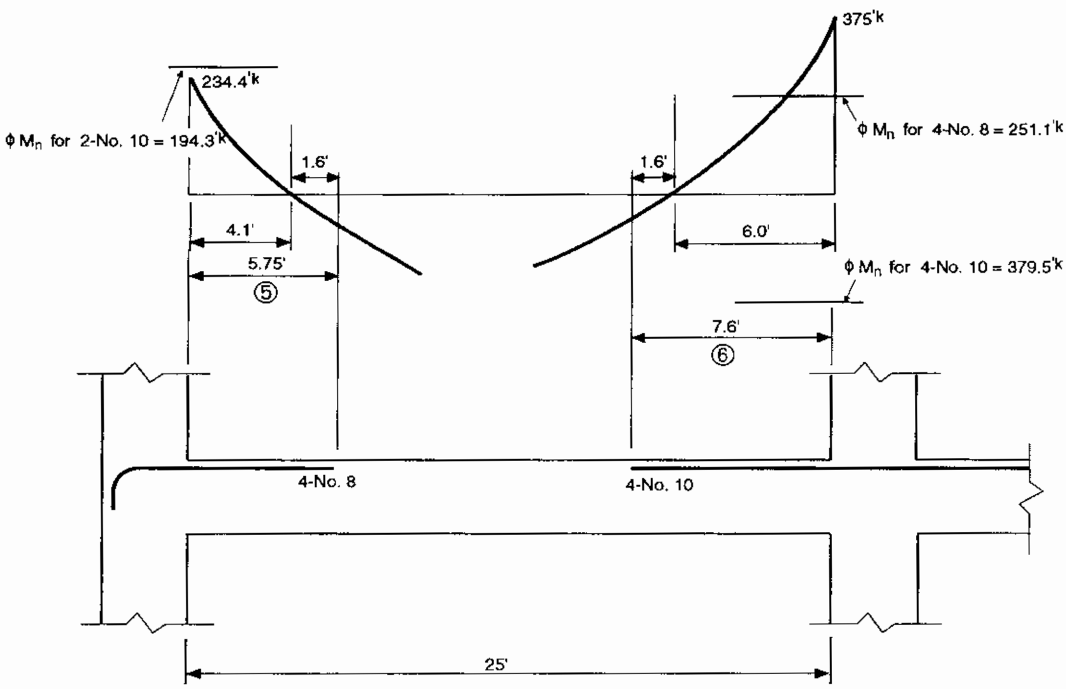
3. Bar lengths for top reinforcement

Shear and moment diagrams for loading condition causing maximum factored negative moments are shown below.

The negative moment portions of the M_u diagram are also shown below at a larger scale, including the design moment strengths ϕM_n for the total negative A_s at each support (4-No. 8 at exterior support and 4-No. 10 at interior support) and for 2-No. 10 bars at the interior support. For 4-No. 8, ϕM_n = 251.1 ft-kips. For 4-No. 10, ϕM_n = 379.5 ft-kips. For 2-No. 10, ϕM_n = 194.3 ft-kips.



W_u = 6 klf
234.4'k
375.0'k
25'
69.4k
80.6k
375.0'k
234.4'k
7.45'
7.45'
4.1'
6.0'
169.9'k



375'k
234.4'k
φ M_n for 2-No. 10 = 194.3'k
φ M_n for 4-No. 8 = 251.1'k
1.6'
1.6'
4.1'
6.0'
5.75'
5
φ M_n for 4-No. 10 = 379.5'k
7.6'
6
4-No. 8
4-No. 10
25'

4. Development requirements for 4-No. 8 bars at exterior support

a. Required number of bars to be extended.

One-third of ($-A_s$) provided at supports must be extended beyond the point of inflection a distance equal to the greater of d, $12d_b$, or $\ell_n/16$. *12.12.3*

$d = 19.4$ in. $= 1.6$ ft (governs)

$12d_b = 12\,(1.0) = 12.0$ in.

$\ell_n/16 = 25 \times 12/16 = 18.75$ in.

Since the inflection point is located only 4.1 ft from the support, total length of the No. 8 bars will be relatively short even with the required 1.6 ft extension beyond the point of inflection. Check required development length ℓ_d for a cutoff location at 5.75 ft from face of support.

Dimension (5) must be at least equal to ℓ_d *12.12.2*

For No. 8 bars, $\ell_d = 47d_b = 47\,(1.0) = 47$ in. *Table 4-2*

With 4-No. 8 bars being developed at same location (face of support):

Including top bar effect, $\ell_d = 1.3\,(47) = 61.1$ in.

For No. 8 top bars, $\ell_d = 61.1$ in. $= 5.1$ ft < 5.75 ft O.K.

b. Anchorage into exterior column.

The No. 8 bars can be anchored into the column with a standard end hook. From Table 4-4, $\ell_{dh} = 19.0$ in. The required ℓ_{dh} for the hook could be reduced if excess reinforcement is considered:

$$\frac{(A_s \text{ required})}{(A_s \text{ provided})} = \frac{2.93}{3.16} = 0.93$$ *12.5.3(c)*

$\ell_{dh} = 19 \times 0.93 = 17.7$ in.

Overall depth of column required would be $17.7 + 2 = 19.7$ in.

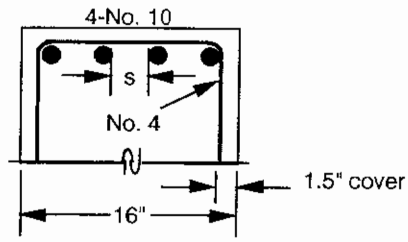
5. Development requirements for 4-No. 10 bars at interior column

a. Required extension for one-third of ($-A_s$) *12.12.3*

$d = 19.4$ in. $= 1.6$ ft (governs)

$12d_b = 12\ (1.27) = 15.24$ in.

$\ell_n/16 = 18.75$ in.



For No. 10 bars, clear spacing $= [16 - 2\ (1.5) - 2\ (0.5) - 4\ (1.27)]/3$
$= 2.31$ in. $= 1.82d_b > d_b$
Center-to-center spacing $= 2.82d_b$

Cover $= 1.5 + 0.5 = 2.0$ in. $= 1.57d_b > d_b$

Distance from center of bar to concrete surface $= 1.57d_b + 0.5d_b = 2.07d_b$

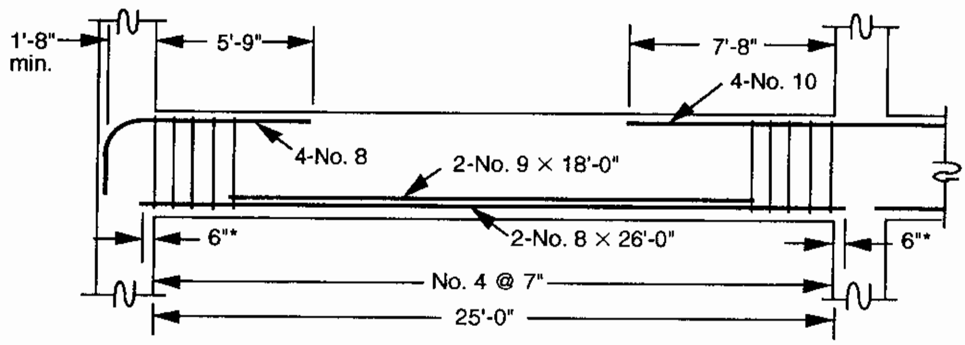
With minimum shear reinforcement provided and including top bar effect

$\ell_d = 1.3\ (47d_b)$ *Table 4-2*

$= 1.3\ (47)\ (1.27) = 77.6$ in.

Dimension (6) $= 6.0$ ft $+ 1.6$ ft $= 7.6$ ft $> \ell_d = 77.6$ in $= 6.5$ ft O.K.

6. Summary: Selected bar lengths for the top and bottom reinforcement shown below.



*see 12.11.1

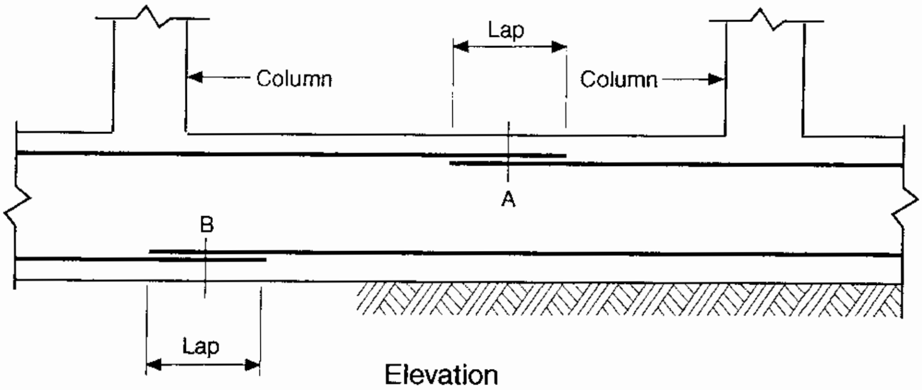
7. Supplementary Requirements

If the beam were part of a primary lateral load resisting system, the 2-No. 8 bottom bars extending into the supports would have to be anchored to develop the bar yield strength at the face of supports. At the exterior column, anchorage can be provided by a standard end hook. Minimum width of support (overall column depth) required for anchorage of the No. 8 bar with a standard hook is a function of the development length ℓ_{dh} from Table 4-4, and the appropriate modification factors (12.5.3). *12.11.2*

At the interior column, the 2-No. 8 bars could be extended ℓ_d distance beyond the face of support into the adjacent span or lap spliced with extended bars from the adjacent span. Consider a Class A lap splice adequate to satisfy the intent of 12.11.2.

Example 4.5—Lap Splices in Tension

Design the tension lap splices for the grade beam shown below.



Elevation

f'_c = 4000 psi
f_y = 60,000 psi, uncoated bars
b = 16 in.
h = 30 in.
Bar cover = 3.0 in.
4-No. 9 bars top and bottom (continuous)
No. 4 stirrups @ 14 in. (entire span)
$+M_u$ @B = 340 ft-kips
$-M_u$ @A = 120 ft-kips

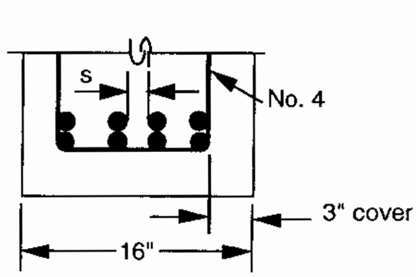
Preferably, splices should be located away from zones of high tension. For a typical grade beam, top bars should be spliced under the columns, and bottom bars about midway between columns. Even though in this example the splice at A is not a preferred location, the moment at A is relatively small. Assume for illustration that the splices must be located as shown.

Calculations and Discussion	Code Reference

Calculations for this example will be performed using 12.2.3.

Nominal diameter of No. 9 bar = 1.128 in.

Assuming all bars are spliced at the same location



Clear spacing = [16 - 2 (cover) - 2 (No. 4 stirrups) - 4 (No. 9 bars)]/3 spaces
= [16 - 2 (3.0) - 2 (0.50) - 4 (1.128)]/3
= 1.50 in.
= $1.33d_b$

Center-to-center spacing of bars being developed = 1.50 + 1.128 = 2.63 in. = $2.33d_b$

Clear cover = 3.0 + 0.5 = 3.5 in. = $3.1d_b$

Distance from center of bar to concrete surface = 3.0 + 0.5 + (1.128/2) = 4.1 in. = $3.6d_b$

c = the smaller of (1) distance from center of bar being developed to the nearest concrete surface and (2) one-half the center-to-center spacing of bars being developed

$c = 3.6d_b$

$= 2.33d_b/2 = 1.17d_b$ (governs)

Lap Splice of Bottom Reinforcement at Section B

$$\ell_d = \left(\frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\psi_t\psi_e\psi_s\lambda}{\left(\frac{c_b + K_{tr}}{d_b}\right)}\right)d_b$$ *Eq. (12-1)*

ψ_t = 1.0 for bottom bar *12.2.4*

ψ_e = 1.0 for uncoated bars

ψ_s = 1.0 for No. 7 and larger bars

λ = 1.0 for normal weight concrete

$c_b = 1.17d_b$ (computed above)

$$K_{tr} = \frac{A_{tr}f_{yt}}{1500sn}$$

A_{tr} = area of 2-No. 4 stirrups = 2 (0.2) = 0.4 in.^{2}
s = 14 in. spacing
n = 4 bars being developed

$$K_{tr} = \frac{0.4\ (60{,}000)}{1500\ (14)\ (4)} = 0.29 \text{ in.} = 0.26d_b$$

$$\left(\frac{c_b + K_{tr}}{d_b}\right) = 1.17 + 0.26 = 1.43 < 2.5 \quad \text{O.K.}$$ *12.2.3*

$$\ell_d = \frac{3\ (60{,}000)\ (1.0)\ (1.0)\ (1.0)\ (1.0)}{40\sqrt{4{,}000}\ (1.43)}(1.128) = 56.1 \text{ in.}$$

A_s required ($+M_u$ @ B = 340 ft-kips) = 3.11 in.^{2}

A_s provided (4 No. 9 bars) = 4.00 in.^{2}

$$\frac{A_s \text{ provided}}{A_s \text{ required}} = \frac{4.00}{3.11} = 1.29 < 2$$

Class B splice required = $1.3\ell_d$ *12.15.1*

12.15.2

Note: Even if lap splices were staggered (A_s spliced = 50%), a Class B splice must be used with (A_s provided/A_s required) < 2

Class B Splice = $1.3\ell_d$ = 1.3 (56.1) = 72.9 in. = 6.1ft

It is better practice to stagger alternate lap splices. As a result, the clear spacing between spliced bars will be increased with a potential reduction of development length.

Clear spacing = 2 (1.50) + 1.128 = 4.13 in. = $3.66d_b$

Center-to-center spacing of bars being developed = $3.66d_b + d_b = 4.66d_b$

Distance from center of bar to concrete surface = $3.6d_b$

Thus, $c = \dfrac{4.66d_b}{2} = 2.33d_b$

$$K_{tr} = \frac{2\,(0.2)\,(60{,}000)}{(1500)\,(14)\,(2)} = 0.57 \text{ in.} = 0.51d_b$$

Therefore, $\left(\dfrac{c_b + K_{tr}}{d_b}\right) = 2.33 + 0.51 = 2.84 > 2.5$ Use 2.5. *12.2.3*

$$\ell_d = \frac{3\,(60{,}000)\,(1.0)\,(1.0)\,(1.0)\,(1.0)}{40\sqrt{4000}\,(2.5)}(1.128) = 32.1 \text{ in.}$$

Class B splice = 1.3 (32.1) = 41.7 in. = 3.5 ft

Use 3 ft-6 in. lap splice @ B and stagger alternate lap splices.

Lap Splice of Top Reinforcement at Section A

As size of top and bottom reinforcement is the same, computed development and splice lengths for top bars will be equal to that of the bottom bars increased by the 1.3 multiplier for top bars. In addition, because positive and negative factored moments are different, the ratio of provided to required reinforcement may affect the type of splice as demonstrated below.

A_s required ($+M_u$ @ A = 120 ft-kips) = 1.05 in.^{2}

A_s provided/A_s required = 4.00/1.05 = 3.81 > 2

If alternate lap splices are staggered at least a lap length (A_s spliced = 50%):

Class A splice may be used = $1.0\ell_d$ *12.15.2*

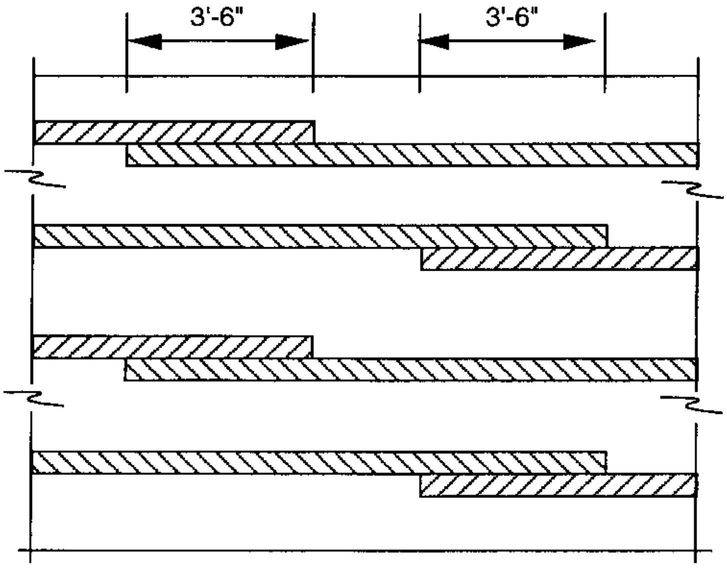
If all bars are lap spliced at the same location (within req'd lap length):

Class B splice must be used = $1.3\ell_d$

Assuming splices are staggered, the top bar multiplier will be 1.3.

Class A splice = 1.3 (1.0) (32.1) = 41.7 in. = 3.5 ft

Use 3 ft-6 in. lap splice @ A also, and stagger alternate lap splices.



Alternate lap splice stagger arrangement
(Note: bar laps are positioned vertically)

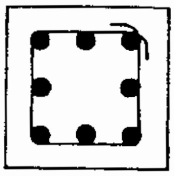
Example 4.6—Lap Splices in Compression

The following two examples illustrate typical calculations for compression lap splices in tied and spirally reinforced columns.

Calculations and Discussion	Code Reference

1. Design a compression lap splice for the tied column shown below. Assume all bars in compression for factored load combinations considered in design (Zone 1 in Fig. 4-21). See also Table 4-8.

   b = 16 in.
   h = 16 in.
   f'_c = 4000 psi
   f_y = 60,000 psi
   8-No. 9 bars



   a. Determine lap splice length: *12.16.1*

   For f_y = 60,000 psi:

   Length of lap = $0.0005f_yd_b$, but not less than 12 in.

   = 0.0005 (60,000) 1.128 = 34 in.

   b. Determine column tie requirements to allow an 0.83 reduced lap length: *12.17.2.4*

   Required column ties: No. 3 @ 16 in. o.c. *7.10.5.2*

   Required spacing of No. 3 ties for reduced lap length:

   effective area of ties $\geq$ 0.0015hs

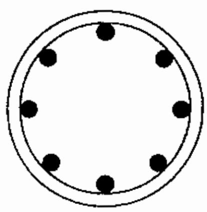
   $(2 \times 0.11) \geq 0.0015 \times 16s$

   s = 9.2 in.

   Spacing of the No. 3 ties must be reduced to 9 in. o.c. throughout the lap splice length to allow a lap length of 0.83 (34 in.) = 28 in.

2. Determine compression lap splice for spiral column shown.

   f'_c = 4000 psi
   f_y = 60,000 psi
   8-No. 9 bars
   No. 3 spirals



Example 4.6 (cont'd)	Calculations and Discussion	Code Reference

a. Determine lap splice length *12.16.1*

For bars enclosed within spirals, "basic" lap splice length may be multiplied by a factor of 0.75. *12.17.2.5*

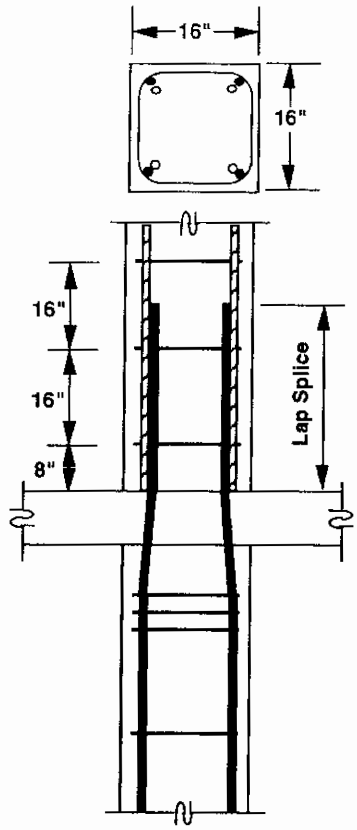
For f_y = 60,000 psi:

lap = 0.75(34) = 26 in.

Note: End bearing, welded, or mechanical connections may also be used. *12.16.3* *12.16.4*

Example 4.7—Lap Splices in Columns

Design the lap splice for the tied column detail shown.



- Continuing bars from column above (4-No. 8 bars)
- Offset bars from column below (4-No. 8 bars)

f'_c = 4000 psi (normal weight)
f_y = 60,000 psi
b = h = 16 in.
4-No. 8 bars (above and below floor level)
No. 3 ties @ 16 in.
Cover = 1.5 in.

Lap splice to be designed for the following factored load combinations:

1. P_u = 465 kips
   M_u = 20 ft-kips

2. P_u = 360 kips
   M_u = 120 ft-kips

3. P_u = 220 kips
   M_u = 100 ft-kips

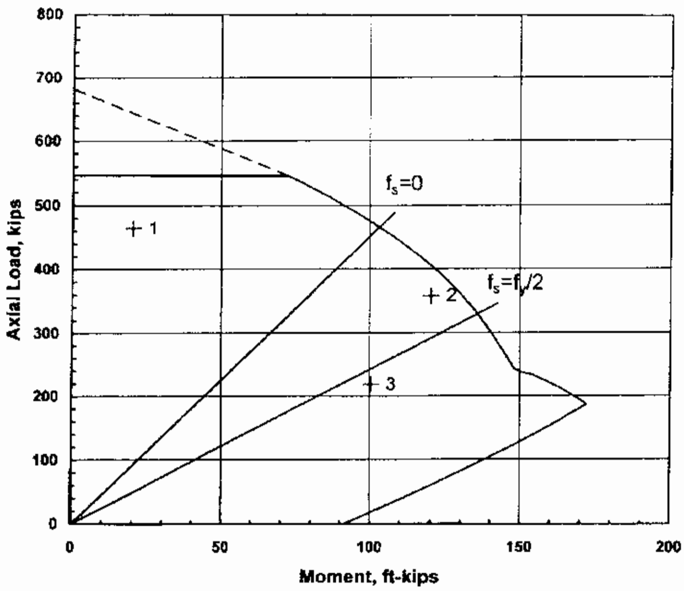
Calculations and Discussion	Code Reference

1. Determine type of lap splice required. *12.17.2*

   Type of lap splice to be used depends on the bar stress at the splice location due to all factored load combinations considered in the design of the column. For design purposes, type of lap splice will be based on which zone, or zones, of bar stress all factored load combinations are located on the column load-moment strength diagram. See discussion for 12.17.2, and Fig. 4-21. The load-moment strength diagram (column design chart) for the 16 × 16 column with 4-No. 8 bars is shown below, with the three factored load combinations considered in the design of the column located on the interaction strength diagram.

   Note that load combination (2) governed the design of the column (selection of 4-No. 8 bars). *Table 4-8*

   For load combination (1), all bars are in compression (Zone 1), and a compression lap splice could be used. For load combination (2), bar stress is not greater than $0.5f_y$ (Zone 2), so a Class B tension lap splice is required; or, a Class A splice may be used if alternate lap splices are staggered. For load combination (3), bar stress is greater than $0.5f_y$ (Zone 3), and a Class B splice must be used.



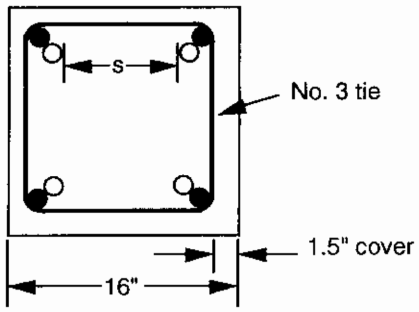
Interaction Diagram for 16 in. × 16 in. Column

Lap splice required for the 4-No. 8 bars must be based on the load combination producing the greatest amount of tension in the bars; for this example, load combination (3) governs the type of lap splice to be used.

Class B splice required = $\ell_d = 1.3\ell_{db}$ *12.15.1*

2. Determine lap splice length

Determine tension development length by 12.2.3.



Nominal diameter of No. 8 bar = 1.00 in.

Clear spacing between bars being developed is large and will not govern.

Clear cover = 1.5 + 0.375 = 1.875 in. = $1.875d_b$

Distance from center of bar to concrete surface = 1.875 + 0.5 = 2.375 in. = $2.375d_b$

c = the smaller of (1) distance from center of bar being developed to the nearest concrete surface, and of (2) one-half the center-to-center spacing of bars being developed

$c = 2.375d_b$

$$\ell_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\psi_t \psi_e \psi_s \lambda}{\left(\frac{c_b + K_{tr}}{d_b} \right)} \right) d_b$$ *Eq. (12-1)*

ψ_t = 1.0 for vertical bar *12.2.4*

ψ_e = 1.0 for uncoated bars

ψ_s = 1.0 for No. 7 and larger bars

λ = 1.0 for normal weight concrete

$c_b = 2.375d_b$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn}$$

A_{tr} = area of 2-No. 3 ties

s = 16 in. spacing

n = 2 bars being developed on one column face

$$K_{tr} = \frac{2\ (0.11)\ (60{,}000)}{1500\ (16)\ (2)} = 0.275 \text{ in.} = 0.275\ d_b$$

$$\left(\frac{c_b + K_{tr}}{d_b} \right) = 2.375 + 0.275 = 2.65 > 2.5 \quad \text{Use } 2.5$$ *12.2.3*

$$\ell_d = \frac{3\ (60{,}000)\ (1.0)\ (1.0)\ (1.0)\ (1.0)}{40\sqrt{4000}\ (2.5)} (1.00) = 28.5 \text{ in.}$$

Class B splice = 1.3(28.5) = 37 in.

Use 37 in. lap splice for the 4 No. 8 bars at the floor level indicated.

Blank

5

Design Methods and Strength Requirements

UPDATE FOR THE '05 CODE

Expressions to determine strength reduction factor ϕ within the transition zone were revised in Figure R.9.3.2. The expressions were modified to resolve the inaccuracy reported from some users of the 2002 code.

In pretensioned members, 9.3.2.7 was revised to allow the linear increase of the ϕ factor from 0.75 to 0.9 for sections in flexural members located between the end of the transfer length and the end of the development length. This revision was introduced to remedy a discontinuity in the calculated flexural strength along the length of prentntioned members.

Spiral transverse reinforcement (10.9.3) has been excluded from the upper limit of 80,000 psi yield strength (9.4). Research shows that 100,000 psi yield strength reinforcement can be used for confinement. This change will help reduce congestion, and allow easier concrete consolidation.

8.1 DESIGN METHODS

Two philosophies of design for reinforced concrete have long been prevalent. Working Stress Design was the principal method used from the early 1900s until the early 1960s. Since publication of the 1963 edition of the ACI code, there has been a rapid transition to Ultimate Strength Design, largely because of its more rational approach. Ultimate strength design, referred to in the code as the Strength Design Method (SDM) is conceptually more realistic in its approach to structural safety and reliability at the strength limit state.

The 1956 ACI code (ACI 318-56) was the first code edition which officially recognized and permitted the ultimate strength method of design. Recommendations for the design of reinforced concrete structures by ultimate strength theories were included in an appendix.

The 1963 ACI code (ACI 318-63) treated the working stress and the ultimate strength methods on an equal basis. However, a major portion of the working stress method was modified to reflect ultimate strength behavior. The working stress provisions of the 1963 code, relating to bond, shear and diagonal tension, and combined axial compression and bending, had their basis in ultimate strength.

The 1971 ACI code (ACI 318-71) was based entirely on "ultimate strength design" for proportioning reinforced concrete members, except for section (8.10) devoted to what was called the Alternate Design Method (ADM). The ADM was not applicable to the design of prestressed concrete members. Even in that section, the service load capacities (except for flexure) were given as various percentages of the ultimate strength capacities of other parts of the code. The transition to ultimate strength methods for reinforced concrete design was essentially complete in the 1971 ACI code, with ultimate strength design definitely established as being preferred.

In the 1977 ACI code (ACI 318-77) the ADM was relegated to Appendix B. The appendix location served to separate and clarify the two methods of design, with the main body of the code devoted exclusively to the SDM. The ADM was retained in all editions of the code from 1977 to the 1999 edition, where it was found in Appendix A. In 2002, the code underwent the most significant revisions since 1963. The ADM method was deleted from the 2002 code (ACI 318-02). It is still referenced in Commentary Section R1.1 of the 2002 code. The general serviceability requirements of the main body of the code, such as the provisions for deflection and crack control, must always be satisfied.

A modification to the SDM, referred to as the Unified Design Provisions, was added to the '95 edition of the code. In keeping with tradition, the method was added as Appendix B. The provisions apply to the design of nonprestressed and prestressed members subject to flexure and axial loads. The Unified Design Provisions were incorporated into the body starting with the 2002 code. See 8.1.2 below.

8.1.1 Strength Design Method

The Strength Design Method requires that the design strength of a member at any section should equal or exceed the required strength calculated by the code-specified factored load combinations. In general,

Design Strength ≥ Required Strength (U)

where

Design Strength = Strength Reduction Factor (ϕ) x Nominal Strength

ϕ = Strength reduction factor that accounts for (1) the probability of understrength of a member due to variations in material strengths and dimensions, (2) inaccuracies in the design equations, (3) the degree of ductility and required reliability of the loaded member, and (4) the importance of the member in the structure (see 9.3.2).

Nominal Strength = Strength of a member or cross-section calculated using assumptions and strength equations of the Strength Design Method before application of any strength reduction factors.

Required Strength (U) = Load factors × Service load effects. The required strength is computed in accordance with the load combinations in 9.2.

Load Factor = Overload factor due to probable variation of service loads.

Service Load = Load specified by general building code (unfactored).

<u>Notation</u>

Required strength:

M_u = factored moment (required flexural strength)
P_u = factored axial force (required axial load strength) at given eccentricity
V_u = factored shear force (required shear strength)
T_u = factored torsional moment (required torsional strength)

Nominal strength:

M_n = nominal flexural strength
M_b = nominal flexural moment strength at balanced strain conditions
P_n = nominal axial strength at given eccentricity

P_o = nominal axial strength at zero eccentricity
P_b = nominal axial strength at balanced strain conditions
V_n = nominal shear strength
V_c = nominal shear strength provided by concrete
V_s = nominal shear strength provided by shear reinforcement
T_n = nominal torsional moment strength

Design Strength:

ϕM_n = design flexural strength
ϕP_n = design axial strength at given eccentricity
ϕV_n = design shear strength = $\phi (V_c + V_s)$
ϕT_n = design torsional moment strength

Section R2.2 gives an in-depth discussion on many of the concepts in the Strength Design Method.

8.1.2 Unified Design Provisions

A modification to the Strength Design Method for nonprestressed and prestressed concrete flexural and compression members was introduced in 1995 in Appendix B. This appendix introduced substantial changes in the design for flexure and axial loads. Reinforcement limits, strength reduction factors ϕ, and moment redistribution were affected.

The Unified Design method is similar to the Strength Design Method in that it uses factored loads and strength reduction factors to proportion the members. The main difference is that in the Unified Design Provisions, a concrete section is defined as either compression-controlled or tension-controlled, depending on the magnitude of the net tensile strain in the reinforcement closest to the tension face of a member. The ϕ factor is then determined by the strain conditions at a section at nominal strength. Prior to these provisions, the ϕ factors were specified for cases of axial load or flexure or both in terms of the type of loading.

It is important to note that the Unified Design Provisions do not alter nominal strength calculations. The major differences occur in checking reinforcement limits for flexural members, determining the ϕ factor for columns with small axial load, and computing redistributed moments. Most other applicable provisions in the body of the 1999 code apply to design using the current code.

The code sections displaced by the Unified Design Provisions are now located in Appendix B. These former provisions are still permitted to be used.

In general, the Unified Design Provisions provide consistent means for designing nonprestressed and prestressed flexural and compression members, and produce results similar to those obtained from the Strength Design Method. The examples in Part 6 and Ref. 5.1 illustrate the use of this new design method.

9.1 STRENGTH AND SERVICEABILITY—GENERAL

9.1.1 Strength Requirements

The basic criterion for strength design as indicated in 9.1.1 is as follows:

Design Strength ≥ Required Strength

Strength Reduction Factor (ϕ) × Nominal Strength ≥ Load Factor × Service Load Effects

All structural members and sections must be proportioned to meet the above criterion under the most critical load combination for all possible actions (flexure, axial load, shear, etc.):

$$\phi P_n \geq P_u$$

$$\phi M_n \geq M_u$$

$$\phi V_n \geq V_u$$

$$\phi T_n \geq T_u$$

The above criterion provides for the margin of structural safety in two ways:

1. It decreases the strength by multiplying the nominal strength with the appropriate strength reduction factor ϕ, which is always less than 1. The nominal strength is computed by the code procedures assuming that the member or the section will have the exact dimensions and material properties assumed in the computations. For example, the nominal flexural strength for the singly reinforced section shown in Fig. 5-1 is:

   $$M_n = A_s f_y (d - a/2)$$

   and the design flexural moment strength is

   $$\phi M_n = \phi[A_s f_y (d - a/2)]$$

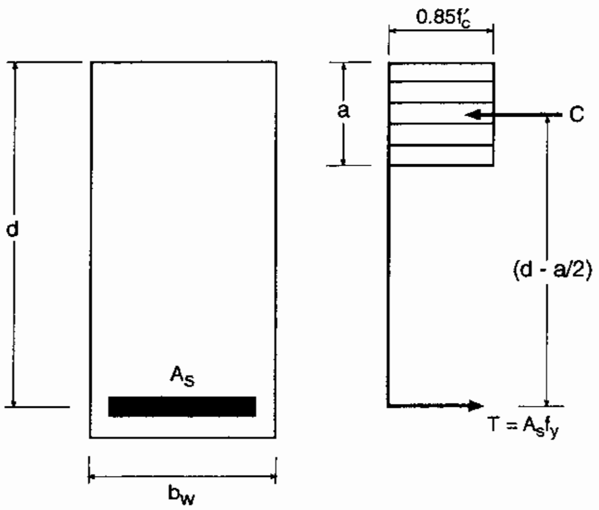


Figure 5-1 Singly Reinforced Section

2. It increases the required strength by using factored loads or the factored internal moments and forces. Factored loads are defined in 2.2 as service loads multiplied by the appropriate load factors. The loads to be used are described in 8.2. Thus, the required flexural strength of the section shown in Fig. 5-1 for dead and live loads is:

   $$M_u = 1.2M_d + 1.6M_\ell \geq 1.4M_d \qquad \textit{Eqs. (9-1) \& (9-2)}$$

   where M_d and M_ℓ are the moments due to service dead and live loads, respectively.

Thus, the design strength requirement for this section becomes:

$$\phi\,[A_s f_y\,(d - a/2)] \geq 1.2M_d + 1.6\,M_\ell \geq 1.4M_d$$

Similarly, for shear acting on the section, the criterion for strength design can be stated as:

$$\phi\,V_n = \phi\,(V_c + V_s) \geq V_u$$

$$\phi\left[2\sqrt{f_c'}\,b_w d + \frac{A_v f_y d}{s}\right] \geq 1.2V_d + 1.6\,V_\ell \geq 1.4V_d$$

The following are the reasons for requiring strength reduction factors and load factors in strength design: [5.2]

1. The strength reduction of materials or elements is required because:

   a. Material strengths may differ from those assumed in design because of:

      - Variability in material strengths—the compression strength of concrete as well as the yield strength and ultimate tensile strength of reinforcement are variable.
      - Effect of testing speed—the strengths of both concrete and steel are affected by the rate of loading.
      - In situ strength vs. specimen strength—the strength of concrete in a structure is somewhat different from the strength of the same concrete in a control specimen.
      - Effect of variability of shrinkage stresses or residual stresses—the variability of the residual stresses due to shrinkage may affect the cracking load of a member, and is significant where cracking is the critical limit state. Similarly, the transfer of compression loading from concrete to steel due to creep and shrinkage in columns may lead to premature yielding of the compression steel, possibly resulting in instability failures of slender columns with small amounts of reinforcement.

   b. Member dimensions may vary from those assumed, due to construction/fabrication tolerances. The following are significant:

      - Formwork tolerances affecting final member dimensions.
      - Rolling and fabrication tolerances in reinforcing bars.
      - Geometric tolerances in cross-section and reinforcement placement tolerances.

   c. Assumptions and simplifications in design equations, such as use of the rectangular stress block and the maximum usable strain of concrete equal to 0.003, introduce both systematic and random inaccuracies.

   d. The use of discrete bar sizes leads to variations in the actual capacity of members. Calculated area of reinforcement has to be rounded up to match the area of an integer number of reinforcing bars.

2. The load factors are required for possible overloading because:

   a. Magnitudes of loads may vary from those determined from building codes. Dead loads may vary because of:

      - Variations in member sizes.
      - Variations in material density.
      - Structural and nonstructural alterations.

   Live loads can vary considerably from time to time and from building to building.

b. Uncertainties exist in the calculation of load effects—the assumptions of stiffnesses, span lengths, etc., and the inaccuracies involved in modeling three-dimensional structures for structural analysis lead to differences between the stresses which actually occur in a building and those estimated in the designer's analysis.

3. Strength reduction and load increase are also required because the consequences of failure may be severe. A number of factors should be considered:

   a. The type of failure, warning of failure, and existence of alternative load paths.
   b. Potential loss of life.
   c. Costs to society in lost time, lost revenue, or indirect loss of life or property due to failure.
   d. The importance of the structural element in the structure.
   e. Cost of replacing the structure.

   By way of background to the numerical values of load factors and strength reduction factors specified in the code, it may be worthwhile reproducing the following paragraph from Ref. 5.2:

   > "The ACI ... design requirements ... are based on an underlying assumption that if the probability of understrength members is roughly 1 in 100 and the probability of overload is roughly 1 in 1000, the probability of overload on an understrength structure is about 1 in 100,000. Load factors were derived to achieve this probability of overload. Based on values of concrete and steel strength corresponding to probability of 1 in 100 of understrength, the strengths of a number of typical sections were computed. The ratio of the strength based on these values to the strength based on nominal strengths of a number of typical sections were arbitrarily adjusted to allow for the consequences of failure and the mode of failure of a particular type of member, and for a number of other sources of variation in strength."

   An Appendix to Ref. 5.2 traces the history of development of the current ACI load and strength reduction factors.

9.1.2 Serviceability Requirements

The provisions for adequate strength do not necessarily ensure acceptable behavior of the member at service load levels. Therefore, the code includes additional requirements to provide satisfactory service load performance.

There is not always a clear separation between the provisions for strength and those for serviceability. For actions other than flexure, the detailing provisions in conjunction with the strength requirements are meant to ensure adequate performance at service loads. For flexural action, there are special serviceability requirements concerning short and long term deflections, distribution of reinforcement, crack control, and permissible stresses in prestressed concrete. A consideration of service load deflections is particularly important in view of the extended use of high-strength materials and more accurate methods of design which result in increasingly slender reinforced concrete members.

9.1.3 Appendix C

Starting with thw 2002 code, the load factors and strength reduction factors used in the 1999 and earlier codes were placed in Appendix C. Use of Appendix C is permitted by 9.1.3. However, it is mandatory that both the load combinations and strength reduction factors of Appendix C are used together.

9.2 REQUIRED STRENGTH

As previously stated, the required strength U is expressed in terms of factored loads, or their related internal moments and forces. Factored loads are the service-level loads specified in the general building code, multiplied by appropriate load factors in 9.2. It is important to recognize that earthquake forces computed in accordance with the latest editions of the model buildings codes in use in the U. S. are strength-level forces. Specifically, seismic forces calculated under the 1993 and later editions of *The BOCA National Building Code*, the 1994 and later editions of the *Standard Building Code*, and the 1997 *Uniform Building Code* are strength-level forces. In addition, the 2000 and 2003 *International Building Code* (IBC) developed by the International Code Council have seismic provisions that are strength-level forces.

This development has created confusion within the structural engineering profession since when designing in concrete one must use some load combinations from ACI 318 and others from the governing building code. To assist the structural engineer in understanding the various load combinations and their proper application to design of concrete structural elements governed by one of these codes, a publication was developed by PCA in 1998. *Strength Design Load Combinations for Concrete Elements*[5.3] provides background on the use of the ACI 318 factored load combinations. In addition, it cites the load combinations in the model codes, including the IBC, that must be used for seismic design.

Section 9.2 prescribes load factors for specific combinations of loads. A list of these combinations is shown below. The numerical value of the load factor assigned to each type of load is influenced by the degree of accuracy with which the load can usually be assessed, the variation which may be expected in the load during the lifetime of a structure and the probability of simultaneous occurrence of different load types. Hence, dead loads, because they can usually be more accurately determined and are less variable, are assigned a lower load factor (1.2) as compared to live loads (1.6). Also, weight and pressure of liquids with well-defined densities and controllable maximum heights are assigned a reduced load factor of 1.2 due the lesser probability of overloading. A higher load factor of 1.6 is required for earth and groundwater pressures due to considerable uncertainty of their magnitude and recurrence. Note that while most usual combinations of loads are included, it should not be assumed that all cases are covered. Section 9.2 contains load combination as follows:

$$U = 1.4(D + F) \qquad \textit{Eq. (9-1)}$$

$$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R) \qquad \textit{Eq. (9-2)}$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W) \qquad \textit{Eq. (9-3)}$$

$$U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R) \qquad \textit{Eq. (9-4)}$$

$$U = 1.2D + 1.0E + 1.0L + 0.2S \qquad \textit{Eq. (9-5)}$$

$$U = 0.9D + 1.6W + 1.6H \qquad \textit{Eq. (9-6)}$$

$$U = 0.9D + 1.0E + 1.6H \qquad \textit{Eq. (9-7)}$$

where:

D = dead loads, or related internal moments and forces

E = load effects of seismic forces, or related internal moments and forces

F = loads due to weight and pressures of fluids with well-defined densities and controllable maximum heights, or related internal moments and forces

H = loads due to weight and pressure of soil, water in soil, or other materials, or related internal moments and forces

L = live loads, or related internal moments and forces
L_r = roof live load, or related internal moments and forces
R = rain load, or related internal moments and forces
S = snow load, or related internal moments and forces
T = cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage-compensating concrete
U = required strength to resist factored loads or related internal moments and forces
W = wind load, or related internal moments and forces

Note that in Eqs. (9-1) through (9-7), the effect of one or more loads not acting simultaneously must also be investigated.

Exceptions to the load combination are as follows:

1. The load factor on L in Eq. (9-3), (9-4), and (9-5) shall be permitted to be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where the live load L is greater than 100 lb/ft^2.
2. Where wind load W has not been reduced by a directionality factor, it shall be permitted to use 1.3W in place of 1.6W in Eq. (9-4) and (9-6). Note that the wind load equation in ASCE 7-98 and IBC 2000 includes a factor for wind directionality that is equal to 0.85 for buildings. The corresponding load factor for wind in the load combination equations was increased accordingly (1.3/0.85 = 1.53 rounded up to 1.6). The code allows use of the previous wind load factor of 1.3 when the design wind load is obtained from other sources that do not include the wind directionality factor.
3. Where earthquake load E is based on service-level seismic forces, 1.4E shall be used in place of 1.0E in Eq. (9-5) and (9-7).
4. The load factor on H shall be set equal to zero in Eq. (9-6) and (9-7) if the structural action due to H counteracts that due to W or E. Where lateral earth pressure provides resistance to structural actions from other forces, it shall not be included in H but shall be included in the design resistance.

Other consideration related to load combination are as follows:

1. Resistance to impact effects, where applicable, shall be included with live load (9.2.2).
2. Differential settlement, creep, shrinkage, expansion of shrinkage-compensating concrete, or temperature change shall be based on a realistic assessment of such effects occurring in service (9.2.3).
3. For a structure in a flood zone, the flood load and load combinations of ASCE 7 shall be used (9.2.4).
4. For post-tensioned anchorage zone design, a load factor of 1.2 shall be applied to the maximum prestressing steel jacking force (9.2.5).

For many members, the loads considered are dead, live, wind, and earthquake. Where the F, H, R, S, and T loads are not considered, the seven equations simplify to those given in Table 5-1 below.

Table 5-1 Required Strength for Simplified Load Combinations

Loads	Required Strength	Code Eq. No.
Dead (D) and Live (L)	$1.4D$	9-1
	$1.2D + 1.6L + 0.5L_r$	9-2
Dead, Live, and Wind (W)	$1.2D + 1.6L_r + 1.0L$	9-3
	$1.2D + 1.6L_r + 0.8W$	9-3
	$1.2D + 1.6W + 1.0L + 0.5L_r$	9-4
	$0.9D + 1.6W$	9-6
Dead, Live, and Earthquake (E)	$1.2D + 1.0L + 1.0E$	9-5
	$0.9D + 1.0E$	9-7

While considering gravity loads (dead and live), a designer using the code moment coefficients (same coefficients for dead and live loads—8.3.3) has three choices: (1) multiplying the loads by the appropriate load factors, adding them into the total factored load, and then computing the forces and moments due to the total load, (2) computing the effects of factored dead and live loads separately, and then superimposing the effects, or (3) computing the effects of unfactored dead and live loads separately, multiplying the effects by the appropriate load factors, and then superimposing them. Under the principle of superposition, all three procedures yield the same answer. For designers performing a more exact analysis using different coefficients for dead and live loads (pattern loading for live loads), choice (1) does not exist. While considering gravity as well as lateral loads, load effects (due to factored or unfactored loads), of course, have to be computed separately before any superposition can be made.

In determining the required strength for combinations of loads, due regard must be given to the proper sign (positive or negative), since one type of loading may produce effects that either add to or counteract the effect of another load. Even though Eqs. (9-6) and (9-7) have a positive sign preceding the wind (W) or earthquake (E) load, the combinations are to be used when wind or earthquake forces or effects counteract those due to dead loads. When the effects of gravity loads and wind (W) or earthquake (E) loads are additive, Eqs. (9-4), (9-5), and (9-6) must be used.

Consideration must be given to various combinations of loads in determining the most critical design combination. This is of particular importance when strength is dependent on more than one load effect, such as strength under combined moment and axial load, or the shear strength of members carrying axial load.

9.3 DESIGN STRENGTH

9.3.1 Nominal Strength vs. Design Strength

The design strength provided by a member, its connections to other members, and its cross-section, in terms of flexure, axial load, shear, and torsion, is equal to the nominal strength calculated in accordance with the provisions and assumptions stipulated in the code, multiplied by a strength reduction factor ϕ, which is less than unity. The rules for computing the nominal strength are based generally on conservatively chosen limit states of stress, strain, cracking or crushing, and conform to research data for each type of structural action. An understanding of all aspects of the strengths computed for various actions can only be obtained by reviewing the background to the code provisions.

9.3.2 Strength Reduction Factors

The ϕ factors prescribed for structural concrete in 9.3.2 are listed in Table 5-2. The reasons for use of strength reduction factors have been given in earlier sections.

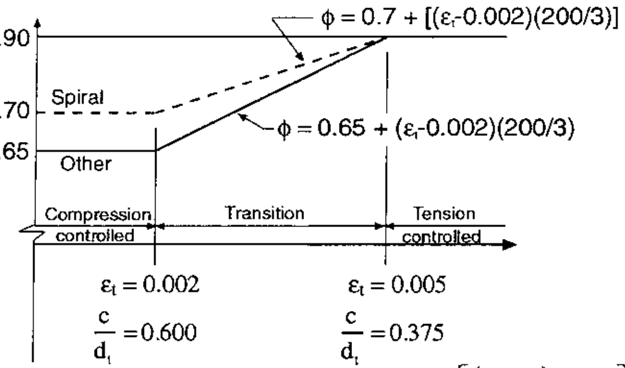
Table 5-2 Strength Reduction Factors ϕ in the Strength Design Method

Tension-controlled sections	0.90
Compression-controlled sections	
Members with spiral reinforcement conforming to 10.9.3	0.70
Other reinforced members	0.65
Shear and torsion	0.75
Bearing on concrete (except for post-tensioned anchorage zones)	0.65
Post-tensioned anchorage zones	0.85
Struts, ties, nodal zones and shearing areas in strut-and-tie models (Appendix A)	0.75

Note that a lower ϕ factor is used for compression-controlled (e.g. columns) sections than for tension-controlled (e.g. beams) sections. This is because compression-controlled sections generally have less ductility and are more sensitive to variations in concrete strength. Additionally, the consequences of failure of a column would generally be more severe than those for failure of a beam. Furthermore, columns with spiral reinforcement are assigned a higher ϕ factor than tied columns because the former have greater toughness and ductility.

Tension-controlled sections and compression-controlled sections are defined in 10.3.3. See Part 6 for detailed discussion.

The code permits a linear transition in ϕ between the limits for tension-controlled and compression-controlled sections. For sections in which the net tensile strain in the extreme tension steel at nominal strength is between the limits for compression-controlled and tension-controlled section, ϕ is permitted to be linearly increased from that for compression-controlled sections to 0.90 as the net tensile strain in the extreme tension steel at nominal strength increases from the compression-controlled strain limit to 0.005. This is best illustrated by Figure 5-2.



Interpolation on c/d_t: Spiral $\phi = 0.70 + 0.20\left[\left(\frac{1}{c/d_t}\right) - \left(\frac{5}{3}\right)\right]$

$$\text{Other } \phi = 0.65 + 0.25\left[\left(\frac{1}{c/d_t}\right) - \left(\frac{5}{3}\right)\right]$$

Figure 5-2 Variation of ϕ with Net Tensile ε_t and c/d_t for Grade 60 Reinforcement and for Prestressing Steel

For members subject to flexure and axial load, the design strengths are determined by multiplying both P_n and M_n by the appropriate single value of ϕ.

9.3.3 Development Lengths for Reinforcement

Development lengths for reinforcement, as specified in Chapter 12, do not require a strength reduction modification. Likewise, ϕ factors are not required for splice lengths, since these are expressed in multiples of development lengths.

9.3.5 Structural Plain Concrete

This section specifies that the strength reduction factor ϕ = 0.55 be used for the nominal strength in flexure, compression, shear, and bearing of plain concrete in Chapter 22 of the code. This is because both the flexural tension strength and the shear strength of plain concrete depend on the tensile strength characteristics of concrete having no reserve strength or ductility in the absence of steel reinforcement.

9.4 DESIGN STRENGTH FOR REINFORCEMENT

An upper limit of 80,000 psi is placed on the yield strength of reinforcing steels other than prestressing steel and spiral transverse reinforcement in 10.9.3. A steel strength above 80,000 psi is not recommended because the yield strain of 80,000 psi steel is about equal to the maximum usable strain of concrete in compression. Currently there is no ASTM specification for Grade 80 reinforcement. However, No. 11, No. 14, and No. 18 deformed reinforcing bars with a yield strength of 75,000 psi (Grade 75) are included in ASTM A615.

In accordance with 3.5.3.2, use of reinforcing bars with a specified yield strength f_y exceeding 60,000 psi requires that f_y be the stress corresponding to a strain of 0.35 percent. ASTM A615 for Grade 75 bars includes the same requirement. The 0.35 percent strain requirement also applies to welded wire reinforcement with wire having a specified yield strength greater than 60,000 psi. Higher-yield-strength wire is available and a value of f_y greater than 60,000 psi can be used in design, provided compliance with the 0.35 percent strain requirement is certified.

There are limitations on the yield strength of reinforcement in other sections of the code:

1. Sections 11.5.2, 11.6.3.4, and 11.7.6: The maximum f_y that may be used in design for shear, combined shear and torsion, and shear friction is 60,000 psi, except that f_y up to 80,000 psi may be used only for shear reinforcement consisting of welded deformed wire reinforcement meeting the requirements of ASTM A497.

2. Sections 19.3.2 and 21.2.5: The maximum specified f_y is 60,000 psi in shells, folded plates and structures governed by the special seismic provisions of Chapter 21.

In addition, the deflection provisions of 9.5 and the limitations on distribution of flexural reinforcement of 10.6 will become increasingly critical as f_y increases.

REFERENCES

5.1 Mast, R. F., "Unified Design Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members," *ACI Structural Journal*, Vol. 89, No. 2, March-April 1992, pp. 185-199.

5.2. MacGregor, J. G., "Safety and Limit States Design for Reinforced Concrete," *Canadian Journal of Civil Engineering*, Vol. 3, No. 4, December 1976, pp. 484-513.

5.3 *Strength Design Load Combinations for Concrete Elements*, Publication IS521, Portland Cement Association, Skokie, Illinois, 1998.

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6

General Principles of Strength Design

UPDATE TO THE '05 CODE

A minor editorial change was made in 2005 in 10.3.5 to clarify that the axial load limit of $0.10 f'_c A_g$ corresponds to the "factored axial load" for nonprestressed flexural members.

GENERAL CONSIDERATIONS

Historically, ultimate strength was the earliest method used in design, since the ultimate load could be measured by test without a knowledge of the magnitude or distribution of internal stresses. Since the early 1900s, experimental and analytical investigations have been conducted to develop ultimate strength design theories that would predict the ultimate load measured by test. Some of the early theories that resulted from the experimental and analytical investigations are reviewed in Fig. 6-1.

Structural concrete and reinforcing steel both behave inelastically as ultimate strength is approached. In theories dealing with the ultimate strength of reinforced concrete, the inelastic behavior of both materials must be considered and must be expressed in mathematical terms. For reinforcing steel with a distinct yield point, the inelastic behavior may be expressed by a bilinear stress-strain relationship (Fig. 6-2). For concrete, the inelastic stress distribution is more difficult to measure experimentally and to express in mathematical terms.

Studies of inelastic concrete stress distribution have resulted in numerous proposed stress distributions as outlined in Fig. 6-1. The development of our present ultimate strength design procedures has its basis in these early experimental and analytical studies. Ultimate strength of reinforced concrete in American design specifications is based primarily on the 1912 and 1932 theories (Fig. 6-1).

INTRODUCTION TO UNIFIED DESIGN PROVISIONS

The Unified Design Provisions introduced in the main body of the code in 2002 do not alter nominal strengths. The nominal strength of a section subject to flexure, axial load, or combinations thereof is the same as it was in previous codes. However, the Unified Design Provisions do alter the calculations of design strengths, which are reduced from nominal strengths by the strength reduction factor ϕ.

The following definitions are related to the Unified Design Provisions, and are given in Chapter 2 of the code. These definitions are briefly explained here, with further detailed discussion under the relevant code sections.

1. Net tensile strain: The tensile strain at nominal strength exclusive of strains due to effective prestress, creep, shrinkage, and temperature. The phrase "at nominal strength" in the definition means at the time the concrete in compression reaches its assumed strain limit of 0.003 (10.2.3). The "net tensile strain" is the strain caused by bending moments and axial loads, exclusive of strain caused by prestressing and by volume changes. The net tensile strain is that normally calculated in nominal strength calculations.

2. Extreme tension steel: The reinforcement (prestressed or nonprestressed) that is the farthest from the extreme compression fiber. The symbol d_t denotes the depth from the extreme compression fiber to the extreme tensile steel. The net tensile strain in the extreme tension steel is simply the maximum tensile steel strain due to external loads.

3. Compression-controlled strain limit: The net tensile strain at balanced strain conditions; see 10.3.2. The definition of balanced strain conditions in 10.3.2 is unchanged from previous editions of the code. Thus, the concrete reaches a strain of 0.003 as the tension steel reaches yield strain. However, 10.3.3 permits the compression-controlled strain limit for Grade 60 reinforcement and for prestressed reinforcement to be set equal to 0.002.

4. Compression-controlled section: A cross-section in which the net tensile strain in the extreme tension steel at nominal strength is less than or equal to the compression-controlled strain limit. The strength reduction factor ϕ for compression-controlled sections is set at 0.65 or 0.7 in 9.3.2.2.

5. Tension-controlled section: A cross-section in which the net tensile strain in the extreme tension steel at nominal strength is greater than or equal to 0.005. The strength reduction factor ϕ for tension-controlled sections is set at 0.9 in 9.3.2.1. However, ACI 318-99 and earlier editions of the code permitted a ϕ of 0.9 to be used for flexural members with reinforcement ratios not exceeding 0.75 of the balanced reinforcement ration ρ_b. For rectangutions, with one layer of tension reinforcement, $0.75\rho_b$ corresponds to a net tensile strain ε_t of 0.00376. The use of ϕ of 0.9 is now permitted only for less heavily reinforced sections with $\varepsilon_t \geqslant 0.005$.

The use of these definitions is described under 8.4, 9.2, 10.3, and 18.8.

10.2 DESIGN ASSUMPTIONS

10.2.1 Equilibrium of Forces and Compatibility of Strains

Computation of the strength of a member or cross-section by the Strength Design Method requires that two basic conditions be satisfied: (1) static equilibrium and (2) compatibility of strains.

The first condition requires that the compressive and tensile forces acting on the cross-section at "ultimate" strength be in equilibrium, and the second condition requires that compatibility between the strains in the concrete and the reinforcement at "ultimate" conditions must also be satisfied within the design assumptions permitted by the code (see 10.2).

The term "ultimate" is used frequently in reference to the Strength Design Method; however, it should be realized that the "nominal" strength computed under the provisions of the code may not necessarily be the actual ultimate value. Within the design assumptions permitted, certain properties of the materials are neglected and other conservative limits are established for practical design. These contribute to a possible lower "ultimate strength" than that obtained by test. The computed nominal strength should be considered a code-defined strength only. Accordingly, the term "ultimate" is not used when defining the computed strength of a member. The term "nominal" strength is used instead.

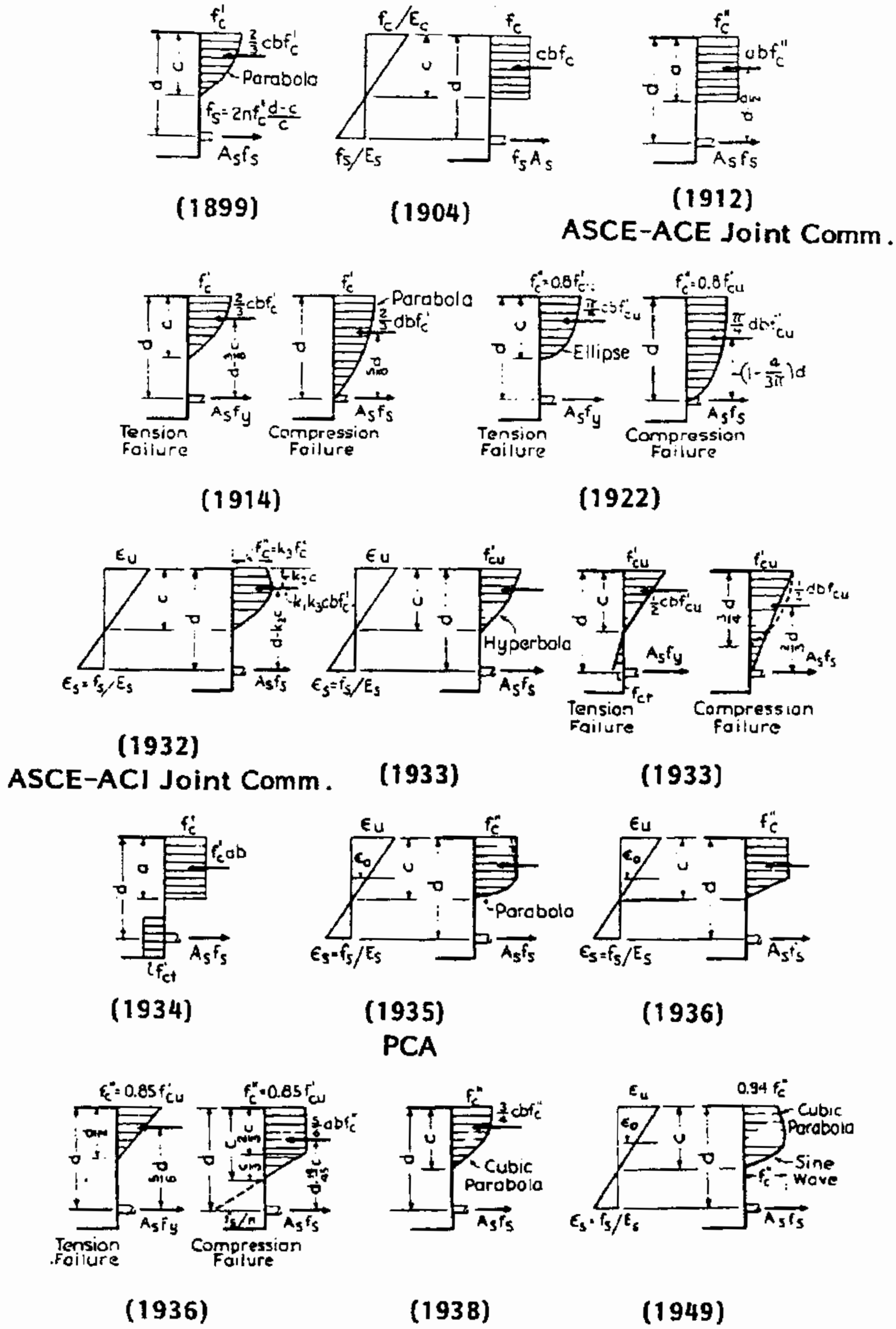


Figure 6-1 Development of Ultimate Strength Theories of Flexure

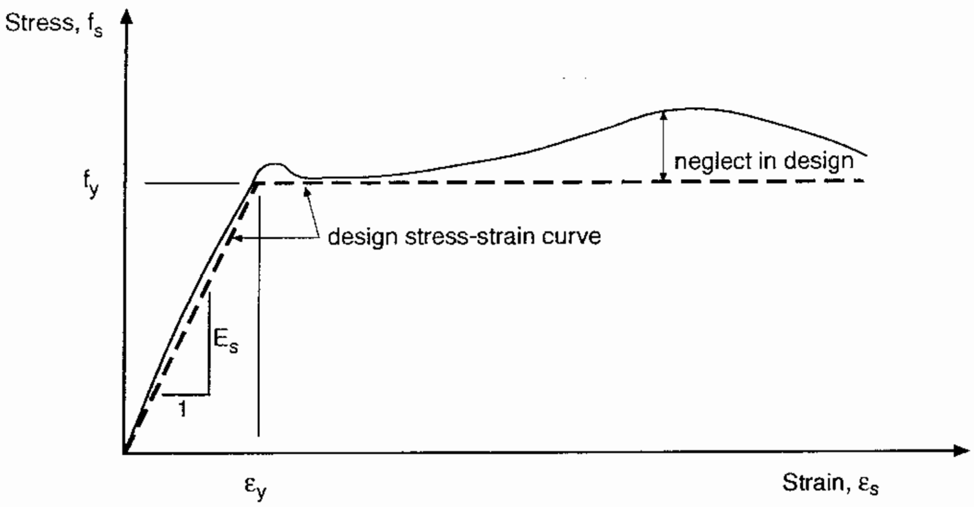


Figure 6-2 Stress-Strain Relationship for Reinforcement

Furthermore, in discussing the strength method of design for reinforced concrete structures, attention must be called to the difference between loads on the structure as a whole and load effects on the cross-sections of individual members. Elastic methods of structural analysis are used first to compute service load effects on the individual members due to the action of service loads on the entire structure. Only then are the load factors applied to the service load effects acting on the individual cross-sections. Inelastic (or limit) methods of structural analysis, in which design load effects on the individual members are determined directly from the ultimate test loads acting on the whole structure, are not considered. Section 8.4, however, does permit a limited redistribution of negative moments in continuous members. The provisions of 8.4 recognize the inelastic behavior of concrete structures and constitute a move toward "limit design." This subject is presented in Part 8.

The computed "nominal strength" of a member must satisfy the design assumptions given in 10.2.

10.2.2 Design Assumption #1

Strain in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis.

In other words, plane sections normal to the axis of bending are assumed to remain plane after bending. Many tests have confirmed that the distribution of strain is essentially linear across a reinforced concrete cross-section, even near ultimate strength. This assumption has been verified by numerous tests to failure of eccentrically loaded compression members and members subjected to bending only.

The assumed strain conditions at ultimate strength of a rectangular and circular section are illustrated in Fig. 6-3. Both the strain in the reinforcement and in the concrete are directly proportional to the distance from the neutral axis. This assumption is valid over the full range of loading—zero to ultimate. As shown in Fig. 6-3, this assumption is of primary importance in design for determining the strain (and the corresponding stress) in the reinforcement.

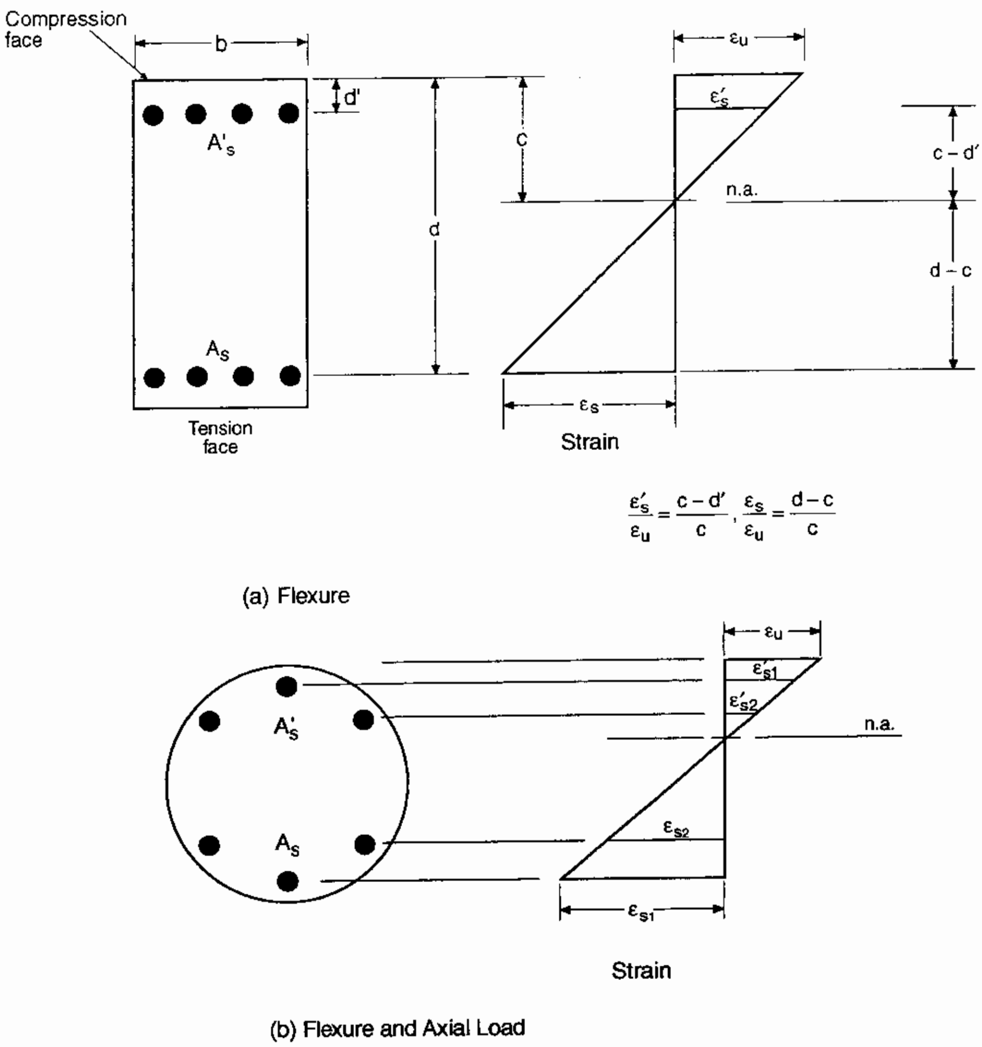


Figure 6-3 Assumed Strain Variation

10.2.3 Design Assumption #2

Maximum usable strain at extreme concrete compression fiber shall be assumed equal to $\varepsilon_u = 0.003$.

The maximum concrete compressive strain at crushing of the concrete has been measured in many tests of both plain and reinforced concrete members. The test results from a series of reinforced concrete beam and column specimens, shown in Fig. 6-4, indicate that the maximum concrete compressive strain varies from 0.003 to as high as 0.008. However, the maximum strain for practical cases is 0.003 to 0.004; see stress-strain curves in Fig. 6-5. Though the maximum strain decreases with increasing compressive strength of concrete, the 0.003 value allowed for design is reasonably conservative. The codes of some countries specify a value of 0.0035 for design, which makes little difference in the computed strength of a member.

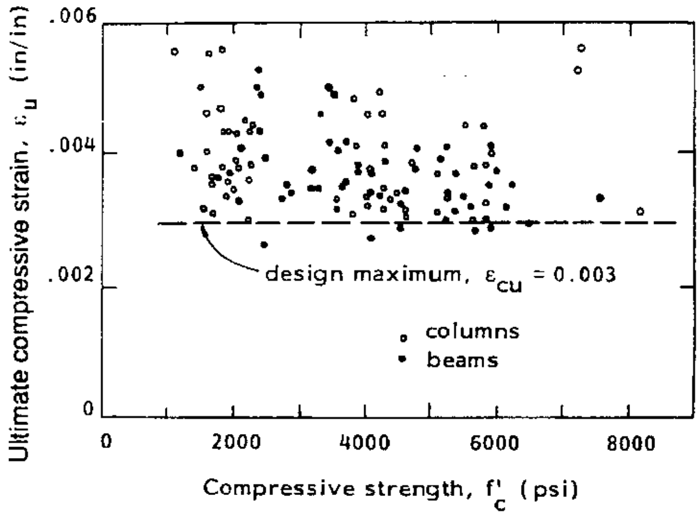


Figure 6-4 Maximum Concrete Compressive Strain, ε_u from Tests of Reinforced Concrete Members

10.2.4 Design Assumption #3

Stress in reinforcement f_s below the yield strength f_y shall be taken as E_s times the steel strain ε_s. For strains greater than f_y/E_s, stress in reinforcement shall be considered independent of strain and equal to f_y.

For deformed reinforcement, it is reasonably accurate to assume that below the yield stress, the stress in the reinforcement is proportional to strain. For practical design, the increase in strength due to the effect of strain hardening of the reinforcement is neglected for strength computations; see actual vs. design stress-strain relationship of steel in Fig. 6-2.

The force developed in the tensile or compressive reinforcement is a function of the strain in the reinforcement ε_s, such that:

when $\varepsilon_s \leq \varepsilon_y$ (yield strain):

$$f_s = E_s \varepsilon_s$$

$$A_s f_s = A_s E_s \varepsilon_s$$

when $\varepsilon_s \geq \varepsilon_y$:

$$f_s = E_s \varepsilon_y = f_y$$

$$A_s f_s = A_s f_y$$

where ε_s is the value from the strain diagram at the location of the reinforcement; see Fig. 6-3. For design, the modulus of elasticity of steel reinforcement, E_s, is taken as 29,000,000 psi (see 8.5.2).

10.2.5 Design Assumption #4

Tensile strength of concrete shall be neglected in flexural calculations of reinforced concrete.

The tensile strength of concrete in flexure, known as the modulus of rupture, is a more variable property than the compressive strength, and is about 8% to 12% of the compressive strength. The generally accepted value for design is $7.5\sqrt{f'_c}$ (9.5.2.3) for normal-weight concrete. This tensile strength in flexure is neglected in strength design. For practical percentages of reinforcement, the resulting computed strengths are in good agreement with test results. For very small percentages of reinforcement, neglecting the tensile strength of concrete is conservative. It should be realized, however, that the strength of concrete in tension is important in cracking and deflection (serviceability) considerations.

10.2.6 Design Assumption #5

Relationship between concrete compressive stress distribution and concrete strain shall be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.

This assumption recognizes the inelastic stress distribution in concrete at high stresses. As maximum stress is approached, the stress-strain relationship of concrete is not a straight line (stress is not proportional to strain). The general stress-strain behavior of concrete is shown in Fig. 6-5. The shape of the curves is primarily a function of concrete strength and consists of a rising curve from zero stress to a maximum at a compressive strain between 0.0015 and 0.002, followed by a descending curve to an ultimate strain (corresponding to crushing of the concrete) varying from 0.003 to as high as 0.008. As discussed under Design Assumption #2, the code sets the maximum usable strain at 0.003 for design. The curves show that the stress-strain behavior for concrete becomes notably nonlinear at stress levels exceeding $0.5f'_c$.

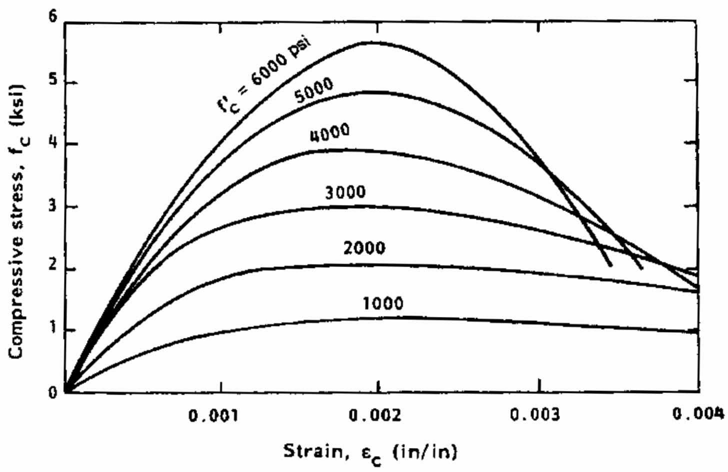


Figure 6-5 Typical Stress-Strain Curves for Concrete

The actual distribution of concrete compressive stress in a practical case is complex and usually not known. However, research has shown that the important properties of the concrete stress distribution can be approximated closely using any one of several different forms of stress distributions (see Fig. 6-1). The three most common stress distributions are the parabolic, the trapezoidal, and the rectangular, each giving reasonable results. At the theoretical ultimate strength of a member in flexure (nominal strength), the compressive stress distribution should conform closely to the actual variation of stress, as shown in Fig. 6-6. In this figure, the maximum stress is indicated by $k_3f'_c$, the average stress is indicated by $k_1k_3f'_c$, and the depth of the centroid of the approximate parabolic distribution from the extreme compression fiber by k_2c, where c is the neutral axis depth.

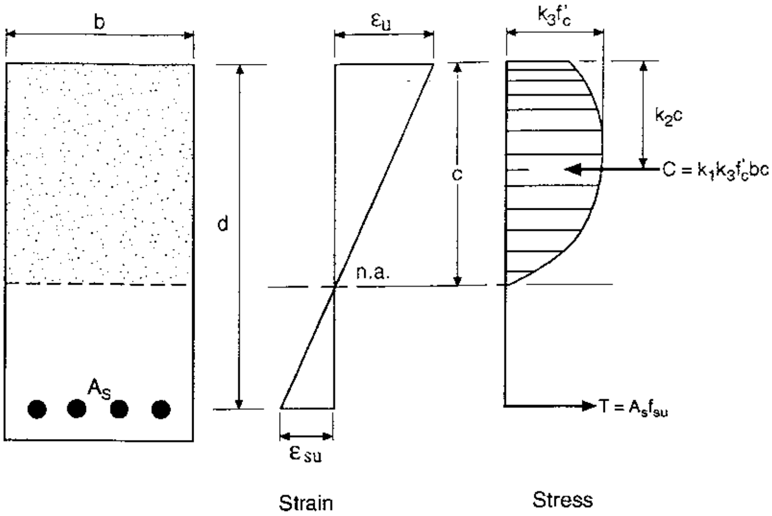


Figure 6-6 Actual Stress-Strain Conditions at Nominal Strength in Flexure

For the stress conditions at ultimate, the nominal moment strength, M_n, may be computed by equilibrium of forces and moments in the following manner:

From force equilibrium (Fig. 6-6):

$$C = T$$

or, $$k_1 k_3 f'_c bc = A_s f_{su}$$

so that $$c = \frac{A_s f_{su}}{k_1 k_3 f'_c b}$$

From moment equilibrium:

$$M_n = (C \text{ or } T)(d - k_2 c) = A_s f_{su}\left(d - \frac{k_2}{k_1 k_3}\frac{A_s f_{su}}{f'_c b}\right) \qquad (1)$$

The maximum strength is assumed to be reached when the strain in the extreme compression fiber is equal to the crushing strain of the concrete, ε_u. When crushing occurs, the strain in the tension reinforcement, ε_{su}, may be either larger or smaller than the yield strain, $\varepsilon_y = f_y / E_s$, depending on the relative proportion of reinforcement to concrete. If the reinforcement amount is low enough, yielding of the steel will occur prior to crushing of the concrete (ductile failure condition). With a very large quantity of reinforcement, crushing of the concrete will occur first, allowing the steel to remain elastic (brittle failure condition). The code has provisions which are intended to ensure a ductile mode of failure by limiting the amount of tension reinforcement. For the ductile failure condition, f_{su} equals f_y, and Eq. (1) becomes:

$$M_n = A_s f_y\left(d - \frac{k_2}{k_1 k_3}\frac{A_s f_y}{f'_c b}\right) \qquad (2)$$

If the quantity $k_2/(k_1 k_3)$ is known, the moment strength can be computed directly from Eq. (2). It is not necessary to know the values of k_1, k_2, and k_3 individually. Values for the combined term, as well as the individual k_1 and

k_2 values, have been established from tests and are shown in Fig. 6-7. As shown in the figure, $k_2/(k_1k_3)$ varies from about 0.55 to 0.63. Computation of the flexural strength based on the approximate parabolic stress distribution of Fig. 6-6 may be done using Eq. (2) with given values of $k_2/(k_1k_3)$. However, for practical design purposes, a method based on simple static equilibrium is desirable.

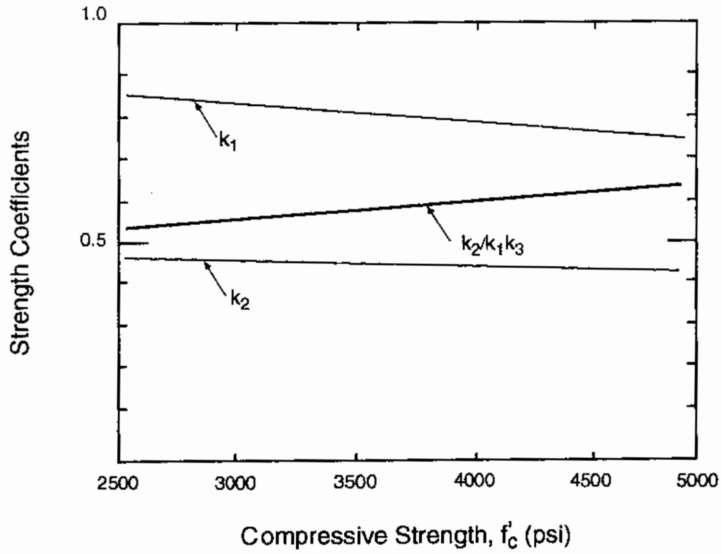


Figure 6-7 Stress-Block Parameters

During the last century, the Portland Cement Association adopted the parabolic stress-strain relationship shown in Fig. 6-8 for much of its experimental and analytical research work. "More exact" stress distributions such as this one have their greatest application with computers and are not recommended for longhand calculations. Recent PCA publications and computer software related to structural concrete design are based entirely on the rectangular stress block.

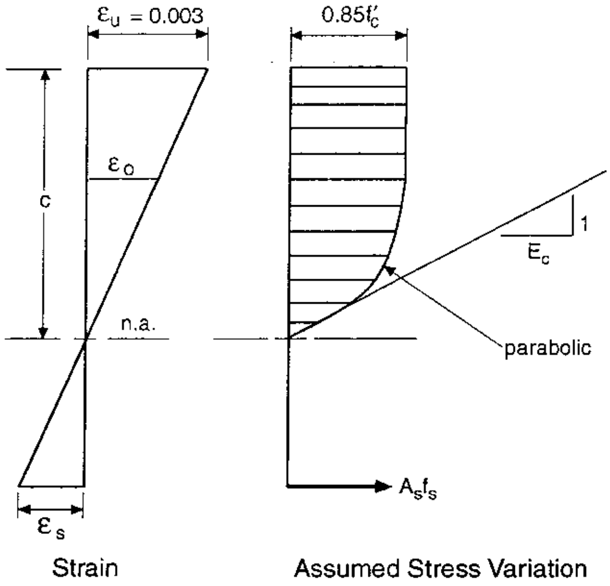
10.2.7 Design Assumption #6

Requirements of 10.2.6 may be considered satisfied by an equivalent rectangular concrete stress distribution defined as follows: A concrete stress of $0.85 f'_c$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain. Distance c from the fiber of maximum compressive strain to the neutral axis shall be measured in a direction perpendicular to that axis. Fraction β_1 shall be taken as 0.85 for strengths f'_c up to 4000 psi and shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi, but β_1 shall not be taken less than 0.65.

The code allows the use of a rectangular compressive stress block to replace the more exact stress distributions of Fig. 6-6 (or Fig. 6-8). The equivalent rectangular stress block, shown in Fig. 6-9, assumes a uniform stress of $0.85 f'_c$ over a depth $a = \beta_1 c$, determined so that $a/2 = k_2 c$. The constant β_1 is equal to 0.85 for concrete with $f'_c \leq 4000$ psi and reduces by 0.05 for each additional 1000 psi of f'_c in excess of 4000 psi. For high-strength concretes, above 8000 psi, a lower limit of 0.65 is placed on the β_1 factor. Variation in β_1 vs. concrete strength f'_c is shown in Fig. 6-10.

$$\varepsilon_o = \frac{2(0.85f'_c)}{E_c}$$

$$E_c = 57{,}000\sqrt{f'_c}$$



Concrete

$$0 < \varepsilon_c < \varepsilon_o \;:\; f_c = 0.85f'_c\left[2\left(\frac{\varepsilon_c}{\varepsilon_o}\right) - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2\right]$$

$$\varepsilon_c \geq \varepsilon_o \;:\; f_c = 0.85f'_c$$

Steel

$$\varepsilon_s \leq \varepsilon_y \;:\; f_s = \varepsilon_s E_s$$

$$\varepsilon_s > \varepsilon_y \;:\; f_s = f_y$$

$$E_s = 29{,}000{,}000 \text{ psi}$$

Figure 6-8 PCA Stress-Strain Relationship

The need for a β_1 factor is caused by the variation in shape of the stress-strain curve for different concrete strengths, as shown in Fig. 6-5. For concrete strengths up to 4,000 psi, the shape and centroid of the actual concrete stress block can reasonably be approximated by a rectangular stress block with a uniform stress of 0.85 f'_c and a depth of 0.85 times the depth to the neutral axis. That is to say, with a β_1 of 0.85.

Higher strength concretes have a more linear shape, with less inelastic behavior. For a good approximation of the stress block for concretes with strengths above 4,000 psi, the ratio β_1 of rectangular stress block depth to neutral axis depth needs to be reduced. Thus, the 1963 code required that β_1"shall be reduced continuously at a rate of 0.05 for each 1,000 psi of strength in excess of 4,000 psi.

As time went by and much higher concrete strengths came into use, it was realized that this reduction in β_1 should not go on indefinitely. Very high strengths have a stress block that approaches a triangular shape. This almost-triangular stress block is best approximated by a rectangular stress block with β_1 = 0.65. Thus, in the 1977 and later codes, β_1 was set at 0.65 for concrete strengths of 8,000 psi and above.

Using the equivalent rectangular stress distribution (Fig. 6-9), and assuming that the reinforcement yields prior to crushing of the concrete ($\varepsilon_s > \varepsilon_y$), the nominal moment strength M_n may be computed by equilibrium of forces and moments.

From force equilibrium:

$$C = T$$

or, $$0.85 f'_c ba = A_s f_y$$

so that $$a = \frac{A_s f_y}{0.85 f'_c b}$$

From moment equilibrium:

$$M_n = (C \text{ or } T)\,(d - \frac{a}{2}) = A_s f_y \,(d - \frac{a}{2})$$

Substituting a from force equilibrium,

$$M_n = A_s f_y \left(d - 0.59 \frac{A_s f_y}{f'_c b} \right) \qquad (3)$$

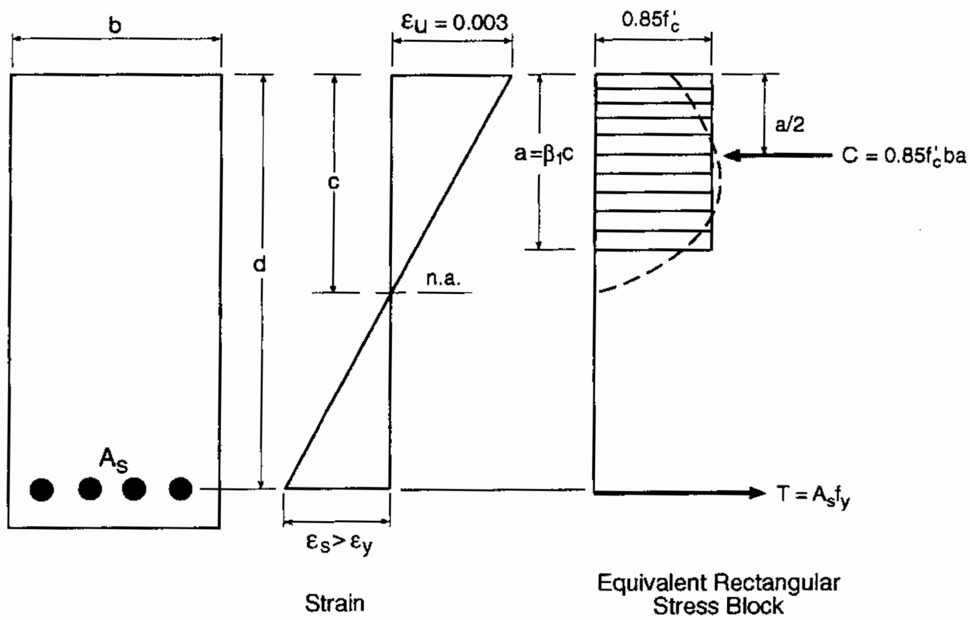


Figure 6-9 Equivalent Rectangular Concrete Stress Distribution (ACI)

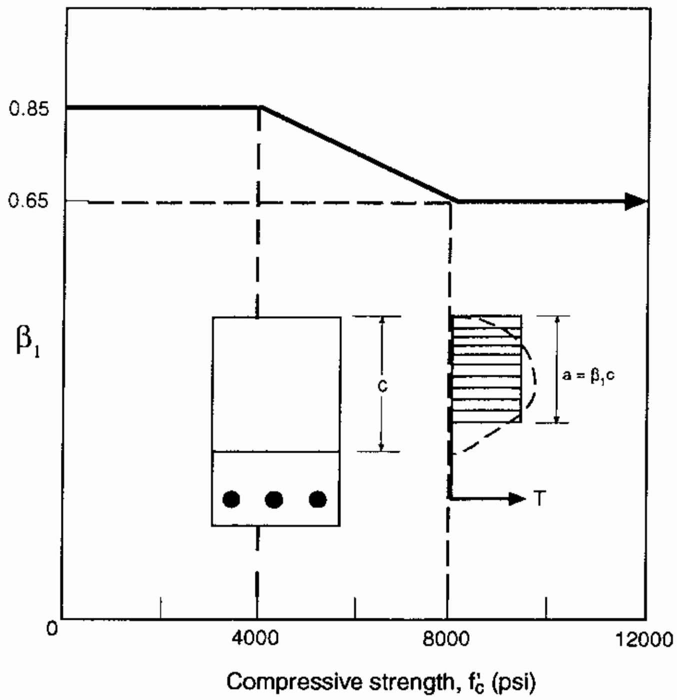


Figure 6-10 Strength Factor β_1

Note that the 0.59 value corresponds to $k_2/(k_1k_3)$ of Eq. (2). Substituting $A_s = \rho bd$, Eq. (3) may be written in the following nondimensional form:

$$\text{let } \omega = \rho \frac{f_y}{f'_c}$$

$$\frac{M_n}{bd^2 f'_c} = \rho \frac{f_y}{f'_c}\left(1 - 0.59\,\rho \frac{f_y}{f'_c}\right) \qquad (4)$$

$$= \omega(1 - 0.59\omega)$$

As shown in Fig. 6-11, Eq. (4) is "in substantial agreement with the results of comprehensive tests." However, it must be realized that the rectangular stress block does not represent the actual stress distribution in the compression zone at ultimate, but does provide essentially the same strength results as those obtained in tests. Computation of moment strength using the equivalent rectangular stress distribution and static equilibrium is illustrated in Example 6.1.

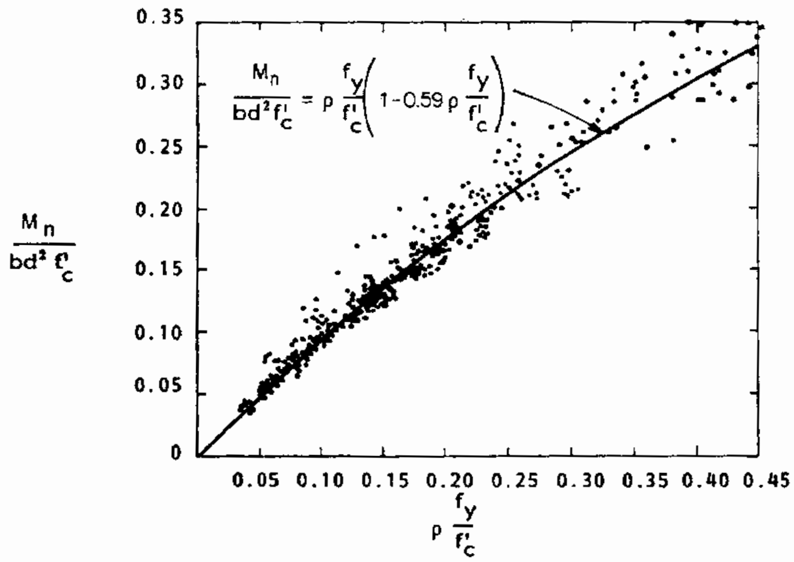


Figure 6-11 Tests of 364 Beams Controlled by Tension ($\varepsilon_s > \varepsilon_y$)

10.3 GENERAL PRINCIPLES AND REQUIREMENTS

10.3.1 Nominal Flexural Strength

Nominal strength of a member or cross-section subject to flexure (or to combined flexure and axial load) must be based on equilibrium and strain compatibility using the design assumptions of 10.2. Nominal strength of a cross-section of any shape, containing any amount and arrangement of reinforcement, is computed by applying the force and moment equilibrium and strain compatibility conditions in a manner similar to that used to develop the nominal moment strength of the rectangular section with tension reinforcement only, as illustrated in Fig. 6-9. Using the equivalent rectangular concrete stress distribution, expressions for nominal moment strength of rectangular and flanged sections (typical sections used in concrete construction) are summarized as follows:

a. Rectangular section with tension reinforcement only (see Fig. 6-9):

   Expressions are given above under Design Assumption #6 (10.2.7).

b. Flanged section with tension reinforcement only:

   When the compression flange thickness is equal to or greater than the depth of the equivalent rectangular stress block a, moment strength M_n is calculated by Eq. (3), just as for a rectangular section with width equal to the flange width. When the compression flange thickness h_f is less than a, the nominal moment strength M_n is (see Fig. 6-12):

$$M_n = (A_s - A_{sf})\, f_y \left(d - \frac{a}{2}\right) + A_{sf} f_y \left(d - \frac{h_f}{2}\right) \qquad (5)$$

where

A_{sf} = area of reinforcement required to equilibrate compressive strength of overhanging flanges

$= 0.85 f_c' (b - b_w) h_f/f_y$

$a = (A_s - A_{sf}) f_y / 0.85 f_c' b_w$

b = width of effective flange (see 8.10)

b_w = width of web

h_f = thickness of flange

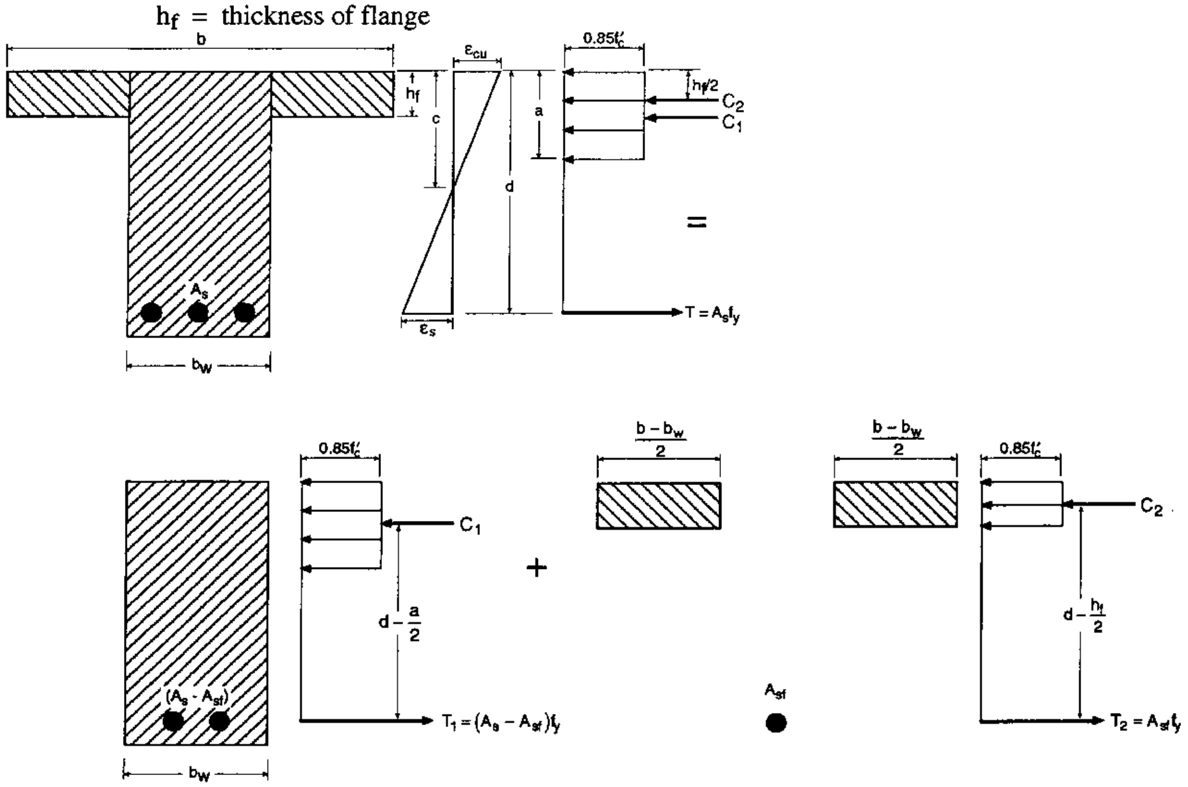


Figure 6-12 Strain and Equivalent Stress Distribution for Flanged Section

c. Rectangular section with compression reinforcement:

For a doubly reinforced section with compression reinforcement A_s', two possible situations can occur (see Fig. 6-13):

i. Compression reinforcement A_s' yields:

$$f_s' = f_y$$

$$a = \frac{(A_s - A_s') f_y}{0.85 f_c' b} \qquad (6)$$

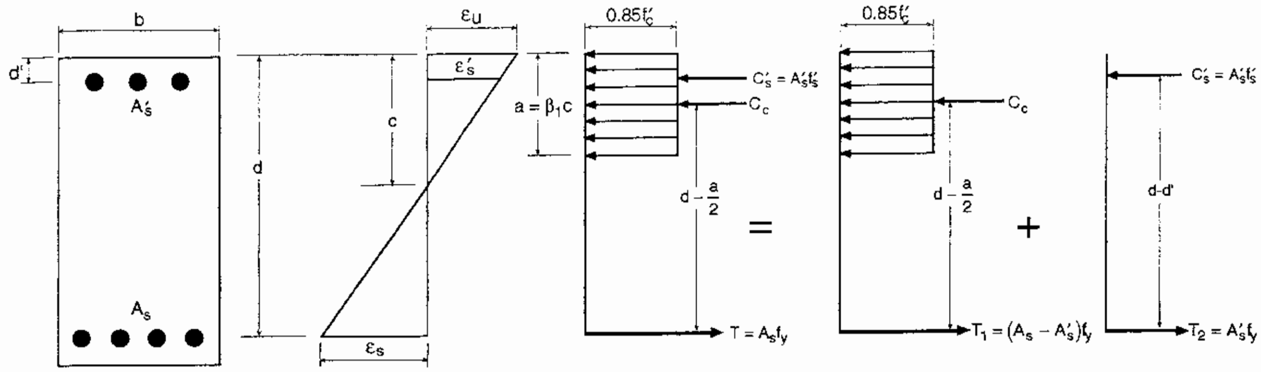


Figure 6-13 Strain and Equivalent Stress Distribution of Doubly Reinforced Rectangular Section

The nominal moment strength is:

$$M_n = (A_s - A'_s)\, f_y \left(d - \frac{a}{2}\right) + A'_s f_y (d - d') \tag{7}$$

Note that A'_s yields when the following (for Grade 60 reinforcement, with $\varepsilon_y = 0.00207$) is satisfied:

$$d'/c \le 0.31$$

$$\text{where } c = \frac{a}{\beta_1}$$

ii. Compression reinforcement does not yield:

$$f'_s = E_s \varepsilon'_s = E_s \varepsilon_u \left(\frac{c - d'}{c}\right) < f_y \tag{8}$$

The neutral axis depth c can be determined from the following quadratic equation:

$$c^2 - \frac{\left(A_s f_y - 87A'_s\right) c}{0.85\beta_1 f'_c b} - \frac{87A'_s d'}{0.85\beta_1 f'_c b} = 0$$

where f'_c and f_y have the units of ksi. The nominal moment strength is:

$$M_n = 0.85 f'_c ab \left(d - \frac{a}{2}\right) + A'_s f'_s (d - d') \tag{9}$$

where

$$a = \beta_1 c$$

Alternatively, the contribution of compression reinforcement may be neglected and the moment strength calculated by Eq. (3), just as for a rectangular section with tension reinforcement only.

d. For other cross-sections, the nominal moment strength M_n is calculated by a general analysis based on equilibrium and strain compatibility using the design assumptions of 10.2.

e. Nominal flexural strength M_n of a cross-section of a composite flexural member consisting of cast-in-place and precast concrete is computed in a manner similar as that for a regular reinforced concrete section. Since the "ultimate" strength is unrelated to the sequence of loading, no distinction is made between shored and unshored members in strength computations (see 17.2.4).

10.3.2 Balanced Strain Condition

A balanced strain condition exists at a cross-section when the maximum strain at the extreme compression fiber just reaches $\varepsilon_u = 0.003$ simultaneously with the first yield strain of $\varepsilon_s = \varepsilon_y = f_y/E_s$ in the tension reinforcement. This balanced strain condition is shown in Fig. 6-14.

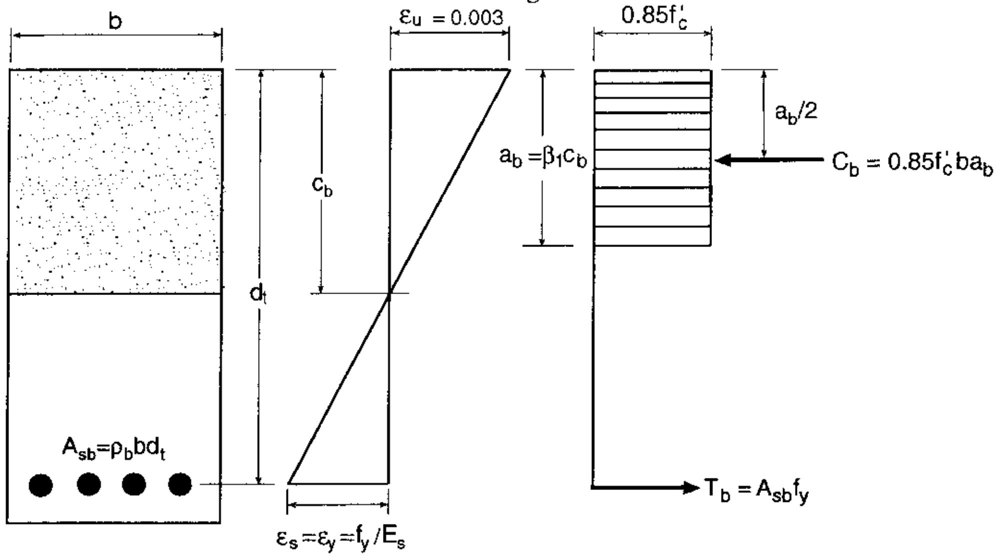


Figure 6-14 Balanced Strain Condition in Flexure

The ratio of neutral axis depth c_b to extreme depth d_t to produce a balanced strain condition in a section with tension reinforcement only may be obtained by applying strain compatibility conditions. Referring to Fig. 6-14, for the linear strain condition:

$$\frac{c_b}{d_t} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

$$= \frac{0.003}{0.003 + f_y/29{,}000{,}000} = \frac{0.003}{0.003 + \varepsilon_y}$$

Note that for Grade 60 steel, 10.3.3 permits the steel strain ε_y to be rounded to 0.002. Substituting into the above equation, the ratio $c_b/d_t = 0.6$. This value applies to all sections with Grade 60 steel, not just to rectangular sections.

10.3.3 Compression-Controlled Sections

Sections are compression-controlled when the net tensile strain in the extreme tension steel is equal to or less than the compression-controlled strain limit at the time the concrete in compression reaches its assumed strain limit of 0.003. The compression-controlled strain limit is the net tensile strain in the reinforcement at balanced strain conditions. For Grade 60 reinforcement, and for all prestressed reinforcement, it is permitted to set the compression-controlled strain limit equal to 0.002.

Note that when other grades of reinforcement are used, the compression-controlled strain limit is not 0.002. This changes the compression-controlled strain limit, and that changes the "transition" equations for the strength reduction factor given in Fig. 5-2 in Part 5.

10.3.4 Tension-Controlled Sections and Transition

Sections are tension-controlled when the net tensile strain in the extreme tension steel is equal to or greater than 0.005 just as the concrete in compression reaches its assumed strain limit of 0.003. Sections with net tensile strain in the extreme tension steel between the compression-controlled strain limit and 0.005 constitute a transition region between compression-controlled and tension-controlled sections.

Figure 6-15 shows the stress and strain conditions at the limit for tension-controlled sections. This limit is important because it is the limit for the use of $\phi = 0.9$ (9.3.2.1). Critical parameters at this limit are given a subscript t. Referring to Fig. 6-15, by similar triangles:

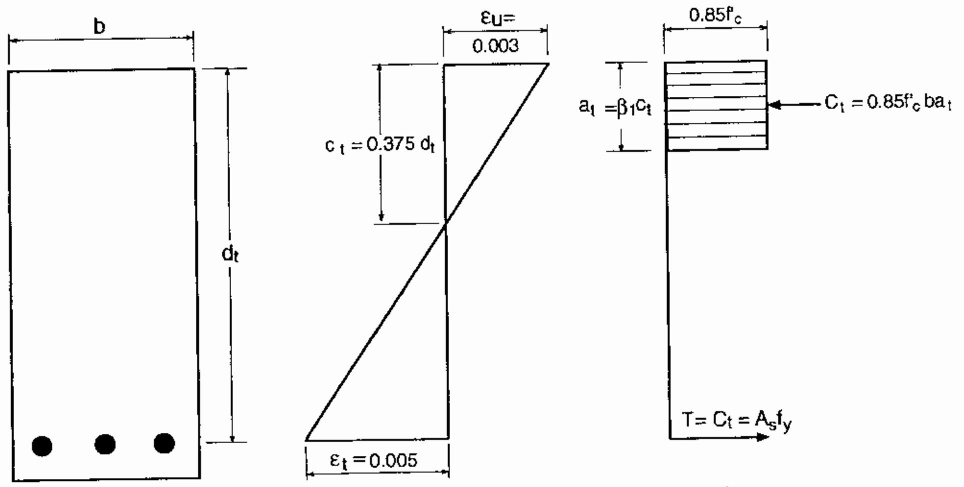


Figure 6-15 Strains at Tension-Controlled Limit

$$c_t = 0.375d_t$$

$$a_t = \beta_1 c_t = 0.375\beta_1 d_t$$

$$C_t = 0.85f'_c ba_t = 0.319\beta_1 f'_c bd_t$$

$$T = A_s f_y = C_t$$

$$A_s = 0.319\beta_1 f'_c bd_t / f_y$$

$$\rho_t = A_s/(bd_t) = 0.319\beta_1 f'_c / f_y \tag{10}$$

$$\omega_t = \frac{\rho_t f_y}{f'_c} = 0.319\beta_1 \tag{11}$$

$$M_{nt} = \omega_t(1 - 0.59\omega_t)f'_c bd_t^2 \qquad \textit{from Eq. (4)}$$

$$R_{nt} = \frac{M_{nt}}{bd_t^2} = \omega_t(1 - 0.59\omega_t)f'_c \tag{12}$$

Values for ρ_t, ω_t, and R_{nt} are given in Table 6-1.

Table 6-1 Design Parameters at Strain Limit of 0.005 for Tension-Controlled Sections

	$f'_c = 3000$ $\beta_1 = 0.85$	$f'_c = 4000$ $\beta_1 = 0.85$	$f'_c = 5000$ $\beta_1 = 0.80$	$f'_c = 6000$ $\beta_1 = 0.75$	$f'_c = 8000$ $\beta_1 = 0.65$	$f'_c = 10,000$ $\beta_1 = 0.65$
R_{nt}	683	911	1084	1233	1455	1819
ϕR_{nt}	615	820	975	1109	1310	1637
ω_t	0.2709	0.2709	0.2550	0.2391	0.2072	0.2072
ρ_t, Grade 40	0.02032	0.02709	0.03187	0.03586	0.04144	0.05180
ρ_t, Grade 60	0.01355	0.01806	0.02125	0.02391	0.02762	0.03453
ρ_t, Grade 75	0.01084	0.01445	0.01700	0.01912	0.02210	0.02762

10.3.5 Maximum Reinforcement for Flexural Members

Since 2002, the body of the code defines reinforcement limits in terms of net tensile strain, ε_t, instead of the balanced ratio ρ/ρ_b that was used formerly. For rectangular sections with one layer of Grade 60 steel, a simple relationship between ε_t and ρ/ρ_b exists (see Fig. 6-16):

$$c = \frac{0.003 d_t}{\varepsilon_t + 0.003}$$

$$a = \beta_1 c = \frac{0.003 \beta_1 d_t}{\varepsilon_t + 0.003}$$

At balanced:

$$a_b = \frac{0.003 \beta_1 d_t}{(60/29{,}000) + 0.003} = 0.592\, \beta_1 d_t$$

$$\frac{\rho}{\rho_b} = \frac{a}{a_b} = \frac{0.00507}{\varepsilon_t + 0.003}$$

or,

$$\varepsilon_t = \frac{0.00507}{\rho/\rho_b} - 0.003$$

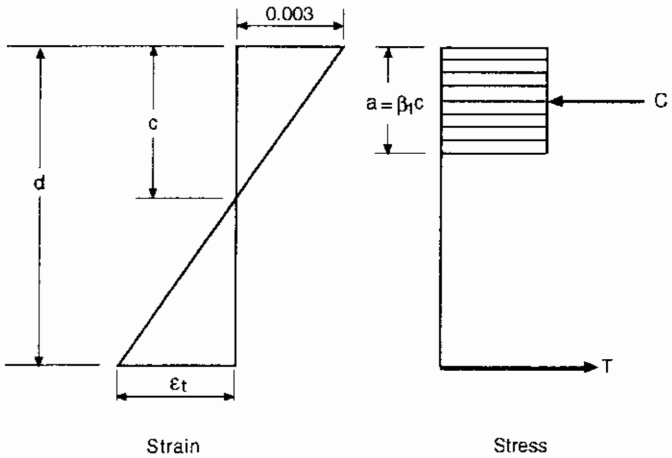


Figure 6-16 Strain and Stress Relationship

This relationship is shown graphically in Fig. 6-17.

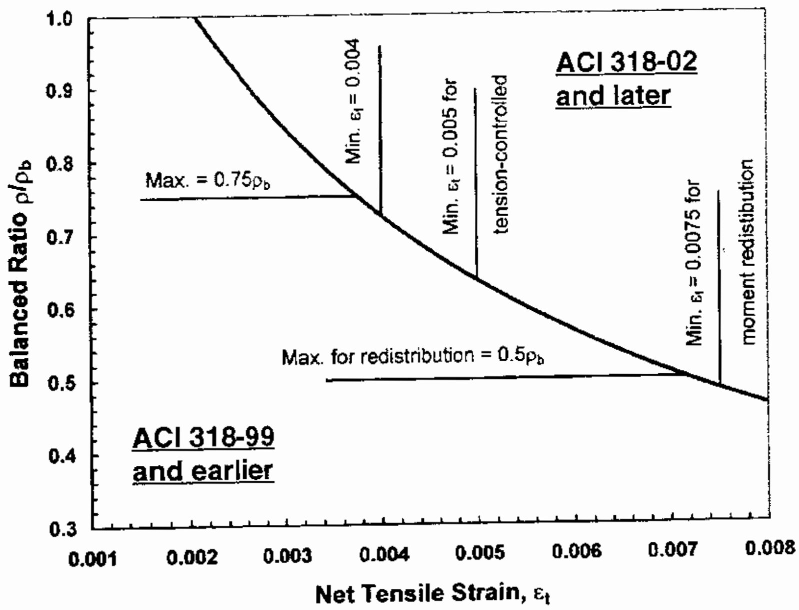


Figure 6-17 Relationship Between Balanced Ratio and Net Tensile Strain

Since 2002, the code limits the maximum reinforcement in a flexural member (with axial load less than $0.1\ f'_c A_g$) to that which would result in a net tensile strain ε_t at nominal strength not less than 0.004. This compares to the former code limit of $0.75\,\rho_b$, which results in an ε_t of 0.00376. Furthermore, at the net tensile strain limit of 0.004, the ϕ factor is reduced to 0.812. For heavily reinforced members, the overall safety margin (load factor/ϕ) is about the same as by 318-99, despite the reduced load factors. See Fig. 6-18.

The strength of tension-controlled sections is clearly controlled by steel strength, which is less variable than concrete strength and this offers greater reliability. For tension-controlled flexural members, since 2002, the ACI code permits a ϕ of 0.9 to be used, despite the reduced load factors introduced in 2002. As Fig. 6-18 shows, the new code reduces the strength requirement by about 10 percent for tension-controlled sections.

As discussed in Part 7, it is almost always advantageous to limit the net tensile strain in flexural members to a minimum of 0.005, even though the code permits higher amounts of reinforcement producing lower net tensile strains. Where member size is limited and extra strength is needed, it is best to use compression reinforcement to limit the net tensile strain so that the section is tension-controlled.

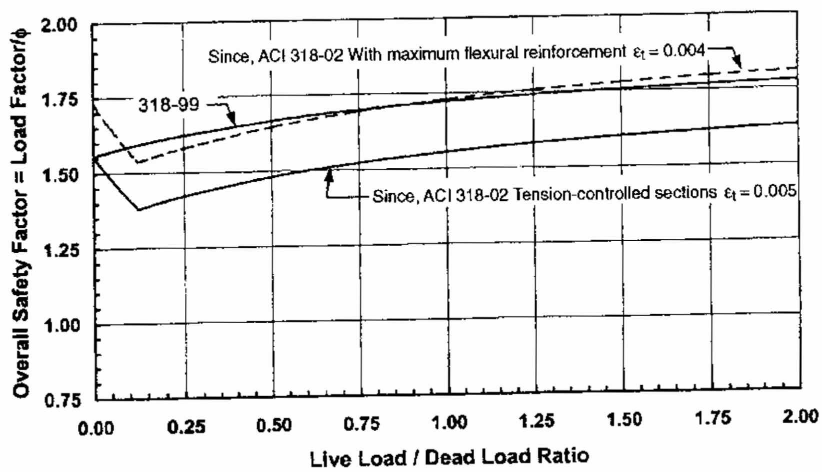


Figure 6-18 Overall Safety Factor for Flexural Members

10.3.6 Maximum Axial Strength

The strength of a member in pure compression (zero eccentricity) is computed by:

$$P_o = 0.85 f_c' A_g + f_y A_{st}$$

where A_{st} is the total area of reinforcement and A_g is the gross area of the concrete section. Refinement in concrete area can be considered by subtracting the area of concrete displaced by the steel:

$$P_o = 0.85 f_c' (A_g - A_{st}) + f_y A_{st} \tag{13}$$

Pure compression strength P_o represents a hypothetical loading condition. Prior to the 1977 ACI code, all compression members were required to be designed for a minimum eccentricity of 0.05h for spirally reinforced members or 0.10h for tied reinforced members (h = overall thickness of member). The specified minimum eccentricities were originally intended to serve as a means of reducing the axial design load strength of a section in pure compression and were included to: (1) account for accidental eccentricities, not considered in the analysis, that may exist in a compression member, and (2) recognize that concrete strength is less than f_c' at sustained high loads.

Since the primary purpose of the minimum eccentricity requirement was to limit the axial strength for design of compression members with small or zero computed end moments, the 1977 code was revised to accomplish this directly by limiting the axial strength to 85% and 80% of the axial strength at zero eccentricity (P_o), for spiral and tied reinforcement columns, respectively.

For spirally reinforced members,

$$P_{n(max)} = 0.85P_o = 0.85\,[0.85 f_c' (A_g - A_{st}) + f_y A_{st}] \tag{14}$$

For tied reinforced members,

$$P_{n(max)} = 0.80P_o = 0.80\,[0.85 f_c' (A_g - A_{st}) + f_y A_{st}] \tag{15}$$

The maximum axial strength, $P_{n(max)}$, is illustrated in Fig. 6-19. In essence, design within the cross-hatched portion of the load-moment interaction diagram is not permitted. The 85% and 80% values approximate the axial strengths at e/h ratios of 0.05 and 0.10 specified in the 1971 code for spirally reinforced and tied reinforced members, respectively (see Example 6.3). The designer should note that R10.3.6 and R10.3.7 state that "Design aids and computer programs based on the minimum eccentricity requirement of the 1963 and 1971 ACI Building Codes may be considered equally applicable for usage."

The current provisions for maximum axial strength also eliminate the concerns expressed by engineers about the excessively high minimum design moments required for large column sections, and the often asked question as to whether the minimum moments were required to be transferred to other interconnecting members (beams, footings, etc.).

Note that a minimum moment (minimum eccentricity requirement) for slender compression members in a braced frame is given in 10.12.3.2. If factored column moments are very small or zero, the design of these columns must be based on the minimum moment P_u (0.6 + 0.03h).

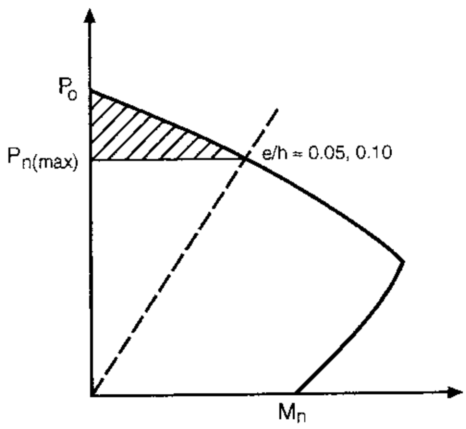
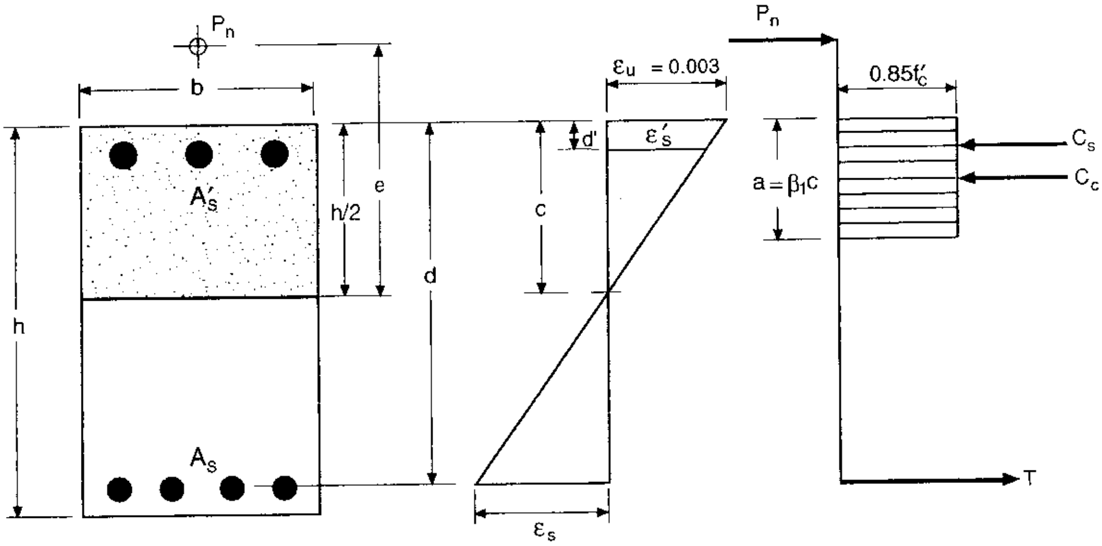


Figure 6-19 Maximum Axial Strength (10.3.6)

10.3.7 Nominal Strength for Combined Flexure and Axial Load

The strength of a member or cross-section subject to combined flexure and axial load, M_n and P_n, must satisfy the same two conditions as required for a member subject to flexure only: (1) static equilibrium and (2) compatibility of strains. Equilibrium between the compressive and tensile forces includes the axial load P_n acting on the cross-section. The general condition of the stress and the strain in concrete and steel at nominal strength of a member under combined flexure and axial compression is shown in Fig. 6-20. The tensile or compressive force developed in the reinforcement is determined from the strain condition at the location of the reinforcement.



$\varepsilon_s \le \varepsilon_y$ (Compression Controls)

$\varepsilon_s = \varepsilon_y$ (Balanced Condition)

$0.005 > \varepsilon_s > \varepsilon_y$ (Transition)

$\varepsilon_s \ge 0.005\varepsilon_y$ (Tension Controls)

Figure 6-20 Strain and Equivalent Stress Distribution for Section Subject to Combined Flexure and Axial Load

Referring to Fig. 6-20,

$$T = A_s f_s = A_s (E_s \varepsilon_s) \qquad \text{when } \varepsilon_s < \varepsilon_y$$

or

$$T = A_s f_y \qquad \text{when } \varepsilon_s \geq \varepsilon_y$$

$$C_s = A'_s f'_s = A'_s (E_s \varepsilon'_s) \qquad \text{when } \varepsilon'_s < \varepsilon_y$$

or

$$C_s = A'_s f_y \qquad \text{when } \varepsilon'_s \geq \varepsilon_y$$

$$C_c = 0.85 f'_c ba$$

The combined load-moment strength (P_n and M_n) may be computed by equilibrium of forces and moments.

From force equilibrium:

$$P_n = C_c + C_s - T \tag{16}$$

From moment equilibrium about the mid-depth of the section:

$$M_n = P_n e = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} - d'\right) + T\left(d - \frac{h}{2}\right) \tag{17}$$

For a known strain condition, the corresponding load-moment strength, P_n and M_n, can be computed directly. Assume the strain in the extreme tension steel, A_s, is at first yield ($\varepsilon_s = \varepsilon_y$). This strain condition with simultaneous strain of 0.003 in the extreme compression fiber defines the "balanced" load-moment strength, P_b and M_b, for the cross-section.

For the linear strain condition:

$$\frac{c_b}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} = \frac{0.003}{0.003 + f_y/29{,}000{,}000} = \frac{87{,}000}{87{,}000 + f_y}$$

so that

$$a_b = \beta_1 c_b = \left(\frac{87{,}000}{87{,}000 + f_y}\right)\beta_1 d$$

Also

$$\frac{c_b}{c_b - d'} = \frac{\varepsilon_u}{\varepsilon'_s}$$

so that

$$\varepsilon'_s = 0.003\left(1 - \frac{d'}{c_b}\right) = 0.003\left[1 - \frac{d'}{d}\left(\frac{87{,}000 + f_y}{87{,}000}\right)\right]$$

and

$$f'_{sb} = E_s \varepsilon'_s = 87{,}000\left[1 - \frac{d'}{d}\left(\frac{87{,}000 + f_y}{87{,}000}\right)\right] \text{ but not greater than } f_y$$

From force equilibrium:

$$P_b = 0.85 f'_c b a_b + A'_s f'_{sb} - A_s f_y \tag{18}$$

From moment equilibrium:

$$M_b = P_b e_b = 0.85 f'_c b a_b \left(\frac{h}{2} - \frac{a}{2}\right) + A'_s f'_{sb} \left(\frac{h}{2} - d'\right) + A_s f_y \left(\frac{d}{2} - h\right) \tag{19}$$

The "balanced" load-moment strength defines only one of many load-moment combinations possible over the full range of the load-moment interaction relationship of a cross-section subject to combined flexure and axial load. The general form of a strength interaction diagram is shown in Fig. 6-21. The load-moment combination may be such that compression exists over most or all of the section, so that the compressive strain in the concrete reaches 0.003 before the tension steel yields ($\varepsilon_s < \varepsilon_y$) (compression-controlled segment); or the load combination may be such that tension exists over a large portion of the section, so that the strain in the tension steel is greater than the yield strain ($\varepsilon_s > \varepsilon_y$) when the compressive strain in the concrete reaches 0.003 (transition or tension-controlled segment). The "balanced" strain condition ($\varepsilon_s = \varepsilon_y$) divides these two segments of the strength curve. The linear strain variation for the full range of the load-moment interaction relationship is illustrated in Fig. 6-21.

Under pure compression, the strain is uniform over the entire cross-section and equal to 0.003. With increasing load eccentricity (moment), the compressive strain at the "tension face" gradually decreases to zero, then becomes tensile, reaching the yield strain ($\varepsilon_s = \varepsilon_y$) at the balanced strain condition. For this range of strain variations, the strength of the section is governed by compression (ε_s = -0.003 to ε_y). Beyond the balanced strain condition, the steel strain gradually increases up to the state of pure flexure corresponding to an infinite load eccentricity ($e = \infty$). For this range of strain variations, strength is governed by tension ($\varepsilon_s > \varepsilon_y$). With increasing eccentricity, more and more tension exists over the cross-section. Each of the many possible strain conditions illustrated in Fig. 6-22 describes a point, P_n and M_n, on the load-moment curve (Fig. 6-21). Calculation of P_n and M_n for four different strain conditions along the load-moment strength curve is illustrated in Example 6.4.

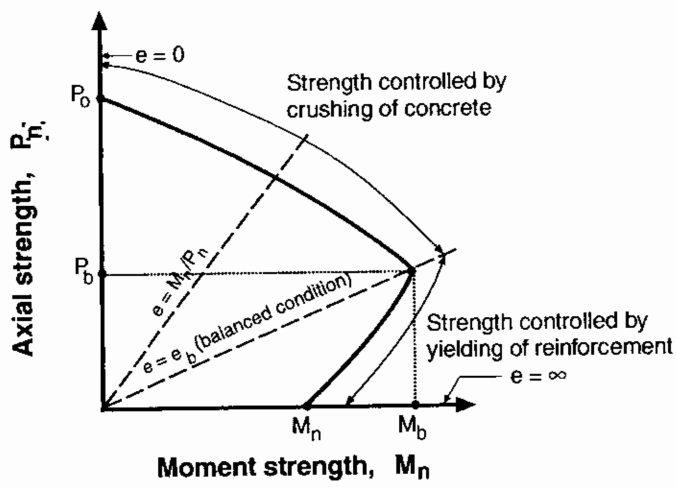


Figure 6-21 Axial Load-Moment Interaction Diagram

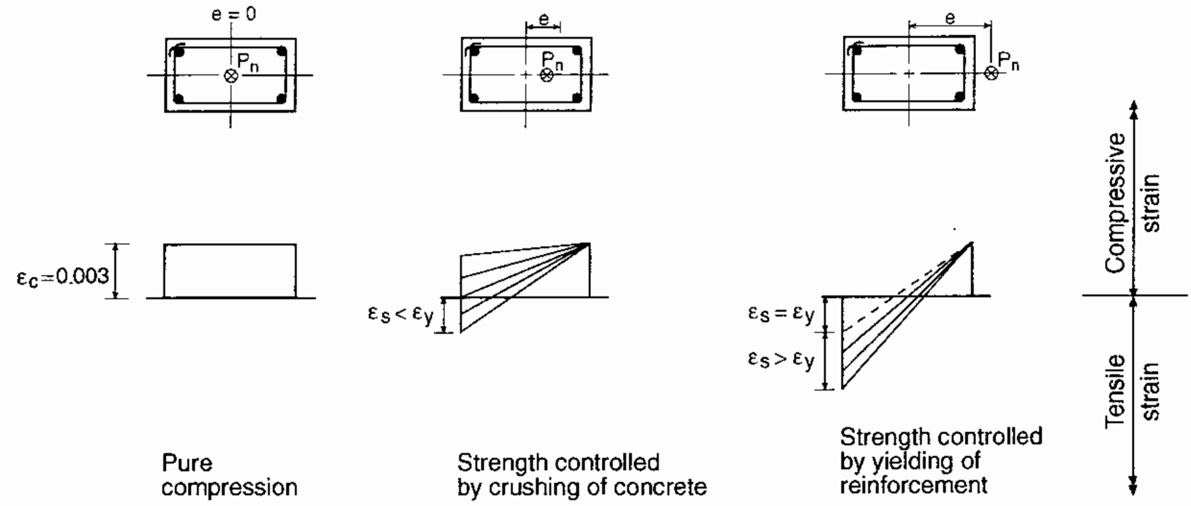


Figure 6-22 Strain Variation for Full Range of Load-Moment Interaction

10.5 MINIMUM REINFORCEMENT OF FLEXURAL MEMBERS

Members with cross-sections much larger than required for strength, for architectural or other reasons, could fail suddenly because of small amounts of tensile reinforcement. The computed moment strength of such sections, assuming reinforced concrete behavior and using cracked section analyses, could become less than that of a corresponding unreinforced concrete section computed from its modulus of rupture. To prevent failure in such situations, a minimum amount of tensile reinforcement is specified in 10.5.

The minimum reinforcement ratio $\rho_{min} = 200/f_y$ was originally derived to provide the same 0.5% minimum (for mild steel grade) as required in earlier versions of the ACI code. This minimum reinforcement is adequate for concrete strengths of about 4000 psi and less. The '95 version of the code recognizes that $\rho_{min} = 200/f_y$ may not be sufficient for f'_c greater than about 5000 psi. The code has accordingly revised 10.5.1 and 10.5.2 to specify the following minimum amounts of steel:

At every section of flexural members where tensile reinforcement is required,

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200}{f_y} b_w d \qquad \textit{Eq. (10-3)}$$

Note that $3\sqrt{f'_c}$ and 200 are equal when $f'_c = 4444$ psi. Thus, $3\sqrt{f'_c} b_w d / f_y$ controls when $f'_c > 4444$ psi; otherwise, $200 b_w d / f_y$ controls.

Equation (10-4) of ACI 318-99 was removed and replaced with the following statement, which says the same thing as the former Eq. (10-4):

> 10.5.2 For statically determinate members with a flange in tension, the area $A_{s,min}$ shall be equal to or greater than the value given by Eq. (10-3) with b_w replaced by either $2b_w$ or the width of the flange, whichever is smaller.

Note that the requirements of 10.5.1 and 10.5.2 need not be applied if at every section the area of tensile reinforcement provided is at least one-third greater than that required by analysis (see 10.5.3). For structural slabs and footings (10.5.4), the flexural reinforcement cannot be less than that required for temperature and shrinkage (7.12).

10.15 TRANSMISSION OF COLUMN LOADS THROUGH FLOOR SYSTEM

When the column concrete strength does not exceed the floor concrete strength by more than 40 percent, no special precautions need be taken in computing the column strength (10.15). For higher column concrete strengths, ACI provisions limit the assumed column strength unless concrete puddling is used in the slab at, and around the column (10.15.1), see Figure 6-23. For columns laterally supported on four sides by beams of approximately equal depth or by slabs, the code permits the strength of the column to be based on an assumed concrete strength in the column joint equal to 75 percent of column concrete strength plus 35 percent of floor concrete strength (10.15.3). In the application of 10.15.3, the ratio of column concrete strength to slab concrete strength was limited to 2.5 for design. This effectively limits the assumed column strength to a maximum of 2.225 times the floor concrete strength.

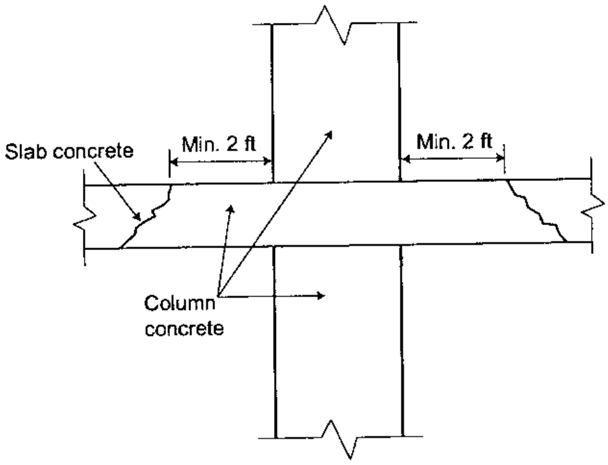


Figure 6-23 Puddling at Slab-Column

Puddling is an intricate procedure that requires coordination between the engineer and the contractor. Special attention should be paid to avoid cold joints and to ensure that the specified column concrete is placed where it is intended. Current industry practice indicates that it is frequently avoided in mainstream high-rise construction since it requires additional time to properly execute in the field. When utilized, a procedure for proper placing and blending of the two concrete types should be clearly called out in the project documents.

10.17 BEARING STRENGTH ON CONCRETE

Code-defined bearing strength (P_{nb}) of concrete is expressed in terms of an average bearing stress of $0.85 f'_c$ over a bearing area (loaded area) A_1. When the supporting concrete area is wider than the loaded area on all sides, the surrounding concrete acts to confine the loaded area, resulting in an increase in the bearing strength of the supporting concrete. With confining concrete, the bearing strength may be increased by the factor $\sqrt{A_2 / A_1}$, but not greater than 2, where $\sqrt{A_2 / A_1}$ is a measure of the confining effect of the surrounding concrete. Evaluation of the strength increase factor $\sqrt{A_2 / A_1}$ is illustrated in Fig. 6-24.

For the usual case of a supporting concrete area considerably greater than the loaded area $\left(\sqrt{A_2 / A_1} > 2\right)$, the nominal bearing stress is $2\,(0.85 f'_c)$.

Referring to Fig. 6-25,

a. For the supported surface (column):

$$P_{nb} = 0.85 f'_c A_1$$

where f'_c is the specified strength of the column concrete.

b. For supporting surface (footing):

$$P_{nb} = 0.85f'_cA_1\sqrt{\frac{A_2}{A_1}} \text{ and } \sqrt{\frac{A_2}{A_1}} \leq 2.0$$

where f'_c is the specified strength of the footing concrete.

The design bearing strength is ϕP_{nb}, where, for bearing on concrete, $\phi = 0.65$. When the bearing strength is exceeded, reinforcement must be provided to transfer the excess load.

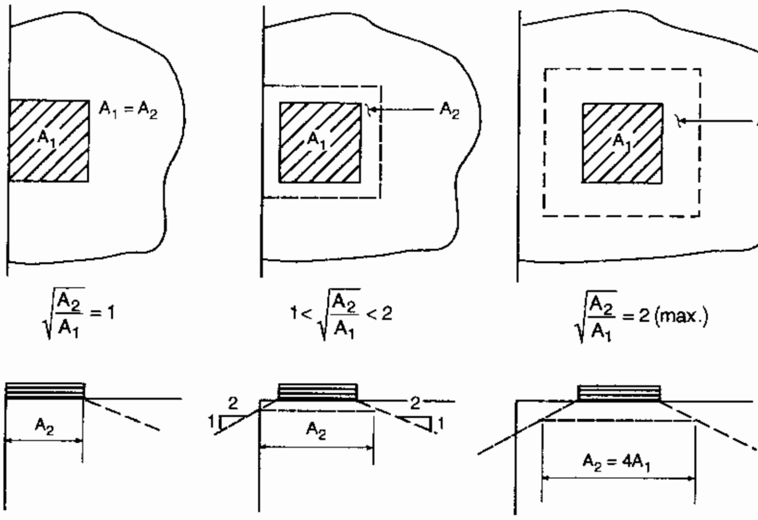


Figure 6-24 Measure of Confinement $\sqrt{A_2/A_1} \leq 2$ Provided by Surrounding Concrete

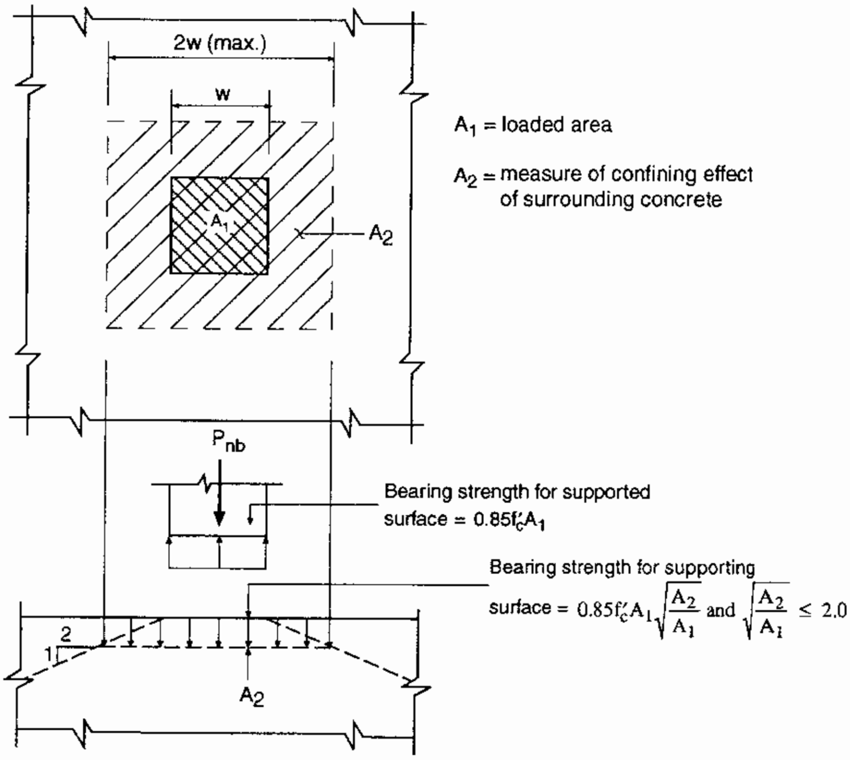


Figure 6-25 Nominal Bearing Strength of Concrete (10.17)

REFERENCES

6.1 Hognestad, E., Hanson, N.W., and McHenry, D., "Concrete Stress Distribution in Ultimate Strength Design," *ACI Journal, Proceedings* Vol. 52, December 1955, pp. 455-479; also *PCA Development Department Bulletin D6.*

6.2 Hognestad, E., "Ultimate Strength of Reinforced Concrete in American Design Practice," Proceedings of a Symposium on the Strength of Concrete Structures, London, England, May 1955; also *PCA Development Department Bulletin D12.*

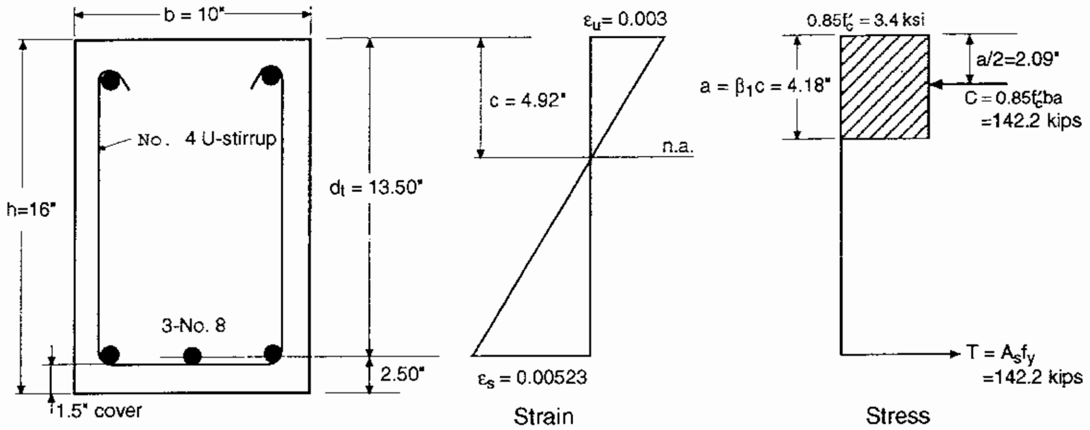
6.3 Hognestad, E., "Confirmation of Inelastic Stress Distribution in Concrete," *Journal of the Structural Division, Proceedings ASCE,* Vol. 83, No. ST2, March 1957; pp. 1189-1—1189-17 also *PCA Development Department Bulletin D15.*

6.4 Mattock, A.H., Kriz, L.B., and Hognestad, E., "Rectangular Concrete Stress Distribution in Ultimate Strength Design," *ACI Journal, Proceedings*, Vol. 57, February 1961, pp. 875-928; also *PCA Development Department Bulletin D49.*

6.5 Wang, C.K., and Salmon, C.G., *Reinforced Concrete Design*, Fourth Edition, Harper & Row Publishers, New York, N.Y. 1985.

Example 6.1—Moment Strength Using Equivalent Rectangular Stress Distribution

For the beam section shown, calculate moment strength based on static equilibrium using the equivalent rectangular stress distribution shown in Fig. 6-9. Assume $f'_c = 4000$ psi and $f_y = 60{,}000$ psi. For simplicity, neglect hanger bars.



Calculations and Discussion	Code Reference

1. Define rectangular concrete stress distribution. *10.2.7*

$d = d_t = 16 - 2.5 = 13.50$ in. *10.0*

$A_s = 3 \times 0.79 = 2.37$ in.^{2}

Assuming $\varepsilon_s > \varepsilon_y$,

$T = A_s f_y = 2.37 \times 60 = 142.2$ kips *10.2.4*

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{142.2}{0.85 \times 4 \times 10} = 4.18 \text{ in.}$$

2. Determine net tensile strain ε_s and ϕ

$$c = \frac{a}{\beta_1} = \frac{4.18}{0.85} = 4.92 \text{ in.}$$

$$\varepsilon_s = \left(\frac{d_t - c}{c}\right)0.003 = \left(\frac{13.50 - 4.92}{4.92}\right)0.003 = 0.00523 > 0.005$$

Therefore, section is tension-controlled *10.3.4*

$\phi = 0.9$ *9.3.2.1*

$\varepsilon_s = 0.00523 > 0.004$ which is minimum for flexural members *10.3.5*

This also confirms that $\varepsilon_s > \varepsilon_y$ at nominal strength.

3. Determine nominal moment srength, M_n, and design moment strength, ϕM_n.

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = 142.2\ (13.50 - 2.09) = 1{,}622.5\ \text{in.-kips} = 135.2\ \text{ft-kips}$$

$$\phi M_n = 0.9(135.2) = 121.7\ \text{ft-kips}$$ *9.3.2.1*

4. Minimum reinforcement.

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200 b_w d}{f_y}$$ *Eq. (10-3)*

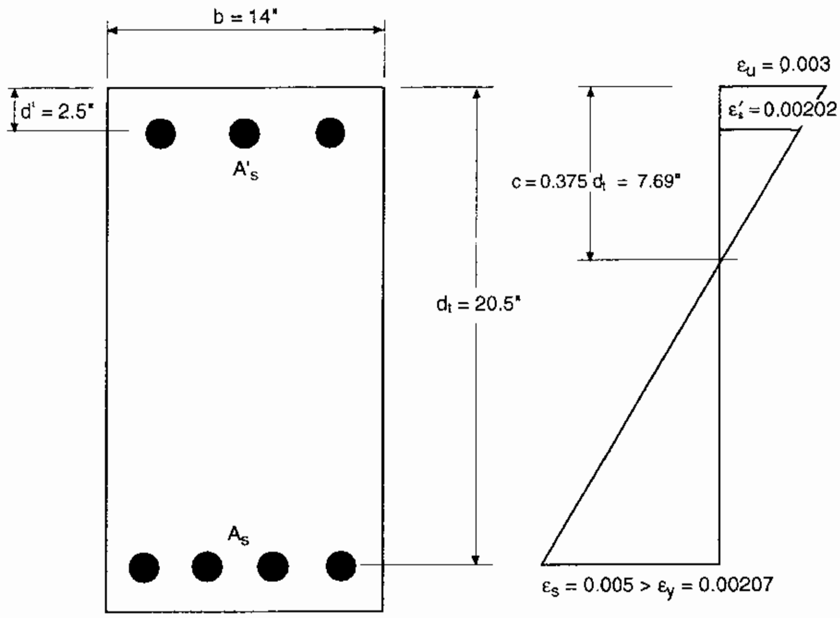
Since $f'_c < 4444$ psi, $200 b_w d / f_y$ governs:

$$\frac{200 b_w d}{f_y} = \frac{200 \times 10 \times 13.50}{60{,}000} = 0.45\ \text{in.}^2$$

A_s (provided) = 2.37 in.^{2} > $A_{s,min}$ = 0.45 in.^{2} O.K.

Example 6.2—Design of Beam with Compression Reinforcement

A beam cross-section is limited to the size shown. Determine the required area of reinforcement for a factored moment $M_u = 516$ ft-kips. $f'_c = 4000$ psi, $f_y = 60{,}000$ psi.



Calculations and Discussion	Code Reference

1. Check if compression reinforcement is required, using $\phi = 0.9$

$$M_n = M_u / \phi = 516/0.9 = 573 \text{ ft-kips}$$

$$R_n = \frac{M_n}{bd_t^2} = \frac{573 \times 12 \times 1000}{214 \times 20.5^2} = 1169$$

This exceeds the maximum R_{nt} of 911 for tension-controlled sections of 4000 psi concrete. (see Table 6-1.) Also, it appears likely that two layers of tension reinforcement will be necessary. But, for simplicity, assume that $d_t = d$.

2. Find the nominal strength moment M_{nt} resisted by the concrete section, without compression reinforcement, and M'_n to be resisted by the compression reinforcement.

$$M_{nt} = R_n bd^2 = 911 \times 14 \times 20.5^2 / (1000 \times 12) = 447 \text{ ft-kips}$$

$$M'_n = M_n - M_{nt} = 573 - 447 = 126 \text{ ft-kips}$$

3. Determine the required compression steel

The strain in the compression steel at nominal strength is just below yield strain, as shown in the strain diagram above.

$$f'_s = E_s\varepsilon_s = 29{,}000 \times 0.00202 = 58.7 \text{ psi} = 58.7 \text{ ksi}$$

$$A'_s = \frac{M'_n}{f'_s(d-d')} = \frac{126 \times 12}{58.7\,(20.5-2.5)} = 1.43 \text{ in.}^2$$

4. Determine the required tension steel

$$A_s = \rho_t(bd) + A'_s(f'_s/f_y)$$

From Table 6-1, $\rho_t = 0.01806$, so that

$$A_s = 0.01806(14)(20.5) + 1.43(58.7/60)$$

$$= 5.18 + 1.40 = 6.58 \text{ in.}^2$$

5. Alternative solution

Required nominal strength

$$M_n = \frac{M_u}{\phi} = \frac{516}{0.9} = 573 \text{ ft-kips}$$

a. Determine maximum moment without compression reinforcement M_{nt}, using $\phi = 0.9$:

$c = 0.375d_t = 0.375 \times 20.5 = 7.69$ in.

$a = \beta_1 c = 0.85 \times 7.69 = 6.54$ in.

$C = T = 3.4 \times 6.54 \times 14 = 311.3$ kips

$$M_{nt} = T\left(d_t - \frac{a}{2}\right) = 311.3\left(20.5 - \frac{6.54}{2}\right) = 5363.7 \text{ kip-in.} = 447.0 \text{ kip-ft}$$

b. Required area of tension steel to develop M_{nt}:

$$A_{s,nt} = \frac{311.3}{60} = 5.19 \text{ in.}^2$$

c. Additional moment (573-447 = 126 ft-kips) must be developed in T-C couple between tension steel and compression steel.

Additional tension steel required: $\Delta A_s = \dfrac{126 \times 12}{(20.5-2.5) \times 60} = 1.40 \text{ in.}^2$

Total tension steel required: $A_s = 5.19 + 1.40 = 6.59 \text{ in.}^2$

Compression steel required: $$A'_s = \frac{126 \times 12}{(20.5-2.5) \times 58.7} = 1.43 \text{ in.}^2$$

6. Comparison to Example 6.2 of *Notes on ACI 318-99* designed by ACI 318-99:

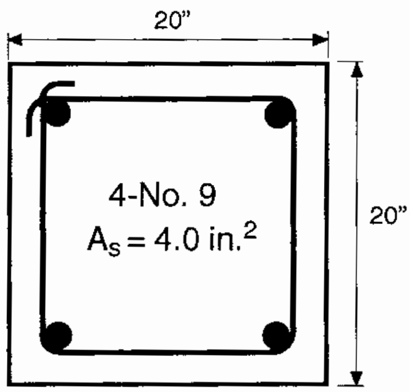
Example 6.2 of *Notes on ACI 318-99* was designed by the 1999 code for an M_u of 580 ft-kips. By current code, assuming a live-to-dead load ratio of 0.5 for this beam, the beam could be designed as a tension-controlled section for an M_u of 516 ft-kips. The results for the required reinforcement are

	by 318-99	Since 2002
Compression reinforcement A'_s	1.49 in.^{2}	1.43 in.^{2}
Tension reinforcement A_s	7.63 in.^{2}	6.58 in.^{2}

The reduction in tension reinforcement is a result of the lower load factors in the current code. However, the compression reinforcement requirement is about the same. This is caused by the need for ductility in order to use the ϕ of 0.9 for flexure.

Example 6.3—Maximum Axial Load Strength vs. Minimum Eccentricity

For the tied reinforced concrete column section shown below, compare the nominal axial load strength P_n equal to $0.80P_o$ with P_n at 0.1h eccentricity. f'_c = 5000 psi, f_y = 60,000 psi.



Calculations and Discussion	Code Reference

Prior to ACI 318-77, columns were required to be designed for a minimum eccentricity of 0.1h (tied) or 0.05h (spiral). This required tedious computations to find the axial load strength at these minimum eccentricities. With the 1977 ACI code, the minimum eccentricity provision was replaced with a maximum axial load strength: $0.80P_o$ (tied) or $0.85P_o$ (spiral). The 80% and 85% values were chosen to approximate the axial load strengths at e/h ratios of 0.1 and 0.05, respectively.

1. In accordance with the minimum eccentricity criterion:

   At e/h = 0.10: P_n = 1543 kips (computer solution)

2. In accordance with maximum axial load strength criterion: *10.3.5.2*

   $P_{n(max)} = 0.80P_o = 0.80\,[0.85 f'_c\,(A_g - A_{st}) + f_yA_{st}]$ *Eq. (10-2)*

   $= 0.80\,[0.85 \times 5\,(400 - 4.0) + (60 \times 4.0)] = 1538$ kips

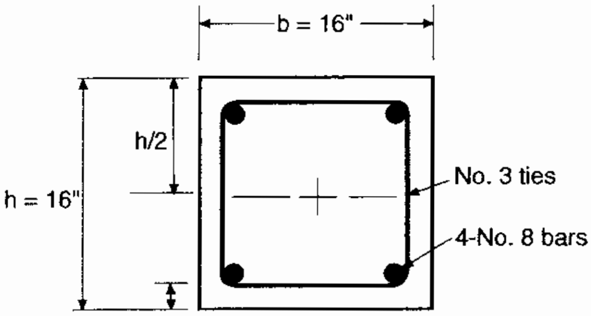
Depending on material strengths, size, and amount of reinforcement, the comparison will vary slightly. Both solutions are considered equally acceptable.

Example 6.4—Load-Moment Strength, P_n and M_n, for Given Strain Conditions

For the column section shown, calculate the load-moment strength, P_n and M_n, for four strain conditions:

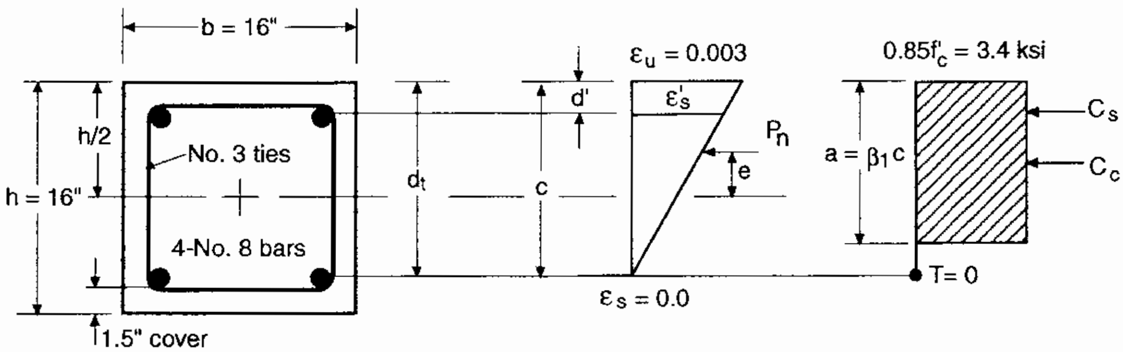
1. Bar stress near tension face of member equal to zero, $f_s = 0$
2. Bar stress near tension face of member equal to $0.5f_y$ ($f_s = 0.5f_y$)
3. At limit for compression-controlled section ($\varepsilon_t = 0.002$)
4. At limit for tension-controlled sections ($\varepsilon_t = 0.005$).

Use $f'_c = 4000$ psi, and $f_y = 60,000$ psi.



Calculations and Discussion	Code Reference

1. Load-moment strength, P_n and M_n, for strain condition 1: $\varepsilon_s = 0$



$A_s = A'_s = 2(0.79) = 1.58$ in.^{2} Strain Condition - 1 Stress

a. Define stress distribution and determine force values. *10.2.7*

$$d' = \text{Cover} + \text{No. 3 tie dia.} + \frac{d_b}{2} = 1.5 + 0.375 + 0.5 = 2.38 \text{ in.}$$

$$d_t = 16 - 2.38 = 13.62 \text{ in.}$$

Since $\varepsilon_s = 0,\ c = d_t = 13.62$ in. *10.2.7.2*

$a = \beta_1 c = 0.85\,(13.62) = 11.58$ in. *10.2.7.1*

where $\beta_1 = 0.85$ for $f'_c = 4000$ psi *10.2.7.3*

$C_c = 0.85 f'_c ba = 0.85 \times 4 \times 16 \times 11.58 = 630.0$ kips *10.2.7*

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$ *10.2.4*

From strain compatibility:

$$\varepsilon'_s = \varepsilon_u\left(\frac{c - d'}{c}\right) = 0.003\left(\frac{13.62 - 2.38}{13.62}\right) = 0.00248 > \varepsilon_y = 0.00207$$ *10.2.2*

Compression steel has yielded.

$C_s = A'_s f_y = 1.58\,(60) = 94.8$ kips

b. Determine P_n and M_n from static equilibrium.

$P_n = C_c + C_s = 630.0 + 94.8 = 724.8$ kips *Eq. (16)*

$$M_n = P_n e = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} - d'\right)$$ *Eq. (17)*

$$= 630\,(8.0 - 5.79) + 94.8\,(8.0 - 2.38) = 1925.1 \text{ in.-kips} = 160.4 \text{ ft-kips}$$

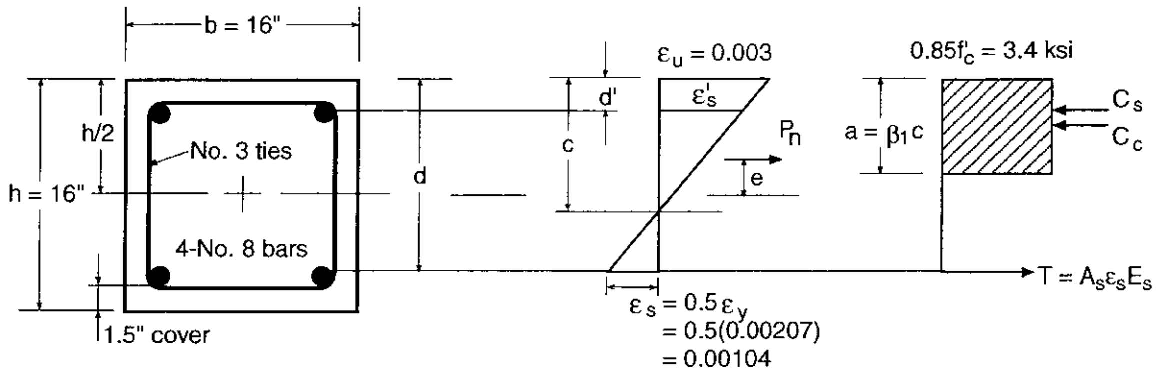
$$e = \frac{M_n}{P_n} = \frac{1925.1}{724.8} = 2.66 \text{ in.}$$

Therefore, for strain condition $\varepsilon_s = 0$:

Design axial load strength, $\phi P_n = 0.65\,(724.8) = 471.1$ kips *9.3.2.2*

Design moment strength, $\phi M_n = 0.65\,(160.4) = 104.3$ ft-kip

2. Load-moment strength, P_n and M_n, for strain condition 2: $\varepsilon_s = 0.5\varepsilon_y$



Strain Condition - 2 Stress

a. Define stress distribution and determine force values. *10.2.7*

$d' = 2.38$ in., $d_t = 13.62$ in.

From strain compatibility:

$$\frac{c}{0.003} = \frac{d_t - c}{0.5\varepsilon_y}$$

$$c = \frac{0.003d_t}{0.5\varepsilon_y + 0.003} = \frac{0.003 \times 13.62}{0.00104 + 0.003} = 10.13 \text{ in.}$$

Strain in compression reinforcement:

$$\varepsilon'_s = \varepsilon_u\left(\frac{c - d'}{c}\right) = 0.003\left(\frac{10.13 - 2.38}{10.13}\right) = 0.00230 > \varepsilon_y = 0.00207$$

Compression steel has yielded.

$a = \beta_1 c = 0.85\,(10.13) = 8.61$ in. *10.2.7.1*

$C_c = 0.85 f'_c ba = 0.85 \times 4 \times 16 \times 8.61 = 468.4$ kips *10.2.7*

$C_s = A'_s f_y = 1.58\,(60) = 94.8$ kips

$T = A_s f_s = A_s\,(0.5f_y) = 1.58\,(30) = 47.4$ kips

b. Determine P_n and M_n from static equilibrium.

$P_n = C_c + C_s - T = 468.4 + 94.8 - 47.4 = 515.8$ kips *Eq. (16)*

$$M_n = P_n e = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} + d'\right) + T\left(d - \frac{h}{2}\right)$$ *Eq. (17)*

$= 468.4\,(8.0 - 4.31) + 94.8\,(8.0 - 2.38) + 47.4\,(13.62 - 8.0)$

$= 2527.6$ in.-kips $= 210.6$ ft-kips

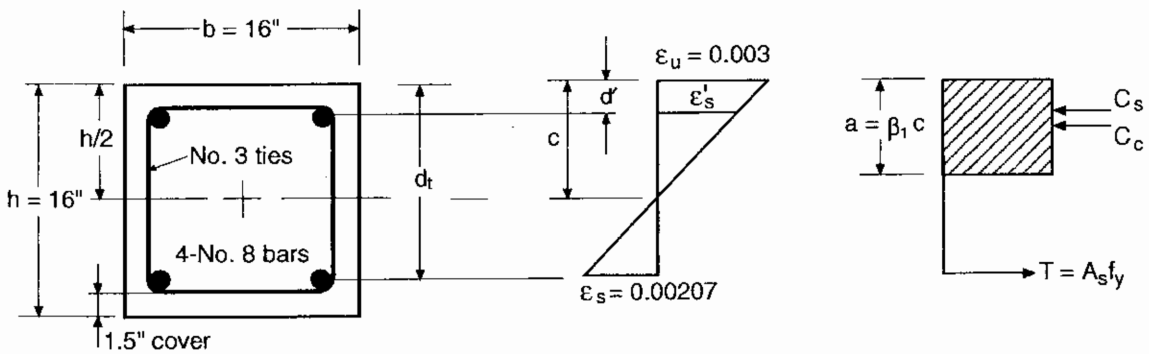
$$e = \frac{M_n}{P_n} = \frac{2527.6}{515.8} = 4.90 \text{ in.}$$

Therefore, for strain condition $\varepsilon_s = 0.5\,\varepsilon_y$:

Design axial load strength, $\phi P_n = 0.65\,(515.8) = 335.3$ kips *9.3.2.2*

Design moment strength, $\phi M_n = 0.65\,(210.6) = 136.9$ ft-kips

3. Load-moment strength, P_n and M_n, for strain condition 3: $\varepsilon_s = \varepsilon_y$



Strain Condition - 3

a. Define stress distribution and determine force values. *10.2.7*

$d' = 2.38$ in., $d_t = 13.62$ in.

From strain compatibility:

$$\frac{c}{0.003} = \frac{d_t - c}{\varepsilon_y}$$

$$c = \frac{0.003d_t}{\varepsilon_y + 0.003} = \frac{0.003 \times 13.62}{0.00207 + 0.003} = 8.06 \text{ in.}$$

Note: The code permits the use of 0.002 as the strain limit for compression-controlled sections with Grade 60 steel. It is slightly conservative, and more consistent, to use the yield strain of 0.00207.

Strain in compression reinforcement:

$$\varepsilon_s' = \varepsilon_u \left(\frac{c - d'}{c}\right) = 0.003 \left(\frac{8.06 - 2.38}{8.06}\right) = 0.00211 > \varepsilon_y = 0.00207$$

Compression steel has yielded.

$a = \beta_1 c = 0.85\,(8.06) = 6.85$ in. *10.2.7.1*

$C_c = 0.85 f_c' ba = 0.85 \times 4 \times 16 \times 6.85 = 372.7$ kips *10.2.7*

$C_s = A_s' f_y = 1.58\,(60) = 94.8$ kips

$T = A_s f_s = A_s f_y = 1.58\,(60) = 94.8$ kips

b. Determine P_n and M_n from static equilibrium.

$P_n = C_c + C_s - T = 372.7 + 94.8 - 94.8 = 372.7$ kips *Eq. (16)*

$$M_n = P_n e = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} + d'\right) + T\left(d - \frac{h}{2}\right)$$ *Eq. (17)*

$= 372.7\,(8.0 - 3.43) + 94.8\,(8.0 - 2.38) + 94.8\,(13.62 - 8.0)$

$= 2770.5$ in.-kips $= 230.9$ ft-kips

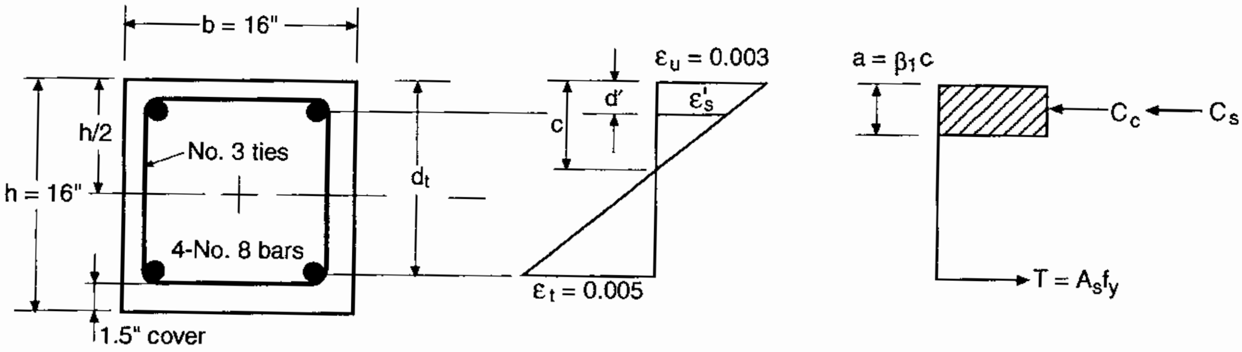
$$e = \frac{M_n}{P_n} = \frac{2770.5}{372.7} = 7.43 \text{ in.}$$

Therefore, for strain condition $\varepsilon_s = \varepsilon_y$:

Design axial load strength, $\phi P_n = 0.65\,(372.7) = 242.3$ kips *9.3.2.2*

Design moment strength, $\phi M_n = 0.65\,(230.9) = 150.1$ ft-kips

4. Load-moment strength, P_n and M_n, for strain condition 4: $\varepsilon_s = 0.005$



Strain Condition - 4

a. Define stress distribution and determine force values. *10.2.7*

$d' = 2.38$ in., $d_t = 13.62$ in.

From strain compatibility:

$$\frac{c}{0.003} = \frac{d-c}{0.005}$$

$$c = \frac{0.003d}{0.005+0.003} = \frac{0.003 \times 13.62}{0.005+0.003} = 5.11 \text{ in.}$$

Strain in compression reinforcement:

$$\varepsilon_s' = \varepsilon_u\left(\frac{c-d'}{c}\right) = 0.003\left(\frac{5.11-2.38}{5.11}\right) = 0.00160 < \varepsilon_y = 0.00207$$

Compression steel has not yielded.

$$f_s' = \varepsilon_s' E_s' = 0.00160\ (29{,}000) = 46.5 \text{ ksi}$$

$$a = \beta_1 c = 0.85\ (5.11) = 4.34 \text{ in.}$$ *10.2.7.1*

$$C_c = 0.85 f_c' ba = 0.85 \times 4 \times 16 \times 4.34 = 236.2 \text{ kips}$$ *10.2.7*

$$C_s = A_s' f_y = 1.58\ (46.5) = 73.5 \text{ kips}$$

$$T = A_s f_s = A_s(f_y) = 1.58\ (60) = 94.8 \text{ kips}$$

b. Determine P_n and M_n from static equilibrium.

$$P_n = C_c + C_s - T = 236.2 + 73.5 - 94.8 = 214.9 \text{ kips}$$ *Eq. (16)*

$$M_n = P_n e = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_s\left(\frac{h}{2} + d'\right) + T\left(d - \frac{h}{2}\right)$$ *Eq. (17)*

$$= 236.2\,(8.0 - 2.17) + 73.5\,(8.0 - 2.38) + 94.8\,(13.62 - 8.0)$$

$$= 2322.9 \text{ in.-kips} = 193.6 \text{ ft-kips}$$

$$e = \frac{M_n}{P_n} = \frac{2322.9}{214.9} = 10.81 \text{ in.}$$

Therefore, for strain condition $\varepsilon_s = 0.005$:

Design axial load strength, $\phi P_n = 0.9\,(214.9) = 193.4$ kips *9.3.2.2*

Design moment strength, $\phi M_n = 0.9\,(193.6) = 174.2$ ft-kips

A complete interaction diagram for this column is shown in Fig. 6-25. In addition, Fig. 6-26 shows the interaction diagram created using the Portland Cement Association computer program pcaColumn.

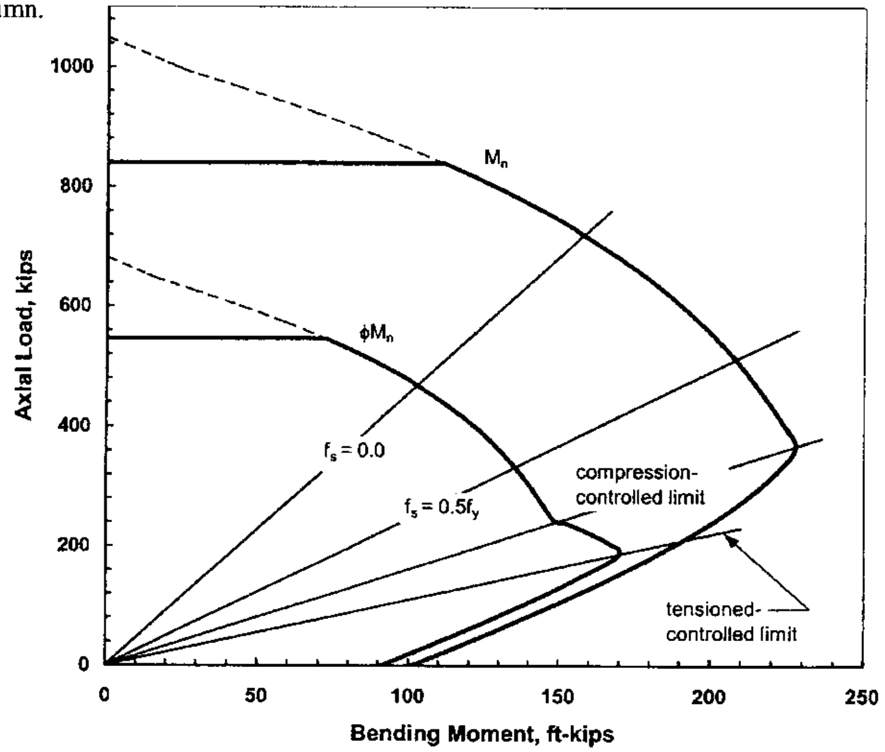


Figure 6-25 Interaction Diagram

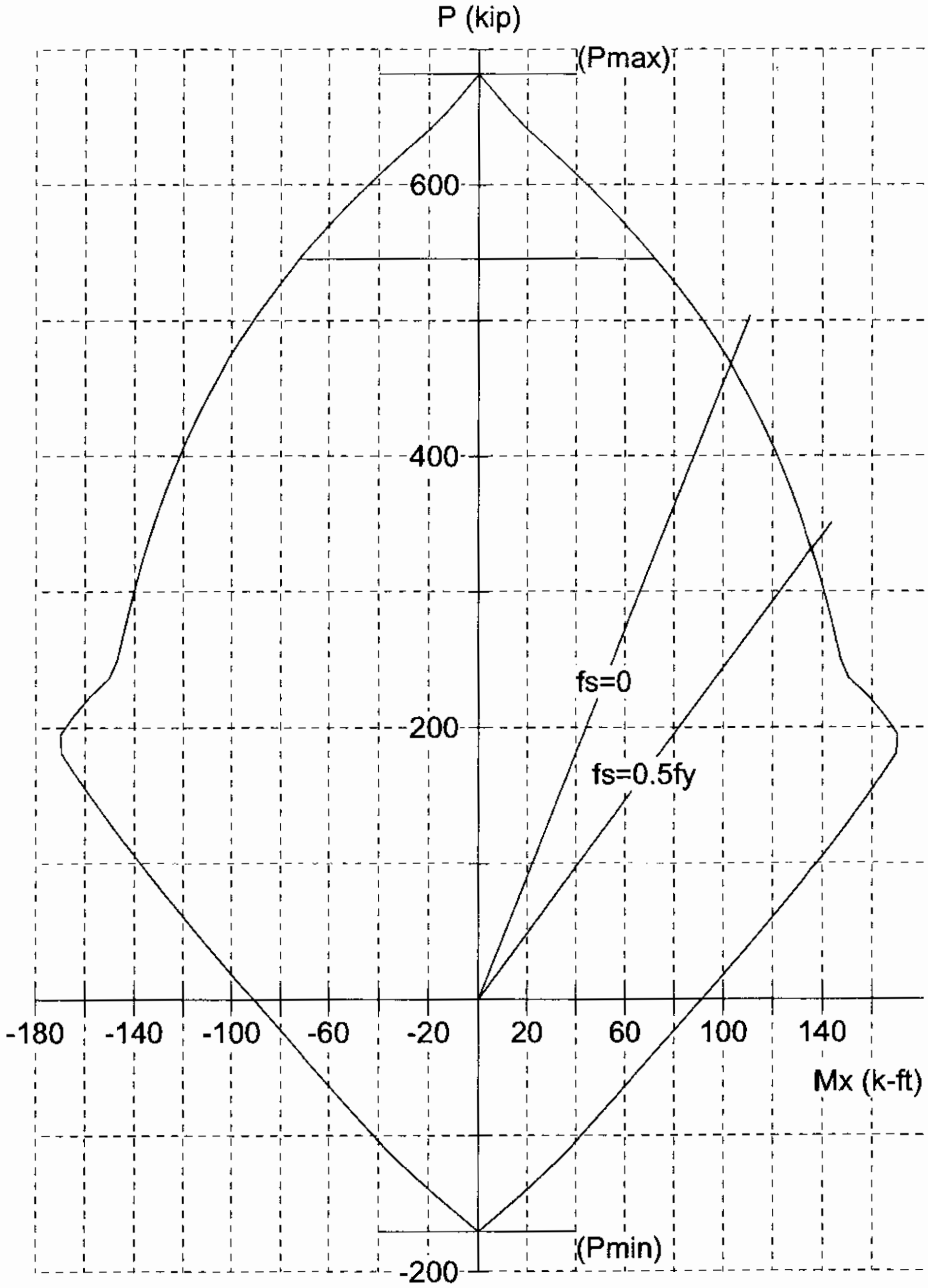


Figure 6-26 Interaction Diagram from pcaColumn

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7

Design for Flexure and Axial Load

GENERAL CONSIDERATIONS—FLEXURE

For design or investigation of members subjected to flexure (beams and slabs), the nominal strength of the member cross-section (M_n) must be reduced by the strength reduction factor ϕ to obtain the design strength (ϕM_n) of the section. The design strength (ϕM_n) must be equal to or greater than the required strength (M_u). In addition, the serviceability requirements for deflection control (9.5) and distribution of reinforcement for crack control (10.6) must also be satisfied.

Examples 7.1 through 7.7 illustrate proper application of the various code provisions that govern design of members subject to flexure. The design examples are prefaced by step-by-step procedures for design of rectangular sections with tension reinforcement only, rectangular sections with multiple layers of steel, rectangular sections with compression reinforcement, and flanged sections with tension reinforcement only.

DESIGN OF RECTANGULAR SECTIONS WITH TENSION REINFORCEMENT ONLY[7.1]

In the design of rectangular sections with tension reinforcement only (Fig. 7-1), the conditions of equilibrium are:

1. Force equilibrium:

$$C = T \tag{1}$$

$$0.85f'_c ba = A_s f_y = \rho b d f_y$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{\rho d f_y}{0.85 f'_c}$$

2. Moment equilibrium:

$$M_n = (C \text{ or } T)\left(d - \frac{a}{2}\right)$$

$$M_n = \rho b d f_y \left[d - \frac{0.5\rho d}{0.85}\frac{f_y}{f'_c}\right] \tag{2}$$

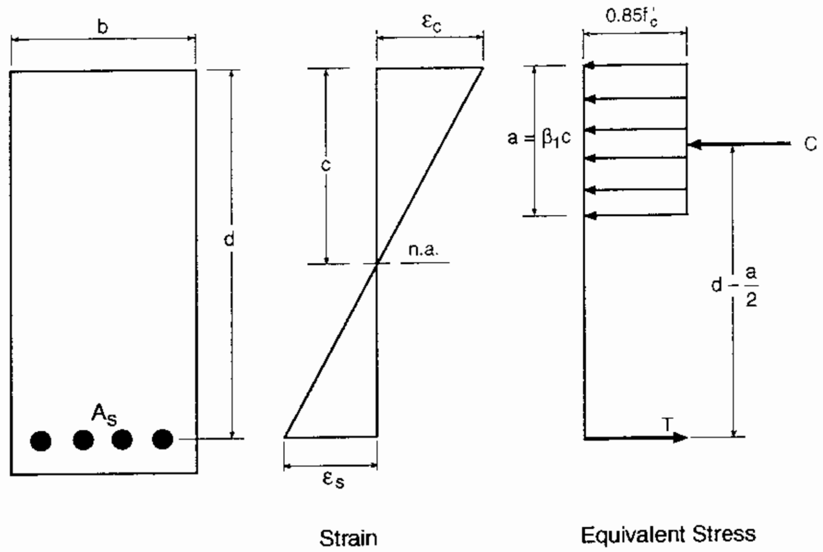


Figure 7-1 Strain and Equivalent Stress Distribution in Rectangular Section

A nominal strength coefficient of resistance R_n is obtained when both sides of Eq. (2) are divided by bd^2:

$$R_n = \frac{M_n}{bd^2} = \rho f_y\left(1 - \frac{0.5\rho f_y}{0.85f'_c}\right) \tag{3}$$

When b and d are preset, ρ is obtained by solving the quadratic equation for R_n:

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85f'_c}}\right) \tag{4}$$

The relationship between ρ and R_n for Grade 60 reinforcement and various values of f'_c is shown in Fig. 7-2.

Equation (3) can be used to determine the steel ratio ρ given M_u or vice-versa if the section properties b and d are known. Substituting $M_n = M_u/\phi$ into Eq. (3) and dividing each side by f'_c:

$$\frac{M_u}{\phi f'_c bd^2} = \frac{\rho f_y}{f'_c}\left(1 - \frac{0.5\rho f_y}{0.85f'_c}\right)$$

Define $\omega = \dfrac{\rho f_y}{f'_c}$

Substituting ω into the above equation:

$$\frac{M_u}{\phi f'_c bd^2} = \omega\,(1 - 0.59\omega) \tag{5}$$

Table 7-1, based on Eq. (5), has been developed in order to serve as a design aid for either design or investigation of sections having tension reinforcement only where b and d are known.

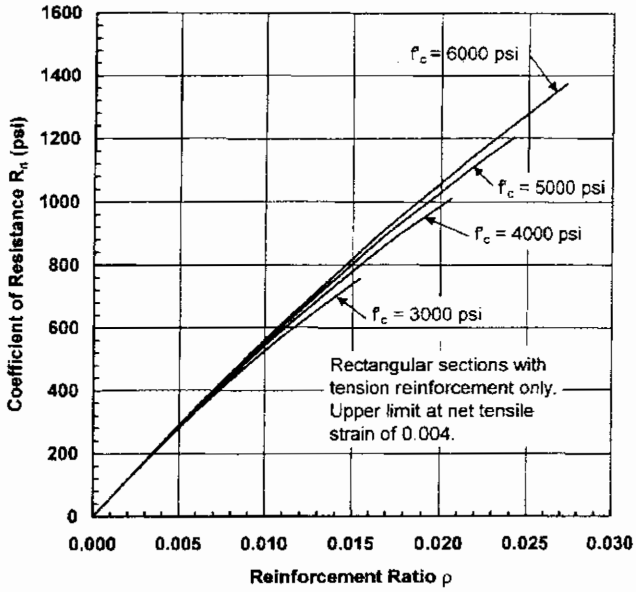


Figure 7-2 Strength Curves (R_n vs. ρ) for Grade 60 Reinforcement

Table 7-1 Flexural Strength $M_u / \phi f'_c bd^2$ or $M_n / f'_c bd^2$ of Rectangular Sections with Tension Reinforcement Only

ω	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
0.00	0	.0010	.0020	.0030	.0040	.0050	.0060	.0070	.0080	0090
0.01	.0099	.0109	.0119	.0129	.0139	.0149	.0159	.0168	.0178	.0188
0.02	.0197	.0207	.0217	.0226	.0236	.0246	.0256	.0266	.0275	.0285
0.03	.0295	.0304	.0314	.0324	.0333	.0343	.0352	.0362	.0372	.0381
0.04	.0391	.0400	.0410	.0420	.0429	.0438	.0448	.0457	.0467	.0476
0.05	.0485	.0495	.0504	.0513	.0523	.0532	.0541	.0551	.0560	.0569
0.06	.0579	.0588	.0597	.0607	.0616	.0626	.0634	.0643	.0653	.0662
0.07	.0671	.0680	.0689	.0699	.0708	.0717	.0726	.0735	.0744	.0753
0.08	.0762	.0771	.0780	.0789	.0798	.0807	.0816	.0825	.0834	.0843
0.09	.0852	.0861	.0870	.0879	.0888	.0897	.0906	.0915	.0923	.0932
0.10	.0941	.0950	.0959	.0967	.0976	.0985	.0994	.1002	.1001	.1020
0.11	.1029	.1037	.1046	.1055	.1063	.1072	.1081	.1089	.1098	.1106
0.12	.1115	.1124	.1133	.1141	.1149	.1158	.1166	.1175	.1183	.1192
0.13	.1200	.1209	.1217	.1226	.1234	.1243	.1251	.1259	.1268	.1276
0.14	.1284	.1293	.1301	.1309	.1318	.1326	.1334	.1342	.1351	.1359
0.15	.1367	.1375	.1384	.1392	.1400	.1408	.1416	.1425	.1433	.1441
0.16	.1449	.1457	.1465	.1473	.1481	.1489	.1497	.1506	.1514	.1522
0.17	.1529	.1537	.1545	.1553	.1561	.1569	.1577	.1585	.1593	.1601
0.18	.1609	.1617	.1624	.1632	.1640	.1648	.1656	.1664	.1671	.1679
0.19	.1687	.1695	.1703	.1710	.1718	.1726	.1733	.1741	.1749	.1756
0.20	.1764	.1772	.1779	.1787	.1794	.1802	.1810	.1817	.1825	.1832
0.21	.1840	.1847	.1855	.1862	.1870	.1877	.1885	.1892	.1900	.1907
0.22	.1914	.1922	.1929	.1937	.1944	.1951	.1959	.1966	.1973	.1981
0.23	.1988	.1995	.2002	.2010	.2017	.2024	.2031	.2039	.2046	.2053
0.24	.2060	.2067	.2075	.2082	.2089	.2096	.2103	.2110	.2117	.2124
0.25	.2131	.2138	.2145	.2152	.2159	.2166	.2173	.2180	.2187	.2194
0.26	.2201	.2208	.2215	.2222	.2229	.2236	.2243	.2249	.2256	.2263
0.27	.2270	.2277	.2284	.2290	.2297	.2304	.2311	.2317	.2324	.2331
0.28	.2337	.2344	.2351	.2357	.2364	.2371	.2377	.2384	.2391	.2397
0.29	.2404	.2410	.2417	.2423	.2430	.2437	.2443	.2450	.2456	.2463
0.30	.2469	.2475	.2482	.2488	.2495	.2501	.2508	.2514	.2520	.2527

$M_n / f'_c bd^2 = \omega(1 - 0.59\omega)$, *where* $\omega = \rho f_y / f'_c$

For design: Using factored moment M_u, enter table with $M_u/\phi f'_c bd^2$; find ω and compute steel percentage $\rho = \omega f'_c / f_y$

For investigation: Enter table with $\omega = \rho f_y / f'_c$; find value of $M_n / f'_c bd^2$ and solve for nominal strength, M_n.

Figure 7-3 shows the effect of the strength reduction factor ϕ. In particular, it shows what happens when the limit for tension-controlled sections with a ϕ of 0.9 is passed. As can be seen from Fig. 7-3, there is no benefit in designing a flexural member that is below the tension-controlled strain limit of 0.005. Any gain in strength with higher reinforcement ratios is offset by the reduction in the strength reduction factor ϕ at higher reinforcement ratios. Therefore, flexural members are more economical when designed as tension-controlled sections.

One might wonder "why even permit higher amounts of reinforcement and lower net tensile strains if there is no advantage?" In many cases, the provided steel is above the optimum at the limit for tension-controlled sections. The "flat" portion of the curve in Fig. 7-3 allows the designer to provide excess reinforcement above that required (considering discrete bar sizes) without being penalized for "being above a code limit."

Although flexural members should almost always be designed as tension-controlled sections with $\varepsilon_t \geq 0.005$, it often happens that columns with small axial load and large bending moments are in the "transition region" with ε_t between 0.002 and 0.005, and ϕ is somewhere between that for compression-controlled sections and that for tension-controlled sections.

Columns are normally designed using interaction charts or tables. The "breakpoint" for ε_t of 0.005 and $\phi = 0.9$ may fall above or below the zero axial load line on the interaction diagrams.

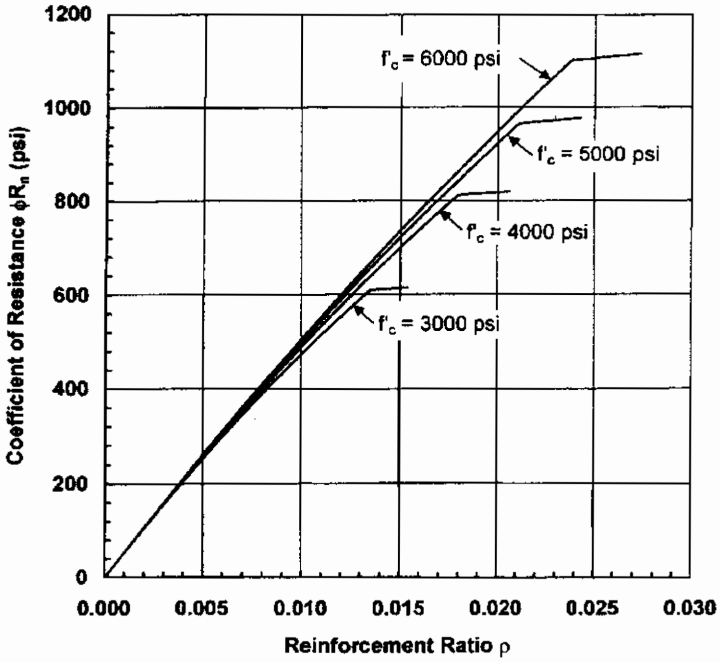


Figure 7-3 Design Strength Curves (ϕR_n vs. ρ) for Grade 60 Reinforcement

DESIGN PROCEDURE FOR SECTIONS WITH TENSION REINFORCEMENT ONLY

Step 1: Select an approximate value of tension reinforcement ratio ρ equal to or less than ρ_t, but greater than the minimum (10.5.1), where the reinforcement ratio ρ_t is given by:

$$\rho_t = \frac{0.319\beta_1 f'_c}{f_y}$$

where $\beta_1 = 0.85$ for $f'_c \leq 4000$ psi

$$= 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \text{ for } 4000 \text{ psi} < f'_c < 8000 \text{ psi}$$

$= 0.65$ for $f'_c \geq 8000$ psi

Values of ρ_t are given in Table 6-1.

Step 2: With ρ preset ($\rho_{min} \leq \rho \leq \rho_t$) compute bd^2 required:

$$bd^2 \text{ (required)} = \frac{M_u}{\phi R_n}$$

where $R_n = \rho f_y \left(1 - \frac{0.5\rho f_y}{0.85 f'_c}\right)$, $\phi = 0.90$ for flexure with $\rho \leq \rho_t$, and M_u = applied factored moment (required flexural strength)

Step 3: Size the member so that the value of bd^2 provided is greater than or equal to the value of bd^2 required.

Step 4: Based on the provided bd^2, compute a revised value of ρ by one of the following methods:

1. By Eq. (4) where $R_n = M_u/\phi bd^2$ (exact method)

2. By strength curves such as those shown in Fig. 7-2 and Fig. 7-3. Values of ρ are given in terms of $R_n = M_u/\phi bd^2$ for Grade 60 reinforcement.

3. By moment strength tables such as Table 7-1. Values of $\omega = \rho f_y/f'_c$ are given in terms of moment strength $M_u/\phi f'_c bd^2$.

4. By approximate proportion

$$\rho \approx (\text{original } \rho) \frac{(\text{revised } R_n)}{(\text{original } R_n)}$$

Note from Fig. 7-2 that the relationship between R_n and ρ is approximately linear.

Step 5: Compute required A_s:

A_s = (revised ρ) (bd provided)

When b and d are preset, the required A_s is computed directly from:

$A_s = \rho$ (bd provided)

where ρ is computed using one of the methods outlined in Step 4.

DESIGN PROCEDURE FOR SECTIONS WITH MULTIPLE LAYERS OF STEEL

The simple and conservative way to design a beam with two layers of tension steel is to take d_t equal to d, the depth to the centroid of all the tension steel. However, the code does permit the designer to take advantage of the fact that d_t, measured to the center of the layer farthest from the compression face, is greater than d. The only time this would be necessary is when designing at or very close to the strain limit of 0.005 for tension-controlled sections.

Figure 7-4 shows strain and stress diagrams for a section with multiple layers of steel with the extreme steel layer at the tension-controlled strain limit of 0.005. Let ρ_2 stand for the maximum ρ (based on d) for this section.

$$\rho_2 = \frac{C}{f_y b d}$$

However,

$$\rho_t = \frac{C}{f_y b d_t}$$

Therefore,

$$\frac{\rho_2}{\rho_t} = \frac{d_t}{d}$$

$$\rho_2 = \rho_t\left(\frac{d_t}{d}\right) \qquad (6)$$

Additional information can be found in the strain diagram of Fig. 7-4. The yield strain of Grade 60 reinforcement is 0.00207. By similar triangles, any Grade 60 steel that is within 0.366 d_t of the bottom layer will be at yield. This is almost always the case, unless steel is distributed on the side faces. Also, compression steel will be at yield if it is within $0.116d_t$ (or, 0.31c) of the compression face.

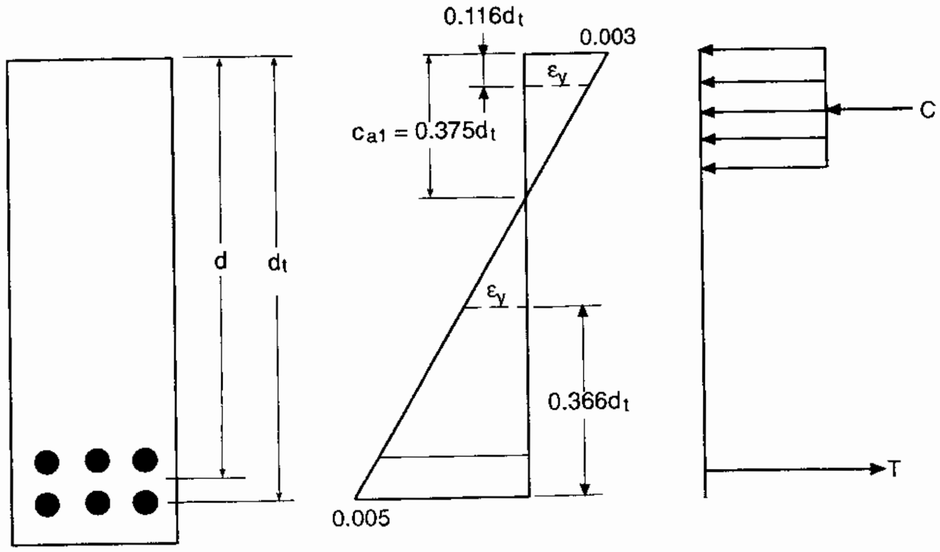


Figure 7-4 Multiple Layers of Reinforcement

DESIGN PROCEDURE FOR RECTANGULAR SECTIONS WITH COMPRESSION REINFORCEMENT (see Part 6)

Steps are summarized for the design of rectangular beams (with b and d preset) requiring compression reinforcement (see Example 7.3).

Step 1. Check to see if compression reinforcement is needed. Compute

$$R_n = \frac{M_n}{bd^2}$$

Compare this to the maximum R_n for tension-controlled sections given in Table 6-1. If R_n exceeds this, use compression reinforcement.

If compression reinforcement is needed, it is likely that two layers of tension reinforcement will be needed. Estimate d_t/d ratio.

Step 2. Find the nominal moment strength resisted by a section without compression reinforcement, and the additional moment strength M_n' to be resisted by the compression reinforcement and by added tension reinforcement.

From Table 6-1, find ρ_t. Then, using Eq. (6);

$$\rho = \rho_t\left(\frac{d_t}{d}\right)$$

$$\omega = \rho\frac{f_y}{f_c'}$$

Determine M_{nt} from Table 7-1.

Compute moment strength to be resisted by compression reinforcement:

$$M_n' = M_n - M_{nt}$$

Step 3. Check yielding of compression reinforcement

If $d'/c < 0.31$, compressive reinforcement has yielded and $f_s' = f_y$

See Part 6 to determine f_s' when the compression reinforcement does not yield.

Step 4. Determine the total required reinforcement, A_s' and A_s

$$A_s' = \frac{M_n'}{(d-d')f_s'}$$

Step 5: Check moment capacity

$$\phi M_n = \phi\left[(A - A_s')\,f_y\left(d - \frac{a}{2}\right) + A_s' f_y\,(d - d')\right] \geq M_u$$

where

$$a = \frac{(A_s - A_s')\,f_y}{0.85f_c'b}$$

DESIGN PROCEDURE FOR FLANGED SECTIONS WITH TENSION REINFORCEMENT (see Part 6)

Steps are summarized for the design of flanged sections with tension reinforcement only (see Examples 7.4 and 7.5).

Step 1: Determine effective flange width b according to 8.10.

Using Table 7-1, determine the depth of the equivalent stress block a, assuming rectangular section behavior with b equal to the flange width (i.e., $a \leq h_f$):

$$a = \frac{A_s f_y}{0.85f_c'b} = \frac{\rho d f_y}{0.85f_c'} = 1.18\omega d$$

where ω is obtained from Table 7-1 for $M_u/\phi f_c'bd^2$. Assume tension-controlled section with ϕ = 0.9.

Step 2: If $a \leq h_f$, determine the reinforcement as for a rectangular section with tension reinforcement only. If $a > h_f$, go to step 3.

Step 3: If $a > h_f$, compute the required reinforcement A_{sf} and the moment strength ϕM_{nf} corresponding to the overhanging beam flange in compression:

$$A_{sf} = \frac{C_f}{f_y} = \frac{0.85f_c'(b - b_w)h_f}{f_y}$$

$$\phi M_{nf} = \phi\left[A_{sf} f_y\left(d - \frac{h_f}{2}\right)\right]$$

Step 4: Compute the required moment strength to be carried by the beam web:

$$M_{uw} = M_u - \phi M_{nf}$$

Step 5: Using Table 7-1, compute the reinforcement A_{sw} required to develop the moment strength to be carried by the web:

$$A_{sw} = \frac{0.85f_c'b_w a_w}{f_y}$$

where $a_w = 1.18\omega_w d$ with ω_w obtained from Table 7-1 for $M_{uw}/\phi f'_c b_w d^2$.

Alternatively, obtain A_{sw} from the following:

$$A_{sw} = \frac{\omega_w f'_c b_w d}{f_y}$$

Step 6: Determine the total required reinforcement:

$$A_s = A_{sf} + A_{sw}$$

Step 7: Check to see if section is tension-controlled, with $\phi = 0.9$.

$$c = a_w / \beta_1$$

If $c/d_t \leq 0.375$, section is tension-controlled
If $c/d_t > 0.375$, add compression reinforcement

Step 8: Check moment capacity:

$$\phi M_n = \phi\left[(A_s - A_{sf})\, f_y \left(d - \frac{a_w}{2}\right) + A_{sf} f_y \left(d - \frac{h_f}{2}\right)\right] \geq M_u$$

where $A_{sf} = \dfrac{0.85 f'_c (b - b_w)\, h_f}{f_y}$

$$a_w = \frac{(A_s - A_{sf})\, f_y}{0.85 f'_c b_w}$$

GENERAL CONSIDERATIONS—FLEXURE AND AXIAL LOAD

Design or investigation of a short compression member (without slenderess effect) is based primarily on the strength of its cross-section. Strength of a cross-section under combined flexure and axial load must satisfy both force equilibrium and strain compatibility (see Part 6). The combined nominal axial load and moment strength (P_n, M_n) is then multiplied by the appropriate strength reduction factor ϕ to obtain the design strength (ϕP_n, ϕM_n) of the section. The design strength must be equal to or greater than the required strength:

$$(\phi P_n, \phi M_n) \geq (P_u, M_u)$$

All members subjected to combined flexure and axial load must be designed to satisfy this basic criterion. Note that the required strength (P_u, M_u) represents the structural effects of the various combinations of loads and forces to which a structure may be subjected; see Part 5 for discussion on 9.2.

A "strength interaction diagram" can be generated by plotting the design axial load strength ϕP_n against the corresponding design moment strength ϕM_n; this diagram defines the "usable" strength of a section at different eccentricities of the load. A typical design load-moment strength interaction diagram is shown in Fig. 7-5, illustrating the various segments of the strength curve permitted for design. The "flat-top" segment of the design strength curve defines the limiting axial load strength $P_{n,max}$; see Part 5 for a discussion on 10.3.6. As the design axial load strength ϕP_n decreases, a transition occurs between the compression-controlled limit and the tension-controlled limit, as shown in the figure. Example 6.4 illustrates the construction of an interaction diagram.

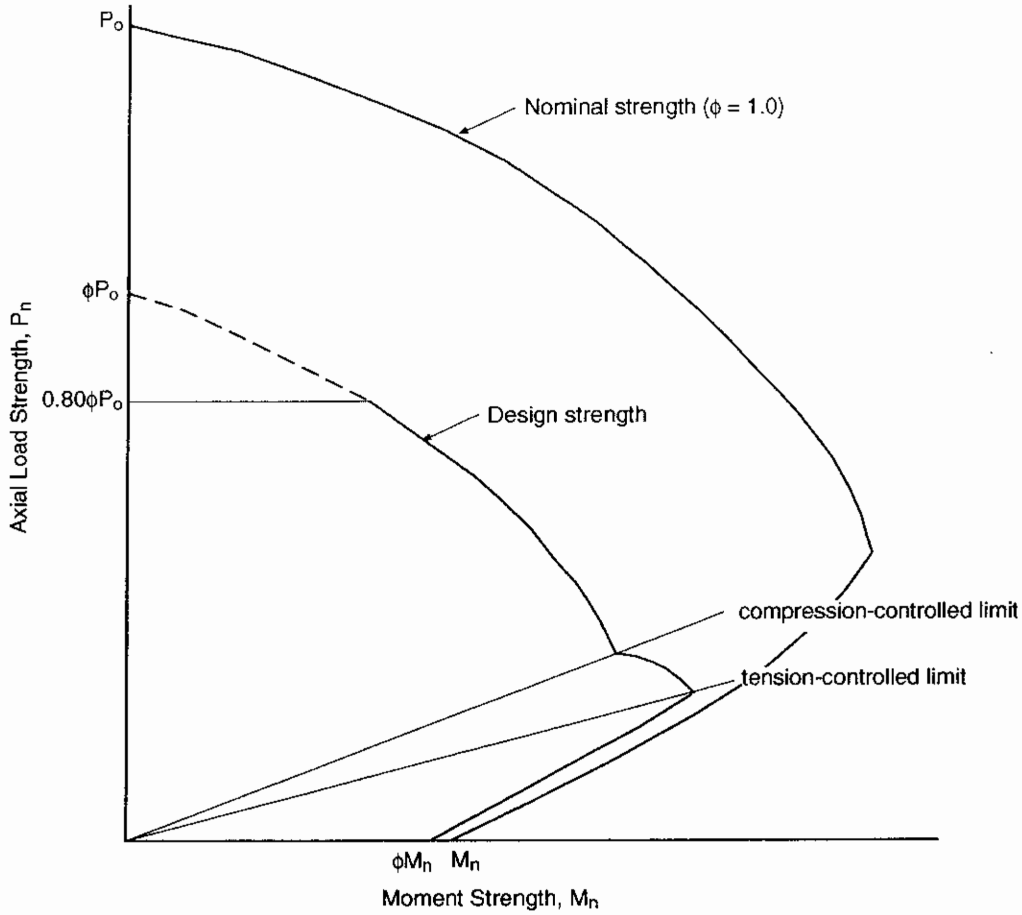


Figure 7-5 Design Load-Moment Strength Diagram (tied column)

GENERAL CONSIDERATIONS—BIAXIAL LOADING

Biaxial bending of columns occurs when the loading causes bending simultaneously about both principal axes. The commonly encountered case of such loading occurs in corner columns. Design for biaxial bending and axial load is mentioned in R10.3.6 and R10.3.7. Section 10.11.6 addresses moment magnifiers for slenderness consideration of compression members under biaxial loading. Section R10.3.6 states that "corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load." Two methods are recommended for combined biaxial bending and axial load design: the Reciprocal Load Method and the Load Contour Method. Both methods, and an extension of the Load Contour Method (PCA Load Contour Method), are presented below.

BIAXIAL INTERACTION STRENGTH

A uniaxial interaction diagram defines the load-moment strength along a single plane of a section under an axial load P and a uniaxial moment M. The biaxial bending resistance of an axially loaded column can be represented schematically as a surface formed by a series of uniaxial interaction curves drawn radially from the P axis (see

Fig. 7-6). Data for these intermediate curves are obtained by varying the angle of the neutral axis (for assumed strain configurations) with respect to the major axes (see Fig. 7-7).

The difficulty associated with the determination of the strength of reinforced columns subject to combined axial load and biaxial bending is primarily an arithmetic one. The bending resistance of an axially loaded column about a particular skewed axis is determined through iterations involving simple but lengthy calculations. These extensive calculations are compounded when optimization of the reinforcement or cross-section is sought.

For uniaxial bending, it is customary to utilize design aids in the form of interaction curves or tables. However, for biaxial bending, because of the voluminous nature of the data and the difficulty in multiple interpolations, the development of interaction curves or tables for the various ratios of bending moments about each axis is impractical. Instead, several approaches (based on acceptable approximations) have been developed that relate the response of a column in biaxial bending to its uniaxial resistance about each major axis.

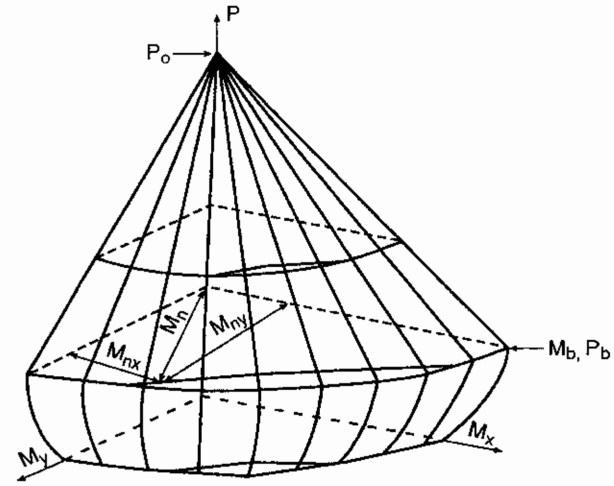


Figure 7-6 Biaxial Interaction Surface

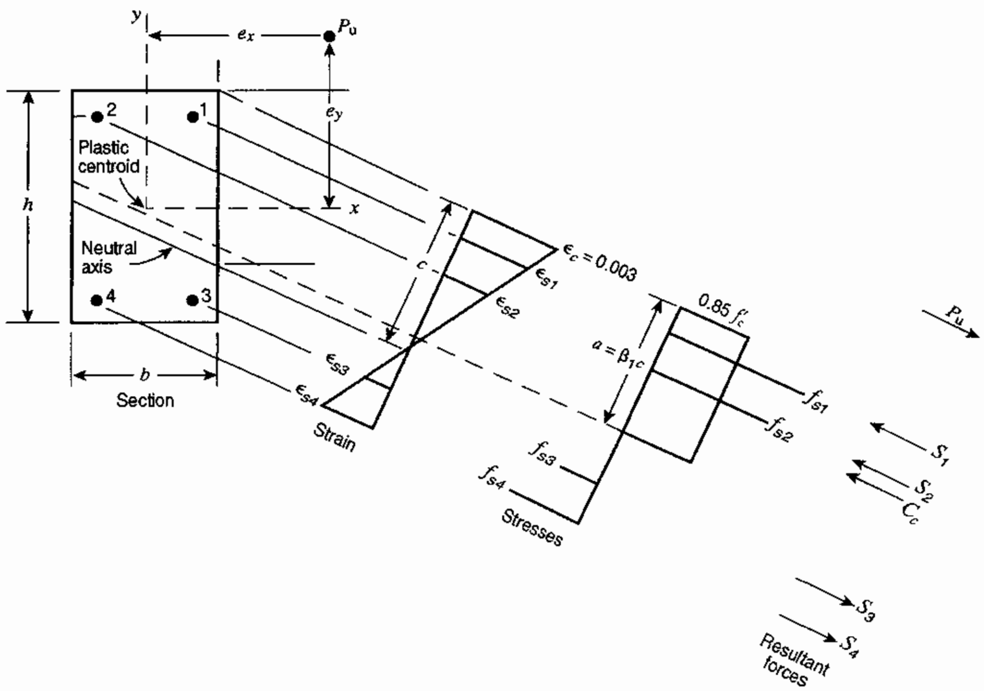


Figure 7-7 Neutral Axis at an Angle to Major Axes

FAILURE SURFACES

The nominal strength of a section under biaxial bending and compression is a function of three variables P_n, M_{nx} and M_{ny} which may be expressed in terms of an axial load acting at eccentricities $e_x = M_{ny}/P_n$ and $e_y = M_{nx}/P_n$ as shown in Fig. 7-8. A failure surface may be described as a surface produced by plotting the failure load P_n as a function of its eccentricities e_x and e_y, or of its associated bending moments M_{ny} and M_{nx}. Three types of failure surfaces have been defined.[7.4, 7.5, 7.6] The basic surface S_1 is defined by a function which is dependent upon the variables P_n, e_x and e_y, as shown in Fig. 7-9(a). A reciprocal surface can be derived from S_1 in which the reciprocal of the nominal axial load P_n is employed to produce the surface S_2 ($1/P_n$, e_x, e_y) as illustrated in Fig. 7-9(b). The third type of failure surface, shown in Fig. 7-9(c), is obtained by relating the nominal axial load P_n to the moments M_{nx} and M_{ny} to produce surface S_3 (P_n, M_{nx}, M_{ny}). Failure surface S_3 is the three-dimensional extension of the uniaxial interaction diagram previously described.

A number of investigators have made approximations for both the S_2 and S_3 failure surfaces for use in design and analysis.[7.6 - 7.10] An explanation of these methods used in current practice, along with design examples, is given below.

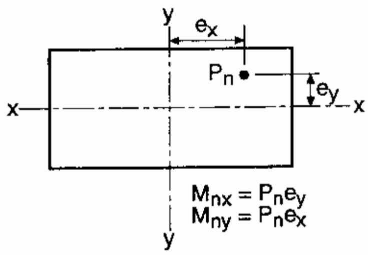
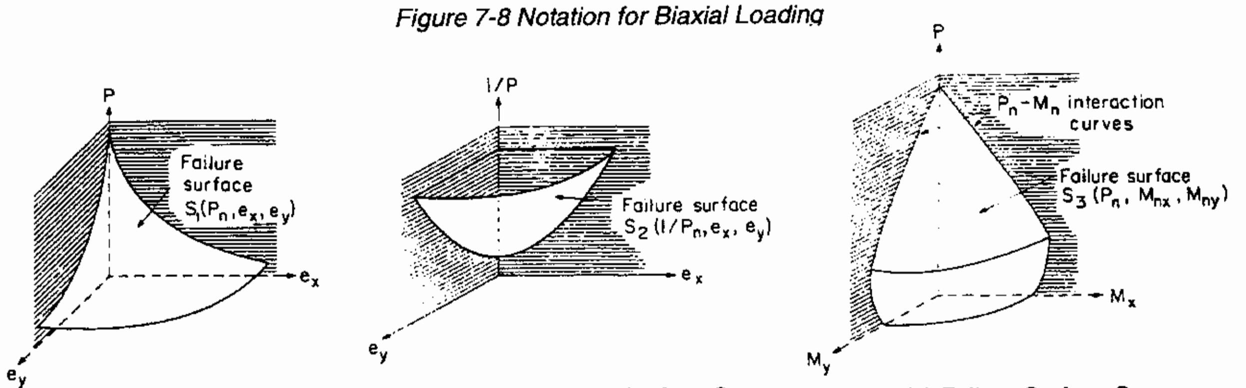


Figure 7-8 Notation for Biaxial Loading



(a) Failure Surface S_1 (b) Reciprocal Failure Surface S_2 (c) Failure Surface S_3

Figure 7-9 Failure Surfaces

A. Bresler Reciprocal Load Method

This method approximates the ordinate $1/P_n$ on the surface S_2 ($1/P_n$, e_x, e_y) by a corresponding ordinate $1/P'_n$ on the plane $S'_2(1/P'_n, e_x, e_y)$, which is defined by the characteristic points A, B and C, as indicated in Fig. 7-10. For any particular cross-section, the value P_o (corresponding to point C) is the load strength under pure axial compression; P_{ox} (corresponding to point B) and P_{oy} (corresponding to point A) are the load strengths under uniaxial eccentricities e_y and e_x, respectively. Each point on the true surface is approximated by a different plane; therefore, the entire surface is approximated using an infinite number of planes.

The general expression for axial load strength for any values of e_x and e_y is as follows:[7.6]

$$\frac{1}{P_n} \approx \frac{1}{P'_n} = \frac{1}{P_{ox}} + \frac{1}{P_{oy}} - \frac{1}{P_o}$$

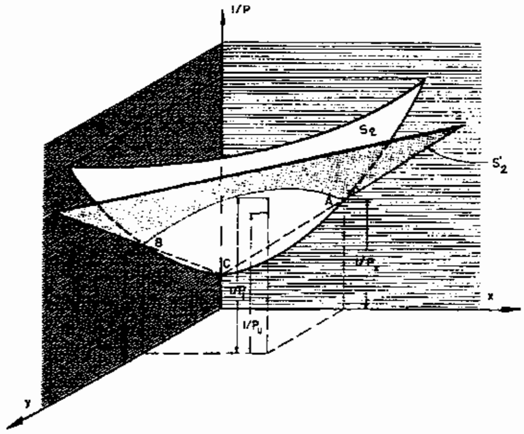


Figure 7-10 Reciprocal Load Method

Rearranging variables yields:

$$P_n \approx \frac{1}{\dfrac{1}{P_{ox}} + \dfrac{1}{P_{oy}} - \dfrac{1}{P_o}} \tag{7}$$

where

P_{ox} = Maximum uniaxial load strength of the column with a moment of $M_{nx} = P_n e_y$

P_{oy} = Maximum uniaxial load strength of the column with a moment of $M_{ny} = P_n e_x$

P_o = Maximum axial load strength with no applied moments

This equation is simple in form and the variables are easily determined. Axial load strengths P_o , P_{ox} , and P_{oy} are determined using any of the methods presented above for uniaxial bending with axial load. Experimental results have shown the above equation to be reasonably accurate when flexure does not govern design. The equation should only be used when:

$$P_n \geq 0.1 f'_c A_g \tag{8}$$

B. Bresler Load Contour Method

In this method, the surface S_3 (P_n , M_{nx}, M_{ny}) is approximated by a family of curves corresponding to constant values of P_n. These curves, as illustrated in Fig. 7-11, may be regarded as "load contours."

The general expression for these curves can be approximated[7.6] by a nondimensional interaction equation of the form

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\beta} = 1.0 \tag{9}$$

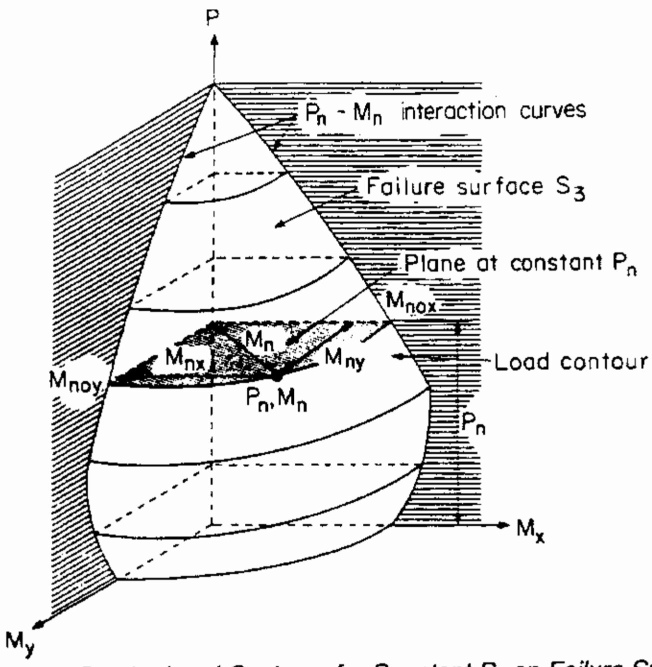


Figure 7-11 Bresler Load Contours for Constant P_n on Failure Surface S_3

where M_{nx} and M_{ny} are the nominal biaxial moment strengths in the direction of the x and y axes, respectively. Note that these moments are the vectorial equivalent of the nominal uniaxial moment M_n. The moment M_{nox} is the nominal uniaxial moment strength about the x-axis, and M_{noy} is the nominal uniaxial moment strength about the y-axis. The values of the exponents α and β are a function of the amount, distribution and location of reinforcement, the dimensions of the column, and the strength and elastic properties of the steel and concrete. Bresler[7.6] indicates that it is reasonably accurate to assume that $\alpha = \beta$; therefore, Eq. (9) becomes

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\alpha} = 1.0 \qquad (10)$$

which is shown graphically in Fig. 7-12.

When using Eq. (10) or Fig. 7-12, it is still necessary to determine the α value for the cross-section being designed. Bresler indicated that, typically, α varied from 1.15 to 1.55, with a value of 1.5 being reasonably accurate for most square and rectangular sections having uniformly distributed reinforcement.

With α set at unity, the interaction equation becomes linear:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} = 1.0 \qquad (11)$$

Equation (11), as shown in Fig. 7-12, would always yield conservative results since it underestimates the column capacity, especially for high axial loads or low percentages of reinforcement. It should only be used when

$$P_n < 0.1 f_c' A_g \qquad (12)$$

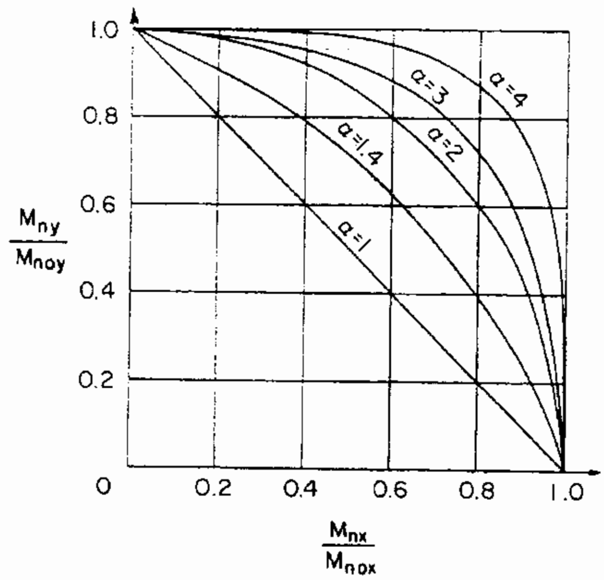


Figure 7-12 Interaction Curves for Bresler Load Contour Method (Eq.(9))

C. PCA Load Contour Method

The PCA approach described below was developed as an extension of the Bresler Load Contour Method. The Bresler interaction equation [Eq. (10)] was chosen as the most viable method in terms of accuracy, practicality, and simplification potential.

A typical Bresler load contour for a certain P_n is shown in Fig. 7-13(a). In the PCA method,[7.11] point B is defined such that the nominal biaxial moment strengths M_{nx} and M_{ny} at this point are in the same ratio as the uniaxial moment strengths M_{nox} and M_{noy}. Therefore, at point B

$$\frac{M_{nx}}{M_{ny}} = \frac{M_{nox}}{M_{noy}} \qquad (13)$$

When the load contour of Fig. 7-13(a) is nondimensionalized, it takes the form shown in Fig. 7-13(b), and the point B will have x and y coordinates of β. When the bending resistance is plotted in terms of the dimensionless parameters P_n/P_o, M_{nx}/M_{nox}, M_{ny}/M_{noy} (the latter two designated as the relative moments), the generated failure surface S_4 (P_n/P_o, M_{nx}/M_{nox}, M_{ny}/M_{noy}) assumes the typical shape shown in Fig. 7-13(c). The advantage of expressing the behavior in relative terms is that the contours of the surface (Fig. 7-13(b))—i.e., the intersection formed by planes of constant P_n/P_o and the surface—can be considered for design purposes to be symmetrical about the vertical plane bisecting the two coordinate planes. Even for sections that are rectangular or have unequal reinforcement on the two adjacent faces, this approximation yields values sufficiently accurate for design.

The relationship between α from Eq. (10) and β is obtained by substituting the coordinates of point B from Fig. 7-13(a) into Eq. (10), and solving for α in terms of β. This yields:

$$\alpha = \frac{\log 0.5}{\log \beta}$$

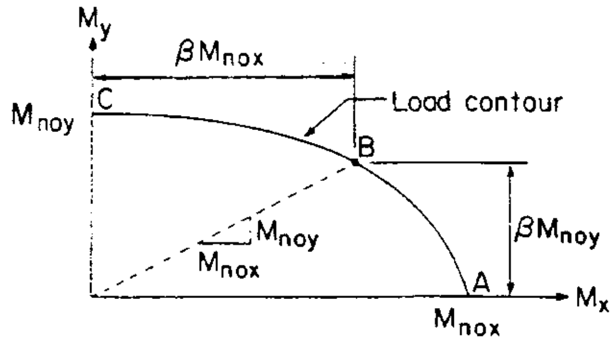


Figure 7-13(a) Load Contour of Failure Surface s_3 along Plane of Constant P_n

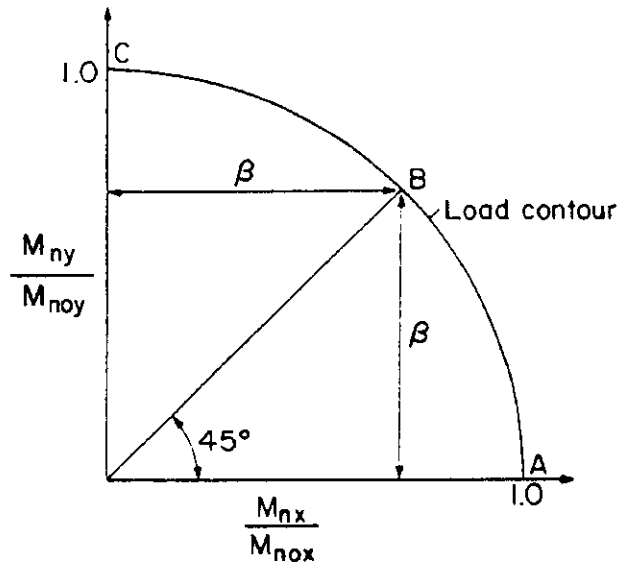


Figure 7-13(b) Nondimensional Load Contour at Constant P_n

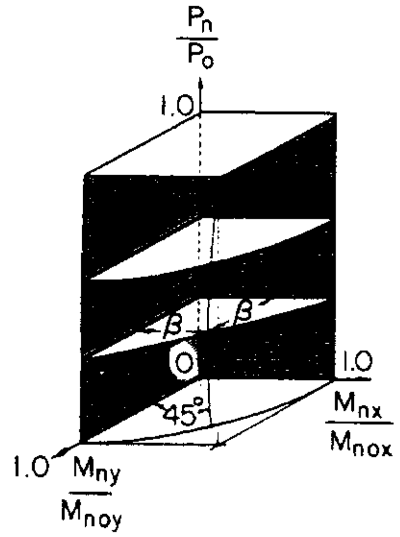


Figure 7-13(c) Failure Surface $S_4\left(\frac{P_n}{P_o}, \frac{M_{nx}}{M_{nox}}, \frac{M_{ny}}{M_{noy}}\right)$

Thus, Eq. (10) may be written as:

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\left(\frac{\log 0.5}{\log \beta}\right)} = 1.0 \tag{14}$$

For design convenience, a plot of the curves generated by Eq. (14) for nine values of β are given in Fig. 7-14. Note that when $\beta = 0.5$, its lower limit, Eq. (14) is a straight line joining the points at which the relative moments equal 1.0 along the coordinate planes. When $\beta = 1.0$, its upper limit, Eq. (14) is two lines, each of which is parallel to one of the coordinate planes.

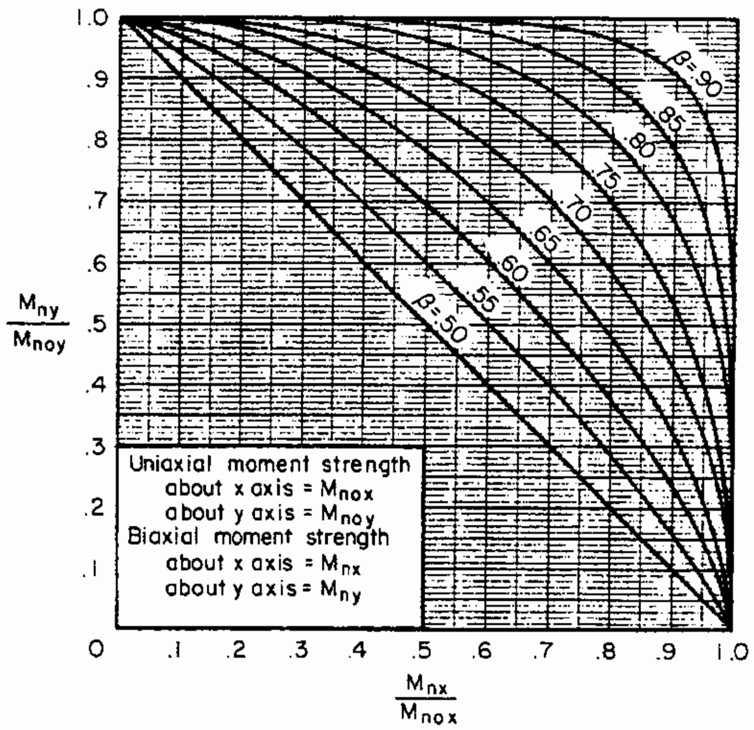
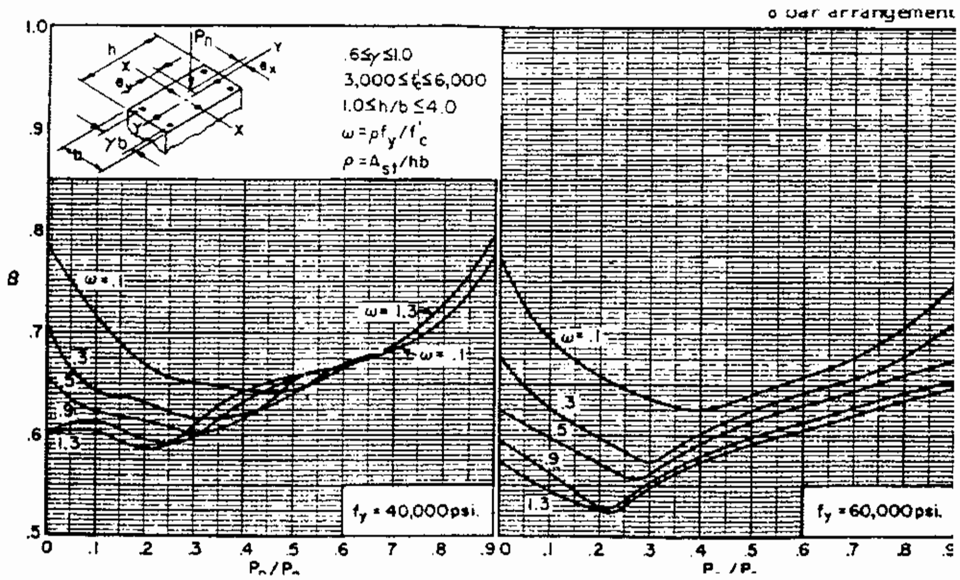


Figure 7-14 Biaxial Moment Strength Relationship

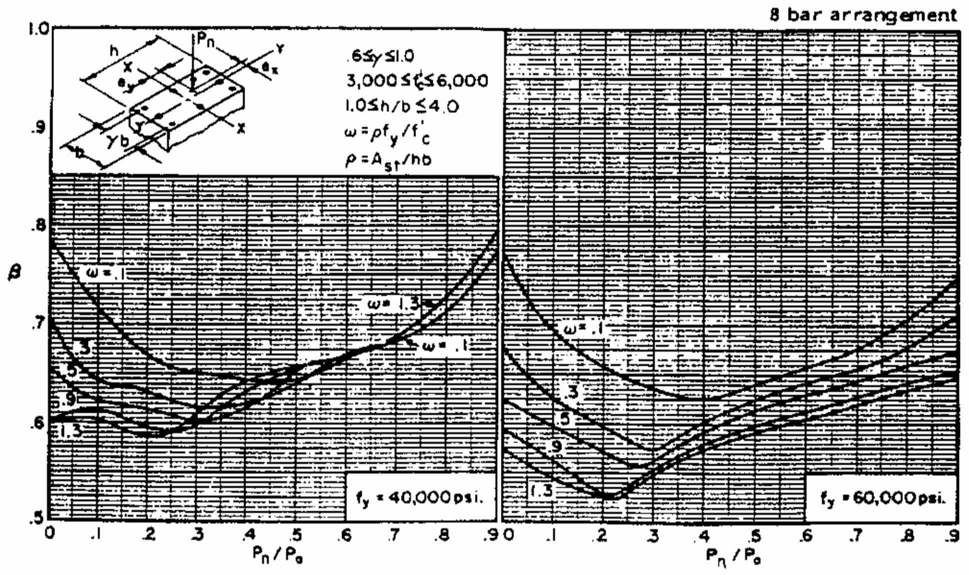
Values of β were computed on the basis of 10.2, utilizing a rectangular stress block and the basic principles of equilibrium. It was found that the parameters γ, b/h, and f'_c had minor effect on the β values. The maximum difference in β was about 5% for values of P_n/P_o ranging from 0.1 to 0.9. The majority of the β values, especially in the most frequently used range of P_n/P_o, did not differ by more than 3%. In view of these small differences, only envelopes of the lowest β values were developed for two values of f_y and different bar arrangements, as shown in Figs. 7-15 and 7-16.

As can be seen from Figs. 7-15 and 7-16, β is dependent primarily on the ratio P_n/P_o and to a lesser, though still significant extent, on the bar arrangement, the reinforcement index ω and the strength of the reinforcement.

Figure 7-14, in combination with Figs. 7-15 and 7-16, furnish a convenient and direct means of determining the biaxial moment strength of a given cross-section subject to an axial load, since the values P_o, M_{nox}, and M_{noy} can be readily obtained by methods described above.



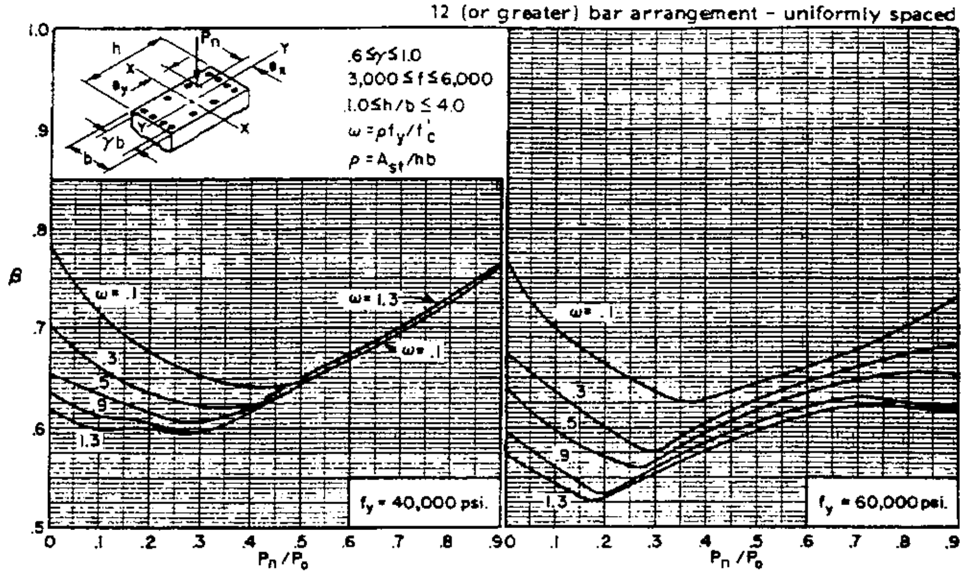
(a) 4 Bar Arrangement



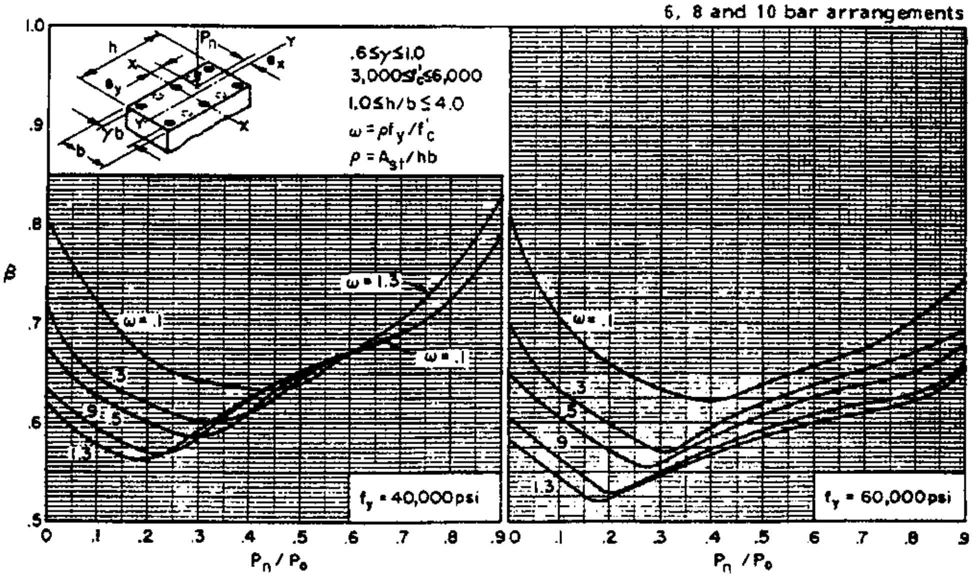
(b) 8 Bar Arrangement

Figure 7-15 Biaxial Design Constants

While investigation of a given section has been simplified, the determination of a section which will satisfy the strength requirements imposed by a load eccentric about both axes can only be achieved by successive analyses of assumed sections. Rapid and easy convergence to a satisfactory section can be achieved by approximating the curves in Fig. 7-14 by two straight lines intersecting at the 45 degree line, as shown in Fig. 7-17.



(a) 6, 8, and 10 Bar Arrangement



(b) 12 (or greater) Bar Arrangement

Figure 7-16 Biaxial Design Constants

By simple geometry, it can be shown that the equation of the upper lines is:

$$\frac{M_{nx}}{M_{nox}}\left(\frac{1-\beta}{\beta}\right) + \frac{M_{ny}}{M_{noy}} = 1 \text{ for } \frac{M_{ny}}{M_{nx}} > \frac{M_{noy}}{M_{nox}} \qquad (15)$$

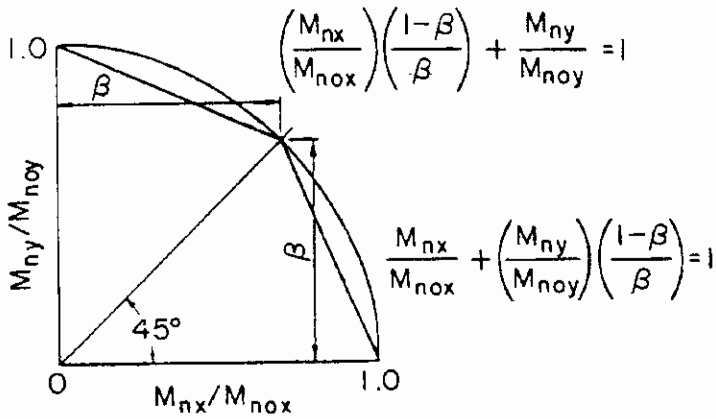


Figure 7-17 Bilinear Approximation of Nondimensionalized Load Contour (Fig. 7-13(b))

which can be restated for design convenience as follows:

$$M_{nx}\left(\frac{M_{noy}}{M_{nox}}\right)\left(\frac{1-\beta}{\beta}\right) + M_{ny} = M_{noy} \qquad (16)$$

For rectangular sections with reinforcement equally distributed on all faces, Eq. (16) can be approximated by:

$$M_{nx}\,\frac{b}{h_a}\left(\frac{1-\beta}{\beta}\right) + M_{ny} \approx M_{noy} \qquad (17)$$

The equation of the lower line of Fig. 7-17 is:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}}\left(\frac{1-\beta}{\beta}\right) = 1 \text{ for } \frac{M_{ny}}{M_{nx}} < \frac{M_{noy}}{M_{nox}} \qquad (18)$$

or $$M_{nx} + M_{ny}\left(\frac{M_{nox}}{M_{noy}}\right)\left(\frac{1-\beta}{\beta}\right) = M_{nox} \qquad (19)$$

For rectangular sections with reinforcement equally distributed on all faces,

$$M_{nx} + M_{ny}\,\frac{h_a}{b}\left(\frac{1-\beta}{\beta}\right) \approx M_{nox} \qquad (20)$$

In design Eqs. (17) and (20), the ratio b/h_a or h_a/b must be chosen and the value of β must be assumed. For lightly loaded columns, β will generally vary from 0.55 to about 0.70. Hence, a value of 0.65 for β is generally a good initial choice in a biaxial bending analysis.

MANUAL DESIGN PROCEDURE

To aid the engineer in designing columns for biaxial bending, a procedure for manual design is outlined below:

1. Choose the value of β at 0.65 or use Figs. 7-15 and 7-16 to make an estimate.

2. If M_{ny}/M_{nx} is greater than b/h, use Eq. (17) to calculate an approximate equivalent uniaxial moment strength M_{noy}. If M_{ny}/M_{nx} is less than b/h_a, use Eq. (20) to calculate an approximate equivalent uniaxial moment strength M_{nox}.

3. Design the section using any of the methods presented above for uniaxial bending with axial load to provide an axial load strength P_n and an equivalent uniaxial moment strength M_{noy} or M_{nox}.

4. Verify the section chosen by any one of the following three methods:

   a. Bresler Reciprocal Load Method:

$$P_n \leq \frac{1}{\frac{1}{P_{ox}} + \frac{1}{P_{oy}} - \frac{1}{P_o}} \tag{7}$$

   b. Bresler Load Contour Method:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} \leq 1.0 \tag{11}$$

   c. PCA Load Contour Method: Use Eq. (14) or,

$$\frac{M_{nx}}{M_{nox}}\left(\frac{1-\beta}{\beta}\right) + \frac{M_{ny}}{M_{noy}} \leq 1.0 \quad \text{for} \quad \frac{M_{ny}}{M_{nx}} > \frac{M_{noy}}{M_{nox}} \tag{15}$$

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}}\left(\frac{1-\beta}{\beta}\right) \leq 1.0 \quad \text{for} \quad \frac{M_{ny}}{M_{nx}} < \frac{M_{noy}}{M_{nox}} \tag{18}$$

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Example 7.1—Design of Rectangular Beam with Tension Reinforcement Only

Select a rectangular beam size and required reinforcement A_s to carry service load moments $M_D = 56$ ft-kips and $M_L = 35$ ft-kips. Select reinforcement to control flexural cracking.

f'_c = 4000 psi
f_y = 60,000 psi

Calculations and Discussion	Code Reference

1. To illustrate a complete design procedure for rectangular sections with tension reinforcement only, a minimum beam depth will be computed using the maximum reinforcement permitted for tension-controlled flexural members, ρ_t. The design procedure will follow the method outlined on the preceding pages. *10.3.4*

Step 1. Determine maximum tension-controlled reinforcement ratio for material strengths $f'_c = 4000$ psi and $f_y = 60,000$ psi.

ρ_t = 0.01806 from Table 6-1

Step 2. Compute bd^2 required.

Required moment strength:

$$M_u = (1.2 \times 56) + (1.6 \times 35) = 123.2 \text{ ft-kips}$$ *Eq. (9-2)*

$$R_n = \rho f_y \left(1 - \frac{0.5\rho f_y}{0.85 f'_c}\right)$$

$$= (0.01806 \times 60{,}000)\left(1 - \frac{0.5 \times 0.01806 \times 60{,}000}{0.85 \times 4000}\right) = 911 \text{ psi}$$

$$bd^2 \text{ (required)} = \frac{M_u}{\phi R_n} = \frac{123.2 \times 12 \times 1000}{0.90 \times 911} = 1803 \text{ in.}^3$$

Step 3. Size member so that bd^2 provided $\geq bd^2$ required.

Set b = 10 in. (column width)

$$d = \sqrt{\frac{1803}{10}} = 13.4 \text{ in.}$$

Minimum beam depth $\approx 13.4 + 2.5 = 15.9$ in.

For moment strength, a 10 × 16 in. beam size is adequate. However, deflection is an essential consideration in designing beams by the Strength Design Method. Control of deflection is discussed in Part 10.

Step 4. Using the 16 in. beam depth, compute a revised value of ρ. For illustration, ρ will be computed by all four methods outlined earlier.

$d = 16 - 2.5 = 13.5$ in.

1. By Eq. (4) (exact method):

$$R_n = \frac{M_u}{\phi\left(bd^2 \text{ provided}\right)} = \frac{123.2 \times 12 \times 1000}{0.90\left(10 \times 13.5^2\right)} = 901 \text{ psi}$$

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2R_n}{0.85f'_c}}\right)$$

$$= \frac{0.85 \times 4}{60}\left(1 - \sqrt{1 - \frac{2 \times 901}{0.85 \times 4000}}\right) = 0.0178$$

2. By strength curves such as shown in Fig. 7-2:

for $R_n = 901$ psi, $\rho \approx 0.0178$

3. By moment strength tables such as Table 7-1:

$$\frac{M_u}{\phi f'_c bd^2} = \frac{123.2 \times 12 \times 1000}{0.90 \times 4000 \times 10 \times 13.5^2} = 0.2253$$

$\omega \approx 0.2676$

$$\rho = \frac{\omega f'_c}{f_y} = 0.2676 \times \frac{4}{60} = 0.0178$$

4. By approximate proportion:

$$\rho \approx (\text{original } \rho)\frac{(\text{revised } R_n)}{(\text{original } R_n)}$$

$$\rho = 0.01806 \times \frac{901}{911} = 0.0179$$

Step 5. Compute A_s required.

$$A_s = (\text{revised } \rho)\ (\text{bd provided})$$

$$= 0.0178 \times 10 \times 13.5 = 2.40 \text{ in.}^2$$

2. A review of the correctness of the computations can be made by considering statics.

$$T = A_s f_y = 2.40 \times 60 = 144.0 \text{ kips}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{144.0}{0.85 \times 4 \times 10} = 4.24 \text{ in.}$$

Design moment strength:

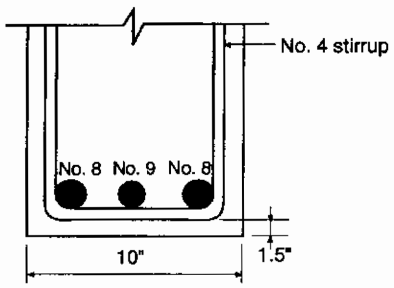
$$\phi M_n = \phi\left[A_s f_y\left(d - \frac{a}{2}\right)\right] = 0.9\left[144.0\left(13.5 - \frac{4.24}{2}\right)\right]$$

$$= 1{,}475 \text{ in.-kips} = 122.9 \text{ ft-kips} \approx \text{required } M_u = 123.2 \text{ ft-kips} \quad \text{O.K.}$$

3. Select reinforcement to satisfy distribution of flexural reinforcement requirements of 10.6. *10.6*

A_s required $= 2.40$ in.^{2}

For illustrative purposes, select 1-No. 9 and 2-No. 8 bars ($A_s = 2.40$ in.^{2}). For practical design and detailing, one bar size for total A_s is preferable.



$c_c = 1.5 + 0.5 = 2.0$ in.

Maximum spacing allowed,

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \le 12\left(\frac{40{,}000}{f_s}\right)$$ *Eq. (10-4)*

Use $f_s = \frac{2}{3} f_y = 40$ ksi

$$s = 15\left(\frac{40{,}000}{40{,}000}\right) - 2.5 \times 2 = 10 \text{ in. (governs)}$$

or, $s = 12\left(\frac{40{,}000}{40{,}000}\right) = 12$ in.

or, refer to Table 9-1: for $f_s = 40$ ksi and $c_c = 2$, $s = 10$ in.

$$\text{Spacing provided } = \frac{1}{2}\left\{10 - 2\left(1.5 + 0.5 + \frac{1.0}{2}\right)\right\}$$

$$= 2.50 \text{ in.} < 10 \text{ in.} \quad \text{O.K.}$$

Example 7.2—Design of One-Way Solid Slab

Determine required thickness and reinforcement for a one-way slab continuous over two or more equal spans. Clear span ℓ_n = 18 ft.

f'_c = 4000 psi
f_y = 60,000 psi
Service loads: w_d = 75 psf (assume 6-in. slab), w_ℓ = 50 psf

Calculations and Discussion	Code Reference

1. Compute required moment strengths using approximate moment analysis permitted by 8.3.3. Design will be based on end span.

   Factored load $q_u = (1.2 \times 75) + (1.6 \times 50) = 170$ psf — *Eq. (9-2)*

   Positive moment at discontinuous end integral with support:

   $+M_u = q_u\ell_n^2/14 = 0.170 \times 18^2/14 = 3.93$ ft-kips/ft — *8.3.3*

   Negative moment at exterior face of first interior support:

   $-M_u = q_u\ell_n^2/14 = 0.170 \times 18^2/10 = 5.51$ ft-kips/ft — *8.3.3*

2. Determine required slab thickness. — *10.3.3*

   Choose a reinforcement percentage ρ equal to about $0.5\rho_t$, or one-half the maximum permitted for tension-controlled sections, to have reasonable deflection control.

   From Table 6-1, for f'_c = 4000 psi and f_y = 60,000 psi: $\rho_t = 0.01806$

   Set $\rho = 0.5\,(0.01806) = 0.00903$

   Design procedure will follow method outlined earlier:

$$R_n = \rho f_y\left(1 - \frac{0.5\rho f_y}{0.85f'_c}\right)$$

$$= (0.00903 \times 60{,}000)\left(1 - \frac{0.5 \times 0.00903 \times 60{,}000}{0.85 \times 4000}\right) = 499 \text{ psi}$$

$$\text{Required } d = \sqrt{\frac{M_u}{\phi R_n b}} = \sqrt{\frac{5.51 \times 12{,}000}{0.90 \times 499 \times 12}} = 3.50 \text{ in.}$$

   Assuming No. 5 bars, required $h_a = 3.50 + 0.31/2 + 0.75 = 4.41$ in.

   The above calculations indicate a slab thickness of 4.5 in. is adequate. However, Table 9-5(a) indicates a minimum thickness of $\ell/24 \geq 9$ in., unless deflections are computed. Also note that Table 9-5(a) is applicable only to "members in one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections." Otherwise deflections must be computed.

For purposes of illustration, the required reinforcement will be computed for $h_a = 4.5$ in., $d = 3.59$ in.

3. Compute required negative moment reinforcement.

$$R_n = \frac{M_u}{\phi bd^2} = \frac{5.51 \times 12 \times 1000}{0.9 \times 12 \times 3.59^2} = 475$$

$$\rho \approx 0.00903\left(\frac{475}{499}\right) = 0.00860$$

$-A_s$ (required) $= \rho bd = 0.00860 \times 12 \times 3.59 = 0.37$ in.^{2}/ft

Use No. 5 @ 10 in. ($A_s = 0.37$ in.^{2}/ft)

4. For positive moment, use Table 7-1:

$$\frac{M_u}{\phi f'_c bd^2} = \frac{3.93 \times 12{,}000}{0.9 \times 4000 \times 12 \times 3.59^2} = 0.0847$$

From Table 7-1, $\omega \approx 0.090$

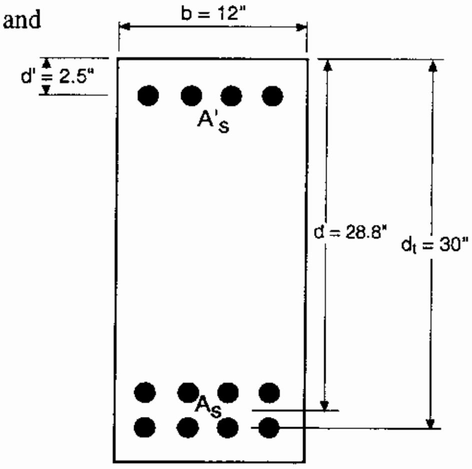
$$\rho = \frac{\omega f'_c}{f_y} = 0.090 \times \frac{4}{60} = 0.006$$

$+A_s$ (required) $= \rho bd = 0.006 \times 12 \times 3.59 = 0.258$ in.^{2}/ft

Use No. 4 @ 9 in. ($A_s = 0.27$ in.^{2}/ft) or No. 5 @ 12 in. ($A_s = 0.31$ in.^{2}/ft)

Example 7.3—Design of Rectangular Beam with Compression Reinforcement

A beam cross-section is limited to the size shown. Determine the required area of reinforcement for service load moments M_D = 430 ft-kips and M_L = 175 ft-kips. Check crack control requirements of 10.6.



f'_c = 4000 psi
f_y = 60,000 psi

Calculations and Discussion	Code Reference

1. Determine required reinforcement.

Step 1. Determine if compression reinforcement is needed.

$$M_u = 1.2M_D + 1.6M_L = 796 \text{ ft-kips}$$ Eq. (9-2)

$$M_n = M_u/\phi = 796/0.9 = 884 \text{ ft-kips}$$

$$R_n = \frac{M_n}{bd^2} = \frac{884 \times 12 \times 1000}{12 \times 30^2} = 982$$

This exceeds the maximum R_n of 911 for tension-controlled sections of 4000 psi concrete, without compression reinforcement. (see Table 6-1.) Also, it appears likely that two layers of tension reinforcement will be necessary. Estimate d = d_t - 1.2 in. = 28.8 in.

Step 2. Find the nominal strength moment resisted by the concrete section, without compression reinforcement.

$$\rho_t = 0.01806 \text{ from Table 6-1}$$

$$\rho = \rho_t\left(\frac{d_t}{d}\right) = 0.01806\left(\frac{30}{28.8}\right) = 0.01881$$ (6)

$$\omega = \rho\frac{f_y}{f'_c} = 0.01881 \times \frac{60}{4} = 0.282$$

$$\frac{M_{nt}}{f'_c bd^2} = 0.2351 \text{ from Table 7-1}$$

$M_{nt} = 0.2351 \times 4 \times 12 \times 28.8^2 = 9{,}360$ in.-kips $= 780$ ft-kips resisted by the concrete

Required moment strength to be resisted by the compression reinforcement:

$M'_n = 884 - 780 = 104$ ft-kips

Step 3. Determine the compression steel stress f'_s.

Check yielding of compression reinforcement. Since the section was designed at the tension-controlled net tensile strain limit $\varepsilon_t = 0.005$, $c_{al}/d_t = 0.375$

$c_{al} = 0.375d_t = 0.375 \times 30 = 11.25$ in.

$d'/c_{al} = 2.5/11.25 = 0.22 < 0.31$

Compression reinforcement yields at the nominal strength ($f'_s = f_y$)

Step 4. Determine the total required reinforcement:

$$A'_s = \frac{M'_n}{f_y(d-d')}$$

$$= \frac{104 \times 12 \times 1000}{60{,}000\left(28.8-2.5\right)} = 0.79 \text{ in.}^2$$

$A_s = 0.79 + \rho bd$

$= 0.79 + (0.01881 \times 12 \times 28.8) = 7.29$ in.^{2}

Step 5. Check moment capacity.

When the compression reinforcement yields:

$$a = \frac{(A_s - A'_s)f_y}{0.85f'_c b} = \frac{6.50 \times 60}{0.85 \times 4 \times 12} = 9.56 \text{ in.}$$

$$\phi M_n = \phi\left[(A_s - A'_s)\, f_y \left(d - \frac{a}{2}\right) + A'_s f_y \,(d - d')\right]$$

$$= 0.9\left[6.50 \times 60\left(28.8 - \frac{9.56}{2}\right) + (0.79 \times 60)\,(28.8 - 2.5)\right]/12$$

$= 796\text{ ft-kips} = M_u = 796\text{ ft-kips}$ O.K.

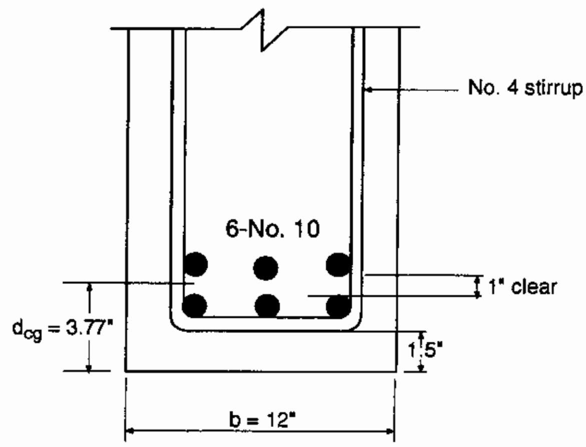
2. Select reinforcement to satisfy control of flexural cracking criteria of 10.6.

Compression reinforcement:

Select 2-No. 6 bars ($A'_s = 0.88\text{ in.}^2 > 0.79\text{ in.}^2$)

Tension reinforcement:

Select 6-No. 10 bars in two layers ($A_s = 7.62\text{ in.}^2 > 7.29\text{ in.}^2$)



Maximum spacing allowed,

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \le 12\left(\frac{40{,}000}{f_s}\right)$$ Eq. (10-4)

$c_c = 1.5 + 0.5 = 2.0$ in.

Use $f_s = \frac{2}{3} f_y = 40$ ksi

$$s = 15\left(\frac{40{,}000}{40{,}000}\right) - (2.5 \times 2) = 10\text{ in. (governs)}$$

$$\text{or, } = 12\left(\frac{40{,}000}{40{,}000}\right) = 12\text{ in.}$$

$$\text{Spacing provided } = \frac{1}{2}\left\{12 - 2\left(1.5 + 0.5 + \frac{1.27}{2}\right)\right\}$$

$$= 4.68\text{ in.} < 10\text{ in.} \quad \text{O.K.}$$

4. Stirrups or ties are required throughout distance where compression reinforcement is required for strength. *7.11.1*

   Max. spacing = 16 × long. bar dia. = 16 × 0.75 = 12 in. (governs) *7.10.5.2*

   = 48 × tie bar dia. = 48 × 0.5 = 24 in.

   = least dimension of member = 12 in.

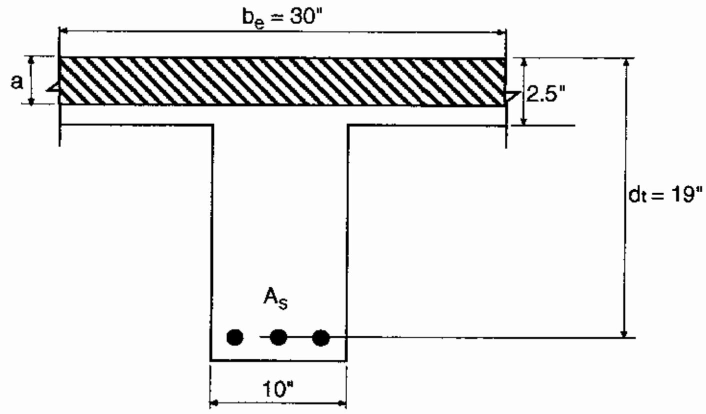
   Use s_{max} = 12 in. for No. 4 stirrups

   Using the simplified assumption of $d = d_t$, the extra steel is only 1.2 percent (calculations are not shown).

Example 7.4—Design of Flanged Section with Tension Reinforcement Only

Select reinforcement for the T-section shown, to carry service dead and live load moments of $M_D = 72$ ft-kips and $M_L = 88$ ft-kips.

f'_c = 4000 psi
f_y = 60,000 psi



Calculations and Discussion	Code Reference
1. Determine required flexural strength.	
$M_u = (1.2 \times 72) + (1.6 \times 88) = 227$ ft-kips	*Eq. (9-2)*

2. Using Table 7-1, determine depth of equivalent stress block a, as for a rectangular section. Assume $\phi = 0.9$.

$$\frac{M_u}{\phi f'_c b d^2} = \frac{227 \times 12}{0.9 \times 4 \times 30 \times 19^2} = 0.0699$$

From Table 7-1, $\omega \approx 0.073$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{\rho d f_y}{0.85 f'_c} = 1.18\omega d = 1.18 \times 0.073 \times 19 = 1.64 \text{ in.} < 2.5 \text{ in.}$$

With $a < h_f$, determine A_s as for a rectangular section (see Ex. 7.5 for the case when $a > h_f$).

Check ϕ:

$c_{a1} = a/\beta_1 = 1.64/0.85 = 1.93$ in.

$c_{a1}/d_t = 1.93/19 = 0.102 < 0.375$

Section is tension-controlled, and $\phi = 0.9$.

3. Compute A_s required.

$$A_sf_y = 0.85f'_cba$$

$$A_s = \frac{0.85 \times 4 \times 30 \times 1.64}{60} = 2.78 \text{ in.}^2$$

Alternatively,

$$A_s = \rho bd = \omega\frac{f'_c}{f_y}bd$$

$$= 0.073 \times \frac{4}{60} \times 30 \times 19 = 2.77 \text{ in.}^2$$

Try 3-No. 9 bars ($A_s = 3.0$ in.^{2}).

4. Check minimum required reinforcement. *10.5*

For $f'_c < 4444$ psi, *Eq. (10-3)*

$$\rho_{min} = \frac{200}{f_y} = \frac{200}{60{,}000} = 0.0033$$

$$\frac{A_s}{b_wd} = \frac{3.0}{10 \times 19} = 0.0158 > 0.0033 \quad \text{O.K.}$$

5. Check distribution of reinforcement. *10.6*

Maximum spacing allowed,

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40{,}000}{f_s}\right)$$ *Eq. (10-4)*

$c_c = 1.5 + 0.5 = 2.0$ in.

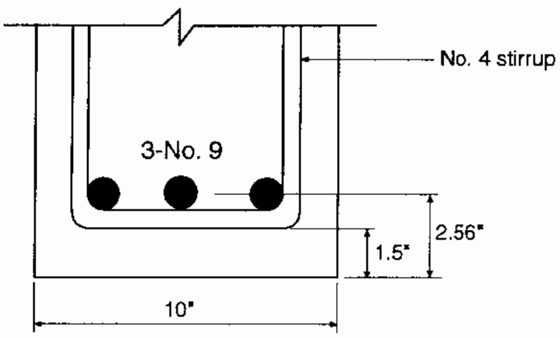
Use $f_s = \frac{2}{3}f_y = 40$ ksi

$$s = 15\left(\frac{40{,}000}{40{,}000}\right) - (2.5 \times 2) = 10 \text{ in. (governs)}$$

$$s = 12\left(\frac{40{,}000}{40{,}000}\right) = 12 \text{ in.}$$

$$\text{Spacing provided} = \frac{1}{2}\left\{10 - 2\left(1.5 + 0.5 + \frac{1.128}{2}\right)\right\}$$

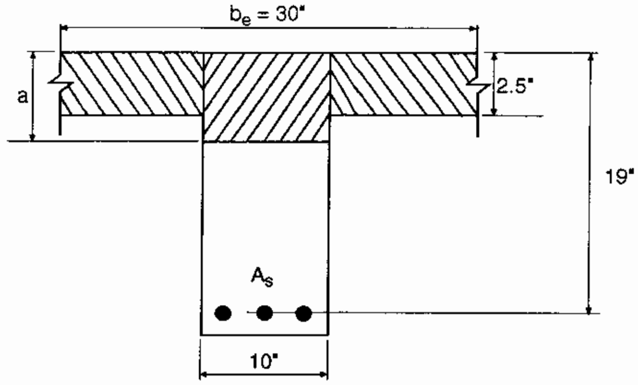
$$= 2.44 \text{ in.} < 10 \text{ in.} \quad \text{O.K.}$$



Example 7.5—Design of Flanged Section with Tension Reinforcement Only

Select reinforcement for the T-section shown, to carry a factored moment of M_u = 400 ft-kips.

f'_c = 4000 psi
f_y = 60,000 psi



Calculations and Discussion	Code Reference

1. Determine required reinforcement.

Step 1. Using Table 7-1, determine depth of equivalent stress block a, as for a rectangular section.

Assume tension-controlled section, $\phi = 0.9$.

$$M_n = M_u/\phi = 400/0.9 = 444 \text{ ft-kips}$$

Assume a < 2.5 in.

$$\frac{M_n}{f'_c bd^2} = \frac{444 \times 12}{4 \times 30 \times 19^2} = 0.123$$

From Table 7-1, $\omega \approx 0.134$

$$a = \frac{A_s f_y}{0.85 f'_c b} = 1.18\omega d$$

$$= 1.18 \times 0.134 \times 19 = 3.0 \text{ in.} > 2.5 \text{ in.}$$

Step 2. Since the value of a as a rectangular section exceeds the flange thickness, the equivalent stress block extends in the web, and the design must be based on T-section behavior. See Example 7.4 when a is less than the flange depth.

Step 3. Compute required reinforcement A_{sf} and nominal moment strength M_{nf} corresponding to the overhanging beam flange in compression (see Part 6).

Compressive strength of flange

$C_f = 0.85f'_c\ (b - b_w)\ h_f$

$= 0.85 \times 4\ (30 - 10)\ 2.5 = 170$ kips

Required A_{sf} to equilibrate C_f :

$$A_{sf} = \frac{C_f}{f_y} = \frac{170}{60} = 2.83 \text{ in.}^2$$

Nominal moment strength of flange:

$$M_{nf} = \left[A_{sf}f_y\left(d - \frac{h_f}{2}\right)\right]$$

$= [2.83 \times 60\ (19 - 1.25)]/12 = 251$ ft-kips

Step 4. Required nominal moment strength to be carried by beam web:

$M_{nw} = M_n - M_{nf} = 444 - 251 = 193$ ft-kips

Step 5. Using Table 7-1, compute reinforcement A_{sw} required to develop moment strength to be carried by the web.

$$\frac{M_{nw}}{f'_c bd^2} = \frac{193 \times 12}{4 \times 10 \times 19^2} = 0.1604$$

From Table 7-1, $\omega_w \approx 0.179$

$$\rho_w = 0.179 \times \frac{4}{60} = 0.01193$$

Step 6. Check to see if section is tension-controlled, with $\phi = 0.9$:

$\rho_t = 0.01806$ from Table 6-1

Therefore, $\rho_w < \rho_t$ and section is tension-controlled ($\phi = 0.9$)

$A_{sw} = \rho_w bd = 0.01193 \times 10 \times 19 = 2.27 \text{ in.}^2$

Step 7. Total reinforcement required to carry factored moment $M_u = 400$ ft-kips:

$A_s = A_{sf} + A_{sw} = 2.83 + 2.27 = 5.10 \text{ in.}^2$

Step 8. Check moment capacity.

$$\phi M_n = \phi\left[(A_s - A_{sf})\,f_y\left(d - \frac{a_w}{2}\right) + A_{sf}f_y\left(d - \frac{h_f}{2}\right)\right]$$

$$a_w = \frac{(A_s - A_{sf})\,f_y}{0.85 f_c' b_w}$$

$$= \frac{(5.10 - 2.83) \times 60}{0.85 \times 4 \times 10} = 4.01 \text{ in.}$$

$$\phi M_n = 0.9\,[(5.10-2.83)\,60\left(19 - \frac{4.01}{2}\right) + (2.83 \times 60)\left(19 - \frac{2.5}{2}\right)]/12$$

$$= 400 \text{ ft-kips} = M_u = 400 \text{ ft-kips} \quad \text{O.K.}$$

2. Select reinforcement to satisfy crack control criteria. *10.6*

Try 5-No. 9 bars in two layers ($A_s = 5.00$ in.^{2}) (2% less than required, assumed sufficient)

Maximum spacing allowed,

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \le 12\left(\frac{40{,}000}{f_s}\right)$$ *Eq. (10-4)*

$c_c = 1.5 + 0.5 = 2.0$ in.

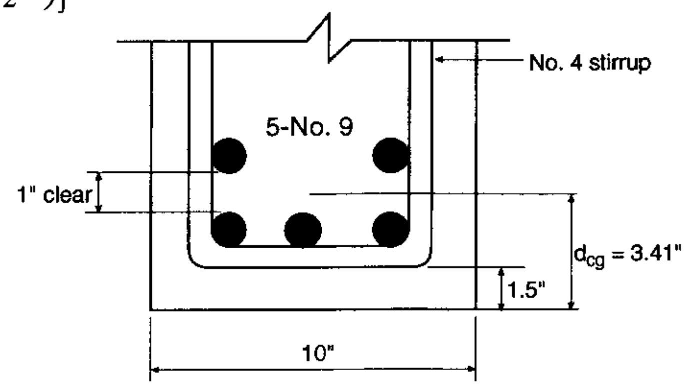
Use $f_s = \frac{2}{3} f_y = 40$ ksi

$$s = 15\left(\frac{40{,}000}{40{,}000}\right) - (2.5 \times 2) = 10 \text{ in. (governs)}$$

$$s = 12\left(\frac{40{,}000}{40{,}000}\right) = 12 \text{ in.}$$

$$\text{Spacing provided} = \frac{1}{2}\left\{10 - 2\left(1.5 + 0.5 + \frac{1.128}{2}\right)\right\}$$

$$= 2.44 \text{ in.} < 10 \text{ in.} \quad \text{O.K.}$$



Note: Two layers of reinforcement are required, which may not have been recognized when d was assumed to be 19 in. Also, the provided steel is slightly less than required. Therefore, the overall height should be a little more than $d + d_{cg} = 22.41$ in., or the steel should be increased.

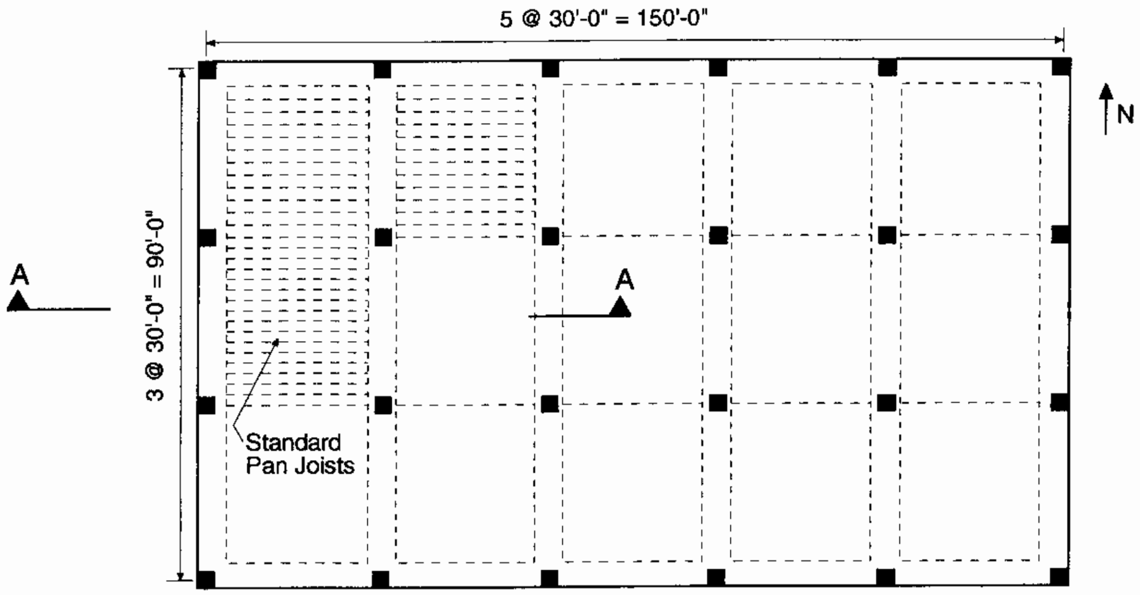
Example 7.6—Design of One-Way Joist

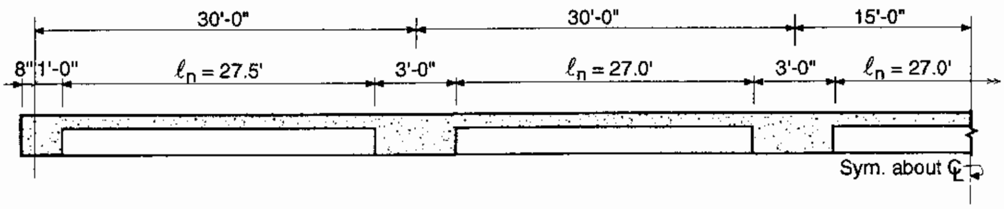
Determine the required depth and reinforcement for the one-way joist system shown below. The joists are 6 in. wide and are spaced 36 in. o.c. The slab is 3.5 in. thick.

f'_c = 4000 psi
f_y = 60,000 psi
Service DL = 130 psf (assumed total for joists and beams plus superimposed dead loads)
Service LL = 60 psf

Width of spandrel beams = 20 in.
Width of interior beams = 36 in.

Columns: interior = 18 × 18 in.
exterior = 16 × 16 in.
Story height (typ.) = 13 ft





Section A-A

Example 7.6 (cont'd)	Calculations and Discussion	Code Reference

1. Compute the factored moments at the faces of the supports and determine the depth of the joists.

$w_u = [(1.2 \times 0.13) + (1.6 \times 0.06)] \times 3 = 0.756$ kips/ft — *Eq. (9-2)*

Using the approximate coefficients, the factored moments along the span are summarized in the table below. — *8.3.3*

Location	M_u (ft-kips)
End span	
Ext. neg.	$w_u\ell_n^2/24 = 0.756 \times 27.5^2/24 = 23.8$
Pos.	$w_u\ell_n^2/14 = 0.756 \times 27.5^2/14 = 40.8$
Int. neg.	$w_u\ell_n^2/10 = 0.756 \times 27.25^2/10 = 56.1$
Interior span	
Pos.	$w_u\ell_n^2/16 = 0.756 \times 27^2/16 = 34.4$
Neg.	$w_u\ell_n^2/11 = 0.756 \times 27^2/11 = 50.1$

For reasonable deflection control, choose a reinforcement ratio ρ equal to about one-half ρ_t. From Table 6-1, $\rho_t = 0.01806$.

Set $\rho = 0.5 \times 0.01806 = 0.00903$

Determine the required depth of the joist based on $M_u = 56.1$ ft-kips:

$$\omega = \frac{\rho f_y}{f'_c} = \frac{0.00903 \times 60}{4} = 0.1355$$

From Table 7-1, $M_u/\phi f'_c bd^2 = 0.1247$

$$d = \sqrt{\frac{M_u}{\phi f'_c b_w (0.1247)}} = \sqrt{\frac{56.1 \times 12}{0.9 \times 4 \times 6 \times 0.1247}} = 15.8 \text{ in.}$$

$h_a \approx 15.8 + 1.25 = 17.1$ in.

From Table 9-5(a), the minimum required thickness of the joist is

$$h_{min} = \frac{\ell}{18.5} = \frac{30 \times 12}{18.5} = 19.5 \text{ in.}$$

Use a 19.5-in. deep joist (16 + 3.5).

2. Compute required reinforcement.

   a. End span, exterior negative

$$\frac{M_u}{\phi f'_c bd^2} = \frac{23.8 \times 12}{0.9 \times 4 \times 6 \times 18.25^2} = 0.0397$$

From Table 7-1, $\omega \approx 0.041$

$$A_s = \frac{\omega bdf'_c}{f_y} = \frac{0.041 \times 6 \times 18.25 \times 4}{60} = 0.30 \text{ in.}^2$$

For $f'_c < 4444$ psi, use

$$A_{s,\,min} = \frac{200b_w d}{f_y} = \frac{200 \times 6 \times 18.25}{60{,}000} = 0.37 \text{ in.}^2 > A_s$$ *Eq. (10-3)*

Distribute bars uniformly in top slab:

$$A_s = \frac{0.37}{3} = 0.123 \text{ in.}^2\text{/ft}$$

Use No. 3 @ 10 in. ($A_s = 0.13$ in.^{2}/ft)

   b. End span, positive

$$\frac{M_u}{\phi f'_c bd^2} = \frac{40.8 \times 12}{0.9 \times 4 \times 36 \times 18.25^2} = 0.0113$$

From Table 7-1, $\omega \approx 0.012$

$$A_s = \frac{\omega bdf'_c}{f_y} = \frac{0.012 \times 36 \times 18.25 \times 4}{60} = 0.53 \text{ in.}^2$$

Check rectangular section behavior:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.53 \times 60}{0.85 \times 4 \times 36} = 0.26 \text{ in.} < 3.5 \text{ in. O.K.}$$

Use 2-No. 5 bars ($A_s = 0.62$ in.^{2})

   c. End span, interior negative

$$\frac{M_u}{\phi f'_c bd^2} = \frac{56.1 \times 12}{0.9 \times 4 \times 6 \times 18.25^2} = 0.0936$$

From Table 7-1, $\omega \approx 0.100$

$$A_s = \frac{\omega b d f'_c}{f_y} = \frac{0.100 \times 6 \times 18.25 \times 4}{60} = 0.73 \text{ in.}^2$$

Distribute reinforcement uniformly in slab:

$$A_s = \frac{0.73}{3} = 0.24 \text{ in.}^2/\text{ft}$$

Use No. 5 @ 12 in. for crack control considerations in slabs (see Table 9-1).

d. The reinforcement for the other sections is obtained in a similar fashion. The following table summarizes the results. Note that at all sections, the requirements in 10.6 for crack control are satisfied.

Location End span	M_u (ft-kips)	A_s (in.^{2})	Reinforcement
Ext. neg.	23.8	0.37	No. 3@10 in.
Pos.	40.8	0.53	2-No. 5
Int. neg.	56.1	0.73	No. 5@12 in.*
Interior span			
Pos.	34.4	0.42	2-No. 5
Neg.	50.1	0.65	No. 5@12 in.*

**Maximum 12 in. spacing required for crack control in slab.*

e. The slab reinforcement normal to the ribs is often located at mid-depth of the slab to resist both positive and negative moments.

$$\text{Use } M_u = \frac{w_u \ell_n^2}{12} = \frac{0.185 \times 2.5^2}{12} = 0.096 \text{ ft-kips}$$

$$\text{where } w_u = 1.2\,(44 + 30) + 1.6\,(60)$$

$$= 185 \text{ psf} = 0.185 \text{ kips/ft}^2$$

$$\frac{M_u}{\phi f'_c b d^2} = \frac{0.096 \times 12}{0.9 \times 4 \times 12 \times 1.75^2} = 0.0087$$

From Table 7-1, $\omega \approx 0.0087$

$$A_s = \frac{\omega b d f'_c}{f_y} = \frac{0.0087 \times 12 \times 1.75 \times 4}{60} = 0.01 \text{ in.}^2/\text{ft}$$

For slabs, minimum reinforcement is governed by the provisions in 7.12.2.1:

$A_{s,min} = 0.0018 \times 12 \times 3.5 = 0.08 \text{ in.}^2\text{/ft}$

$s_{max} = 5h = 5 \times 3.5 = 17.5 \text{ in.}$ (governs) *7.12.2.2*

$= 18 \text{ in.}$

Use No. 3 @ 16 in. ($A_s = 0.08 \text{ in.}^2\text{/ft}$)

3. Shear at supports must be checked. Since the joists meet the requirements in 8.11, the contribution of the concrete to shear strength V_c is permitted to be 10% more than that specified in Chapter 11. *8.11.8*

Example 7.7—Design of Continuous Beams

Determine the required depth and reinforcement for the support beams along the interior column line in Example 7.6. The width of the beams is 36 in.

f'_c = 4000 psi
f_y = 60,000 psi
Service DL = 130 psf (assumed total for joists and beams plus superimposed dead loads)
Service LL = 60 psf

Columns: interior = 18 × 18 in.
exterior = 16 × 16 in.
Story height (typ.) = 13 ft

Calculations and Discussion	Code Reference

1. Compute the factored moments at the faces of the supports and determine the depth of the beam.

$w_u = [(1.2 \times 0.13) + (1.6 \times 0.06)] \times 30 = 7.56$ kips/ft — *Eq. (9-2)*

Using the approximate coefficients, the factored moments along the span are summarized in the table below. — *8.3.3*

Location	M_u (ft-kips)
End span	
Ext. neg.	$w_u \ell_n^2/16 = 7.56 \times 28.58^2/16 = 385.9$
Pos.	$w_u \ell_n^2/14 = 7.56 \times 28.58^2/14 = 441.1$
Int. neg.	$w_u \ell_n^2/10 = 7.56 \times 28.54^2/10 = 615.8$
Interior span	
Pos.	$w_u \ell_n^2/16 = 7.56 \times 28.50^2/16 = 383.8$

For overall economy, choose a beam depth equal to the joist depth used in Example 7.6.

Check the 19.5-in. depth for M_u = 615.8 ft-kips:

From Table 6-2,

$$\phi R_{nt} = 820 = \frac{M_{ut}}{bd^2}$$

$$M_{ut} = 820 \times 36 \times 17^2/1000 = 8531 \text{ in.-kips} = 711 \text{ ft-kips}$$

$M_u < M_{ut}$

Section will be tension-controlled without compresion reinforcement.

Check beam depth based on deflection criteria in Table 9.5(a):

$$h_{min} = \frac{\ell}{18.5} = \frac{30 \times 12}{18.5} = 19.5 \text{ in.} \quad \text{O.K.}$$

Use a 36 × 19.5 in. beam.

2. Compute required reinforcement:

   a. End span, exterior negative

$$\frac{M_u}{\phi f'_c bd^2} = \frac{385.9 \times 12}{0.9 \times 4 \times 36 \times 17^2} = 0.1236$$

From Table 7-1, $\omega \approx 0.134$

$$A_s = \frac{\omega bdf'_c}{f_y} = \frac{0.134 \times 36 \times 17 \times 4}{60} = 5.47 \text{ in.}^2$$

For $f'_c < 4444$ psi, use

$$A_{s,min} = \frac{200b_w d}{f_y} = \frac{200 \times 36 \times 17}{60{,}000} = 2.04 \text{ in.}^2$$ Eq. (10-3)

Use 7-No. 8 bars ($A_s = 5.53$ in.^{2})

Check distribution of flexural reinforcement requirements of 10.6.

Maximum spacing allowed,

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40{,}000}{f_s}\right)$$ Eq. (10-4)

$c_c = 1.5 + 0.5 = 2.0$ in.

Use $f_s = \frac{2}{3} f_y = 40$ ksi

$$s = 15\left(\frac{40{,}000}{40{,}000}\right) - 2.5 \times 2 = 10 \text{ in. (governs)}$$

$$s = 12\left(\frac{40{,}000}{40{,}000}\right) = 12 \text{ in.}$$

$$\text{Spacing provided} = \frac{1}{6}\left\{36 - 2\left(1.5 + 0.5 + \frac{1.0}{2}\right)\right\}$$

$$= 5.17 \text{ in.} < 10 \text{ in.} \quad \text{O.K.}$$

b. End span, positive

$$\frac{M_u}{\phi f'_c bd^2} = \frac{441.1 \times 12}{0.9 \times 4 \times 36 \times 17^2} = 0.1413$$

From Table 7-1, $\omega \approx 0.156$

$$A_s = \frac{\omega bdf'_c}{f_y} = \frac{0.156 \times 36 \times 17 \times 4}{60} = 6.37 \text{ in.}^2$$

Use 11-No. 7 bars ($A_s = 6.60$ in.^{2})

Note that this reinforcement satisfies the cracking requirements in 10.6.4, and fits adequately within the beam width. It can also conservatively be used at the midspan section of the interior span.

c. End span, interior negative

$$\frac{M_u}{\phi f'_c bd^2} = \frac{615.8 \times 12}{0.9 \times 4 \times 36 \times 17^2} = 0.1973$$

From Table 7-1, $\omega \approx 0.228$

$$A_s = \frac{\omega bdf'_c}{f_y} = \frac{0.228 \times 36 \times 17 \times 4}{60} = 9.30 \text{ in.}^2$$

Use 10-No. 9 bars ($A_s = 10.0$ in.^{2})

This reinforcement is adequate for cracking and spacing requirements as well.

Example 7.8—Design of a Square Column for Biaxial Loading

Determine the required square tied column size and reinforcement for the factored load and moments given. Assume the reinforcement is equally distributed on all faces.

$P_u = 1200$ kips, $M_{ux} = 300$ ft-kips, $M_{uy} = 125$ ft-kips

$f'_c = 5000$ psi, $f_y = 60{,}000$ psi

Calculations and Discussion	Code Reference

1. Determine required nominal strengths, assuming compression-controlled behavior: *9.3.2.2(b)*

$$P_n = \frac{P_u}{\phi} = \frac{1200}{0.65} = 1846 \text{ kips}$$

$$M_{nx} = \frac{M_{ux}}{\phi} = \frac{300}{0.65} = 461.5 \text{ ft-kips}$$

$$M_{ny} = \frac{M_{uy}}{\phi} = \frac{125}{0.65} = 192.3 \text{ ft-kips}$$

2. Assume $\beta = 0.65$

3. Determine an equivalent uniaxial moment strength M_{nox} or M_{noy}.

$$\frac{M_{ny}}{M_{nx}} = \frac{192.3}{465.1} = 0.42 \text{ is less than } \frac{b}{h_a} = 1.0 \text{ (square column)}$$

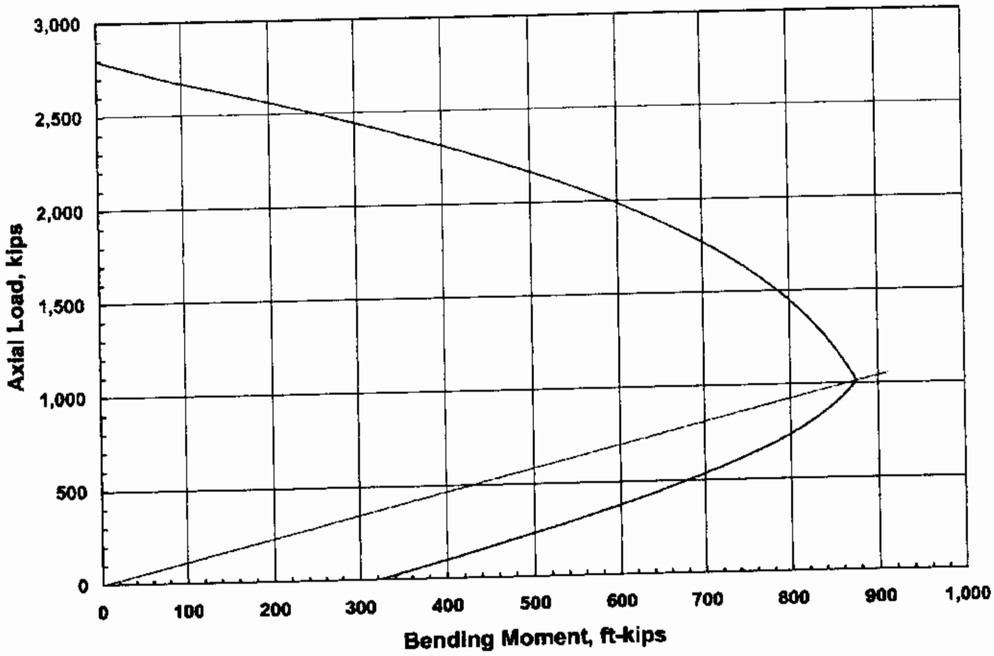
Therefore, using Eq. (20)

$$M_{nox} \approx M_{nx} + M_{ny}\frac{h_a}{b}\left(\frac{1-\beta}{\beta}\right)$$

$$= 461.5 + \left[192.3 \times (1.0)\left(\frac{1-0.65}{0.65}\right)\right] = 565.1 \text{ ft-kips}$$

4. Assuming a 24 in. square column, determine the reinforcement required to provide an axial load strength $P_n = 1846$ kips and an equivalent uniaxial moment strength $M_{nox} = 565.1$ ft-kips

The figure below is an interaction diagram for this column with 4-No. 11 bars. The section is adequate with this reinforcement for (P_n, M_{nox}).



5. Selected section will now be checked for biaxial strength by each of the three methods presented in the discussion.

a. Bresler Reciprocal Load Method

Check $P_n \geq 0.1 f'_c A_g$ (8)

1714 kips > 0.1 (5) (576) = 288 kips O.K.

To employ this method, P_o, P_{ox}, and P_{oy} must be determined.

$$P_o = 0.85 f'_c (A_g - A_{st}) + A_{st} f_y$$

$$= 0.85 (5) (576 - 6.24) + 6.24 (60) = 2796 \text{ kips}$$

P_{ox} is the uniaxial load strength when only M_{nx} acts on the column. From the interaction diagram, P_{ox} = 2225 kips when M_{nx} = 461.5 ft-kips.

Similarly, P_{oy} = 2575 kips when M_{ny} = 192.3 ft-kips. Note that both P_{ox} and P_{oy} are greater than the balanced axial force, so that the section is compression-controlled.

Using the above values, Eq. (7) can now be evaluated:

$$P_n = 1846 \text{ kips} \leq \frac{1}{\frac{1}{P_{ox}} + \frac{1}{P_{oy}} - \frac{1}{P_o}}$$

$$< \frac{1}{\frac{1}{2225} + \frac{1}{2575} - \frac{1}{2796}} = 2083 \text{ kips O.K}$$

b. Bresler Load Contour Method

Due to a lack of available data, a conservative α value of 1.0 is chosen. Although $P_u > 0.1 f'_c A_g$, the necessary calculations will be carried out for example purposes. Since the section is symmetrical, M_{nox} is equal to M_{noy}.

From the interaction diagram, M_{nox} = 680 ft-kips for P_n = 1846 kips.

Using the above value, Eq. (11) can now be evaluated:

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}} = \frac{461.5}{680} + \frac{192.3}{680} = 0.68 + 0.28 = 0.96 < 1.0 \text{ O.K.}$$

c. PCA Load Contour Method

To employ this method, P_o, M_{nox}, M_{noy} and the true value of β must first be found.

$$P_o = 0.85 f'_c (A_g - A_{st}) + A_{st} f_y$$

$$= 0.85 (5) (576 - 6.24) + 6.24 (60) = 2796 \text{ kips}$$

Since the section is symmetrical, M_{nox} and M_{noy} are equal.

From the interaction diagram, M_{nox} = 680 ft-kips for P_n = 1846 kips.

Having found P_o and using ρ_g (actual), the true β value is determined as follows:

$$\frac{P_n}{P_o} = \frac{1846}{2796} = 0.66, \ \omega = \frac{\rho_g f_y}{f'_c} = \frac{(6.24/24^2)}{5} = 0.13$$

From Fig. 7-15(a), read β = 0.66

Using the above values, Eq. (13) can now be evaluated:

$$\left(\frac{M_{nx}}{M_{nox}}\right)^{\left(\frac{\log 0.5}{\log\beta}\right)} + \left(\frac{M_{ny}}{M_{noy}}\right)^{\left(\frac{\log 0.5}{\log\beta}\right)} \le 1.0$$

$\log 0.5 = -0.3$

$\log \beta = \log 0.66 = -0.181$

$$\frac{\log\ 0.5}{\log\ \beta} = 1.66$$

$$\left(\frac{461.5}{680}\right)^{1.66} + \left(\frac{192.3}{680}\right)^{1.66} = 0.53 + 0.12 = 0.65 < 1.0 \quad \text{O.K.}$$

This section can also be checked using the bilinear approximation.

Since $\dfrac{M_{ny}}{M_{nx}} < \dfrac{M_{noy}}{M_{nox}}$, Eq. (17) should be used.

$$\frac{M_{nx}}{M_{nox}} + \frac{M_{ny}}{M_{noy}}\left(\frac{1-\beta}{\beta}\right) = \frac{461.5}{680} + \frac{192.3}{680}\left(\frac{1-0.66}{0.66}\right)$$

$$= 0.68 + 0.15 = 0.83 < 1.0 \quad \text{O.K.}$$

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8

Redistribution of Negative Moments in Continuous Flexural Members

UPDATE FOR THE '05 CODE

BACKGROUND

The behavior of a concrete member is affected by its reinforcement layout. For example, consider a three span reinforced concrete beam built monolithically with reinforcement provided only at the bottom of the beam. Prior to cracking, the beam behaves as three continuous spans. After cracking over the interior supports, the three-span beam will behave as three simply supported spans. Therefore, after cracking, redistribution of internal forces occurs in the system. However, the cracks over the interior supports may become large and unacceptable from a serviceability point of view. Section 8.4 sets rules for redistribution of negative moments in continuous beams provided they have sufficient ductility. The redistribution provisions allow for adequate serviceability.

The provisions of 8.4 are beneficial when evaluating existing structures or during the design of new structures. The procedure recognizes that the moment envelop is the result of different transient load patterns (8.9). For example, when considering the pattern that produces the largest negative moment, the designer can reduce that negative moment. This reduction, however, will cause an increase of the concurrent positive moment is the midspan. Similarly, increasing the negative moment over supports will reduce the positive moment in the midpsan. By increasing and decreasing negative moments over supports of continuous members, the negative and positive moments can be optimized and the required amount of flexural reinforcement can be economized. This procedure is illustrated in Examples 8.1 and 8.2.

8.4 REDISTRIBUTION OF NEGATIVE MOMENTS IN CONTINUOUS FLEXURAL MEMBERS

Section 8.4 permits a redistribution of negative moments in continuous flexural members if the net tensile strain exceeds a specified amount. This provision recognizes the inelastic behavior of concrete structures and constitutes a move toward "limit design." Application of moment redistribution, in many cases, results in substantial decrease in total required reinforcement, which allows avoiding reinforcement congestion or reduction of concrete dimensions.

A maximum 10 percent adjustment of negative moments was first permitted in the 1963 ACI Code. Experience with the use of that provision, though satisfactory, was still conservative. The 1971 code increased the maximum adjustment percentage up to 20 percent depending on the reinforcement indices. The increase was justified by additional knowledge of ultimate and service load behavior obtained from tests and analytical studies. Moment redistribution was allowed for both nonprestressed and prestressed members but different specifications were used for each type of member. Starting with the 2002 revision of the code, 8.4 specified the negative moment redistribution factor in terms of the net tensile strain, ε_t. This unified provision applies equally to both nonprestressed and prestressed members. Former provisions involving reinforcement indices may still be used as prescribed in B.8.4 and B.18.10.4.

According to 8.9, continuous members must be designed to resist more than one configuration of live loads. An elastic analysis is performed for each loading configuration, and an envelope moment value is obtained for the design of each section. Thus, for any of the loading conditions considered, certain sections in a given span will reach the ultimate moment while others will have reserve capacity. Tests have shown that a structure can continue to carry additional loads if the sections that reached their moment capacities continue to rotate as plastic hinges and redistribute the moments to other sections until a collapse mechanism forms.

Recognition of this additional load capacity beyond the intended original design suggests the possibility of redesign with resulting savings in material. Section 8.4 allows a redesign by decreasing or increasing the elastic negative moments for each loading condition (with the corresponding changes in positive moment required by statics). These moment changes may be such as to reduce both the maximum positive and negative moments in the final moment envelope. In order to ensure proper rotation capacity, the net tensile strain in the sections at the support must conform to 8.4. Example 8.1 illustrates this requirement.

Limits of applicability of 8.4 may be summarized as follows:

1. Provisions apply to continuous nonprestressed and prestressed flexural members.
2. Provisions do not apply to members designed by the approximate moments of 8.3.3, or to slab systems designed by the Direct Design Method (13.6.1.7).
3. Bending moments must be determined by analytical methods, such as moment distribution, slope deflection, etc. Redistribution is not allowed for moments determined through approximate methods.
4. Redistribution is only permitted when the net tensile strain is not less than 0.0075 (8.4.3).
5. Maximum allowable percentage increase or decrease of negative moment is equal to 1000 ε_t, but not more than 20 percent (8.4.1).
6. Adjustment of negative moments is made for each loading configuration considered. Members are then proportioned for the maximum adjusted moments resulting from all loading conditions.
7. Adjustment of negative support moments for any span requires adjustment of positive moments in the same span (8.4.2). A decrease of a negative support moment requires a corresponding increase in the positive span moment for equilibrium.
8. Static equilibrium must be maintained at all joints before and after moment redistribution.
9. In the case of unequal negative moments on the two sides of a fixed support (i.e., where adjacent spans are unequal), the difference between these two moments is taken into the support. Should either or both of these negative moments be adjusted, the resulting difference between the adjusted moments is taken into the support.
10. Moment redistribution may be carried out for additional cycles. After each cycle of redistribution, a new allowable percentage increase or decrease in negative moment is calculated. After the first iteration, the reduction is typically 15 percent off its final value, which is usually reached after three cycles.

The permissible percentage redistribution is defined in terms of the net tensile strain ε_t. In general, the design procedures outlined in Part 7 of the Notes can be used to determine the location of the neutral axis, c, which allows calculating ε_t from the expression

$$\varepsilon_t = 0.003\left(\frac{d_t}{c} - 1\right) \tag{1}$$

However, in the case of a section with a rectangular compression block and one layer of tension reinforcement only ($d_t = d$), an explicit relation between the net tensile strain, ε_t, and the nondimensional coefficient of resistance,

$$R_n / f_c' = M_n / \left(f_c' b d^2\right) = M_u / \left(\phi f_c' b d^2\right) \tag{2}$$

can be derived as follows (see Fig. 8-1).

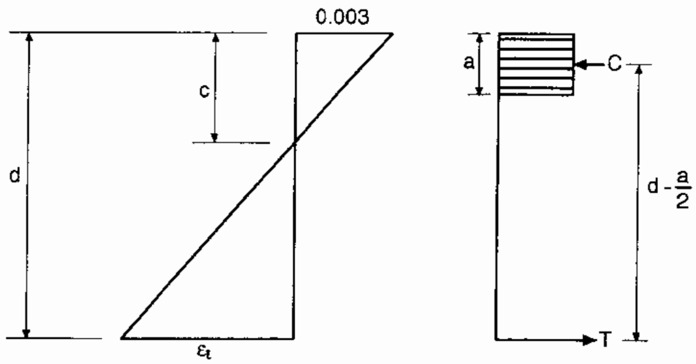


Figure 8-1 Strains and Stresses

Setting $r = c / d_t$, the depth of the concrete stress block, a, and the concrete stress block resultant, C, can respectively be expressed as:

$$a = \beta_1 c = \beta_1 r d \tag{3}$$

$$C = 0.85 f_c' b a = 0.85 f_c' b \beta_1 r d \tag{4}$$

Substituting Eq. (3) and Eq. (4) into the equilibrium condition for internal and external moments:

$$M_n = C\left(d - \frac{a}{2}\right) \tag{5}$$

results in:

$$\frac{M_n}{f_c' b d^2} = 0.85\, \beta_1 r \left(1 - \frac{\beta_1 r}{2}\right) \tag{6}$$

with the nondimensional coefficient of resistance [see Eq. (2)] on the left hand side. Solving Eq. (6) with respect to r yields:

$$r = \frac{1 - \sqrt{1 - \frac{40}{17}\frac{R_n}{f_c'}}}{\beta_1} \tag{7}$$

Substituting r into Eq. (1) gives

$$\varepsilon_t = 0.003\left(\frac{\beta_1}{1 - \sqrt{1 - \frac{40}{17}\frac{R_n}{f_c'}}} - 1\right) \tag{8}$$

Note that Eq. (8) does not involve steel strength and is valid for use with all types of steel, including prestressing steel. Figure 8-2 shows the relationship between permissible redistribution, net tensile strain, and coefficient of resistance.

The following procedure may be utilized to determine the permissible moment redistribution.

1. Determine factored bending moments at supports by analytical elastic methods. Compute coefficients of resistance using Eq (2). Use $\phi = 0.90$ because the assumption $\varepsilon_t \geq 0.0075$ implies a tension-controlled section.
2. Use Eq. (8) to calculate ε_t, and if it satisfies $\varepsilon_t \geq 0.0075$ then determine the corresponding permissible percent redistribution $1000\,\varepsilon_t \leq 20\%$.

   Alternatively enter Fig. 8-2 with value of R_n / f_c'. Move up to intersect the appropriate curve, and move left to find the permissible percent redistribution. Interpolate between curves if needed.
3. Adjust support moments, and corresponding positive moments to satisfy equilibrium.

It usually happens that the steel provided using discrete bar sizes is somewhat more than that required. This reduces ε_t and the permissible percent redistribution slightly. However, the excess steel increases the strength far more than the change in percent redistribution. For example, referring to Fig. 8-2, the curve for 4,000 psi concrete shows a coefficient of resistance of 0.112 when $\varepsilon_t = 0.015$ and a 15 percent redistribution. If so much extra steel were provided that ε_t was reduced to 0.010, with a permissible redistribution of 10 percent, the coefficient of resistance increases from 0.112 to 0.150. Thus, a 5 percent reduction in permissible redistribution is accompanied by a 34 percent increase in strength. Consequently, it is not necessary to calculate the slight reduction in permissible redistribution, because it is offset by a far greater increase in strength.

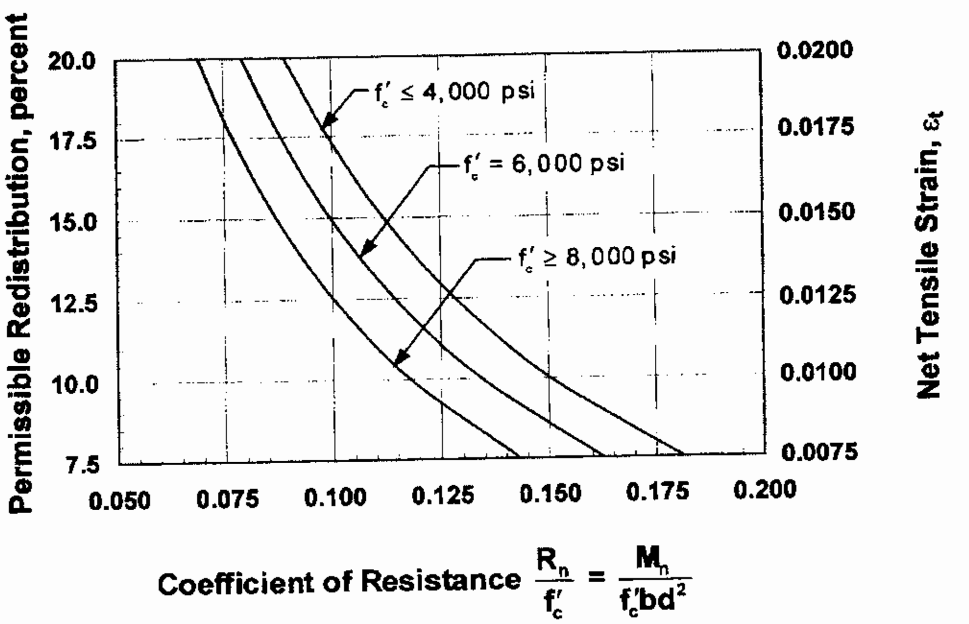


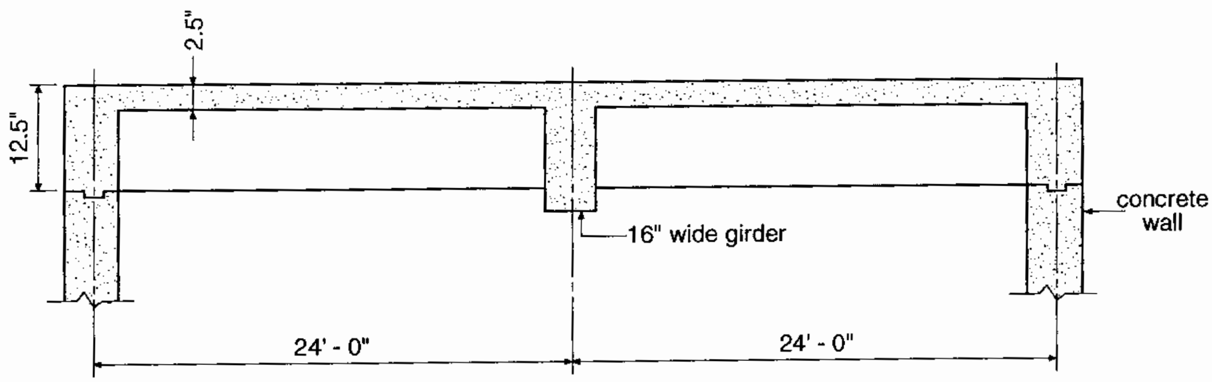
Figure 8-2 Permissible Moment Redistribution

REFERENCE

8.1 Mast, R.F., (1992), "Unified Design Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members,""*ACI Structural Journal*, V. 89, pp. 185-199.

Example 8.1—Moment Redistribution

Determine required reinforcement for the one-way joist floor shown, using moment redistribution to reduce total reinforcement.



Joist-slab: 10 + 2.5 × 5 + 25 (10-in. deep form + 2.5-in. slab, 5-in. wide form spaced @ 25 in. o.c.)

f'_c = 4000 psi
f_y = 60,000 psi
DL = 80 psf
LL = 100 psf

For simplicity, continuity at concrete walls is not considered.

Calculations and Discussion	Code Reference

1. Determine factored loads.

$U = 1.2D+1.6L$ *Eq. (9-2)*

$w_d = 1.2 \times 0.08 \times 25/12 = 0.200$ kips/ft

$w_\ell = 1.6 \times 0.10 \times 25/12 = 0.333$ kips/ft

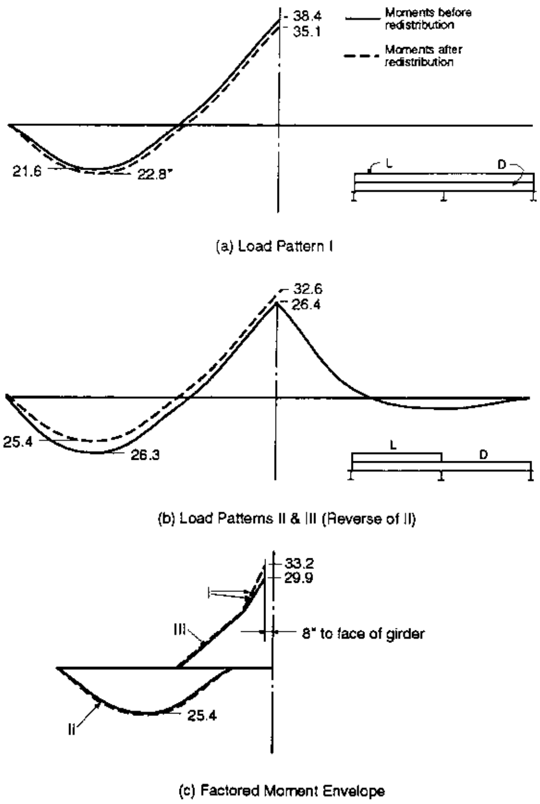
$w_u = 0.533$ kips/ft per joist

2. Obtain moment diagrams by elastic analysis.

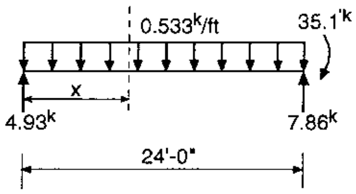
Consider three possible load patterns:

Load pattern I: Factored DL and LL on both spans.
Load pattern II: Factored DL and LL on one span and factored DL only on the other span.
Load pattern III: Reverse of load pattern II.

The elastic moment diagrams for these load cases are shown in the figure (moments shown in ft-kips).



*Calculation of $22.8^{'k}$ adjusted positive moment



$V = 0$ @ $x = 9.25$ ft

$$M \text{ @ } x = 9.25 \text{ ft} = (4.93 \times 9.25) - 0.533 \times \frac{9.25^2}{2}$$

$$= 22.8^{'k}$$

Figure 8-3 Redistribution of Moments for Example 8.1

3. Redistribution of negative moments.

   a. Load pattern I:

   The intent is to decrease the negative moment at the support to obtain a new moment envelope.

   From load pattern I: $M_u = -33.2$ ft-kips at face of girder.

   For b = 5 in., and d = 11.5 in.:

$$\frac{R_n}{f_n} = \frac{33.2 \times 12}{0.9 \times 4 \times 5 \times (11.5)^2} = 0.167 \text{ and the permissible reduction}$$

$$1000\,\varepsilon_t = 3\left(\frac{0.85}{1-\sqrt{\frac{40}{17}} \times 0.167} - 1\right) = 8.5\%$$

   Decreasing the negative moment M_u = -38.4 ft-kips in Fig. 8-3(a) by 8.5%, redistributed moment diagrams are obtained as shown by the dashed lines in Fig. 8-3(a).

   The maximum span moment correspondingly increases to 22.8 ft-kips by equilibrium (see calculation in figure).

b. Load pattern II:

The elastic moment diagram of Load Pattern II is compared with the redistributed moment diagram of Load Pattern I. For savings in span positive moment reinforcement, it is desirable to reduce the span positive moment of 26.3 ft-kips. This can be achieved through redistribution of the negative moment at the support by increasing it by 8.5%, to 26.4 × 1.085 = 28.6 ft-kips. As a result the positive moment is reduced from 26.3 to 25.4 ft-kips.

4. Design factored moments.

From the redistributed moment envelope, factored moments and required reinforcement are determined as shown in the following table.

Table 8-1 Summary of Final Design

Section	Load Pattern		Required Steel		Provided Steel		Redistribution, percent
	I	I	A_s (in.^{2})	ρ	A_s (in.^{2})	ρ**	
Support Moment* (ft-kips)	-29.9	—	0.52	0.0092	2-No.5	0.0103 (b = 5 in.)	-8.5
Midspan Moment (ft-kips)	—	25.4	0.50	0.0017	2-No. 5	0.0021*** (b = 25 in.)	+8.5 at support

**Calculated at face of support.*

**Check $\rho_{min} = \dfrac{3\sqrt{f'_c}}{f_y} = 0.0032$

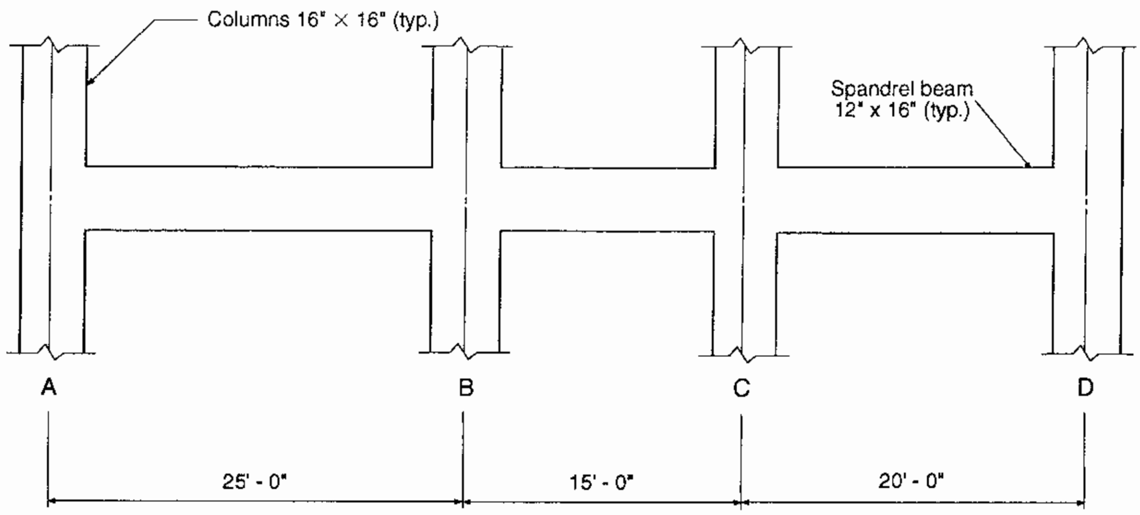
$$\rho_{min} = \frac{200}{60{,}000} = 0.0033 \text{ (governs)}$$

*** $\rho = \dfrac{A_s}{b_w d} = \dfrac{0.51}{5 \times 11.5} = 0.0089 > \rho_{min}$

Final note: Moment redistribution has permitted a reduction of 8.5% in the negative moment. Similarly, the positive span moment has been reduced through redistribution of the negative support moment.

Example 8.2—Moment Redistribution

Determine the required reinforcement areas for the spandrel beam at an intermediate floor level as shown, using moment redistribution to reduce total reinforcement required.



Columns = 16 × 16 in.
Story height = 10 ft
Spandrel beam = 12 × 16 in.
f'_c = 4000 psi
f_y = 60,000 psi
DL = 1167 lb/ft
LL = 450 lb/ft

Calculations and Discussion	Code Reference

1. Determine factored loads.

$U = 1.2D + 1.6L$ — *Eq. (9-2)*

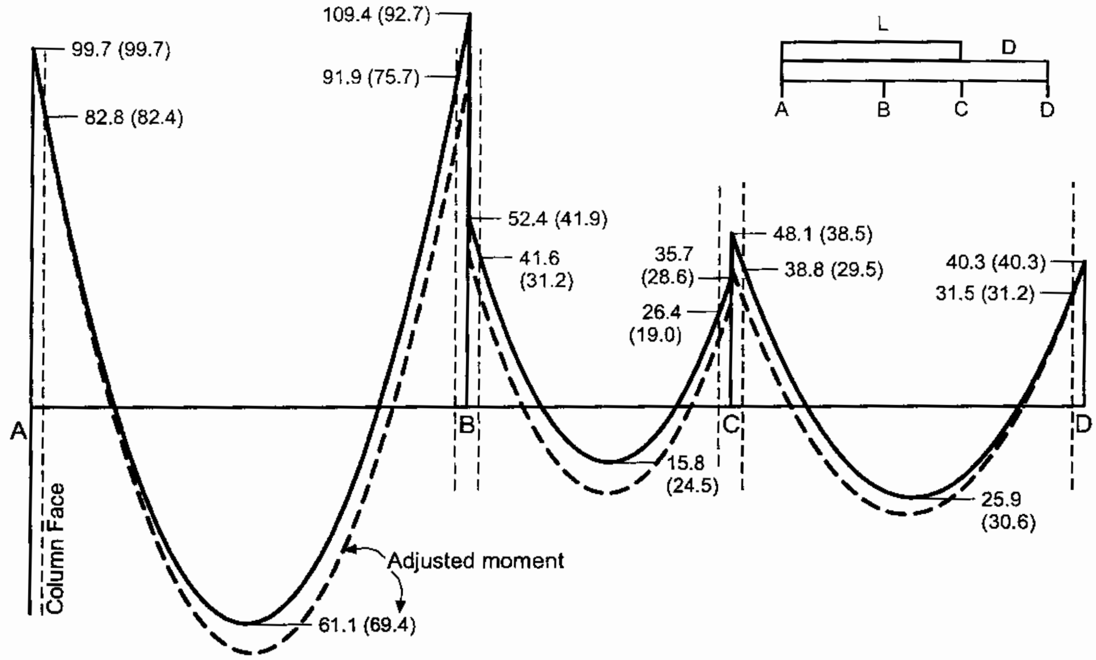
$w_d = 1.2 \times 1.167 = 1.4$ kips/ft

$w_\ell = 1.6 \times 0.45 = 0.72$ kips/ft

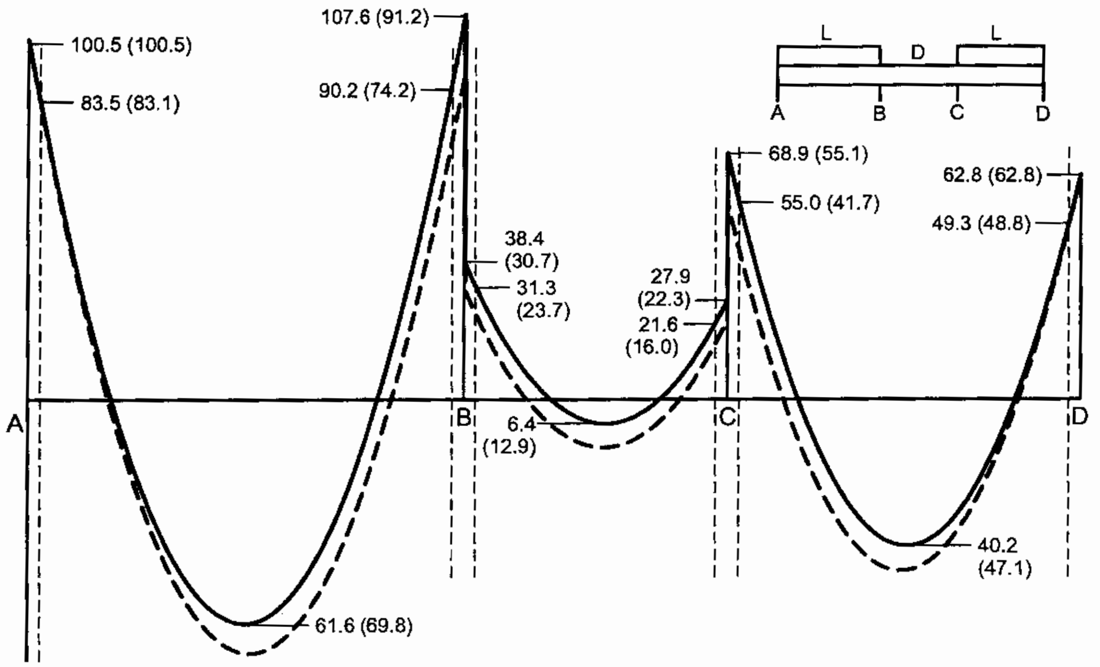
$w_u = 2.12$ kips/ft

2. Determine the elastic bending moment diagrams for the five load patterns shown in Figs. 8-4 (a) to (e) and the maximum moment envelope values for all load patterns. — *8.9.2*

Maximum negative moments at column counterlines and column faces, and positive mid-span moments were determined by computer analysis using pcaBeam program for each of the five loading configurations. Adjusted moments after redistribution are also shown by dashed lines. The values of the adjusted moments are given in parentheses.

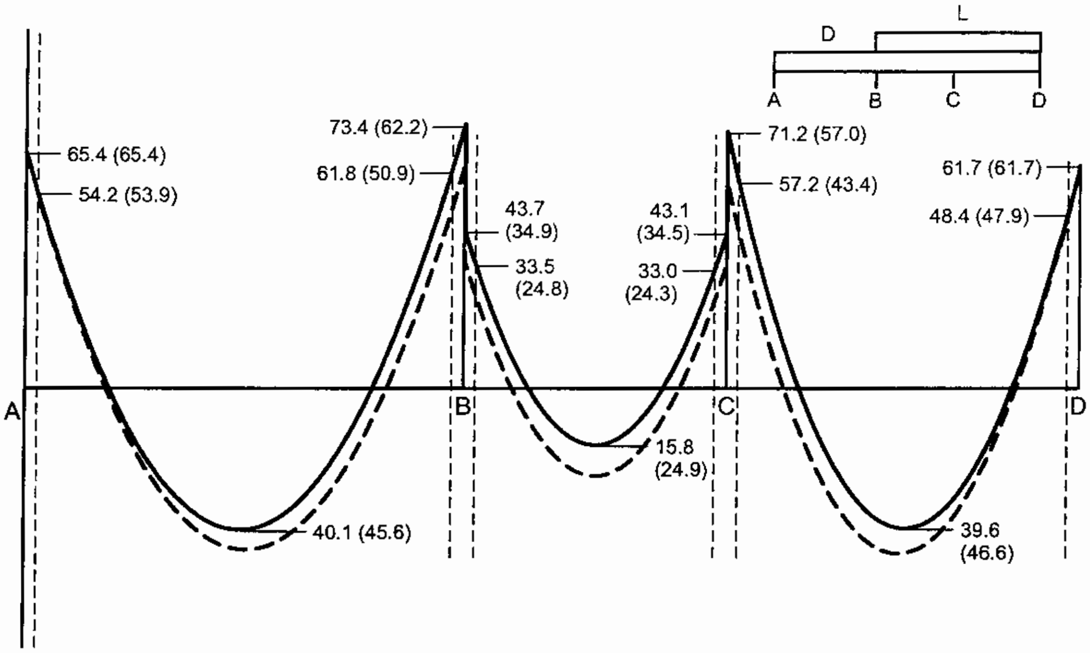


(a) Load Pattern I (moments in ft-kips)

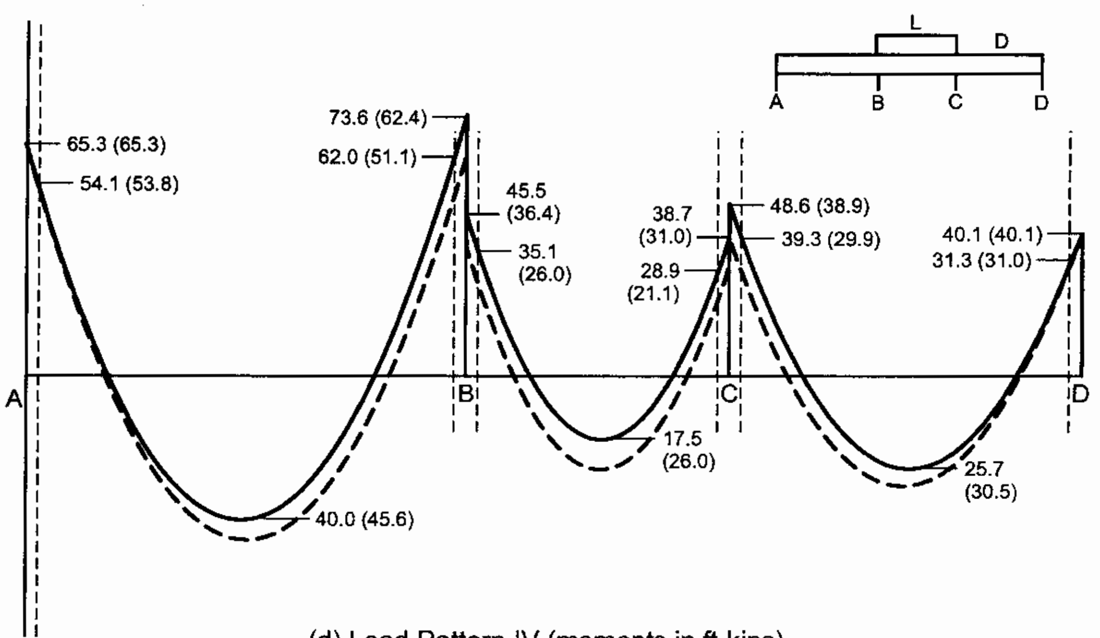


(b) Load Pattern II (moments in ft-kips)

Figure 8-4 Redistribution of Moments for Example 8.2

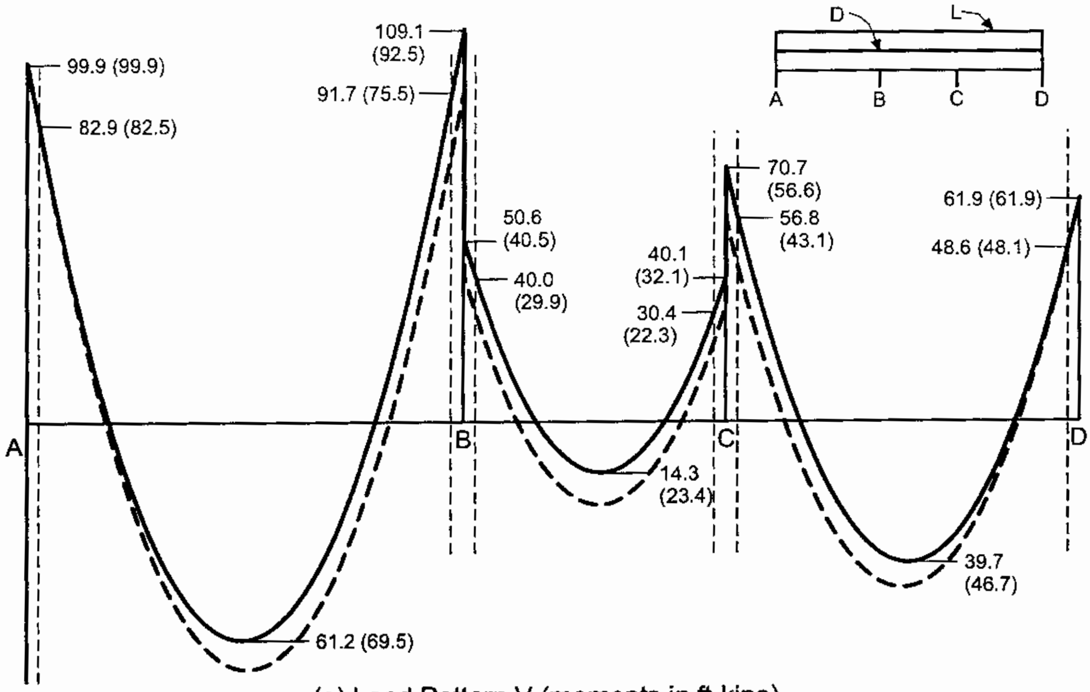


(c) Load Pattern III (moments in ft-kips)

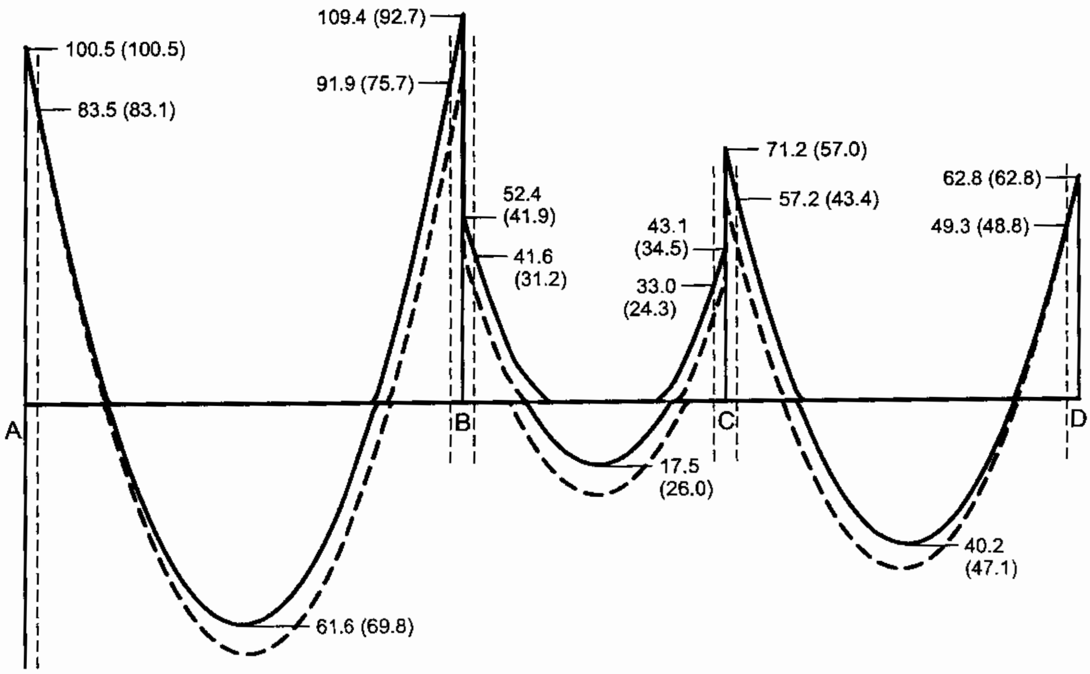


(d) Load Pattern IV (moments in ft-kips)

Figure 8-4 (continued) Redistribution of Moments for Example 8.2



(e) Load Pattern V (moments in ft-kips)



(f) Maximum Moment Envelopes for Pattern Loading (moments in ft-kips)

Figure 8-4 (continued) Redistribution of Moments for Example 8.2

3. Determine maximum allowable percentage increase or decrease in negative moments:

use d = 14.0 in.; cover = 1.5 in. *7.7.1*

Calculate $\frac{R_n}{f_c'} = \frac{M_u}{\phi f_c' bd^2}$ and corresponding $\varepsilon_t = 0.003\left(\frac{\beta_1}{1-\sqrt{1-\frac{40}{17}\frac{R_n}{f_c'}}}-1\right)$.

For M_u use envelope value at support face. Based on ε_t calculate the adjustment. Iterate until the adjusted moments converge (starts repeating). See Table 8-2.

Table 8-2 Moment Adjustments at Supports

		Support					
		A	B		C		D
		Right	Left	Right	Left	Right	Left
Iteration 1	M_u (ft-kips)	83.5	91.9	41.6	33.0	57.2	49.3
	R_n/f'_c	0.1184	0.1303	0.0589	0.0467	0.0811	0.0699
	ε_t	0.0139	0.0122	0.0325	0.0421	0.0224	0.0267
	Adjustment (%)	13.9	12.2	20.0	20.0	20.0	20.0
Iteration 2	M_u (ft-kips)	71.9	80.7	33.3	26.4	45.8	39.4
	R_n/f'_c	0.1019	0.1143	0.0471	0.0374	0.0649	0.0559
	ε_t	0.0169	0.0146	0.0417	0.0537	0.0291	0.0345
	Adjustment (%)	16.9	14.6	20.0	20.0	20.0	20.0
Iteration 3	M_u (ft-kips)	69.4	78.5				
	R_n/f'_c	0.0984	0.1113				
	ε_t	0.0177	0.0151				
	Adjustment (%)	17.7	15.1				
Iteration 4	M_u (ft-kips)	68.8	78.0				
	R_n/f'_c	0.0975	0.1106				
	ε_t	0.0179	0.0152				
	Adjustment (%)	17.9	15.2				
Iteration 5	M_u (ft-kips)	68.6	77.9				
	R_n/f'_c	0.0972	0.1104				
	ε_t	0.0179	0.0153				
	Adjustment (%)	17.9	15.3				
Iteration 6	M_u (ft-kips)		77.9				
	R_n/f'_c		0.1104				
	ε_t		0.0153				
	Adjustment (%)		15.3				
Final Allowable Adjustment (%)		17.9	15.3	20.0	20.0	20.0	20.0

4. Adjustment of moments.

Note: Adjustment of negative moments, either increase or decrease, is a decision to be made by the engineer. In this example, it was decided to reduce the negative moments on both sides of supports B and C and accept the increase in the corresponding positive moments, and not to adjust the negative moments at the exterior supports A and D.

Referring to Figs. 8-4(a) through (e), the following adjustments in moments are made.

Load Pattern I — Fig. (a)
$M_{B,Left}$ = 109.4 ft-kips (adjustment = 15.3%)
Reduction to $M_{B,Left}$= $-109.44 \times 0.153 = 16.7$ ft-kips
AAdjusted $M_{B,Left}$ = $-109.4-(-16.7) = -92.7$ ft-kips

Increase in positive moment in span A-B
$M_A = -99.7$ ft-kips
Adjusted $M_{B,Left} = -92.7$ ft-kips

Mid-span ordinate on line M_A to $M_{B,Left} = \dfrac{-99.7+(-92.7)}{2} = -96.2\text{ft}-\text{kips}$

Moment due to uniform load = $w_u\ell^2/8 = 2.12 \times 25^2/8 = 165.6$ ft-kips
Adjusted positive moment at mid-span = $-96.2+165.6 = 69.4$ ft-kips

Decrease in negative moment at the left face of support B

Ordinate on line M_A to $M_{B,Left} = -99.7 + \dfrac{-92.7-(-99.7)}{25.0} \times 24.33 = 92.9$ ft-kips

Moment due to uniform load $= \dfrac{1}{2}\,w_u x(\ell - x) = \dfrac{1}{2} \times 2.12 \times 24.33 \times (25.0-24.33) = -17.2$ ft-kips

Adjusted negative moment at the left face of support B = $-92.9+17.2$=-75.7ft-kips

Similar calculations are made to determine the adjusted moment at other locations and for other load patterns. Results of the additional calculations are shown in Table 8-3.

5. After the adjusted moments have been determined analytically, the adjusted bending moment diagrams for each loading pattern can be determined. The adjusted moment curves were determined graphically and are indicated by the dashed lines in Figs. 8-4 (a) to (e).

6. An adjusted maximum moment envelope can now be obtained from the adjusted moment curves as shown in Fig. 8-4 (f) by dashed lines.

7. Final steel ratios ρ can now be obtained on the basis of the adjusted moments.

   From the redistributed moment envelopes of Fig. 8-4 (f), the design factored moments and the required reinforcement area are obtained as shown in Table 8-4.

Table 8-3 Moments Before and After Redistribution (moments in ft-kips)

Location	Load Pattern I		Load Pattern II		Load Pattern III		Load Pattern IV		Load Pattern IV	
	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}	M_u	M_{adj}
A	-99.7	-99.7	-100.5	-100.5	-65.4	-65.4	-65.3	-65.3	-99.9	-99.9
A Right Face	-82.8	-82.4	-83.5	-83.1	-54.2	-53.9	-54.1	-53.8	-82.9	-82.5
Mid-Span A-B	+61.1	+69.4	+61.6	+69.8	+40.1	+45.6	+40.0	+45.6	+61.2	+69.5
B Left Face	-91.9	-75.7	-90.2	-74.2	-61.8	-50.9	-62.0	-51.1	-91.7	-75.5
B Left Center	-109.4	-92.7	-107.6	-91.2	-73.4	-62.2	-73.6	-62.4	-109.1	-92.5
B Right Center	-52.4	-41.9	-38.4	-30.7	-43.7	-34.9	-45.5	-36.4	-50.6	-40.5
B Right Face	-41.6	-31.2	-31.3	-23.7	-33.5	-24.8	-35.1	-26.0	-40.0	-29.9
Mid-Span B-C	+15.8	+24.5	+6.4	+12.9	+15.8	+24.9	+17.5	+26.0	+14.3	+23.4
C Left Face	-26.4	-19.0	-21.6	-16.0	-33.0	-24.3	-28.9	-21.1	-30.4	-22.3
C Left Center	-35.7	-28.6	-27.9	-22.3	-43.1	-34.5	-38.7	-31.0	-40.1	-32.1
C Right Center	-48.1	-38.5	-68.9	-55.1	-71.2	-57.0	-48.6	-38.9	-70.7	-56.6
C Right Face	-38.8	-29.5	-55.0	-41.7	-57.2	-43.4	-39.3	-29.9	-56.8	-43.1
Mid-Span C-D	+25.9	+30.6	+40.2	+47.1	+39.6	+46.6	+25.7	+30.5	+39.7	+46.7
D Left Face	-31.5	-31.2	-49.3	-48.8	-48.4	-47.9	-31.3	-31.0	-48.6	-48.1
D	-40.3	-40.3	-62.8	-62.8	-61.7	-61.7	-40.1	-40.1	-61.9	-61.9

Final design moments after redistribution

Table 8-4 Summary of Finasl Design

Location		Moment (ft-kips)	Load Case	Required	
				A_s (in^2)	ρ
Support A	Right Face	-83.1	II	1.43	0.0085
Midspan A-B		69.8	II	1.18	0.0070
Support B	Left Face	-75.7	I	1.29	0.0077
	Right Face	-31.2	I	0.51	0.0030
Midspan B-C		26	IV	0.42	0.0025
Support C	Left Face	-24.3	III	0.39	0.0023
	Right Face	-43.4	III	0.72	0.0043
Midspan C-D		47.1	II	0.78	0.0046
Support D	Left Face	-48.8	II	0.81	0.0048

Use $A_{s,min} = 200\frac{b_w d}{f_y} = 200 \times \frac{12 \times 14}{60{,}000} = 0.56 \text{ in.}^2$

9

Distribution of Flexural Reinforcement

UPDATE FOR THE '05 CODE

Equation 10-4, for maximum bar spacing to control cracking, was modified to provide results consistent with previous editions of the code while maintaining similar level of crack control. The default steel stress at service load in the equation was increased from $0.6f_y$ to $(2/3)f_y$. The revised equation is intended to recognize the increase in service load stress level in flexural reinforcement resulting from the use of the load combinations introduced in the 2002 code.

The provisions for skin reinforcement in Section 10.6.7 were simplified and made consistent with the requirement for flexural tension reinforcement in 10.6.4. Research [Ref. 9.3] has shown that control of side face cracking can be achieved through proper spacing of the skin reinforcement for selected cover dimension. The research also confirmed that the reinforcement spacing requirements in Section 10.6.4 are sufficient to control side face cracking. To eliminate the confusion regarding the definition of effective depth for multi-layer reinforced members, 10.6.7 is simplified to require skin reinforcement based on the overall depth of the member instead of the effective depth.

GENERAL CONSIDERATIONS

Provisions of 10.6 require proper distribution of tension reinforcement in beams and one-way slabs to control flexural cracking. Structures built in the past using Working Stress Design methods and reinforcement with a yield strength of 40,000 psi or less had low tensile stresses in the reinforcement at service loads. Laboratory investigations have shown that cracking is generally in proportion to the steel tensile stress. Thus, with low tensile stresses in the reinforcement at service loads, these structures exhibited few flexural cracking problems.

With the advent of high-strength steels having yield stresses of 60,000 psi and higher, and with the use of Strength Design methods which allow higher stresses in the reinforcement, control of flexural cracking has assumed more importance. For example, if a beam were designed using Working Stress Design and a steel yield strength of 40,000 psi, the stress in the reinforcement at service loads would be about 20,000 psi. Using Strength Design and a steel yield strength of 60,000 psi, the stress at service loads could be as high as 40,000 psi. If flexural cracking is indeed proportional to steel tensile stress, then it is quite evident that the criteria for crack control must be included in the design process.

Early investigations of crack width in beams and members subject to axial tension indicated that crack width was proportional to steel stress and bar diameter, but was inversely proportional to reinforcement percentage. More recent research using deformed bars has confirmed that crack width is proportional to steel stress. However, other variables such as the quality of concrete and concrete cover were also found to be important. It should be kept in mind that there are large variations in crack widths, even in careful laboratory-controlled work.

For this reason, only a simple crack control expression, designed to give reasonable reinforcing details that are in accord with laboratory work and practical experience, is presented in the code.

10.6 BEAMS AND ONE-WAY SLABS

10.6.4 Distribution of Tension Reinforcement

There are three perceived reasons that were identified early on for limiting the crack widths in concrete. These are appearance, corrosion, and water tightness. The three seldom apply simultaneously in a particular structure. Appearance is important for concrete exposed to view such as wall panels. Corrosion is important for concrete exposed to aggressive environments. Water tightness may be required for marine/sanitary structures. Appearance requires limiting of crack widths on the surface. This can be ensured by locating the reinforcement as close as possible to the surface (by using small cover) to prevent cracks from widening. Corrosion control, on the other hand, is obtained by using better quality concrete and by increasing the thickness of concrete cover. Water tightness requires severe limits on crack widths, applicable only to specialty structures. Thus, it should be recognized that a single provision, such as Eq. (10-4) of this code, may not be sufficient to address the control of cracking for all the three different reasons of appearance, corrosion, and water tightness.

There is a strong correlation between surface crack width and cover d_c, as shown in Fig. 9-1. For a particular magnitude of strain in the steel, the larger the cover, the larger will be the surface crack width affecting the appearance. From 1971 through 1995, the code specified limiting of z-factors based on the concept that the width of surface cracks needs to be limited. The specified values of $z = 175$ and 145 kips/in. for interior and exterior exposures, respectively, corresponded to the limiting crack widths of 0.016 and 0.013 in. It was assumed that by limiting the crack width to these values, one would achieve corrosion protection. But in order to comply with the specified z-value limits, the method essentially encouraged reduction of the reinforcement cover, which could be detrimental to corrosion protection. Furthermore, the method severely penalized structures with covers more than 2 in. by either reducing the spacing or the service load stress of the reinforcement.

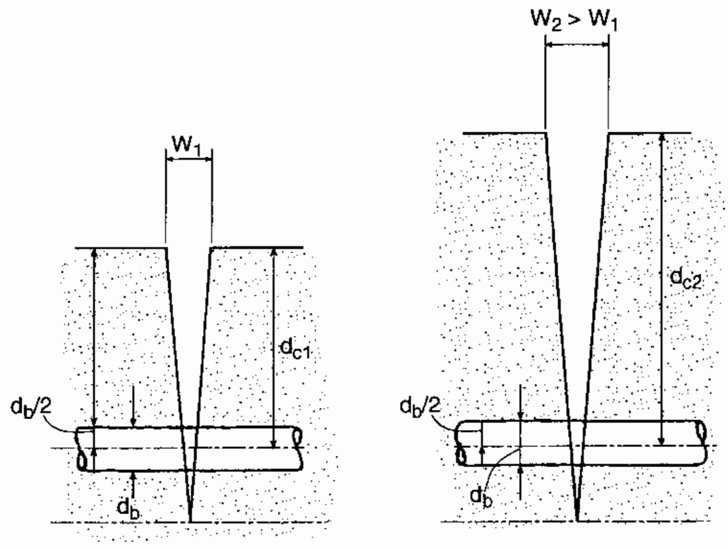


Figure 9-1 Crack Width for Different Cover Thicknesses.

The role of cracks in the corrosion of reinforcement has been found to be controversial. Research [9.1 & 9.2] shows that corrosion is not clearly correlated with surface crack widths in the range normally found with reinforcement stresses at service load level. In fact, it is weakly related to the earlier codes' surface crack width limits of 0.013 to 0.016 in. Further, it has been found that actual crack widths in structures are highly variable. A scatter of the order of ± 50% is observed. This prompted investigation of altermatives to the z factor limits for exterior and interior exposure, as given in the 1995 and earlier editions of the code.

Addressing some of the limitations of the previous approach, a simple and more practical equation has been adopted starting with the 1999 code, which directly limits the maximum reinforcement spacing. The new method is intended to control surface cracks to a width that is generally acceptable in practice but may vary widely in a given structure. The new method, for this reason, does not purport to predict crack widths in the field. According to the new method, the spacing of reinforcement closest to a tension surface shall not exceed that given by

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \qquad \textit{Eq. (10-4)}$$

but not greater than $12(40{,}000 / f_s)$

where $s =$ center-to-center spacing of flexural tension reinforcement nearest to the extreme tension face, in. (where there is only one bar or wire nearest to the extreme tension face, s is the width of the extreme tension face).

$f_s =$ calculated stress (psi) in reinforcement at service load computed as the unfactored moment divided by the product of steel area and internal moment arm. It is permitted to take f_s as $2/3\ f_y$.

$c_c =$ clear cover from the nearest surface in tension to the surface of flexural tension reinforcement, in.

Note, in the 1999 and 2002, codes, the default steel stress at service load was $0.6f_y$. To recognize the increase in service load stress level in the flexural reinforcement resulting from the use of the load combinations introduced in the 2002 code, the default steel stess used in (Eq. 10-4) was adjusted in 2005 by increasing it from $0{,}6f_y$, to $(2/3)f_y$. Note also that contrary to the 1995 provision, this spacing is independent of the exposure condition.

For the usual case of beams with Grade 60 reinforcement with 2 in. clear cover to the tension face and assuming $f_s = 2/3(60{,}000) = 40{,}000$ psi, the maximum bar spacing is 10 in. Using the upper limit of Eq. (10-4), the maximum spacing allowed, irrespective of the cover, is 12 in. for $f_s = 40{,}000$ psi. The spacing limitation is independent of the bar size used. Thus for a required amount of flexural reinforcement, this approach would encourage use of smaller bar sizes to satisfy the spacing criteria of Eq. (10-4).

Although Eq. (10-4) is easy to solve, it is convenient to have a table showing maximum spacing of reinforcement for various amounts of clear cover and different service level steel stress f_s. (see Table 9-1 below).

Table 9-1 Maximum Spacing of Reinforcement

	Clear Cover (in.)							
Steel Stress, f_s, (psi)	3/4	1	1-1/4	1-1/2	1-3/4	2	2-1/2	3
30,000	16	16	16	16	15.63	15	13.75	12.5
40,000	12	12	11.88	11.25	10.63	10	8.75	7.5

* Note, maximum reinforcement spacing is 18 in. (7.6.5, 7.12.2.2, 10.5.4, 14.3.5)

10.6.5 Corrosive Environments

As described under 10.6.4, data are not available regarding crack width beyond which a danger of corrosion exists. Exposure tests indicate that concrete quality, adequate compaction, and ample cover may be of greater importance for corrosion protection than crack width at the concrete surface. The requirements of 10.6.4 do not apply to structures subject to very aggressive exposure or designed to be watertight. Special precautions are required and must be investigated for such cases.

10.6.6 Distribution of Tension Reinforcement in Flanges of T-Beams

For control of flexural cracking in the flanges of T-beams, the flexural tension reinforcement must be distributed over a flange width not exceeding the effective flange width (8.10) or 1/10 of the span, whichever is smaller. If the effective flange width is greater than 1/10 the span, some additional longitudinal reinforcement, as illustrated in Fig. 9-2, must be provided in the outer portions of the flange (see Example 9.2).

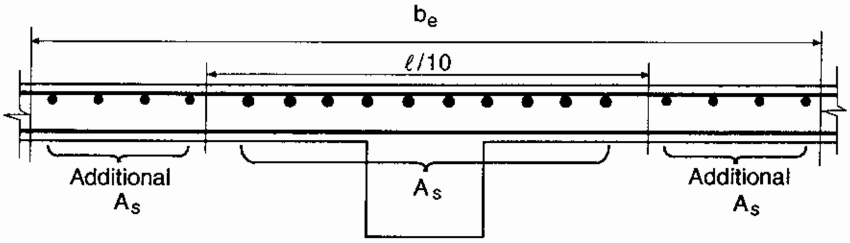
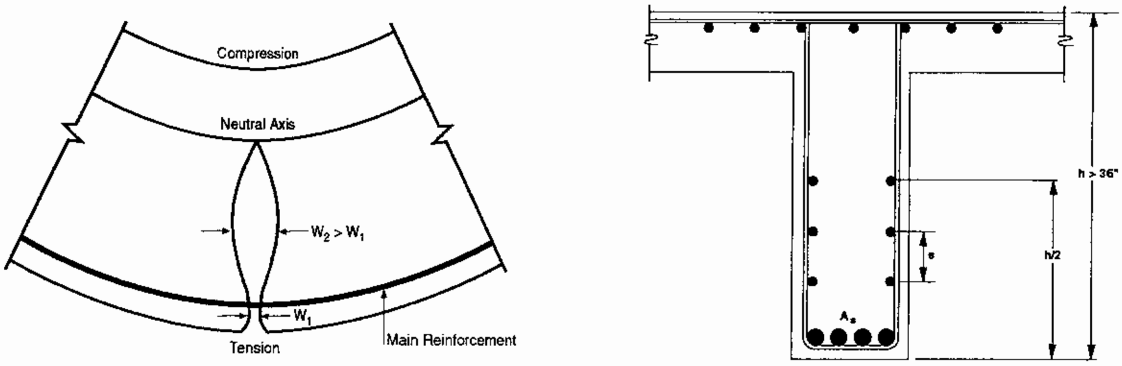


Figure 9-2 Negative Moment Reinforcement for Flanged Floor Beams

10.6.7 Crack Control Reinforcement in Deep Flexural Members

In the past, several cases of wide cracks developing on side faces of deep beams between the main reinforcement and neutral axis (Fig. 9-3(a)) have been observed. These cracks are attributed to the absence of any skin reinforcement, as a result of which cracks in the web widen more as compared to the cracks at the level of flexural tension reinforcement (Fig. 9-3(a)). For flexural members with overall height h exceeding 36 in, the code requires that additional longitudinal skin reinforcement for crack control must be distributed along the side faces of the member. The skin reinforcement must be extended for a distance h/2 from the tension face of the member. The vertical spacing s of the skin reinforcement is computed from 10.6.4 (Eq. 10-4). The code does not specify the size of the skin reinforcement. Research [Ref. 9.3] has shown that control of side face cracking can be achieved through proper spacing of the skin reinforcement for selected cover dimension. The research also confirmed that the reinforcement spacing requirements in Section 10.6.4 are sufficient to control side face cracking. Research has shown that the spacing rather than bar size is of primary importance [Ref. 9.3]. Typically No. 3 to No. 5 bars (or welded wire reinforcement with minimum area of 0.1 in.^{2} per foot of depth) is provided.



a) Side Face Cracking (Exaggerated)   b) Crack Control "Skin Reinforcement for Deep Beams

Figure 9-3 Skin Reinforcement

Note that the provisions of 10.6 do not directly apply to prestressed concrete members, as the behavior of a prestressed member is considerably different from that of a nonprestressed member. Requirements for proper distribution of reinforcement in prestressed members are given in Chapter 18 of the code and Part 24 of this book.

13.4 TWO-WAY SLABS

Control of flexural cracking in two-way slabs, including flat plates and flat slabs, is usually not a problem, and is not specifically covered in the code. However, 13.3.2 restricts spacing of slab reinforcement at critical moment sections to 2 times the slab thickness, and the area of reinforcement in each direction for two-way slab systems must not be less than that required for shrinkage and temperature (7.12). These limitations are intended in part to control cracking. Also, the minimum thickness requirements for two-way construction for deflection control (9.5.3) indirectly serve as a control on excessive cracking.

REFERENCES

9.1 Darwin, David et al, "Debate: Crack Width, Cover and Corrosion," *Concrete International,* Vol. 7, No. 5, May 1985, American Concrete Institute, Farmington Hills, MI, pp. 20-35.

9.2 Oesterle, R.G., "The Role of Concrete Cover in Crack Control Crieria and Corrosion Protection," RD Serial No. 2054, Portland Cement Association, Skokie, IL, 1997.

9.3 Frosch, R.J., "Modeling and Control of Side Face Beam Cracking,," *ACI Structural Journal,* Vol 99, No. 3, May–June 2002, pp. 376-385.

Example 9.1—Distribution of Reinforcement for Effective Crack Control

Assume a 16 in. wide beam with A_s (required) = 3.00 in.^{2}, and f_y = 60,000 psi. Select various bar arrangements to satisfy Eq. (10-4) for control of flexural cracking.

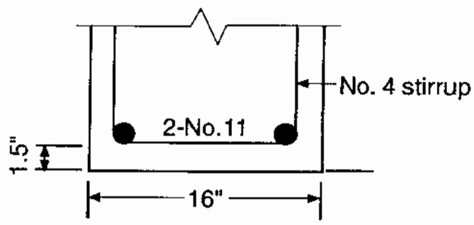
Calculations and Discussion	Code Reference

1. For 2-No. 11 bars (A_s = 3.12 in.^{2})

   c_c = 1.5 + 0.5 = 2.0 in. (No. 4 stirrup)

   use f_s = 2/3 f_y = 40 ksi

   Maximum spacing allowed,



$$s = 15\left(\frac{40{,}000}{40{,}000}\right) - (2.5 \times 2.0) = 10 \text{ in. Eq. (10-4)}$$

$$12(40{,}000/40{,}000) = 12 \text{ in.} > 10 \text{ in.}$$

$$\text{spacing provided} = 16 - 2\left(1.5 + 0.5 + \frac{1.41}{2}\right)$$

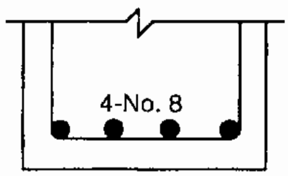
$$= 10.6 \text{ in.} > 10 \text{ in.} \quad \text{N.G.}$$

2. For 4-No. 8 bars (A_s = 3.16 in.^{2})

   c_c = 2.0 in. (No. 4 stirrup)

   Maximum spacing allowed,

   s = 10 in. [Eq. (10-4)]



$$\text{spacing provided} = \frac{1}{3}\left[16 - 2\left(1.5 + 0.5 + \frac{1.0}{2}\right)\right]$$

$$= 3.7 \text{ in.} < 10 \text{ in.} \quad \text{O.K.}$$

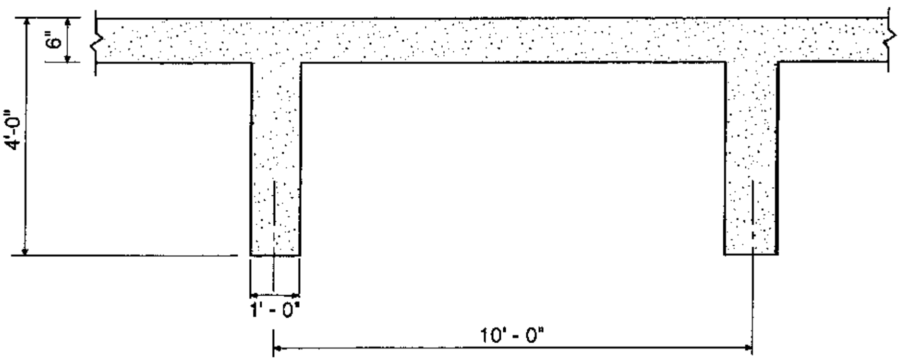
Example 9.2—Distribution of Reinforcement in Deep Flexural Member with Flanges

Select reinforcement for the T-section shown below.

Span: 50 ft continuous $f'_c = 4000$ psi
$f_y = 60{,}000$ psi

Service load moments:

Positive Moment	Negative Moment
$M_d = +265$ ft-kips	$M_d = -280$ ft-kips
$M_\ell = +680$ ft-kips	$M_\ell = -750$ ft-kips



Calculations and Discussion	**Code Reference**

1. Distribution of positive moment reinforcement

a. $M_u = 1.2\,(265) + 1.6(680) = 1406$ ft-kips — *Eq. (9-2)*

Assuming 2 layers of No. 11 bars with 1.5 in. clear cover and No. 4 stirrups,

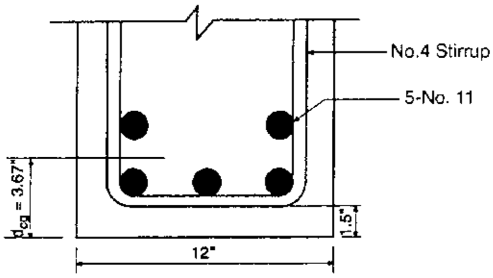
$$d_{cg} = \frac{(3 \times 1.56)\,(2.71) + (2 \times 1.56)\,(5.12)}{(5 \times 1.56)} = 3.67 \text{ in.}$$

$d = 48 - 3.67 = 44.3$ in.

Effective width = 108 in. — *8.10.2*

A_s required = 7.18 in.^{2}

Try 5-No. 11 (A_s = 7.80 in.^{2})



b. Clear cover to the tension reinforcement

$c_c = 1.5 + 0.5 = 2.0$ in. *10.0*

Stress in reinforcement at service load: *10.6.4*

$$f_s = \frac{+M}{jdA_s} = \frac{(265 + 680)\,12}{0.87 \times 44.3 \times 7.80} = 37.7 \text{ ksi}$$

$$s = 15\left(\frac{40{,}000}{37{,}700}\right) - 2.5\,c_c$$ *Eq. (10-4)*

$$= \frac{540}{37.7} - (2.5 \times 2) = 10.9 \text{ in.}$$

$$12\left(\frac{40}{f_s}\right) = 12\left(\frac{40}{37.7}\right)$$

$$= 12.7 \text{ in.} > 10.9 \text{ in.} \quad \text{O.K.}$$

$$\text{Spacing provided} = \frac{1}{2}\left[12 - 2\left(1.5 + 0.5 + \frac{1.41}{2}\right)\right]$$

$$= 3.3 \text{ in.} < 10.9 \text{ in.} \quad \text{O.K.}$$

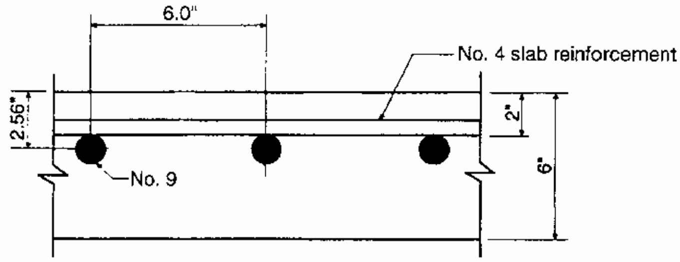
2. Distribution of negative moment reinforcement

a. M_u = 1.2 (280) + 1.6 (750) = 1536 ft-kips

A_s required = 8.76 in.^{2}

Effective width for tension reinforcement = 1/10 × 50 × 12 = 60 in. < 108 in. *10.6.6*

Try 9-No. 9 bars @ ≈ 10 in. (A_s = 9.0 in.^{2})



b. $c_c = 2.0$ in.

In lieu of computations for f_s at service load, use $f_s = 2/3f_y$ as permitted in 10.6.4

Maximum spacing allowed,

$$s = 15\left(\frac{40{,}000}{40{,}000}\right) - (2.5 \times 2.0) = 10 \text{ in.} = 10 \text{ in.} \quad \text{O.K.}$$ *Eq. (10-4)*

c. Longitudinal reinforcement in slab outside 60-in. width. *10.6.6*

For crack control outside the 60-in. width, use shrinkage and temperature reinforcement according to 7.12. *7.12*

For Grade 60 reinforcement, $A_s = 0.0018 \times 12 \times 6 = 0.130$ in.^{2}/ft

Use No. 4 bars @ 18 in. ($A_s = 0.133$ in.^{2}/ft)

3. Skin reinforcement (h > 36 in.) *10.6.7*

The spacing of the skin reinforcement is provided according to equation 10-4. The clear cover of the skin reinforcement is the same as the tension reinforcement; therefore the maximum allowed spacing of the skin reinforcement is 10 in.

Use 3-No. 3 bars uniformly spaced along each face of the beam extending a distance > h/2 beyond the bottom surface of the beam.

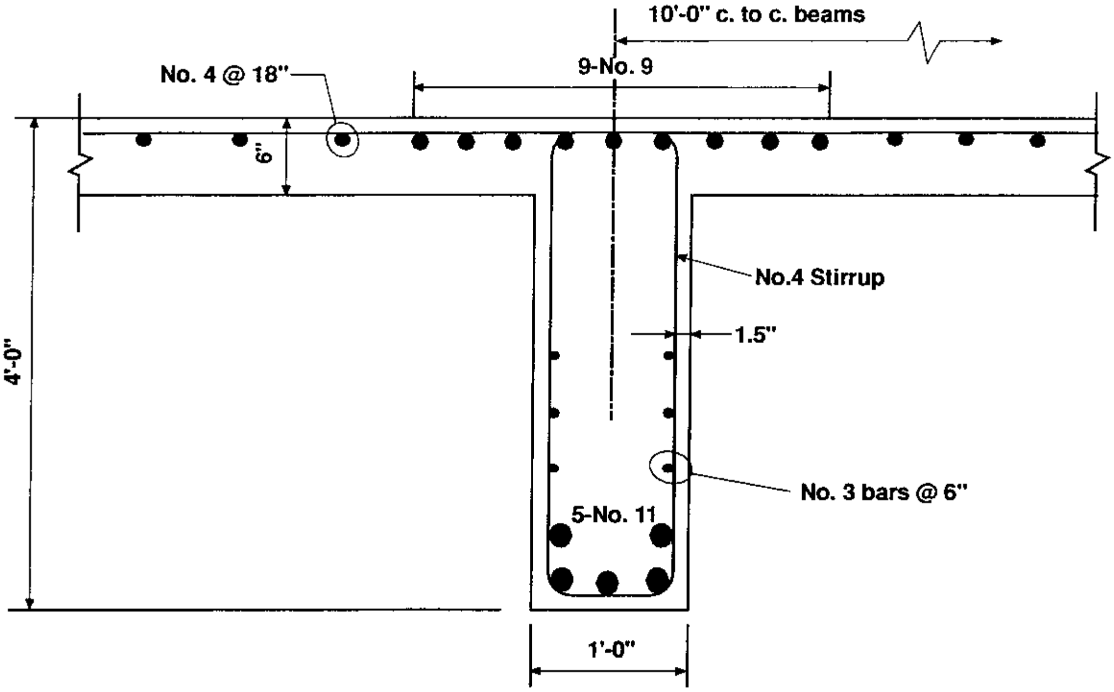
Spacing of the skin reinforcement:

$s = (24 - 1.5 - 0.5 - 1.41 - 1 - 1.41/2)/3 = 6.3$ in. < 10 in. OK

Use skin reinforcement at a spacing of 6.0 in.

Similarly, provide No. 3 @ 6.0 in. in the upper half of the depth in the negative moment region.

4. Detail section as shown below.



10

Deflections

GENERAL CONSIDERATIONS

The ACI code provisions for control of deflections are concerned only with deflections that occur at service load levels under static conditions and may not apply to loads with strong dynamic characteristics such as those due to earthquakes, transient winds, and vibration of machinery. Because of the variability of concrete structural deformations, designers must not place undue reliance on computed estimates of deflections. In most cases, the use of relatively simple procedures for estimating deflections is justified. In-depth treatments of the subject of deflection control, including more refined methods for computing deformations, may be found in Refs. 10.1 and 10.2.

9.5 CONTROL OF DEFLECTIONS

Two methods are given in the code for controlling deflections of one-way and two-way flexural members. Deflections may be controlled directly by limiting computed deflections [see Table 9.5(b)] or indirectly by means of minimum thickness [Table 9.5(a) for one-way systems, and Table 9.5(c) and Eqs. (9-12) and (9-13) for two-way systems.]

9.5.2.1 Minimum Thickness for Beams and One-Way Slabs (Nonprestressed)—Deflections of beams and one-way slabs supporting loads commonly experienced in buildings will normally be satisfactory when the minimum thickness from Table 9.5(a) (reproduced in Table 10-1) are met or exceeded.

The designer should especially note that this minimum thickness requirement is intended only for members **not** supporting or attached to partitions or other construction likely to be damaged by large deflections. For all other members, deflections need to be computed.

9.5.2.2 Immediate Deflection of Beams and One-Way Slabs (Nonprestressed)—Initial or short-term deflections of beams and one-way slabs occur immediately on the application of load to a structural member. The principal factors that affect the immediate deflection (see Ref. 10.3) of a member are:

a. magnitude and distribution of load,
b. span and restraint condition,
c. section properties and steel percentage,
d. material properties, and
e. amount and extent of flexural cracking.

Table 8-1 Minimum Thickness for Nonprestressed Beams and One-Way Slabs (Grade 60 Reinforcement and Normal Weight Concrete)

Member	Minimum Thickness, h			Cantilever
	Simply Supported	One End Continuous	Both Ends Continuous	
One-Way Slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

(1) For f_y other than 60,000 psi, multiply by tabulated values by $(0.4 + f_y/100{,}000)$ e.g., for grade 40 reinforcement, multiply values by 0.80

(2) For structural lightweight concrete, multiply tabulated values by $(1.65 - 0.005w_c)$ but not less than 1.09, where w_c is the unit weight in lb per cu ft.

The following concrete properties strongly influence the behavior of reinforced flexural members under short-time loads: compressive strength (f'_c), modulus of elasticity (E_c) and modulus of rupture (f_r). The modulus of elasticity particularly shows more variation with concrete quality, concrete age, stress level, and rate or duration of load.

The idealized short-term deflection of a typical reinforced concrete beam is shown in Fig. 10-1. There are two distinct phases of behavior: (i) uncracked behavior, when the applied moment (M_a) is less than the cracking moment (M_{cr}); and (ii) cracked behavior, when the applied moment (M_a) is greater than the cracking moment (M_{cr}). Two different values for the moment of inertia would therefore be used for calculating the deflections: the gross moment of inertia (I_g) for the uncracked section, and the reduced moment of inertia for the cracked section (I_{cr}).

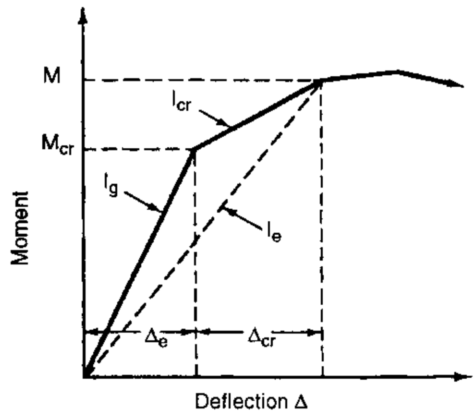


Figure 10-1 Bilinear Moment-Deflection Relationship[10.4]

For the uncracked rectangular beam shown in Fig. 10-2, the gross moment of inertia is used ($I_g = bh^3/12$). The moment of inertia of a cracked beam with tension reinforcement (I_{cr}) is computed in the following manner:

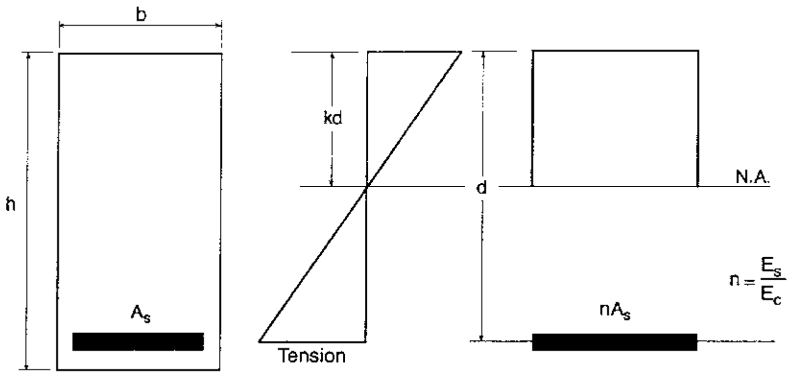


Figure 10-2 Cracked Transformed Section of Singly Reinforced Beam

Taking moment of areas about the neutral axis

$$b \times kd \times \frac{kd}{2} = nA_s\,(d - kd)$$

use $$B = \frac{b}{nA_s}$$

$$kd = \frac{\sqrt{2Bd + 1} - 1}{B}$$

Moment of inertia of cracked section about neutral axis,

$$I_{cr} = \frac{b\,(kd)^3}{3} + nA_s\,(d - kd)^2$$

Expressions for computing the cracked moment of inertia for sections with compression reinforcement and flanged sections, which are determined in a similar manner, are given in Table 10-2.

9.5.2.3, 9.5.2.4 Effective Moment of Inertia for Beams and One-Way Slabs (Nonprestressed)—The flexural rigidity EI of a beam may not be constant along its length because of varying amounts of steel and cracking at different sections along the beam. This, and other material related sources of variability, makes the exact prediction of deflection difficult in practice.

The effective moment of inertia of cantilevers, simple beams, and continuous beams between inflection points is given by

$$I_e = (M_{cr}/M_a)^3\,I_g + [1 - (M_{cr}/M_a)^3]\,I_{cr} \le I_g \qquad \textit{Eq. (9-8)}$$

where $$M_{cr} = f_r\,I_g/y_t \qquad \textit{Eq. (9-9)}$$

M_a = maximum service load moment (unfactored) at the stage for which deflections are being considered

$$f_r = 7.5\sqrt{f'_c} \text{ for normal weight concrete} \qquad \textit{Eq. (9-10)}$$

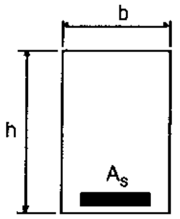
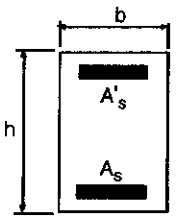
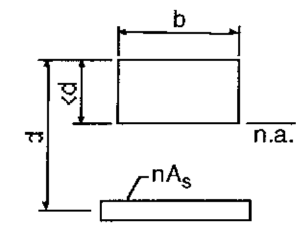
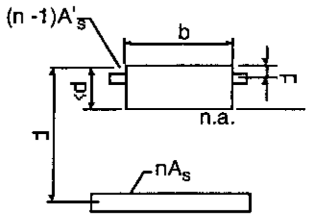
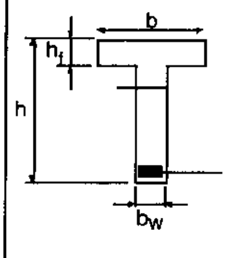
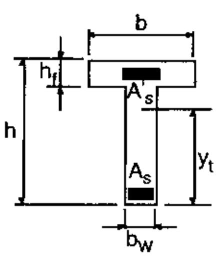
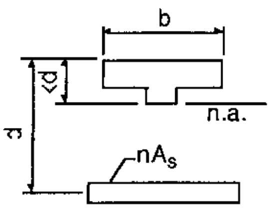
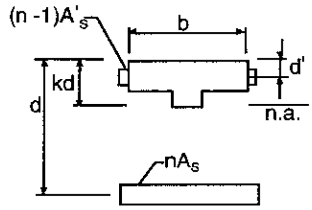
For lightweight concrete, f_r is modified according to 9.5.2.3.

The effective moment of inertia I_e provides a transition between the well-defined upper and lower bounds of I_g and

I_{cr} as a function of the level of cracking represented by M_a/M_{cr}. The equation empirically accounts for the effect of tension stiffening—the contribution of uncracked concrete between cracks in regions of low tensile stress.

For each load combination being considered, such as dead load or dead plus live load, deflections should be calculated using an effective moment of inertia [Eq. (9-8)] computed with the appropriate service load moment, M_a. The incremental deflection caused by the addition of load, such as live load, is then computed as the difference between deflections computed for any two load combinations.

Table 10-2 Gross and Cracked Moment of Inertia of Rectangular and Flanged Section

Gross Section	Cracked Transformed Section	Gross and Cracked Moment of Inertia
 	 Without compression steel  With compression steel	$n = \frac{E_s}{E_c}$ $B = \frac{b}{(nA_s)}$ $I_g = \frac{bh^3}{12}$ Without compression steel $kd = \left(\sqrt{2dB+1}-1\right)/B$ $I_{cr} = b(kd)^3/3 + nA_s(d-kd)^2$ With compression steel $r = (n-1)A'_s/(nA_s)$ $kd = \left[\sqrt{2dB(1+rd'/d)+(1+r)^2}-(1+r)\right]/B$ $I_{cr} = b(kd)^3/3 + nA_s(d-kd)^2 + (n-1)A'_s(kd-d')^2$
 	 Without compression steel  With compression steel	$n = \frac{E_s}{E_c}$ $C = b_w/(nA_s),\ f = h_f(b-b_w)/(nA_s),$ $y_t = h - 1/2\left[(b-b_w)h_f^2 + b_wh^2\right]/\left[(b-b_w)h_f + b_wh\right]$ $I_g = (b-b_w)h_f^3/12 + b_wh^3/12 + (b-b_w)h_f(h-h_f/2-y_t)^2 +$ $b_wh(y_t-h/2)^2$ Without compression steel $kd = \left[\sqrt{C(2d+h_ff)+(1+f)^2}-(1+f)\right]/C$ $I_{cr} = (b-b_w)h_f^3/12 + b_w(kd)^3/3 + (b-b_w)h_f(kd-h_f/2)^2 +$ $nA_s(d-kd)^2$ With compression steel $kd = \left[\sqrt{C(2d+h_ff+2rd')+(f+r+1)^2}-(f+r+1)\right]/C$ $I_{cr} = (b-b_w)h_f^3/12 + b_w(kd)^3/3 + (b-b_w)h_f(kd-h_f/2)^2 +$ $nA_s(d-kd)^2 + (n-1)A'_s(kd-d')^2$

For prismatic members (including T-beams with different cracked sections in positive and negative moment regions), I_e may be determined at the support section for cantilevers and at the midspan section for simple and continuous spans. The use of the midspan section properties for continuous prismatic members is considered satisfactory in approximate calculations primarily because the midspan rigidity has the dominant effect on deflections. Alternatively, for continuous prismatic and nonprismatic members, 9.5.2.4 suggests using the average I_e at the critical positive and negative moment sections. The '83 commentary on 9.5.2.4 suggested the following approach to obtain improved results:

Beams with one end continuous:

$$\text{Avg. } I_e = 0.85 I_m + 0.15 (I_{cont.end}) \tag{1}$$

Beams with both ends continuous:

$$\text{Avg. } I_e = 0.70 I_m + 0.15 (I_{e1} + I_{e2}) \tag{2}$$

where I_m refers to I_e at the midspan section

I_{e1} and I_{e2} refer to I_e at the respective beam ends.

Moment envelopes based on the approximate moment coefficients of 8.3.3 are accurate enough to be used in computing both positive and negative values of I_e (see Example 10.2). For a single heavy concentrated load, only the midspan I_e should be used.

The initial or short-term deflection (Δ_i) for cantilevers and simple and continuous beams may be computed using the following elastic equation given in the '83 commentary on 9.5.2.4. For continuous beams, the midspan deflection may usually be used as an approximation of the maximum deflection.

$$\Delta_i = K (5/48) M_a \ell^2 / E_c I_e \tag{3}$$

where M_a is the support moment for cantilevers and the midspan moment (when K is so defined) for simple and continuous beams

ℓ is the span length as defined in 8.7.

For uniformly distributed loading w, the theoretical values of the deflection coefficient K are shown in Table 10-3.

Since deflections are logically computed for a given continuous span based on the same loading pattern as for maximum positive moment, Eq. (3) is thought to be the most convenient form for a deflection equation.

9.5.2.5 Long-Term Deflection of Beams and One-Way Slabs (Nonprestressed)—Beams and one-way slabs subjected to sustained loads experience long-term deflections. These deflections may be two to three times as large as the immediate elastic deflection that occurs when the sustained load is applied. The long-term deflection is caused by the effects of shrinkage and creep, the formation of new cracks and the widening of earlier cracks. The principal factors that affect long-term deflections (see Ref. 10.3) are:

a. stresses in concrete
b. amount of tensile and compressive reinforcement
c. member size
d. curing conditions
e. temperature
f. relative humidity
g. age of concrete at the time of loading
h. duration of loading

Table 10-3 Deflection Coefficient K

		K
1.	Cantilevers (deflection due to rotation at supports not included)	2.40
2.	Simple beams	1.0
3.	Continuous beams	$1.2\text{-}0.2\,M_o/M_a$
4.	Fixed-hinged beams (midspan deflection)	0.80
5.	Fixed-hinged beams (maximum deflection using maximum moment)	0.74
6.	Fixed-fixed beams	0.60

For other types of loading, K values are given in Ref. 8.2.

M_o = Simple span moment at midspan $\left(\frac{w\ell^2}{8}\right)$

M_a = Net midspan moment.

The effects of shrinkage and creep must be approximated because the strain and stress distribution varies across the depth and along the span of the beam. The concrete properties (strength, modulus of elasticity, shrinkage and creep) also vary with mix composition, curing conditions and time. Two approximate methods for estimating long-term deflection appear below.

ACI 318 Method

According to 9.5.2.5, additional long-term deflections due to the combined effects of shrinkage and creep from sustained loads $\Delta_{(cp+sh)}$ may be estimated by multiplying the immediate deflection caused by the sustained load $(\Delta_i)_{sus}$ by the factor λ_Δ; i.e.

$$\Delta_{(cp+sh)} = \lambda(\Delta_i)_{sus} \qquad (4)$$

where $$\lambda_\Delta = \frac{\xi}{1+50\rho'} \qquad \text{Eq. (9-11)}$$

Values for ξ are given in Table 10-4 for different durations of sustained load. Figure R9.5.2.5 in the commentary to the code shows the variation of ξ for periods up to 5 years. The compression steel $\rho' = A_s'/bd$ is computed at the support section for cantilevers and the midspan section for simple and continuous spans. Note that sustained loads include dead load and that portion of live load that is sustained. See R9.5.1.

Table 10-4 Time-Dependent Factor ξ (9.5.2.5)

Sustained Load Duration	ξ
5 years and more	2.0
12 months	1.4
6 months	1.2
3 months	1.0

Alternate Method

Alternatively, creep and shrinkage deflections may be computed separately using the following expressions from Refs. 10.2, 10.5, and 10.6. The procedure is summarized in Section 2.6.2 of Ref. 10.4.

$$\Delta_{cp} = \lambda_{cp} (\Delta_i)_{sus} \tag{5}$$

$$\Delta_{sh} = K_{sh}\phi_{sh}\ell^2 \tag{6}$$

where

$\lambda_{cp} = k_r C_t;$

$k_r = 0.85/(1 + 50\rho')$

C_t = time dependent creep coefficient (Table 2.1 or Eq. 2.7 of Ref. 10.4)

K_{sh} = shrinkage deflection constant (Table 10-5)

$\phi_{sh} = A_{sh}(\varepsilon_{sh})_t / h$

A_{sh} = shrinkage deflection multiplier (Figure 10-3 or Eq. 6.1 below)

$(\varepsilon_{sh})_t$ = time dependent shrinkage strain (Table 2.1 or Eq. 2.8 & 2.9 of Ref. 10.4)

ℓ = beam span length

h = beam depth

The ultimate value of the creep coefficient C_t, denoted as C_u, is dependent on the factors a through h listed above. Likewise, the ultimate value of the time dependent shrinkage strain depends on the varying conditions and is designated $(\varepsilon_{sh})_u$. Typical values for the two properties are discussed in Section 2.3.4 of ACI 435 (Ref. 10.4).

In Ref. 10.4, the ultimate creep coefficient is dependent on six factors:

a. relative humidity
b. age of concrete at load application
c. minimum member dimension
d. concrete consistency
e. fine aggregate content
f. air content

Standard conditions for these six variables are 40% R.H., 3 days (steam cured) or 7 days (moist cured), 6 in. least dimension, 3 in. slump, 50% fine aggregate and 6% air content. For the case of standard conditions, C_u is equal to 2.35. Correction factors are presented in Fig. 2.1 of Ref. 10.4, to adjust the value of C_u for non-standard conditions.

Two variations from standard conditions that might be encountered in normal construction are for relative humidity of 70% and load application taking place at an age of 20 days. The correction factor for the relative humidity is given by the following:

$$K_h^c = 1.27 - 0.0067H$$

where H is the relative humidity in percent. For the case of 70% relative humidity,

$$K_h^c = 1.27 - 0.0067(70) = 0.80$$

Correction for the time of load application is given in the following two expressions for steam or moist curing conditions:

$$K_{to}^c = 1.13(t^{-0.095}) \quad \text{(Steam Cured)}$$

$$K_{to}^c = 1.25(t^{-0.118}) \quad \text{(Moist Cured)}$$

where t is the age of load application in days. For t = 20 days the two equations give 0.85 and 0.88 respectively. The average is 0.865.

If it is assumed that all other conditions remain constant the ultimate creep coefficient for the condition of 70% relative humidity and load application at 20 days becomes, according to the methodology indicated:

$$C_u = (0.80)(0.865)(2.35) = 1.63$$

By comparison, the value for C_u suggested in the 1978 edition of ACI 435, based on relative humidity of 70%, age at load application of 20 days and minimum dimension of 6 in. (the standard case) was $C_u = 1.60$.

An evaluation of ultimate creep strain can also be made. In Ref. 10.4 it is stated that $(\varepsilon_{sh})_u$ is dependent on a set of factors similar to those that affect the ultimate creep coefficient. In particular, the five conditions, and their standard values, are as follows:

a. relative humidity – 40%
b. minimum member dimension – 6 in.
c. fine aggregate content – 50%
d. cement content – 1200 kg/m^3
e. air content – 6%

For standard conditions, the ultimate shrinkage strain is 780 x 10^{-6}. Keeping all applicable conditions the same as used in evaluation of the ultimate creep and use of a cement factor of 6 bags per cubic yard (335 kg/m^3), calculation of the appropriate correction factors yields:

$$K_h^s = 1.4 - 0.01H = 1.4 - (0.01)(70) = 0.70 \quad \text{(relative humidity)}$$

$$K_b^s = 0.75 + 0.000214B = 0.75 + (0.000214)(335) = 0.82 \quad \text{(cement content)}$$

Application of the product of the two corrections to the standard value gives:

$$(\varepsilon_{sh})_u = (0.70)(0.82)(780 \times 10^{-6}) = 448 \times 10^{-6}$$

This value compares with 400 x 10^{-6} suggested in the 1978 edition of ACI 435.

In summary, an estimate of the values of C_u and $(\varepsilon_{sh})_u$ can be obtained for non-prestressed flexural members using the methodology presented in Section 2.3.4 of Ref. 10.4.

Once the ultimate values for creep and shrinkage are determined, the relationships between these ultimate values and the values at earlier times can be estimated by Eqs. 2.7, 2.8 and 2.9 of ACI 435R[10.4]. The expressions are reproduced below:

$$C_t = \left(\frac{t^{0.6}}{10 + t^{0.6}}\right) C_u \qquad \textit{Eq. (2.7) of ACI 435R}$$

Where t represents time, in days, after application of load.

For moist cured concrete, the shrinkage relationship is:

$$(\varepsilon_{sh})_t = \left(\frac{t}{35 + t}\right)(\varepsilon_{sh})_u \qquad \textit{Eq. (2.8) of ACI 435R}$$

(t is in days minus 7 after placement)

and for steam cured concrete:

$$(\varepsilon_{sh})_t = \left(\frac{t}{55 + t}\right)(\varepsilon_{sh})_u \qquad \textit{Eq. (2.9) of ACI 435R}$$

(t is in days minus 3 after placement)

Comparison of the values for the time dependent creep coefficients and shrinkage strains given in Table 2.1 of ACI 435R and those that result from Eqs. 2.7, 2.8 and 2.9 shows that the values obtained by the two methods vary slightly, particularly for the lower values of time, t. Since the calculation of deflections in concrete structures involves considerable approximation, the use of the time dependent quantities obtained either from the table or from the equations is considered acceptable.

A_{sh} may be taken directly from Fig. 10-3 or computed by the following set of equations which are given in Section 2.6.2 of ACI 435:

$$\begin{aligned} A_{sh} &= 0.7 \cdot (\rho - \rho')^{\frac{1}{3}} \cdot \left(\frac{\rho - \rho'}{\rho}\right)^{\frac{1}{2}} && \text{for } \rho - \rho' \le 3.0 \\ &= 0.7 \cdot \rho^{\frac{1}{3}} && \text{for } \rho' = 0 \qquad \textit{(6.1)} \\ &= 1.0 && \text{for } \rho - \rho' \ge 3.0 \end{aligned}$$

In the above equations, both ρ and ρ' are expressed in percent, not in decimal fraction as is usual. The ratios are also expressed in percent for determination of A_{sh} from Figure 10-3.

Values for the shrinkage deflection coefficient K_{sh} are given in Table 10-5, assuming equal positive and negative shrinkage curvatures with an inflection point at the quarter-point of continuous spans, which is generally satisfactory for deflection computation.

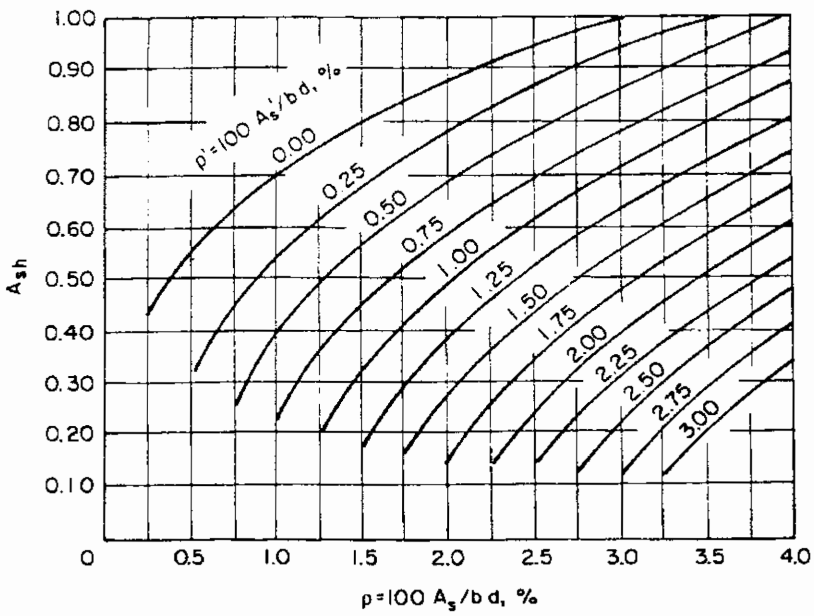


Figure 10-3 Values of A_{sh} for Calculating Shrinkage Deflection

Table 10-5 Shrinkage Deflection Coefficient K_{sh}

	K_{sh}
Cantilevers	0.50
Simple Spans	0.13
Spans with One End Continuous—Multi-Span Beams	0.09
Spans with One End Continuous—Two-Span Beams	0.08
Spans With Both Ends Continuous	0.07

The reinforcement ratios ρ and ρ′ used in determining A_{sh} from Fig. 10-3, refer to the support section of cantilevers and the midspan section of simple and continuous beams. For T-beams, use $\rho = 100\ (\rho + \rho_w)/2$ and a similar calculation for any compression steel ρ′ in determining A_{sh} , where $\rho_w = A_s/b_w d$. See Example 10.2.

As to the choice of computing creep and shrinkage deflections by Eq. (9-11) or separately by Eqs. (5) and (6), the combined ACI calculation is simpler but provides only a rough approximation, since shrinkage deflections are only indirectly related to the loading (primarily by means of the steel content). One case in which the separate calculation of creep and shrinkage deflections may be preferable is when part of the live load is considered as a sustained load.

All procedures and properties for computing creep and shrinkage deflections apply equally to normal weight and lightweight concrete.

9.5.2.6 Deflection Limits—Deflections computed using the preceding methods are compared to the limits given in Table 9.5(b). The commentary gives information for the correct application of these limits, including consideration of deflections occurring prior to installation of partitions.

9.5.3 Two-Way Construction (Nonprestressed)

Deflections of two-way slab systems with and without beams, drop panels, and column capitals need not be computed when the minimum thickness requirements of 9.5.3 are met. The minimum thickness requirements include the effects of panel location (interior or exterior), panel shape, span ratios, beams on panel edges, supporting columns and capitals, drop panels, and the yield strength of the reinforcing steel.

Table 10-6 Minimum Thickness of Slabs without Interior Beams (Table 9.5(c))

Yield strength, f_y psi*	Without drop panels†			With drop panels†		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams††		Without edge beams	With edge beams††	
40,000	$\frac{\ell_n}{33}$ **	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{40}$	$\frac{\ell_n}{40}$
60,000	$\frac{\ell_n}{30}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$
75,000	$\frac{\ell_n}{28}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{34}$	$\frac{\ell_n}{34}$

* *For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.*

** *For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.*

† *Drop panel is defined in 13.2.5.*

†† *Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.*

Section 9.5.3.2 provides minimum thickness requirements for two-way slab systems without beams between interior columns (flat plates and flat slabs). The minimum thickness is determined directly as a function of span length using Table 9.5(c). The section also provides minimum values for slabs with and without drop panels. The values given in Table 9.5(c) represent the upper limit of slab thicknesses given by Eqs. (9-12) and (9-13). The minimum thickness requirements of 9.5.3.2 are illustrated in Fig. 10-4.

Section 9.5.3.3 provides minimum thickness requirements for two-way slab systems with beams supporting all sides of a panel. It should be noted that these provisions are intended to apply only to two-way systems, that is, systems in which the ratio of long to short span is not greater than 2. For slabs that do not satisfy this limitation, Eqs. (9-12) and (9-13) may give unreasonable results. For such cases, 9.5.2 should be used.

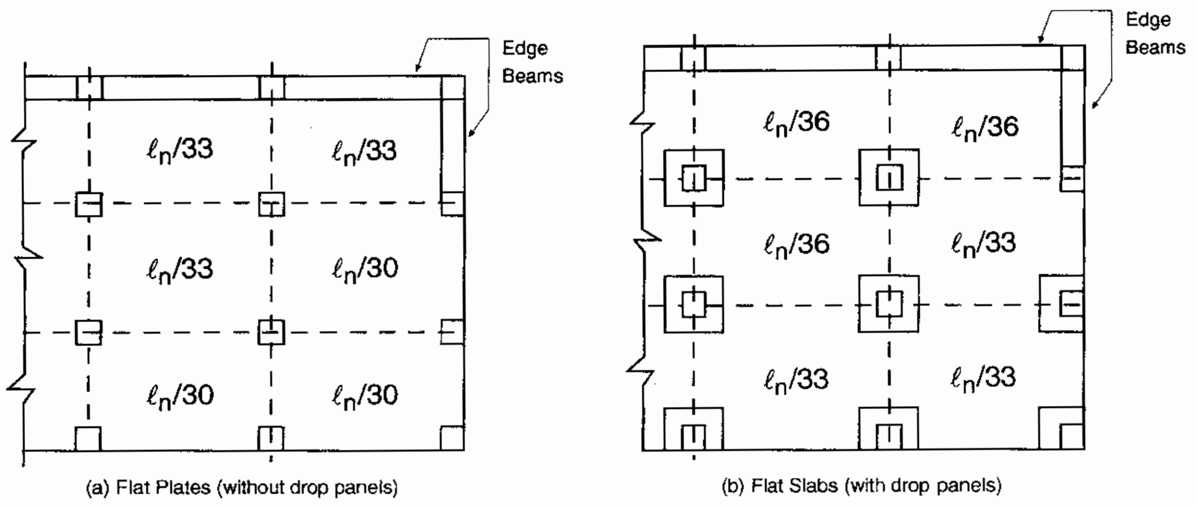


Figure 10-4 Minimum Thickness of Slabs without Interior Beams (Grade 60 Reinforcement)

Figure 10-5 may be used to simplify minimum thickness calculations for two-way slabs. It should be noted in Fig. 10-5 that the difference between the controlling minimum thickness for square panels and rectangular panels having a 2-to-1 panel side ratio is not large.

$$f = \left[\frac{36 + 5\beta(\alpha_{fm} - 0.2)}{0.8 + f_y / 200{,}000}\right] \quad \text{Eq. (9-12)}$$

$$f = \left[\frac{36 + 9\beta}{0.8 + f_y / 200{,}000}\right] \quad \text{Eq. (9-13)}$$

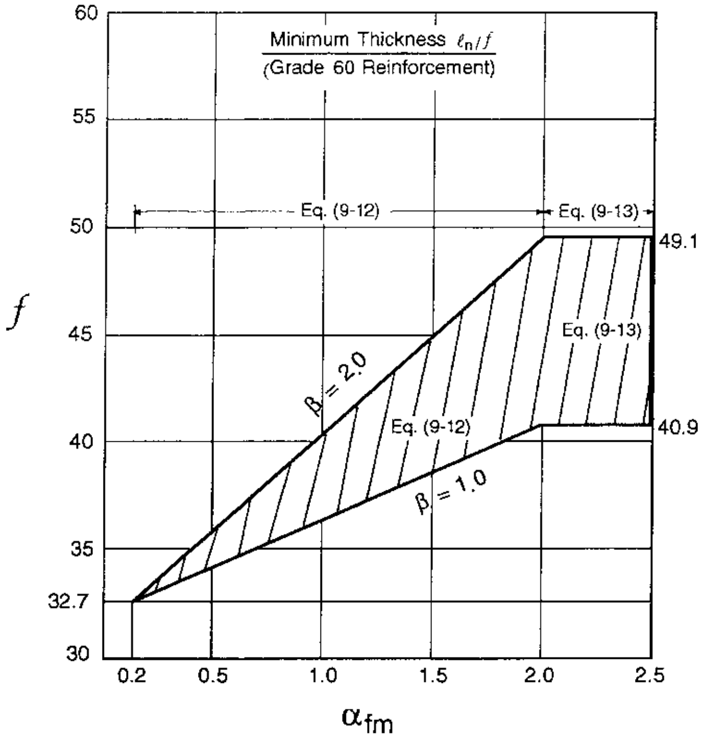


Figure 10-5 Minimum Thickness for Two-Way Beam Supported Slabs

9.5.3.4 Deflection of Nonprestressed Two-Way Slab Systems—

Initial or Short-Term Deflection: An approximate procedure[10.2, 10.7] that is compatible with the Direct Design and Equivalent Frame Methods of code Chapter 13 may be used to compute the initial or short-term deflection of two-way slab systems. The procedure is essentially the same for flat plates, flat slabs, and two-way beam-supported slabs, after the appropriate stiffnesses are computed. The midpanel deflection is computed as the sum of the deflection at midspan of the column strip or column line in one direction, Δ_{cx} or Δ_{cy}, and deflection at midspan of the middle strip in the orthogonal direction, Δ_{mx} or Δ_{my} (see Fig. 10-6). The column strip is the width on each side of column center line equal to 1/4 of the smaller panel dimension. The middle strip is the central portion of the panel which is bounded by two column strips.

For square panels,

$$\Delta = \Delta_{cx} + \Delta_{my} = \Delta_{cy} + \Delta_{mx} \qquad (7)$$

For rectangular panels, or for panels that have different properties in the two directions, the average Δ of the two directions is used:

$$\Delta = \left[\left(\Delta_{cx} + \Delta_{my}\right) + \left(\Delta_{cy} + \Delta_{mx}\right)\right]/2 \qquad (8)$$

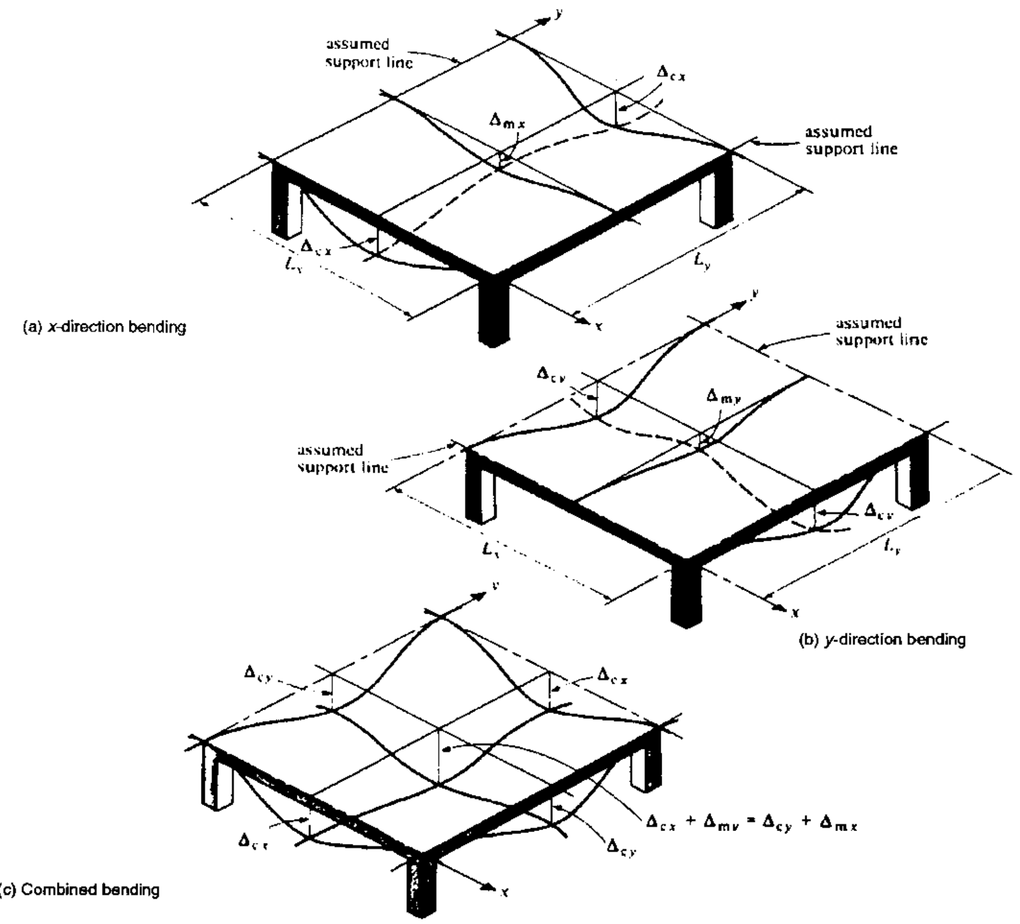


Figure 10-6 Basis for Equivalent Frame Method of Deflection Analysis of Two-Way Slab Systems, with or without Beams

The midspan deflection of the column strip or middle strip in an equivalent frame is computed as the sum of three parts: deflection of panel assumed fixed at both ends, plus deflection of panel due to the rotation at the two support lines. In the x direction, the deflections would be computed using the following expressions:

$$\Delta_{cx} = \text{Fixed}\,\Delta_{cx} + (\Delta\theta_1)_{cx} + (\Delta\theta_2)_{cx} \qquad \text{for column strip} \qquad (9)$$

$$\Delta_{mx} = \text{Fixed}\,\Delta_{mx} + (\Delta\theta_1)_{mx} + (\Delta\theta_2)_{mx} \qquad \text{for middle strip}$$

While these equations and the following discussion address only the computation of deflections in the x direction, similar computations to determine Δ_{cy} and Δ_{my} would be necessary to compute deflections in the y direction.

The first step in the process of computing Fixed Δ_{cx} and Fixed Δ_{mx} is to compute the midspan fixed-end deflection of the full-width equivalent frame under uniform loading, given by

$$\text{Fixed}\,\Delta_{frame} = \frac{w\ell^4}{384\,E_c I_{frame}} \qquad (10)$$

where $\quad w$ = load per unit area × full width

The effect of different stiffnesses in positive and negative moment regions [primarily when using drop panels and/or I_e in Eq. (9-8)] can be included by using an average moment of inertia as given by Eqs. (1) and (2).

The midspan fixed-end deflection of the column and middle strips is then computed by multiplying Fixed Δ_{frame} (Eq. (10)) by the M/EI ratio of the strips (column or middle) to the full-width frame.

$$\text{Fixed } \Delta_{c,m} = (LDF)_{c,m} \text{ Fixed } \Delta_{frame} \frac{(EI)_{frame}}{(EI)_{c,m}} \quad \text{for column or middle strip} \qquad (11)$$

where $\quad (LDF)_{c,m} = \dfrac{M_{c,m}}{M_{frame}}$ = lateral distribution factor

The distribution of the total factored static moment, M_o,, to the column and middle strips is prescribed in 13.6.3 and 13.6.4. In particular, 13.6.4.1, 13.6.4.2 and 13.6.4.4 provide tables which allocate fractions of M_o to the interior and exterior negative moment regions and the positive moment region, respectively, for column strips. The percent of the total not designated for the column strips is allocated to the middle strips. That is, for example, if 75 percent of M_o is designated for the interior negative moment of a column strip, the corresponding moment in the middle strip will be required to sustain 25 percent of M_o. The following expressions provide linear interpolation between the tabulated values given in 13.6.4.1, 13.6.4.2 and 13.6.4.4. Note that all expressions are given as percentages of M_o:

$$M^-_{ext} = 100 - 10\beta_t + 12\beta_t (\alpha_{f1}\ell_2/\ell_1)\ (1 - \ell_2/\ell_1) \qquad \text{(Exterior negative moment, \% } M_o)$$

$$M^-_{int} = 75 + 30(\alpha_{f1}\ell_2/\ell_1)\ (1 - \ell_2/\ell_1) \qquad \text{(Interior negative moment, \% } M_o)$$

$$M^+ = 60 + 30\ (\alpha_{f1}\ell_2/\ell_1)\ (1.5 - \ell_2/\ell_1) \qquad \text{(Positive moment, \% } M_o)$$

In application of the above expressions, if the actual value of $\alpha_{f1}\ell_2/\ell_1$ exceeds 1.0, the value 1.0 is used. Similarly, if β_t exceeds 2.5, the value 2.5 is used.

In order to calculate the lateral distribution factors (LDF), three cases should be considered:

a. strips for interior panels
b. strips in edge panels parallel to the edge
c. strips in edge panels perpendicular to the edge

Note that in corner panels, *Case c* is used for strips in either direction as there is an exterior negative moment at each outer panel edge. In all cases, the strip moment, used in determination of the LDFs, is taken as the average of the positive and negative moment. Thus, the following formulas are obtained for the three cases:

Case a: $LDF = \frac{1}{2}(M^-_{int} + M^+)$
Case b: $LDF = \frac{1}{2}(M^-_{int} + M^+)$
Case c: $LDF = \frac{1}{2}[\frac{1}{2}(M^-_{int} + M^-_{ext}) + M^+]$

These lateral distribution factors apply to column strips and are expressed in percentages of the total panel moment M_o. The corresponding factors for the middle strips are determined, in general, as follows:

$$LDF_{mid} = 100 - LDF_{col}$$

The remaining terms in Eq. (9), the midspan deflection of column strip or middle strip caused by rotations at the ends $\left((\Delta\theta_1)_{cx}, (\Delta\theta_1)_{mx}, \text{etc.}\right)$, must now be computed. If the ends of the column at the floor above and below are assumed fixed (usual case for an equivalent frame analysis) or ideally pinned, the rotation of the column at the floor in question is equal to the net applied moment divided by the stiffness of the equivalent column.

$$\theta_{frame} = \theta_c = \theta_m = \frac{(M_{net})_{frame}}{K_{ec}} \tag{12}$$

where K_{ec} = equivalent column stiffness (see 13.7.4)

The midspan deflection of the column strip subjected to a rotation of θ_1 radians at one end with the opposite end fixed is

$$(\Delta\theta_1)_c = \frac{\theta_1 \ell}{8} \tag{13}$$

The additional deflection terms for the column and middle strips would be computed similarly.

Because θ in Eq. (12) is based on gross section properties, while the deflection calculations are based on I_e, Eq. (14) may be used instead of Eq. (13) for consistency:

$$(\Delta\theta_1)_c = \theta_1 \left(\frac{\ell}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame} \tag{14}$$

Direct Design Method: The deflection computation procedure described above has been expressed in terms of the equivalent frame method for moment analysis. However, it is equally suited for use with the direct design method in which coefficients are used to calculate moments at critical sections instead of using elastic frame analysis as in case of the equivalent frame method. In the direct design method, design moments are computed using clear spans. When determining deflections due to rotations at the ends of a member, these moments should theoretically be corrected to obtain moments at the center of the columns. However, this difference is generally small and may be neglected. In the case of flat plates and flat slabs, the span measured between the column centerlines is thought to be more appropriate than the clear span for deflection computations.

If all spans are equal and are identically loaded, the direct design method will give no unbalanced moments and rotations except at an exterior column. Therefore, in these cases, rotations need be considered only at the exterior columns. When live load is large compared to the dead load (not usually the case), end rotations may be computed by a simple moment-area procedure in which the effect of pattern loading may be included.

Effective Moment of Inertia: The effective moment of inertia given by Eq. (9-8) is recommended for computing deflections of partially cracked two-way construction. An average I_e of the positive and negative regions in accordance with Eqs. (1) and (2) may also be used.

For the typical cracking locations found empirically, the following moment of inertia values have been shown to be applicable in most cases.

	Case	Inertia
a.	Slabs without beams (flat plates, flat slabs)	
	(i) All dead load deflections—	I_g
	(ii) Dead-plus-live load deflections:	
	For the column strips in both directions—	I_e
	For the middle strips in both directions—	I_g
b.	Slab with beams (two-way beam-supported slabs)	
	(i) All dead load deflections—	I_g
	(ii) Dead-plus-live load deflections:	
	For the column strips in both directions—	I_g
	For the middle strips in both directions—	I_e

The I_e of the equivalent frame in each direction is taken as the sum of the column and middle strip I_e values.

Long-Term Deflection: Since the available data on long-term deflections of two-way construction is too limited to justify more elaborate procedures, the same procedures as those used for one-way members are recommended. Equation (9-11) may be used with $\xi = 2.5$ for sustained loading of five years or longer duration.

9.5.4 Prestressed Concrete Construction

Typical span-depth ratios for general use in design of prestressed members are given in the PCI Design Handbook[10.8] and summarized in Ref. 10.2 from several sources. Starting with the 2002 edition of ACI 318, the Building Code classifies prestressed concrete flexural members, in 18.3.3, as Class U (uncracked), Class T (transition), or Class C (cracked.) For Class U flexural members, deflections must be calculated based on the moment of inertia of the gross section I_g. For Classes T and C, deflections must be computed based on a cracked transformed section analysis or on a bilinear moment-deflection relationship. Reference 10.9 provides a procedure to compute deflection of cracked prestressed concrete members.

Deflection of Noncomposite Prestressed Members—The ultimate (in time) camber and deflection of prestressed members may be computed based on a procedure described in Ref. 10.2. The procedure includes the use of I_e for partially prestressed members (Ref. 10.8) as a suggested method of satisfying 9.5.4.2 for deflection analysis when the computed tensile stress exceeds the modulus of rupture, but does not exceed $12\sqrt{f_c'}$. For detailed information on the deflection of cracked prestressed beams and on the deflection of composite prestressed beams, see Refs. 10.2 and 10.9.

The ultimate deflection of noncomposite prestressed members is obtained as (Refs. 10.2 and 10.10):

$$\Delta_u = \underbrace{-\Delta_{po}}_{(1)} + \underbrace{\Delta_o}_{(2)} - \underbrace{\left[-\frac{\Delta P_u}{P_o} + (k_r C_u)\left(1 - \frac{\Delta P_u}{2P_o}\right)\right]}_{(3)}\underbrace{\Delta_{po} + (k_r C_u)\,\Delta_o}_{(4)} + \underbrace{\Delta_s}_{(5)}$$

$$+ \underbrace{(\beta_s k_r C_u)\,\Delta_s}_{(6)} + \underbrace{\Delta_\ell}_{(7)} + \underbrace{(\Delta_{cp})_\ell}_{(8)} \qquad (15)$$

Term (1) is the initial camber due to the initial prestressing moment after elastic loss, $P_o e$. For example, $\Delta_{po} = P_o e\ell^2/8E_{ci}I_g$ for a straight tendon.

Term (2) is the initial deflection due to self-weight of the beam. $\Delta_o = 5M_o\ell^2/48E_{ci}I_g$ for a simple beam, where M_o = midspan self-weight moment.

Term (3) is the creep (time-dependent) camber of the beam due to the prestressing moment. This term includes the effects of creep and loss of prestress; that is, the creep effect under variable stress. Average values of the prestress loss ratio after transfer (excluding elastic loss), $(P_o - P_e)/P_e$, are about 0.18, 0.21, and 0.23 for normal, sand, and all-lightweight concretes, respectively. An average value of $C_u = 2.0$ might be reasonable for the creep factor due to ultimate prestress force and self-weight. The k_r factor takes into account the effect of any nonprestressed tension steel in reducing time-dependent camber, using Eq. (16). It is also used in the PCI Design Handbook[8.8] in a slightly different form.

$$k_r = 1/\left[1 + (A_s/A_{ps})\right] \quad \text{for } A_s/A_{ps} < 2 \qquad (16)$$

When $k_r = 1$, Terms (1) + (3) can be combined as:

$$-\Delta_{po} - \left[-\Delta_{po} + \Delta_{pe} + C_u \left(\frac{\Delta_{po} + \Delta_{pe}}{2} \right) \right] = -\Delta_{pe} - C_u \left(\frac{\Delta_{po} + \Delta_{pe}}{2} \right)$$

Term (4) is the creep deflection due to self-weight of the beam. Use the same value of C_u as in Term (3). Since creep due to prestress and self-weight takes place under the combined stresses caused by them, the effect of any nonprestressed tension steel in reducing the creep deformation is included in both the camber Term (3) and the deflection Term (4).

Term (5) is the initial deflection of the beam under a superimposed dead load. $\Delta_s = 5M_s\ell^2/48E_cI_g$ for a simple beam, where M_s = midspan moment due to superimposed dead load (uniformly distributed).

Term (6) is the creep deflection of the beam caused by a superimposed dead load. k_r is the same as in Terms (3) and (4), and is included in this deflection term for the same reason as in Term (4). An average value of $C_u = 1.6$ is recommended, as in Eq. (7) for nonprestressed members, assuming load application at 20 days after placement. β_s is the creep correction factor for the age of the beam concrete when the superimposed dead load is applied at ages other than 20 days (same values apply for normal as well as lightweight concrete): $\beta_s = 1.0$ for age 3 weeks, 0.96 for age 1 month, 0.89 for age 2 months, 0.85 for age 3 months, and 0.83 for age 4 months.

Term (7) is the initial live load deflection of the beam. $\Delta_\ell = 5M_\ell\ell^2/48E_cI_g$ for a simple beam under uniformly distributed live load, where M_ℓ = midspan live load moment. For uncracked members, $I_e = I_g$. For partially cracked noncomposite and composite members, see Refs. 10.2 and 10.3. See also Example 8.5 for a partially cracked case.

Term (8) is the live load creep deflection of the beam. This deflection increment may be computed as $\left(\Delta_{cp}\right)_\ell = \left(M_s/M_\ell\right) C_u\Delta_\ell$, where M_s is the sustained portion of the live load moment and $C_u = 1.6$, for load application at 20 days or multiplied by the appropriate β_s, as in Term (6).

An alternate method of calculation of long-term camber and deflection is the so-called *PCI Multiplier Method* which is presented in both Ref. 10.4 and Ref.10.8. In that procedure the various instantaneous components of camber or deflection are simply multiplied by the appropriate tabulated coefficients to obtain the additional contributions due to long term effects. The coefficients are given in Table 3.4 of Ref. 10.4 or Table 4.8.2 of Ref. 10.8.

9.5.5 Composite Construction

The ultimate (in time) deflection of unshored and shored composite flexural members may be computed by methods discussed in Refs. 10.2 and 10.10. The methods are reproduced in the following section for both unshored and shored construction. Subscripts 1 and 2 are used to refer to the slab (or effect of the slab, such as under slab dead load) and the precast beam, respectively. Examples 10.6 and 10.7 demonstrate the beneficial effect of shoring in reducing deflections.

9.5.5.1 Shored Construction—For shored composite members, where the dead and live load is resisted by the full composite section, the minimum thicknesses of Table 9.5(a) apply as for monolithic structural members.

The calculation of deflections for shored composite beams is essentially the same as for monolithic beams, except for the deflection due to shrinkage warping of the precast beam, which is resisted by the composite section after the slab has hardened, and the deflection due to differential shrinkage and creep of the composite beam. These effects are represented by Terms (3) and (4) in Eq. (17).

$$\Delta_u = \underbrace{(\Delta_i)_{1+2}}_{(1)} + \underbrace{1.80k_r (\Delta_i)_{1+2}}_{(2)} + \underbrace{\Delta_{sh}\frac{I_2}{I_c}}_{(3)} + \underbrace{\Delta_{ds}}_{(4)} + \underbrace{(\Delta_i)_\ell}_{(5)} + \underbrace{(\Delta_{cp})_\ell}_{(6)} \qquad (17)$$

When $k_r = 0.85$ (neglecting any effect of slab compression steel) and Δ_{ds} is assumed to be equal to $(\Delta_i)_{1+2}$, Eq. (17) reduces to Eq. (18).

$$\Delta_u = \underbrace{3.53\,(\Delta_i)_{1+2}}_{(1+2+4)} + \underbrace{\Delta_{sh}\frac{I_2}{I_c}}_{(3)} + \underbrace{(\Delta_i)_\ell}_{(5)} + \underbrace{(\Delta_{cp})_\ell}_{(6)} \tag{18}$$

Term (1) is the initial or short-term deflection of the composite beam due to slab plus precast beam dead load (plus partitions, roofing, etc.), using Eq. (3), with $M_a = M_1 + M_2$ = midspan moment due to slab plus precast beam dead load. For computing $(I_e)_{1+2}$ in Eq. (1), M_a refers to the moment $M_1 + M_2$, and M_{cr}, I_g, and I_{cr} to the composite beam section at midspan.

Term (2) is the creep deflection of the composite beam due to the dead load in Term (1), using Eq. (5). The value of C_u to be used must be a combination of that for the slab and that for the beam. In the case of the slab, an adjusted value of $C_u = 1.74$, based on the shores being removed at 10 days of age for a moist-cured slab, may be used. The beam may be older than 20 days (the standard condition) when the loads are applied, however C_u = 1.60 may be used conservatively. An average of the two values may be used as an approximation. For other times of load application, the adjustments can be made in similar fashion using the correction factors, β_s, listed previously in the description of Term (6) of Eq. (15). Index ρ' refers to any compression steel in the slab at midspan when computing k_r.

Term (3) is the shrinkage deflection of the composite beam after the shores are removed, due to the shrinkage of the precast beam concrete, but not including the effect of differential shrinkage and creep which is given by Term (4). Equation (6) may be used to compute Δ_{sh}. Assuming the slab is cast at a precast (steam-cured) beam concrete age of 2 months and that shores are removed about 10 days later. At that time, the shrinkage in the beam is approximately 36% of the ultimate, according to Table 2.1 of ACI 435. The shrinkage strain subsequent to that time will be $(\varepsilon_{sh})_{rem} = (1 - 0.36)\,(\varepsilon_{sh})_u$. That value should be used in Eq. (6) to calculate the deflection component in this Term.

Term (4) is the deflection due to differential shrinkage and creep. As an approximation, $\Delta_{ds} = (\Delta_i)_{1+2}$ may be used.

Term (5) is the initial or short-term live load deflection of the composite beam, using Eq. (3). The calculation of the incremental live load deflection follows the same procedure as that for a monolithic beam. This is the same as in the method described in connection with Term (9) of Eq. (19) discussed below.

Term (6) is the creep deflection due to any sustained live load, using Eq. (5). In computing this component of deflection, use of an ultimate creep coefficient, $C_u = 1.6$ is conservative. The creep coefficient may be reduced by the factor β_s defined in Term (6) of Eq. (15).

These procedures suggest using midspan values only, which may normally be satisfactory for both simple composite beams and those with a continuous slab as well. See Ref. 10.10 for an example of a continuous slab in composite construction.

9.5.5.2 Unshored Construction—For unshored composite construction, if the thickness of a nonprestressed precast member meets the minimum thickness requirements, deflections need not be computed. Section 9.5.5.2 also states that, if the thickness of an unshored nonprestressed composite member meets the minimum thickness requirements, deflections occurring after the member becomes composite need not be computed, but the long-term deflection of the precast member should be investigated for the magnitude and duration of load prior to beginning of effective composite action.

$$\Delta_u = \underbrace{(\Delta_i)_2}_{(1)} + \underbrace{0.77k_r(\Delta_i)_2}_{(2)} + \underbrace{0.83k_r(\Delta_i)_2\frac{I_2}{I_c}}_{(3)} + \underbrace{0.36\Delta_{sh}}_{(4)} + \underbrace{0.64\Delta_{sh}\frac{I_2}{I_c}}_{(5)}$$

$$+ \underbrace{(\Delta_i)_1}_{(6)} + \underbrace{1.22k_r(\Delta_i)_1\frac{I_2}{I_c}}_{(7)} + \underbrace{\Delta_{ds}}_{(8)} + \underbrace{(\Delta_i)_\ell}_{(9)} + \underbrace{(\Delta_{cp})_\ell}_{(10)} \qquad (19)$$

With $k_r = 0.85$ (no compression steel in the precast beam) and Δ_{ds} assumed to be equal to $0.50(\Delta_i)_1$, Eq. (19) reduces to Eq. (20).

$$\Delta_u = \underbrace{\left(1.65 + 0.71\frac{I_2}{I_c}\right)(\Delta_i)_2}_{(1+2+3)} + \underbrace{\left(0.36 + 0.64\frac{I_2}{I_c}\right)\Delta_{sh}}_{(4+5)}$$

$$+ \underbrace{\left(1.50 + 1.04\frac{I_2}{I_c}\right)(\Delta_i)_1}_{(6+7+8)} + \underbrace{(\Delta_i)_\ell}_{(9)} + \underbrace{(\Delta_{cp})_\ell}_{(10)} \qquad (20)$$

In Eqs. (19) and (20), the parts of the total creep and shrinkage occurring before and after slab casting are based on the assumption of a precast beam age of 20 days when its dead load is applied and of 2 months when the composite slab is cast.

Term (1) is the initial or short-term dead load deflection of the precast beam, using Eq. (3), with $M_a = M_2$ = midspan moment due to the precast beam dead load. For computing $(I_e)_2$ in Eq. (9-9), M_a refers to the precast beam dead load, and M_{cr}, I_g, and I_{cr} to the precast beam section at midspan.

Term (2) is the dead load creep deflection of the precast beam up to the time of slab casting, using Eq. (5), with $C_t = 0.48 \times 1.60 = 0.77$ (for 20 days to 2 months; Table 2.1 of ACI 435; for slabs cast at other than 60 days, the appropriate values from Table 2.1 should be used), and the ρ' refers to the compression steel in the precast beam at midspan when computing k_r.

Term (3) is the creep deflection of the composite beam following slab casting, due to the precast beam dead load, using Eq. (5), with the long term creep being the balance after the slab is cast, $C_t = 1.60 - 0.77 = 0.83$. As indicated in Term (3), if the slab is cast at time other than 2 months, C_t will be as determined from Table 2.1 of ACI 435 and the value of C_t to be used for this term will be found as the difference between 1.60 and the value used for Term (2). ρ' is the same as in Term (2). The ratio I_2/I_c modifies the initial stress (strain) and accounts for the effect of the composite section in restraining additional creep curvature (strain) after the composite section becomes effective. As a simple approximation, $I_2/I_c = [(I_2/I_c)_g + (I_2/I_c)_{cr}]/2$ may be used.

Term (4) is the deflection due to shrinkage warping of the precast beam up to the time of slab casting, using Eq. (6), with $(\varepsilon_{sh})_t = 0.36(\varepsilon_{sh})_u$ at age 2 months for steam cured concrete (assumed to be the usual case for precast beams) The multiplier 0.36 is obtained from Table 2.1 of Ref. 10.4. As in the previous two terms, if the slab is cast at time different from 2 months after beam manufacture, the percentage of the ultimate shrinkage strain should be adjusted to reflect the appropriate value from Table 2.1 of ACI 435. $(\varepsilon_{sh})_u = 400 \times 10^{-6}$ in./in.

Term (5) is the shrinkage deflection of the composite beam following slab casting, due to the shrinkage of the precast beam concrete, using Eq.(6), with $\varepsilon_{sh} = 0.64(\varepsilon_{sh})_u$. This term does not include the effect of differential shrinkage and creep, which is given by Term (8). I_2/I_c is the same as in Term (3).

Term (6) is the initial or short-term deflection of the precast beam under slab dead load, using Eq. (3), with the incremental deflection computed as follows: $(\Delta_i)_1 = (\Delta_i)_{1+2} - (\Delta_i)_2$, where $(\Delta_i)_2$ is the same as in Term (1). For computing $(I_e)_{1+2}$ and $(\Delta_i)_{1+2}$ in Eqs. (9-8) and (3), $M_a = M_1 + M_2$ due to the precast beam plus slab dead load at midspan, and M_{cr}, I_g, and I_{cr} refer to the precast beam section at midspan. When partitions, roofing, etc., are placed at the same time as the slab, or soon thereafter, their dead load should be included in M_1 and M_a.

Term (7) is the creep deflection of the composite beam due to slab dead load using Eq. (5), with $C_u = \beta_s$ x 1.60. For loading age of 2 months, $\beta_s = 0.89$ is the appropriate correction factor as noted in Term(6) of Eq. (15). For loading at other times, the appropriate value of β_s should be used. In this term, the initial strains, curvatures and deflections under slab dead load were based on the precast section only. Hence the creep curvatures and deflections refer to the precast beam concrete, although the composite section is restraining the creep curvatures and deflections, as mentioned in connection with Term (3). k_r is the same as in Term (2), and I_2/I_c is the same as in Term (3).

Term (8) is the deflection due to differential shrinkage and creep. As an approximation, $\Delta_{ds} = 0.50\,(\Delta_i)_1$, may be used.

Term (9) is the initial or short-term deflection due to live load (and other loads applied to the composite beam and not included in Term (6)) of the composite beam, using Eq. (4), with the incremental deflection estimated as follows: $(\Delta_i)_\ell = (\Delta_i)_{d+\ell} - (\Delta_i)_d$, based on the composite section. This is thought to be a conservative approximation, since the computed $(\Delta_i)_d$ is on the low side and thus the computed $(\Delta_i)_\ell$ is on the high side, even though the incremental loads are actually resisted by different sections (members). This method is the same as for Term (5) of Eq. (17), and the same as for a monolithic beam. Alternatively, Eq. (3) may be used with $M_a = M_l$ and $I_e = (I_c)_{cr}$ as a simple rough approximation. The first method is illustrated in Example 8.7 and the alternative method in Example 8.6.

Term (10) is the creep deflection due to any sustained live load applied to the composite beam, using Eq. (5), with $C_u = \beta_s$ x 1.60. As in the other cases, β_s is given for various load application times in the explanation of Term (6) of Eq. (15). ρ' refers to any compression steel in the slab at midspan when computing k_r. This Term corresponds to Term (6) in Eqs. (17) and (18).

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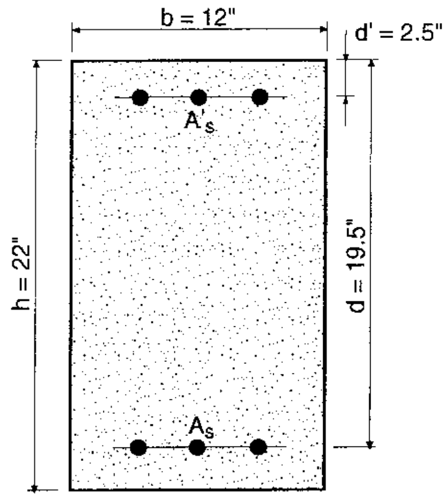
Example 10.1—Simple-Span Nonprestressed Rectangular Beam

Required: Analysis of short-term deflections, and long-term deflections at ages 3 months and 5 years (ultimate value)

Data:



f'_c = 3000 psi (normal weight concrete)
f_y = 40,000 psi
A_s = 3-No. 7 = 1.80 in.^{2}
E_s = 29,000,000 psi
$\rho = A_s/bd = 0.0077$
A'_s = 3-No. 4 = 0.60 in.^{2}
$\rho' = A'_s/bd = 0.0026$

(A'_s not required for strength)
Superimposed dead load (not including beam weight) = 120 lb/ft
Live load = 300 lb/ft (50% sustained)
Span = 25 ft

Calculations and Discussion	Code Reference

1. Minimum beam thickness, for members not supporting or attached to partitions or other construction likely to be damaged by large deflections:

$$h_{min} = \left(\frac{\ell}{16}\right)$$ *Table 9.5(a)*

multiply by 0.8 for f_y = 40,000 psi steel

$$h_{min} = \frac{25 \times 12}{16} \times 0.8 = 15 \text{ in.} < 22 \text{ in.} \quad \text{O.K.}$$

2. Moments:

$$w_d = 0.120 + (12)(22)(0.150)/144 = 0.395 \text{ kips/ft}$$

$$M_d = \frac{w_d\ell^2}{8} = \frac{(0.395)(25)^2}{8} = 30.9 \text{ ft-kips}$$

$$M_\ell = \frac{w_\ell\ell^2}{8} = \frac{(0.300)(25)^2}{8} = 23.4 \text{ ft-kips}$$

$$M_{d+\ell} = 54.3 \text{ ft-kips}$$

$$M_{sus} = M_d + 0.50M_\ell = 30.9 + (0.50)(23.4) = 42.6 \text{ ft-kips}$$

3. Modulus of rupture, modulus of elasticity, modular ratio:

$$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{3000} = 411 \text{ psi}$$ *Eq. (9-10)*

$$E_c = w_c^{1.5}\, 33\sqrt{f'_c} = (150)^{1.5}\, 33\sqrt{3000} = 3.32 \times 10^6 \text{ psi}$$ *8.5.1*

$$n_s = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.32 \times 10^6} = 8.7$$

4. Gross and cracked section moments of inertia, using Table 10-2:

$$I_g = \frac{bh^3}{12} = \frac{(12)\,(22)^3}{12} = 10{,}650 \text{ in.}^4$$

$$B = \frac{b}{(nA_s)} = \frac{12}{(8.7)\,(1.80)} = 0.766 \text{ in.}$$

$$r = \frac{(n-1)\,A'_s}{(nA_s)} = \frac{(7.7)\,(0.60)}{(8.7)\,(1.80)} = 0.295$$

$$kd = \left[\sqrt{2dB\,(1 + rd'/d) + (1 + r)^2} - (1 + r)\right]/B$$

$$= \left[\sqrt{(2)\,(19.5)\,(0.766)\left\{1 + \frac{0.295 \times 2.5}{19.5}\right\} + (1.295)^2} - 1.295\right]/0.766 = 5.77 \text{ in.}$$

$$I_{cr} = \frac{bk^3d^3}{3} + nA_s\,(d - kd)^2 + (n-1)\,A'_s\,(kd - d')^2$$

$$= \frac{(12)\,(5.77)^3}{3} + (8.7)\,(1.80)\,(19.5 - 5.77)^2 + (7.7)\,(0.60)\,(5.77 - 2.5)^2$$

$$= 3770 \text{ in.}^4$$

$$\frac{I_g}{I_{cr}} = 2.8$$

5. Effective moments of inertia, using Eq. (9-8):

$$M_{cr} = \frac{f_r I_g}{y_t} = [(411)\,(10{,}650)/(11)]/(12{,}000) = 33.2 \text{ ft-kips}$$ *Eq. (9-9)*

a. Under dead load only