Determination of Voltage Dependence in High-Voltage Standard Capacitors

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***Abstract*— The voltage coefficient of compressed gas capacitors is a relevant parameter in high-voltage calibrations. These capacitors, used as standards, are calibrated at low voltages so that it is necessary to know their variation when they are used at high voltages. Although several methods have been proposed to determine that coefficient, their implementation is very difficult. In this paper, a simplified method that does not need any special equipment is proposed, so that it is easy to carry out for most high-voltage laboratories. It only requires geometric data on the radius of the electrodes and the mass of the internal ones but it is not necessary to know them with high accuracy.**

***Index Terms*— Calibration, capacitor, high voltage, power frequency.**

I. Introduction

MANY calibrations of high-voltage power equipment depend on standard capacitors. This is the case of voltage transformers, high-voltage dividers [1], and tangent delta bridges, among others [2]. For these devices, high accuracy calibrations are needed. For example, standard high-voltage transformers reach ratio accuracies, in phase and quadrature, of some tens in 10^6 [3]. For their calibration, the standard capacitor uncertainty must be four times lower. To get this uncertainty value, their voltage dependence must be determined because it can be higher than that the limit, in some units.

All high precision capacitors, used at high voltage, are of gas insulated cylindrical coaxial type. In the past, different constructions were proposed, with other isolating materials with high dielectric losses [4], [5], but nowadays the most used standard capacitors are low-loss SF6-compressed, cylindrical construction, in a fiberglass enclosure (see Fig. 1).

Fig. 1. Typical modern SF6-standard capacitor.

Most models have two capacitors, one is the main capacitor and the other is an auxiliary capacitor. The first one is used as standard with values around 50–100 pF. The second one, around 10 pF, forms a voltage divider together with an external low-voltage capacitor. This divider is used for voltage measurement where low accuracy is enough. Generally, the low-voltage capacitor is a medium-precision solid-dielectric type. The schematic of Fig. 2 shows the internal construction. In Fig. 2, 1 is the high voltage electrode, common to all capacitors, 2 is the main-capacitor low-voltage-electrode, 4 is the auxiliary-capacitor low-voltage electrode; and 3 is a shield electrode to avoid influences between electrodes 2 and 4.

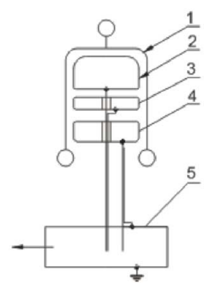

Fig. 2. Internal diagram of a standard capacitor. 1—High voltage electrode. 2—Main-capacitor low-voltage-electrode. 3—Shield electrode. 4—Auxiliary-capacitor low-voltage electrode.

Internal electrodes are connected to the external terminals with coaxial cables. The cable shield of the auxiliary capacitor is connected to ground, while for the main one, it remains isolated. This allows to use different types of guarding in case the low-voltage electrode of the main capacitor is not at ground potential. Fig. 3 shows a picture of the internal electrodes of a 100 pF, 100 kV capacitor of the same type of construction. In this case, there is no auxiliary electrode. Therefore, there

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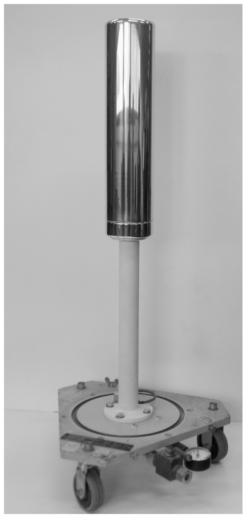

Fig. 3. Internal electrodes of a 100 pF, 100 kV, gas compressed capacitor.

are only two electrodes: the main one and the shield electrode. All cables are inside the white metallic support tube.

The influence of the voltage on the capacitance is due to deformation and displacement of the electrodes, because of forces produced by the electrical field [6]. In modern units, displacement is the main cause of voltage–capacitance dependence. This force increases proportionally to the square of the voltage and can reach significant values at nominal voltages, in some units. The internal electrodes are mounted on a metallic tube that also acts as shielding for the connecting cables. This tube is fixed to the base of the structure but can be slightly tilted by the electrical force. If the axes of internal and external cylindrical electrodes are centered, the net force is nulled. However, if some eccentric displacement exists between axes of the external and internal electrodes, a resulting force appears that tends to increase the eccentricity.

Several methods have been proposed for estimating the voltage coefficient. The simplest one is to compare the capacitor under test with a reference capacitor with a known voltage coefficient but it only postpones the problem. This reference capacitor needs to be characterized by means of an absolute method. More confidence can be obtained by international comparisons [7]–[9], although, they cannot determine by themselves the voltage coefficients.

The method proposed in [10] is based on a precision voltage transformer. Then, as this transformer acts as the standard, it needs linearity characterization. While it is possible to measure the linear error of a voltage transformer without using any standard capacitor (e.g., with step-up method), most laboratories base their voltage-ratio traceability on standard capacitors; that is, calibrate voltage transformers against capacitors.

Other methods are very cumbersome to implement. In [11], two auxiliary capacitors of identical performance and a measuring bridge which null detector must be connected at a high voltage point are needed. This method is based on the calibration of the capacitor under test in a 2:1 voltage ratio. For that purpose, it uses two auxiliary capacitors connected in series. Inverting the position of these capacitor, and taking the average value, a 2:1 absolute ratio can be reached. This is a known method, relatively easy to implement in low-voltage calibrations. Nevertheless, in high voltage applications many drawbacks appear. The first one is that the null detector must be connected to the middle point of the series, which is at 50% of the applied voltage, reaching hundreds of kilovolts. Shielding and guarding becomes very cumbersome. Another problem is the management of the stray capacitances to ground at that middle point, as well as, driving the guard electrode of the upper auxiliary capacitor.

In [12], it is proposed to connect in series two high-voltage generators, with different frequencies, for using them as the voltage source in a modified Schering bridge. The two high voltage arms of the bridge are formed by the capacitor under test and an auxiliary capacitor. The main generator, at power frequency, sets the voltage of the two capacitors. The auxiliary generator (190 Hz in this case), in series only with one arm of the bridge, changes the rms voltage of the capacitor under test. Many auxiliary devices must be included and a very sharp filter must be used to discriminate the auxiliary frequency from the main one in the null detector. All this nonconventional equipment makes the method very complicated to implement.

A similar superposition method that adds a high-voltage dc source with an ac voltage source is proposed in [13]. Among the high voltage dc source, it needs many special components to avoid interactions between ac and dc sources, leading to several difficulties for implementing it. In particular, it needs a high-value capacitor (around 1 μF) with the same rated voltage of the capacitor under test and with a known voltage coefficient.

Other proposals directly measure internal displacements when a force is applied to the electrodes, which requires disassembling the capacitor [14], [15]. This is not possible for operating units. In particular, the proposal [16] requires knowing all mechanical characteristics of the internal parts of the capacitor, such as dimensions and the elasticity coefficient of the electrodes. Dimensions are possible to know from manufacturer information but the value of the elasticity coefficient is very difficult to obtain. It is not available for users, not even for capacitor manufacturers. In that proposal, the elasticity coefficient is measured before the capacitor is assembled, which is not a general case.

Our proposal also requires knowing the diameters of the external and internal electrodes and the weight of the internal electrodes but it does not require the elasticity coefficient. A summary of the proposal was presented in [17] and a detailed description with examples and uncertainty estimation is shown in Sections II and III.

II. Description of the Proposed Method

The main idea of the proposal is to compare the displacement of the electrodes when the electrical force is acting, with the displacement when a mechanical force is applied. The latter is created by tilting the capacitor. In modern constructions, the largest displacement is of the internal electrodes as they are fixed to a relatively long and thin metallic tube, far from the base (see Fig. 3). The displacement is due to the

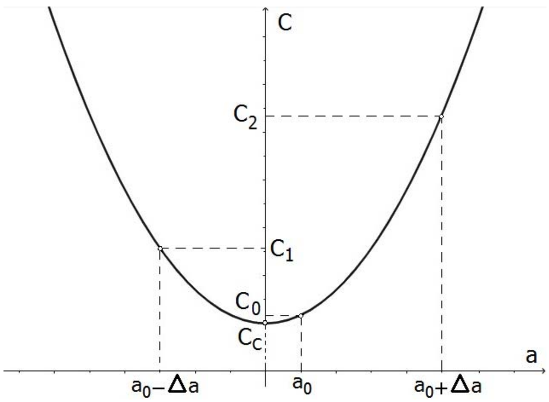

Fig. 4. Relation between capacitance and axes eccentricity.

nonrigid tube structure. The external electrode is fixed to the fiberglass tube, which generally is more than 10-mm thick and has large diameters. This structure is much more rigid than the internal one, so the displacement of the external electrode can be neglected. Some measurements that confirm this premise were shown in [14] and [16]. Although this displacement is not exactly a translation, but it involves a certain angle, it is small enough to be neglected. Therefore, the displacement will be taken as a simple translation of the internal cylinder axis. The capacitance dependence on the eccentricity a (distance between external and internal cylinder axes) [18] is

$$C = \frac{2\pi \varepsilon L}{\ln(Y + \sqrt{Y^2 - 1})} \tag{1}$$

where

$$Y = \frac{R^2 + r^2 - a^2}{2rR} \tag{2}$$

where R and r are the radius of the external and internal electrodes, L their length, and ε the electrical constant of the gas. As a is much smaller than the electrode radii—more than three orders—this relation can be approximated by (see Appendix A)

$$C = 2\pi \varepsilon L \left[\frac{1}{\ln(R/r)} + \frac{a^2}{(R^2 - r^2)\ln^2(R/r)} \right]. \tag{3}$$

The central value C_c with $a = 0$ is

$$C_c = \frac{2\pi \epsilon L}{\ln(R/r)}. \tag{4}$$

This shows, as it is known, that the capacitance increases with a with a parabolic shape and has its minimum value at $a = 0$ (see Fig. 4).

The electrical force between electrodes F_e can be calculated from the variation of the capacitor energy with a. All capacitances, main, and auxiliaries, must be included in C, because all of them contribute to the electrical force

$$F_e = \frac{V^2}{2}\frac{dC}{da} \tag{5}$$

which leads to a linear dependence on a

$$F_e = C_c \alpha a V^2 \tag{6}$$

where

$$\alpha = \frac{1}{(R^2 - r^2)\ln(R/r)}. \tag{7}$$

If $a = 0$, the electrical force nulls, as expected because of symmetry reasons. For other values, the force depends on the voltage square and the geometric parameters of the capacitor. It is expected that the largest variations occur from 30% up to 100% of the rated voltage. At lower values, the electrical force is one order smaller than at rated voltage.

The next step for calculating the eccentricity and the elasticity coefficient of the structure is to measure the capacitance at low voltage when the capacitor is tilted [19]. This is a known technique for detecting the best position, where the capacitance has its minimum value but in our case we use this test for other purpose. The capacitor is tilted an angle θ in all directions. In each one, the capacitances at the vertical position and at $-\theta$ and $+\theta$ are measured. In the direction of the largest variation, the capacitances C_0 (vertical), C_1 ($-\theta$ and left), and C_2 ($+\theta$ and right) are recorded. When the capacitor is tilted to the right (in this main direction), the eccentricity changes from a_0 to $a_0 + \Delta a$; and when is tilted to the left, to $a_0 - \Delta a$. The absolute change Δa is the same because the radial mechanical force: $F_m = m.g.\tan(\theta)$ also has the same magnitude at both positions (m is the mass of the internal electrodes). Then, the problem to solve for a_0 and Δa is reduced to find three points at the parabola $C(a)$ (3), whose differences $\Delta C_1 = C_1 – C_0$ and $\Delta C_2 = C_2 – C_0$ be equal to the recorded ones (see Fig. 4).

From (3)

$$\Delta C_1 = C_c \alpha(-2a_0 \Delta a + \Delta a^2) \tag{8}$$

$$\Delta C_2 = C_c \alpha(2a_0 \Delta a + \Delta a^2). \tag{9}$$

The signs of ΔC_1 and ΔC_2 depend on the values of a_0 and Δa. Then, solving (8) and (9)

$$\Delta a = \sqrt{\frac{\delta C_1 + \delta C_2}{2\alpha}} \tag{10}$$

$$a_0 = \frac{\delta C_2 - \delta C_1}{4\alpha \Delta a} \tag{11}$$

being δC_i the fractional variation of the capacitance

$$\delta C_i = \Delta C_i / C_c. \tag{12}$$

The following step is to calculate the elasticity coefficient k as the ratio between the force (gravitational in this case) and the corresponding displacement

$$k = \frac{mg\tan(\theta)}{\Delta a}. \tag{13}$$

On the other hand, the eccentricity displacement Δa_e, due to the force produced by the electrical field, $\Delta a_e = a_e – a_0$, is the quotient of the force and the elasticity coefficient

$$\Delta a_e = \frac{F_e}{k}. \tag{14}$$

The force can be calculated using (6), with $a = a_e$. Both terms of (14), Δa_e and F_e, depend on a_e, then solving this equation one gets

$$\Delta a_e = \frac{1}{\frac{k}{\alpha C_c V^2} - 1} a_0. \tag{15}$$

Fig. 5. Intercomparison test between UTE standard capacitor (right) and INTI standard capacitor (left), at the INTI HV Laboratory.

In most cases, this equation can be approximated neglecting the factor 1 in the denominator. $k/(\alpha C_c V^2)$ is generally two orders greater than 1.

The next step is to calculate, from (3), the fractional capacitance variation due to the electrical force, $\delta C_e = [C(a_o + \Delta a_e) - C(a_o)]/C_c$

$$\delta C_e = \alpha(2a_0\Delta a_e + \Delta a_e{}^2). \tag{16}$$

From (15) and using the above approximation results that the variation δC_e depends on a_0^2 as (see Appendix B)

$$\delta C_e \cong \frac{2\alpha^2 C_c V^2}{k} a_0^2 \tag{17}$$

which allows to know the voltage dependence of the capacitor under test.

III. Examples of Application

The method was applied to some compressed SF6-isolated capacitors, all of the similar construction but of different rated voltages and from different manufacturers.

A. Capacitor of 50 pF, 400 kV ($C_{50/400}$)

A picture of this capacitor is shown in Fig. 5, on the right, and its specifications, in Table I.

The internal constructive parameters, from manufacturer information, are shown in Table II, and the measured and calculated data are shown in Tables III and IV.

Then, the calculated variation, δCe, of the capacitance between low voltage and 400 kV, was 0.8×10^{-6}.

This capacitor was used as traveling standard for an intercomparison between the National Metrology Institutes of Argentina and Uruguay (INTI and UTE) [9]. The standard of INTI was a 50 pF, 600 kV compressed capacitor of similar construction [Fig. 5 (left)]. Variation less than 3×10^{-6} of the capacitance ratio was detected between both capacitors, from 50 to 400 kV, with an uncertainty of 5.1×10^{-6}, $k = 2$. This uncertainty calculation takes into account only type A uncertainty and the resolution of the bridge. Other type B sources vanish because this test only measures the capacitance ratio variation when the voltage is adjusted between the mentioned limits. Constant deviation of the bridge and other constant influence factors do not affect this test. It takes a very short time, so that temperature change is negligible, even more so considering that the HV hall has temperature control and the capacitors were acclimated for more than 24 h.

As the variation of UTE capacitor, according to the proposed method, was lower than 10^{-6}, it can be concluded that also INTI capacitor has a limited variation of few parts in 10^6, up to 400 kV.

TABLE I
SPECIFICATIONS OF CAPACITOR $C_{50/400}$

Rated voltage (indoor)	400 kV rms
Rated main capacitance	50 pF
Auxiliary capacitances	24 pF
Tan delta C_{12}	$< 1\times10^{-5}$
Partial discharge at rated voltage	< 5 pC
Operating pressure SF6	0.35 MPa ±0.05 MPa
Temperature range	-5 °C to +45 °C
Temperature coefficient	$\leq +3\times10^{-5}$ K^{-1}

TABLE II
CONSTRUCTIVE PARAMETERS OF CAPACITOR $C_{50/400}$

R	190 mm
r	120 mm
L	600 mm
m	15 kg

TABLE III
MEASURED DATA IN THE TILT TEST OF CAPACITOR $C_{50/400}$

θ	10°
$\delta C1$	-23×10^{-6}
$\delta C2$	31×10^{-6}

TABLE IV
CALCULATED DATA OF CAPACITOR $C_{50/400}$

a_0	6.7×10^{-4} m
k	1.30×10^{5} N/m
F_e	0.79 N
Δa_e	6.1×10^{-6} m
δCe	0.8×10^{-6}

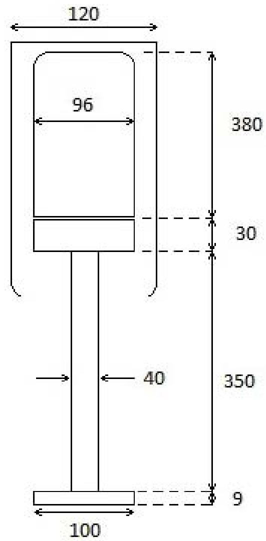

Fig. 6. Electrode dimensions in millimeter of a 100 pF, 100 kV, capacitor.

TABLE V
MEASURED DATA IN THE TILT TEST OF CAPACITOR $C_{100/100}$

θ	10°
$\delta C1$	-9×10^{-6}
$\delta C2$	10×10^{-6}

TABLE VI
CALCULATED DATA OF CAPACITOR $C_{100/100}$

a_0	1.1×10^{-4} m
k	4.7×10^{5} N/m
F_e	0.4 N
Δa_e	0.8×10^{-6} m
δCe	0.7×10^{-6}

B. Capacitor of 100 pF, 100 kV ($C_{100/100}$)

A similar SF6-isolated capacitor of 100 pF and 100 kV was tested. In this case, the capacitor was dismantled for repairing and that opportunity was used to obtain its internal constructing details. Therefore, it was possible to better estimate the internal electrode mass, adding the contribution of the support tube. Fig. 6 shows the dimensions. For the tilt test, it is important to know the effective mass, that is, the mass that cause the same momentum than the actual momentum but applied to the center of the electrode, instead of the actual gravity center. This is because in this point is where the net electrical force is applied. The resulting value was 3.3 kg.

The measured data and calculated results, according to the proposed method, are shown in Tables V and VI. To avoid thermal drift influence on capacitance measurements, an averaging technique was used, repeating the same sequence with a vertical test between each tilted one.

TABLE VII
MEASURED DATA IN THE TILT TEST OF CAPACITOR $C_{100/200}$

θ	7°
$\delta C1$	19×10^{-6}
$\delta C2$	-16×10^{-6}

TABLE VIII
CALCULATED DATA OF CAPACITOR $C_{100/200}$

a_0	-2.5×10^{-4} m
k	2.0×10^{5} N/m
F_e	1.2 N
Δa_e	6.0×10^{-6} m
δCe	2.4×10^{-6}

Then, the calculated variation, δC_e, of the capacitance between low voltage and 100 kV was 0.7×10^{-6}.

C. Capacitor of 100 pF, 200 kV ($C_{100/200}$)

This unit, shown in Fig. 1, has a 100-pF main capacitor, plus 40 pF of the auxiliary ones. In this case, internal data are not known. However, a gross estimation can be done, based on the dielectric strength of the gas and the radii ratio of the electrodes. Most SF6-isolated capacitors are designed to have a maximum electric field around 100 kV/cm and a ratio between the external and internal radius of 1.5. These values are proposed for this estimation. The maximum electric field $E_{\max}$ occurs at the surface of the internal electrode, and the value is

$$E_{\max} = \frac{V}{r\ln(R/r)}. \tag{18}$$

Combining this equation with $R/r = 1.5$, α can be calculated as

$$\alpha = 0.32\left(\frac{E}{V}\right)^2. \tag{19}$$

For this capacitor, $\alpha = 810$ m^{-2}. The mass of the internal electrodes was estimated by using an interpolation between the mass of $C_{100/100}$ and $C_{50/400}$, obtaining 7 kg.

Obviously, all these assumptions have a large amount of uncertainty but the purpose of this estimation is only to have an idea of the order of the voltage coefficient. The measured results in the tilt test are shown in Table VII.

Applying these values to (17), the capacitance variation at 200 kV was $\delta C_e = 2.4 \times 10^{-6}$, as shown in Table VIII.

IV. UNCERTAINTY

For type B uncertainty calculation, Monte Carlo method [20] was applied on (17). In all cases, expanded uncertainty at 95% of probability was used. For all components, uniform distribution was assumed. In addition, a contribution analysis was done to find the influence of

TABLE IX
UNCERTAINTY DATA AND RELATIVE CONTRIBUTION FOR $C_{50/400}$

Variable	Half-width of the interval (%)	Contribution (%)
V	1	0
C_c	1	0
R	5	11
r	5	4
m	20	22
g	0	0
δC_1	13	52
δC_2	10	4
θ	10	6

TABLE X
UNCERTAINTY DATA AND RELATIVE CONTRIBUTION FOR $C_{100/100}$

Variable	half-width of the interval (%)	Contribution (%)
V	1	0
C_c	1	0
R	1	2
r	1	1
m	5	2
g	0	0
δC_1	5	61
δC_2	5	27
θ	10	7

TABLE XI
CONSTRUCTIVE PARAMETERS OF CAPACITOR $C_{50/350}$

R	170 mm
r	120 mm
L	308 mm

each variable on the combined uncertainty. These values indicate, in percentage, the ratio between the square of the uncertainty caused by the analyzed variable and the square of the combined uncertainty. Estimations of these results are shown in the third column of Tables IX and X.

For the capacitor $C_{50/400}$, the resulting uncertainty was 0.3×10^{-6}, which leads to a voltage variation, between low voltage and 400 kV, of $(0.8 \pm 0.3) \times 10^{-6}$. The half-width of the interval of each independent variable is shown in column two of Table IX. The uncertainty of the gravity acceleration can be neglected, and the uncertainty of the mass is relatively high because it took into account the mass, not only of the electrodes but that of the supporting structure which was not accurately known. The uncertainty of C_c is much larger than that of the accuracy of the bridge and the standard capacitor because it includes the unknown stray capacitance between the high voltage electrode and the supporting tube and the capacitance at the top of the electrodes that does not contribute to the electrical force F_e. The difference between C_c and C_0 is about three orders lower than C_c uncertainty. This allows to use C_0 instead of $C_c$in all equations without loss of accuracy,

TABLE XII
MEASURED DATA IN THE TILT TEST OF CAPACITOR $C_{50/350}$

θ	6.6°
$\delta C1$	-5×10^{-6}
$\delta C2$	10×10^{-6}

simplifying the measuring work as it is not necessary to measure C_c.

For this capacitor, the highest contributions come from δC_1, the mass, and the external radius measurements.

For the capacitor $C_{100/100}$, the resulting uncertainty was 0.4×10^{-6}, which leads to a voltage variation, between low voltage and 100 kV, of $(0.7 \pm 0.4) \times 10^{-6}$. The half-width of the interval of each independent variable is shown in column two of Table X, and their contributions, in column three. The largest uncertainty contributions come from δC_1 and δC_2.

V. COMPARISON TO OTHER METHODS

Direct comparisons against a reference capacitor were made for the previously analyzed capacitors.

$C_{100/100}$ was compared against $C_{50/400}$. The latter capacitor has a rated voltage four times greater than $C_{100/100}$, which means that the electrical force of $C_{50/400}$ at 100 kV is 16× less than that at 400 kV. Considering its calculated voltage variation up to 400 kV, around 10^{-6}, it is reasonable to assume that it has negligible variation up to 100 kV.

The measured ratio variation between both capacitors between 30 and 100 kV was 0, with an uncertainty of 1.6×10^{-6}, $k = 2$. As previously mentioned, this uncertainty calculation takes into account only type A uncertainty and the resolution of the bridge (1×10^{-6}). The proposed method gave a calculated variation of 0.7×10^{-6}, with an uncertainty of 0.4×10^{-6}. Then, the difference between both methods, 0.7×10^{-6}, is covered by the combined uncertainty.

A direct comparison between $C_{100/200}$ and $C_{50/400}$, gave a variation of $C_{100/200}$ of 2×10^{-6}, between 30 and 200 kV, with an uncertainty of 1.6×10^{-6}, $k = 2$. The calculated variation (2.4×10^{-6}) was very close to the measured one.

Another case, were not all required data for the proposed method is available, was taken from [16]. The capacitor had rated values of 50 pF, 350 kV ($C_{50/350}$) with the geometric data shown in Table XI; but the internal electrode mass was not informed. An estimation of it was made, assuming the same value than $C_{50/400}$, 15 kg.

A reproduction of the result of the tilt test, shown in that work, is presented in Table XII.

The application of the proposed method to this capacitor leads to the results shown in Table XIII.

As previously mentioned, that paper used a method that need to know the elasticity coefficient, by direct measurement. The published capacitance variation was $\delta C_e = (4\pm1)\times10^{-8}$, coinciding with the result of the proposed method.

In these two last examples, it is not possible to perform an uncertainty calculation on the proposed method due to the lack of some data. Anyway, the estimated results, close to

TABLE XIII
CALCULATED DATA OF CAPACITOR $C_{50/350}$

a_0	1.6×10^{-4} m
k	1.52×10^{5} N/m
F_e	0.16 N
Δa_e	1.0×10^{-6} m
δCe	0.05×10^{-6}

the actual ones, show that it is possible to have an estimation of the voltage influence although the values of some internal dimensions or weight are not available.

VI. Conclusion

A simple method, easy to implement, that can calculate the voltage influence on high-voltage capacitors knowing only a few mechanical characteristics and performing simple tests was proposed. It requires information of the radii of the electrodes and the mass of the internal ones. In general, manufacturers can provide this data, but if they are not available, estimates can be made according to the design criteria of similar capacitors. A validation of the method was performed using a 100 pF, 100 kV standard capacitor in which all internal mechanical characteristics was known. In this case, the result of the proposed method $(0.7 \pm 0.4) \times 10^{-6}$ was compared to direct measurements against a reference capacitor $(0 \pm 1.6) \times 10^{-6}$. The difference between both methods is covered by their uncertainties.

Appendix A

From (2)

$$\sqrt{Y^2-1} = \frac{R^2-r^2}{2Rr}\sqrt{1+\frac{a^4-2a^2(R^2+r^2)}{(R^2-r^2)^2}}. \tag{A1}$$

Disregarding a^4 because $a \ll r$, and approximating the square root by its first two terms of the Taylor series

$$\sqrt{Y^2-1} = \frac{1}{2Rr}\left(R^2-r^2-a^2\frac{R^2+r^2}{R^2-r^2}\right). \tag{A2}$$

Then

$$\ln(Y+\sqrt{Y^2-1}) = \ln\left(\frac{R}{r}\right) + \ln\left(1-\frac{a^2}{R^2-r^2}\right). \tag{A3}$$

Using the same type of Taylor approximation to the logarithm function

$$\ln(Y+\sqrt{Y^2-1}) \cong \ln\left(\frac{R}{r}\right) - \frac{a^2}{R^2-r^2}. \tag{A4}$$

From (1)

$$C \cong 2\pi\varepsilon L\frac{1}{\ln\left(\frac{R}{r}\right)-\frac{a^2}{R^2-r^2}}. \tag{A5}$$

Considering again $a \ll r$, A5 can be expressed as

$$C \cong 2\pi\varepsilon L\left(\frac{1}{\ln\left(\frac{R}{r}\right)}+\frac{a^2}{(R^2-r^2)\ln^2\left(\frac{R}{r}\right)}\right). \tag{A6}$$

Appendix B

From (15) and taking into account that $k/(\alpha C_c V^2) \gg 1$

$$\Delta a_e = \frac{\alpha C_c V^2}{k}a_0. \tag{B1}$$

Combining with (16)

$$\delta C_e \cong \frac{\alpha^2 C_c V^2}{k}a_0^2\left(2+\frac{\alpha C_c V^2}{k}\right). \tag{B2}$$

The second term inside the parenthesis can be neglected because it is much less than 2, arriving to

$$\delta C_e \cong \frac{2\alpha^2 C_c V^2}{k}a_0^2. \tag{B3}$$

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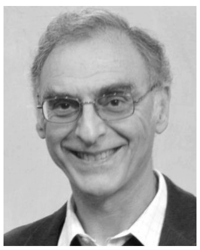

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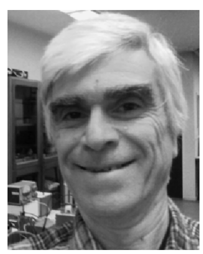

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