Discussion on AASHTO LRFD Load Distribution Factors for Slab-on-Girder Bridges

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Abstract: The present study developed a new set of formulas for load distribution factors that are more rational than the current AASHTO LRFD formulas. A formula to quantify the intermediate diaphragm effect on live load distribution was also proposed. Preliminary coefficients of these formulas were obtained from curve fitting either with the values of AASHTO LRFD formulas or with the results from finite element modeling. The load distribution factors predicted with the proposed formulas were compared with those from AASHTO codes, field measurements, and finite element analysis with or without considering the effects of intermediate diaphragms. While the present study was intended to shed some light in developing future AASHTO LRFD design formulas, development of complete load distribution factors is out of the scope of the present study.

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Introduction

With the development of computational tools, engineers are increasingly using finite element models to analyze the entire bridge. However, there are still many reasons that design engineers need to simplify a bridge system into individual components, which uncouples the longitudinal and transverse load effects with the so-called load distribution factors (LDFs). Single-parameter "S-over" formulas of LDFs for shear and moment have been used for bridge design since the 1930s (AASHTO 1996), partly due to the simplicity of these formulas. The traditional "S-over" LDFs are easy to apply, but can be overly conservative in some parameter ranges while unconservative in others. More accurate and more complex LDF equations were thus developed under National Cooperative Highway Research Program (NCHRP) Project 12-26 (Zokaie et al. 1991). These new equations have been included in the LRFD Bridge Design Specifications (AASHTO 1998). However, designers have found the complexity of the new equations troubling. Simpler and less complex live load distribution factor equations would be welcomed by the design community. As a result, a new study under project NCHRP 12-62 was initiated for this purpose and is on-going. The main objective of the present study is to shed some light on the format of the LDF formulas, which may help develop more rational design formulas for the bridge community.

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Parameters of Load Distribution Factor

Load distribution among girders reflects the bridge response to applied loads and has been studied extensively in the last few decades (Heins and Kuo 1975; Kim and Nowak 1997; Mabsout et al. 1999; Barr et al. 2001; among many others). The LDF is a function of many parameters, such as the bridge geometry, the relative stiffness of the components, and the nature of the loads. Strictly speaking, the load distribution is different in different situations, such as service versus ultimate strength state, moment versus shear, and positive versus negative moment. Ideally, a set of load distribution formulas should include the major parameters that have practical/meaningful effects on load distribution; they need to be simple and more intuitive to help minimize potential mistakes. Each term of the formulas needs also to carry clear physical meaning so that designers can better appreciate the effect of each parameter in order to develop a more rational and economical design.

The current LRFD specifications (AASHTO 1998) recognize that the LDF is a function of girder spacing, span length, slab thickness, and beam stiffness. The LDFs are specified differently for exterior and interior girders, for shear and moment, and for one-lane loaded and two-or-more-lane loaded cases. Those formulas were developed based on a trial-and-error type of curve fitting. In the present study, a discussion of the load distribution of slab-on-girder bridges, based on the beam-on-elastic-foundation theory, will help elaborate these observations and identify the major parameters of LDFs, which will lead to a more rational format of the LDF formulas.

For the analytical derivation, a slab-on-girder bridge system is visualized as a beam-on-elastic foundation as shown in Fig. 1. An unit width of slab is simulated as a beam in the transverse direction and the longitudinal beams underneath the slab provide elastic supports to the slab. By assuming that the discrete stiffness of these supports is evenly distributed across the girder spacing, the stiffness per unit width across the bridge width at the midspan of the bridge is thus calculated using beam theory as

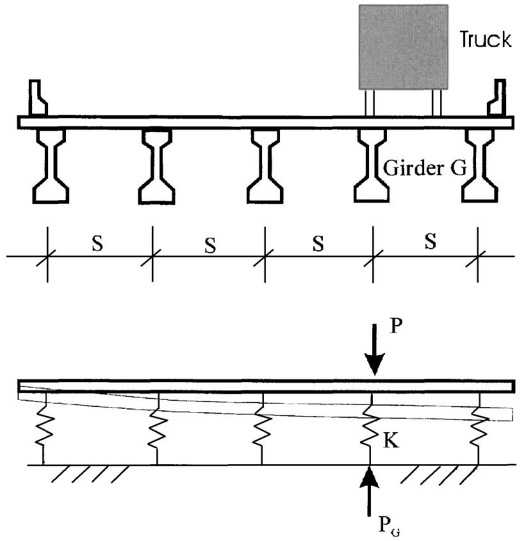

Fig. 1. Illustration model of slab-on-girder bridges

$$k = \frac{K}{S} = \frac{48EI_L}{SL^3} \tag{1}$$

where $K(=48\ EI_L/L^3)$=spring stiffness provided by the longitudinal beam at midspan location; k=distributed stiffness per unit width across bridge width; S=girder spacing; and EI_L=flexural rigidity of the longitudinal beam.

To derive the formula, assuming that a point load P is applied above the *Girder G* as shown in Fig. 1, since P will also be distributed to other girders, only a portion of P, say P_G, will be directly carried by *Girder G*. The deflection of the *Girder G* at the loading point is then calculated from the stiffness $K(=kS)$ as

$$y_{\max} = \frac{P_G}{K} = \frac{P_G}{kS} \tag{2}$$

On the other hand, for a beam on elastic foundation (assume infinite length), the maximum deflection (at the location of P) due to load P can be derived as

$$y_{\max} = \frac{P\beta}{2k} \tag{3}$$

where

$$\beta = \sqrt[4]{\frac{k}{4EI_T}} \tag{4}$$

and where EI_T=flexural rigidity of lateral slab per unit width.

Solving Eqs. (1)–(4) gives

$$\frac{P_G}{P} = \left(\frac{S}{L}\right)^{0.75}\left(\frac{3EI_L}{4EI_T}\right)^{0.25} \tag{5}$$

It is noted that the derived Eq. (5) is not intended to be a complete load distribution formula. It is to show the interaction between lateral and longitudinal stiffness in order to identify possible parameters for load distributions. This simplified derivation shows that the load ratio carried by *Girder G*, P_G/P, depends on the ratio of the girder spacing and the span length, S/L, and the ratio of the longitudinal rigidity and lateral rigidity, EI_L/EI_T. Eq. (5) is used next to form the load distribution formulas.

Proposed Formula for Load Distribution Factor

Based on many studies, the "S-over" formulas have reasonable accuracy in some parameter ranges (but not for all of the ranges) compared with the more complicated LRFD formulas (Arockiasamy et al. 1997; Cai and Shahawy 2004; and many other papers). The "S-over" term reflects the lever or simple beam rule, while Eq. (5) represents the interaction of longitudinal and transverse components. For these reasons, it will be advantageous to keep the traditional "S-over" term in the proposed new formula along with the derived parameters shown in Eq. (5). The format of LDF formulas is thus proposed as

$$\mathrm{LDF} = C_1 + \frac{S}{C_2} + C_3\left(\frac{S}{L}\right)^{0.75}\left(\frac{K_g}{12Lt_s^3}\right)^{0.25} = \left(C_1 + \frac{S}{C_2} + C_3R\right) \tag{6}$$

where

$$R = \left(\frac{S}{L}\right)^{0.75}\left(\frac{K_g}{12Lt_s^3}\right)^{0.25} \tag{7}$$

It is noted that the last term of Eq. (5) represents the ratio of longitudinal and transverse stiffness. It has been expressed in Eq. (6) as $K_g/(12\ Lt_s^3)$ to be consistent with the current LRFD format (AASHTO 1998). Eq. (6) is more rational and has more physical meaning than that used in the current LRFD codes for the reasons:

- The coefficient (or constant) C_1 reflects the fact that the LDF is nonzero even when the girder spacing S approaches zero, as evidenced by many studies and also reflected in the current LRFD codes (AASHTO 1998).
- The C_2 term reflects the linear relationship of the LDF versus girder spacing, which results from the simple beam action and is consistent with the traditional "S-over" term.
- The C_3 term represents the effect of relative longitudinal stiffness and transverse stiffness on load distributions. It will be seen later that C_3 may be in some cases close to zero and Eq. (6) will automatically reduce to the "S-over" type of simple formula.

When loads are placed near piers and abutments for the maximum shear effects, the girders provide larger stiffness to support the slab than near the midspan. The distribution factor for shear may thus be different from that for moments, which may justify using different formulas for shear and moment effects as do the LRFD codes (AASHTO 1998). As will be seen in Table 1 (to be discussed later), for load distributions of shear, the C_3 term automatically reduces to zero during the curve fitting analysis, meaning that the shear distribution near the supports largely depends on the simple beam theory, rather than the interaction between the transverse and longitudinal components.

Comparison with AASHTO LRFD Formulas

It is interesting to see that the derived Eq. (5) and its modified version Eq. (6) include the same parameters as the current LRFD formulas (AASHTO 1998) but with a different format. For example, the power terms "0.75" and "0.25" in the present study were obtained based on analytical derivation instead of being from numerical regression as was done in the code formulas. To confirm the applicability of the proposed formulas and to see how far away the proposed formulas are from the current LRFD formulas, the proposed formulas were curve-fitted with the data gen-

Table 1. Comparison of Current LRFD and Proposed Formulas

Loading cases	AASHTO LRFD	Proposed
Moment: two-or-more-lane loaded	$0.075+(S/9.5)^{0.6}(S/L)^{0.2}(K_g/12Lt_s^3)^{0.1}$	$0.15+(S/22)+0.77(S/L)^{0.75}(K_g/12Lt_s^3)^{0.25}$
Moment: one-lane loaded	$0.06+(S/14)^{0.4}(S/L)^{0.3}(K_g/12Lt_s^3)^{0.1}$	$0.15+(S/42)+0.71(S/L)^{0.75}(K_g/12Lt_s^3)^{0.25}$
Shear: two-or-more-lane loaded	$0.2+(S/12)-(S/35)^2$	$0.25+(S/14)$
Shear: one-lane loaded	$0.36+(S/25)$	$0.36+(S/25)$

erated with the current LRFD LDF formulas. The variables considered in generating these data are AASHTO Type I to Type VI girders, slab thickness from 101 to 303 mm (4 to 12 in.), and girder spacing from 1.22 to 3.66 m (4 to 12 ft). Other parameters such as span lengths were also varied to cover a wide range of practical values. About 3,600 LDF data were generated for the cases of one-lane loaded moment distribution, and the same amount of data for two-or-more-lane loaded cases were also generated. Correspondingly, the same amount of data were obtained for shear distributions. The coefficients of Eq. (6), i.e., C_1, C_2, and C_3, have thus been determined by curve fitting and are given in Table 1 along with the LRFD formulas for comparison.

The statistics of the ratio between the proposed new formula values and the LRFD values is given in Table 2 for four different loading cases (each loading case includes 3,600 data), namely, Moment: two-or-more-lane loaded; Moment: one-lane loaded; Shear: two-or-more-lane loaded; and Shear: one-lane loaded. The maximum difference is about 7%; the average of the difference is about 1%; and maximum standard deviation (SD) is 0.026. It can be concluded from Table 2 that Eq. (6) can well fit the current LRFD formulas and may thus be used to replace the current LRFD formulas if Eq. (6) is more rational. A few notes below will help readers to see the difference and appreciate the advantages of the proposed formulas over the current LRFD ones.

- According to the statistics in Table 2, if we assume that the current LRFD formulas are a good curve fitting of the finite element prediction, then the proposed formulas may have the same accuracy in curve fitting finite element results (final conclusions can only be made after a systematic comparison using finite element analysis).
- The proposed formulas can be regarded as a modification of the traditional "S-over" formulas by adding the interaction term [the third term in Eq. (6)] to represent the effects of the relative longitudinal and lateral stiffness on load distributions.
- The proposed formulas were obtained based on analytical derivation. They may cover a wider range of bridge parameters and are more consistent in format. As a result, all of the proposed formulas will have the same format while they are different just in the coefficients of C_1, C_2, and C_3, which is shown in Table 1 for the one-lane loaded and two-or-more-lane loaded cases. In the case of one-lane loaded shear distribution, as shown in Table 1, C_3 becomes zero and the proposed formula becomes the same as that of the current LRFD codes. The new formulas provide much convenience for routine design, reduce the calculation effort, and, more importantly, reduce potential error in calculation due to the variation in formulas.
- In the proposed formulas, due to the contribution of the "S-over" term [the second term in Eq. (6)], the K_g has less effect on the overall values of LDFs than it does in the current LRFD formulas. Since the K_g is unknown before a typical LDF calculation (with section yet to be designed), the current LRFD code specifications suggest an iteration process to decide the K_g such as by assuming $K_g=0$. The less the contribution of the K_g-related term to the LDFs, the less iteration cycles are required in finding the unknown K_g and LDFs.
- In the present study, the same power terms from derivation are consistently used for different formulas. In the current LRFD code specifications, these power terms are different for different formulas, which is part of the inconvenience (or complaints from users) and sources of potential errors for routine applications.

Diaphragm Effect on Load Distribution

While the overall effect of intermediate diaphragms on bridge performance, justification of their existence, and corresponding practice in each state are controversial, their effects on the improvement of load distributions have been found by many researchers to a different extent (Abendroth et al. 1995; Barker 2001; Green et al. 2002; Cai and Shahawy 2004). The LRFD codes (AASHTO 1998) have also recognized these effects as stated in the commentary. However, this effect is not included in the formulas of load distributions. It is necessary to emphasize that including this effect is not intended to make the already-complicated LDF calculations even more complicated, but to provide an option when designers choose to consider this effect. For example, including this effect may help raise the rating and avoid unnecessary low load posting and/or replacement for existing bridges. For new bridge design, this factor can be simply taken as a unit if designers choose to do so. The present study suggests using a diaphragm modification factor, R_D, to include the intermediate diaphragm effects on the moment load distributions as

$$R_D = 1 - C_{T1}\frac{R_{sk}}{R}\left(\frac{I_T}{I_T+12Lt^3}\right)^{C_{T2}} \qquad (8)$$

where R is defined in Eq. (7); R_{sk}=reduction factor of skew angle effect per LRFD codes (AASHTO 1998); C_{T1} and C_{T2} =coefficients to be determined; and I_T=intermediate diaphragm stiffness at the bridge section considered that is calculated as (or evaluated alternatively to find the actual stiffness)

Table 2. Statistics of $LDF_{Proposed}/LDF_{LRFD}$

Loading cases	Maximum	Minimum	Average	Standard deviation
Moment: two-or-more-lane loaded	1.07	0.95	1.01	0.026
Moment: one-lane loaded	1.07	0.99	1.00	0.015
Shear: two-or-more-lane loaded	1.03	1.00	1.01	0.007
Shear: one-lane loaded	1.0	1.0	1.0	0

Note: LDF=load distribution factor.

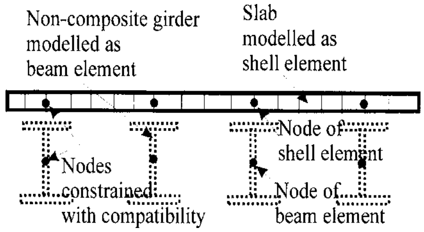

Fig. 2. Finite element model

$$I_T = I_{\text{Diaph}} + A_{\text{Diaph}} \cdot e_{\text{Offset}}^2 \qquad (9)$$

where A_{Diaph} and I_{Diaph}=equivalent cross-sectional area and moment of inertia of intermediate diaphragm at location considered (which will be discussed later); and e_{Offset}=eccentricity offset between the centroid of the intermediate diaphragm and the concrete slab.

Eq. (8), which is applied to Eq. (6), reflects the dependence of the diaphragm modification factor on the two parameters, R and R_{sk}, that reflects the effect of relative stiffness and geometry, respectively. The proposed procedure [Eqs. (8) and (9)] is based on a few observations made from previous studies (Green et al. 2002; Cai and Shahawy 2004). The effect of intermediate diaphragms on the load distribution:

- Decreases (meaning an increase of R_D value) with the increase of skew angles;
- Decreases with the increase of R value due to the increase of the S and K_g values; and
- Is only significant when the intermediate diaphragm is located near the bridge section considered. In other words, it does not significantly affect the load distributions for bridge sections far away from the diaphragm.

By default, the present study is concerned about the load distribution of the bridge section at midspan of the bridge. Before using the proposed Eq. (6), the actual condition of the intermediate diaphragm, such as cracked or noncracked diaphragms, conditions of the connection between diaphragms and girders, and connection of diaphragms and slab, should be considered to evaluate the equivalent (or actual) diaphragm stiffness.

Correlation with Finite Element Modeling

Finite element analyses on six prestressed concrete bridges were conducted using a model shown in Fig. 2 in order to predict LDFs of individual girders. These bridges were field tested and used in previous studies (Cai et al. 2002; Cai and Shahawy 2004). The intermediate diaphragm effects were identified by comparing the model with and without intermediate diaphragms. To investigate the sensitivity of the load distributions to the intermediate diaphragms, diaphragm stiffness was varied from noncomposite (reduced stiffness) to full-composite (full stiffness) actions between the diaphragm and the slab, considering the fact that the diaphragms may crack and have a variable stiffness. A weak connection between the diaphragm and the girders, such as in the case of terminating diaphragm reinforcement at this interface in the current practice of many states, may also actually reduce the equivalent stiffness of the diaphragm.

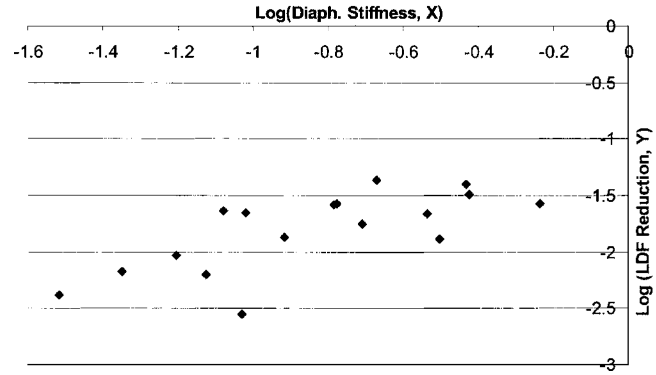

Fig. 3. Effect of diaphragm on load distribution factor

The information of these bridges, including straight and skewed bridges and various parameters, is given in Table 3. The results from finite element analysis were used here to develop the diaphragm effect modification factor. First, Eq. (8) is rearranged into a linear format as

$$y = \log C_{T1} + C_{T2}x \qquad (10)$$

where

$$y = \log\left[(1 - R_D)\frac{R}{R_{sk}}\right] \qquad (11)$$

and

$$x = \log\left[\left(\frac{I_T}{I_T + 12Lt^3}\right)\right] \qquad (12)$$

The correlation between y and x is shown in Fig. 3. The coefficients were obtained based on curve fitting and slightly adjusted to be conservative as C_{T1}=0.03 and C_{T2}=0.6.

The comparison of load distributions between the measured (second column), finite element prediction (third column),

Table 3. Summary of Bridge Information

Bridge number	Span length (L) (m)	Girder spacing (S_G) (m)	Slab thickness (t_s) (mm)	Number of girder	Skew angle (deg.)	Number of lanes	Section modulus (S) (m^3)	E_{Girder} (MPa)	E_{Slab} (MPa)
880083	12.50	3.13	190.50	5	0.00	2	0.103	27,790.36	22,916.52
490023	19.82	2.50	190.50	6	0.00	2	0.171	27,790.36	22,916.52
720408	31.71	1.62	177.80	8	17.48	2	0.255	27,790.36	22,916.52
720252	16.77	1.88	177.80	10	0.00	3	0.096	27,790.36	22,916.52
940115	38.11	2.02	177.80	9	45.26	3	0.353	29,146.77	22,916.52
930378	37.20	2.57	177.80	8	51.75	4	0.362	29,146.77	22,916.52

Table 4. Comparison of Load Distribution Factor (Wheel) without Considering Diaphragm Effect

				AASHTO LRFD (1998)				Proposed		
Bridge number	Measured LDF	FEM LDF	AASHTO standard (1996) LDF	Two or more lane LDF-M	One Lane LDF-M	Two or more lane LDF-S	One lane LDF-S	Two or more lane LDF-M	One lane LDF-M	Two or more lane LDF-S
880083	1.45	1.65	1.86	1.81	1.34	1.94	1.54	1.84	1.35	1.96
490023	1.11	1.18	1.49	1.46	1.06	1.66	1.38	1.44	1.05	1.67
720408	0.73	0.72	0.97	1.02	0.74	1.24	1.15	1.00	0.76	1.26
720252	0.96	0.86	1.12	1.20	0.91	1.37	1.21	1.20	0.91	1.38
940115	0.88	0.86	1.20	1.09	0.76	1.43	1.25	1.05	0.77	1.45
930378	1.08	1.04	1.53	1.23	0.84	1.69	1.39	1.18	0.85	1.70

Note: LDF=load distribution factor; LDF-M=load distribution factor-moment; and LDf-S=load distribution factor-shear.

AASHTO standard code specifications (fourth column), AASHTO LRFD specifications (fifth–eighth columns), and proposed formulas without considering the diaphragm effects (ninth and tenth columns) are listed in Table 4, where LDF-M and LDF-S stand for load distributions for moment and shear, respectively. It can be seen that the proposed formulas and AASHTO LRFD predict very close results since, as discussed earlier, the coefficients of the proposed formulas were curve-fitted from the AASHTO LRFD formulas. Both are conservative compared with the results from finite element analysis and field tests [see Cai and Shahawy (2004) for more details]. The agreement of the results of the proposed formulas and AASHTO LRFD for these field bridges further verifies the suitability in using the proposed formulas to replace the LRFD ones.

By considering the diaphragm reduction factor, the accuracy of the proposed formula is enhanced as shown in Table 5 for the case of moment for two-or-more-lane loaded (there exists a similar observation for the one-lane loaded case). The full EI noncomp represents the stiffness of diaphragms without considering the composite action between the diaphragm and slab, i.e., the eccentricity of the diaphragms and the slab is set to be zero (see Fig. 2). The EI comp represents the stiffness of the diaphragm full composite with the slab by using eccentricity offsetting. The EI/3 comp represents one-third of the stiffness of the EI comp. Variation of the diaphragm stiffness is to investigate the sensitivity of R_D to the diaphragm stiffness. It can be seen that the composite diaphragm has more pronounced effects than the noncomposite diaphragm on load distributions. As discussed earlier, actual diaphragm stiffness of each bridge needs to be evaluated case-by-case in the actual application of the proposed formulas.

Conclusions

The present study proposed a new set of formulas for load distribution factors that are more rational than the current AASHTO LRFD formulas. The proposed formulas are regarded as a modification of the "S-over" formulas by including a constant term C_1 and an interaction term C_3R. If the proposed formulas are used, the difference of the load distribution factors between moment and shear, exterior and interior girders, one-lane loaded, and two-or-more-lane loaded can be reflected by the same set of formulas only with different values of coefficients. Therefore the many tables in the current LRFD codes may be combined into a single table defining the coefficients C_1, C_2, and C_3 for different applications. The many tables for the current LRFD load distribution calculations are one of the major inconveniences that the users have experienced so far.

A formula to quantify the intermediate diaphragm effects on load distributions was proposed based on the results of finite element analysis. When users are willing to find more accurate values, further calculations for the correction factors, such as the intermediate diaphragm modification factor (R_D) proposed in the present study, can be conducted. This will give the users the flexibility to balance the accuracy and simplicity.

While the present study was intended to shed some light in developing future AASHTO LRFD design formulas, development

Table 5. Comparison of Load Distribution Factor (Wheel) Considering Diaphragm Effect

	Proposed diaphragm reduction factor R_D			Proposed LDF (two or more lane, moment)			Proposed/measured		
Bridge number	Full EI noncomposite	EI/3 composite	Full EI composite	Full EI noncomposite	EI/3 composite	Full EI composite	Full EI noncomposite	EI/3 composite	Full EI composite
880083	0.99	0.98	0.97	1.82	1.81	1.79	1.25	1.25	1.23
490023	0.98	0.94	0.92	1.41	1.35	1.32	1.27	1.22	1.18
720408	0.96	0.94	0.90	0.96	0.95	0.90	1.32	1.29	1.24
720252	0.98	0.97	0.95	1.18	1.17	1.14	1.24	1.22	1.20
940115	0.96	0.94	0.91	1.01	0.99	0.95	1.15	1.13	1.08
930378	0.97	0.96	0.93	1.14	1.13	1.10	1.06	1.05	1.02

Note: LDF=load distribution factor; E=elastic modulus; and I=moment of inertia.

of complete load distribution factors is out of the scope of the present study. The values of the coefficients determined from the present study are to show the suitability of the proposed formulas. These coefficients can be refined based on curve fitting to a more comprehensive database of finite element analyses.

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